

# The Universal Structure of Propagators in Open Superstring Field Theory

Shingo TORII (Univ. of Tokyo, Komaba)

- JHEP 04(2012)050 [arXiv:1201.1762]

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## Background of this talk

An important problem in open String Field Theory (SFT)

- Can we describe closed strings in terms of FIELDS of open strings?
  - Can we describe external closed strings in terms of open string fields?

How can we find a clue?

**point:** the open-closed duality

- It is natural to consider the quantization of open SFT.
  - Can open SFT be consistently quantized? Is it anomalous?

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## A project on gauge fixing (of the WZW-type open SuperSFT)

- JHEP 03(2012)030 [arXiv:1201.1761]

Kroyter, Okawa, Schnabl, Zwiebach, S. T.

completed gauge fixing of the **NS sector** of free **open SuperSFT** and computed **propagators** in some gauges.

⇒ However, **what type of gauge we should choose depends on what we would like to investigate.** There will be **no “almighty” gauge.**

⇒ It is important to elucidate the **universal structure** of propagators.

# Plan

1. Gauge Fixing of **Free Bosonic** SFT
2. The Structure of Propagators in **Bosonic** SFT
3. Gauge Fixing of **Free Super**SFT (**NS sector**)
4. The Structure of Propagators in **Super**SFT
5. Comment

# 1. Gauge Fixing of Free **Bosonic** SFT

action:  $S_0 = -\frac{1}{2} \langle \Psi_1, Q\Psi_1 \rangle$  [Witten]

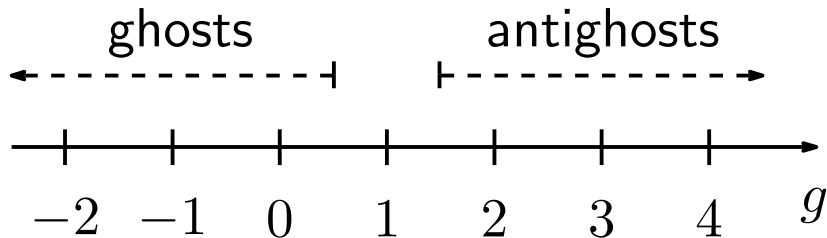
- $Q$ : BRST charge in the 1st quantized theory
- $\Psi_g$ : an open string field of world-sheet ghost number  $g$
- $\langle A, B \rangle$ : BPZ inner product of  $A$  and  $B$ .
- $\langle A, B \rangle = 0$  unless  $g(A) + g(B) = 3$ .
- gauge transf.:  $\delta\Psi_1 = Q\Lambda_0$  ( $Q^2 = 0$ )

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- Gauge fixing requires **infinitely many** ghosts and antighosts.
- However, we can obtain the gauge-fixed action **simply by removing the constraint on the world-sheet ghost number:**

$$S_0 = -\frac{1}{2} \langle \Psi_1, Q\Psi_1 \rangle \implies S = -\frac{1}{2} \langle \Psi, Q\Psi \rangle \text{ with constraints.}$$

$$\Psi := \sum_{n=-\infty}^{\infty} \Psi_n = \underbrace{\cdots + \Psi_{-1} + \Psi_0}_{\text{ghosts}} + \Psi_1 + \underbrace{\Psi_2 + \Psi_3 + \cdots}_{\text{antighosts}}$$



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$$S = -\frac{1}{2} \langle \Psi, Q\Psi \rangle = -\frac{1}{2} \langle \Psi_1, Q\Psi_1 \rangle - \sum_{g \leq 0} \langle \Psi_{2-g}, Q\Psi_g \rangle$$

Let us consider the gauge  $\mathcal{B} \Psi_{g(\leq 1)} = 0$ ,  $\mathcal{B} = \sum_n v_n b_n$   
(a linear combination of  $b_n$ ).

- (Space-time) BRST invariance restricts the conditions on the antighosts  $\Psi_{g(\geq 2)}$  (and they are automatically determined from the conditions on  $\Psi_{g(\leq 1)}$ ).

## 2. The Structure of Propagators in **Bosonic SFT**

gauge-fixed action

$$S = -\frac{1}{2} \langle \Psi_1, Q\Psi_1 \rangle - \sum_{g \leq 0} \langle \Psi_{2-g}, Q\Psi_g \rangle \quad \text{with } \mathcal{B}\Psi_g = 0.$$

Let us consider the  $\Psi_g$ - $\Psi_{2-g}$  propagator  $\Delta_g$

= the inverse of  $Q$  in the restricted subspace:  $\Delta_g Q\Psi_g = \Psi_g$ .

$$\Rightarrow \boxed{\Delta_g = \frac{1}{\mathcal{L}}\mathcal{B}}, \quad \mathcal{L} := \{\mathcal{B}, Q\}.$$

**key relation**  $\underline{\frac{1}{\mathcal{L}}\mathcal{B}Q = 1 - \frac{1}{\mathcal{L}}QB} \Rightarrow \left(\frac{1}{\mathcal{L}}\mathcal{B}\right) Q\Psi_g = \Psi_g$



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## generalization

In the  $g$ -dependent gauges  $\mathcal{B}_{(g)}\Psi_g = 0$  ( $g \leq 0$ ), we have

$$\Delta_g = \frac{\mathcal{B}_{(g)}}{\mathcal{L}_{(g)}} Q \frac{\mathcal{B}_{(g+1)}}{\mathcal{L}_{(g+1)}} \quad \text{with} \quad \mathcal{L}_{(g)} := \{Q, \mathcal{B}_{(g)}\} .$$

$\Rightarrow$  In bosonic SFT, it is manifest How the information about gauge-fixing conditions is reflected in the structure of propagators.

### 3. Gauge Fixing of Free **SuperSFT**

action:  $S_0 = -\frac{i}{2} \langle \Phi_{(0,0)}, Q\eta_0\Phi_{(0,0)} \rangle$  [Berkovits]

$\Phi_{(g,p)}$ : NS string field of world-sheet ghost number  $g$  and picture  $p$ .

- $Q^2 = \eta_0^2 = \{Q, \eta_0\} = 0$ .

- formulated in the large Hilbert space.

$\Rightarrow \langle A, B \rangle = 0$  unless  $(g(A) + g(B), p(A) + p(B)) = (2, -1)$ .

- $(g(Q), p(Q)) = (1, 0), \quad (g(\eta), p(\eta)) = (1, -1)$

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$$\boxed{\text{Gauge Struct.}} \quad S_0 = -\frac{i}{2} \langle \Phi_{0,0}, Q\eta_0\Phi_{(0,0)} \rangle \quad (Q^2 = \eta_0^2 = \{Q, \eta_0\} = 0)$$

$$1) \delta\Phi_{(0,0)} = Q\Lambda_{(-1,0)} + \eta_0\Lambda_{(-1,1)} = \begin{bmatrix} Q & \eta_0 \end{bmatrix} \begin{bmatrix} \Lambda_{(-1,0)} \\ \Lambda_{(-1,1)} \end{bmatrix}$$

(gauge transf.)

$$\boxed{\text{Gauge Struct.}} \quad S_0 = -\frac{i}{2} \langle \Phi_{0,0}, Q\eta_0\Phi_{(0,0)} \rangle \quad (Q^2 = \eta_0^2 = \{Q, \eta_0\} = 0)$$

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$$2) \quad \delta \begin{bmatrix} \Lambda_{(-1,0)} \\ \Lambda_{(-1,1)} \end{bmatrix} = \begin{bmatrix} Q & \eta_0 & 0 \\ 0 & Q & \eta_0 \end{bmatrix} \begin{bmatrix} \Lambda_{(-2,0)} \\ \Lambda_{(-2,1)} \\ \Lambda_{(-2,2)} \end{bmatrix} \quad \left( \begin{bmatrix} Q & \eta_0 \end{bmatrix} \begin{bmatrix} Q & \eta_0 & 0 \\ 0 & Q & \eta_0 \end{bmatrix} = 0 \right)$$

(gauge transf. of gauge transf.)

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$$3) \quad \delta \begin{bmatrix} \Lambda_{(-2,0)} \\ \Lambda_{(-2,1)} \\ \Lambda_{(-2,2)} \end{bmatrix} = \begin{bmatrix} Q & \eta_0 & 0 & 0 \\ 0 & Q & \eta_0 & 0 \\ 0 & 0 & Q & \eta_0 \end{bmatrix} \begin{bmatrix} \Lambda_{(-3,0)} \\ \vdots \\ \Lambda_{(-3,3)} \end{bmatrix}$$

... What does this mean?  $\Rightarrow$  **ghosts, ghosts for ghosts, ....**

It is convenient to introduce the following matrices.

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$$Q_{n,n+1} = \underbrace{\begin{bmatrix} Q & \eta_0 & & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ & & Q & \eta_0 \end{bmatrix}}_{n+1} \Bigg\} n, \quad Q_{n,n+1} Q_{n+1,n+2} = 0. \quad \text{cf.) } Q^2 = 0.$$

$$\Lambda_{-n} = \begin{bmatrix} \Lambda_{(-n,0)} \\ \vdots \\ \Lambda_{(-n,n)} \end{bmatrix} \Bigg\} n+1, \quad \Phi_{-n} = \begin{bmatrix} \Phi_{(-n,0)} \\ \vdots \\ \Phi_{(-n,n)} \end{bmatrix} \Bigg\} n+1, \quad \Phi_{n+1} = \begin{bmatrix} \Phi_{(n+1,-1)} \\ \vdots \\ \Phi_{(n+1,-n)} \end{bmatrix} \Bigg\} n$$

$$1) \quad \delta \Phi_0 = Q_{1,2} \Lambda_{-1}$$

$$2) \quad \delta \Lambda_{-1} = Q_{2,3} \Lambda_{-2}$$

$$3) \quad \delta \Lambda_{-2} = Q_{3,4} \Lambda_{-3}$$

...

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$$1) \quad \delta \Phi_0 = Q_{1,2} \Lambda_{-1} \quad \Rightarrow \quad S_1 = - \langle \Phi_2, Q_{1,2} \Phi_{-1} \rangle$$

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$$Q_{n,n+1} = \underbrace{\begin{bmatrix} Q & \eta_0 & & \mathbf{0} \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ & & Q & \eta_0 \end{bmatrix}}_{n+1} \Bigg\} n, \quad Q_{n,n+1} Q_{n+1,n+2} = 0. \quad \text{cf.) } Q^2 = 0.$$

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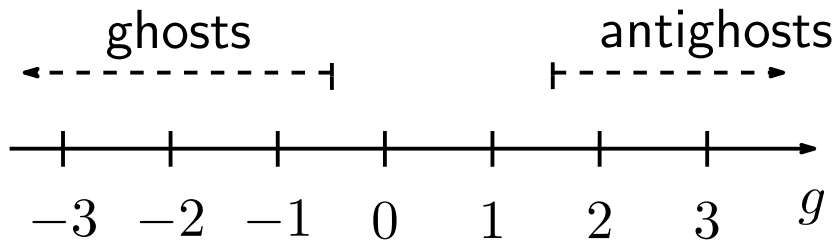
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$$3) \quad \delta \Phi_{-2} = Q_{3,4} \Lambda_{-3} \quad \Rightarrow \quad S_3 = - \langle \Phi_4, Q_{3,4} \Phi_{-3} \rangle$$

...

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$$S = \sum_{n=0}^{\infty} S_n = -\frac{i}{2} \langle \Phi_0, Q\eta_0 \Phi_0 \rangle - \sum_{n=1}^{\infty} \langle \Phi_{n+1}, Q_{n,n+1} \Phi_{-n} \rangle$$



We can consider gauge-fixing conditions of the form

$$B_{n+2,n+1} \Phi_{-n} = 0 \quad (n \geq 0).$$

## 4. The Structure of Propagators in SuperSFT

gauge-fixed action

$$S = -\frac{i}{2} \langle \Phi_0, Q\eta_0\Phi_0 \rangle - \sum_{n=1}^{\infty} \langle \Phi_{n+1}, Q_{n,n+1}\Phi_{-n} \rangle, \quad B_{n+2,n+1}\Phi_{-n} = 0.$$

- $\Phi_{-n}$ - $\Phi_{n+1}$  propagator  $\Delta_{n+1,n}$ :  $\Delta_{n+1,n} Q_{n,n+1}\Phi_{-n} = \Phi_{-n}$

key in bosonic SFT	$\frac{1}{\mathcal{L}}\mathcal{B}Q = 1 - \frac{1}{\mathcal{L}}Q\mathcal{B} \Rightarrow \left(\frac{1}{\mathcal{L}}\mathcal{B}\right)Q\Psi_g = \Psi_g$
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$\implies$  In SuperSFT, we would like to find matrices  $P$  such that

$$PBQ = 1 + MB. \quad (\text{Note: } \Delta \text{ have to satisfy } \Delta Q = 1 + MB.)$$

Correspondence:  $B \leftrightarrow \mathcal{B}$ ,  $P \leftrightarrow \frac{1}{\mathcal{L}}$ . propagator  $\Delta = PB?$

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Such  $P$  exists indeed:

$$(P_{n+1,n+1}B_{n+1,n})Q_{n,n+1} = \mathbf{1}_{n+1} + M_{n+1,n+2}B_{n+2,n+1},$$

$(P_{n+1,n+1}B_{n+1,n})$  is the inverse of  $Q_{n,n+1}$ :

$$(P_{n+1,n+1}B_{n+1,n})Q_{n,n+1}\Phi_{-n} = \Phi_{-n}$$

$$\implies \Delta_{n+1,n} = P_{n+1,n+1}B_{n+1,n} \quad \text{cf.)} \quad \Delta_g = \frac{1}{\mathcal{L}}\mathcal{B}$$

• In general we have

$$\Delta_{n+1,n} = (P_{n+1,n+1}B_{n+1,n})Q_{n,n+1}(P'_{n+1,n+1}B'_{n+1,n}).$$

$$\text{cf.)} \quad \Delta_g = \frac{\mathcal{B}_{(g)}}{\mathcal{L}_{(g)}} Q \frac{\mathcal{B}_{(g+1)}}{\mathcal{L}_{(g+1)}}$$

## 5. Comment

Important Property

$$\Delta_{n,n-1} Q_{n-1,n} + Q_{n,n+1} \Delta_{n+1,n} = \mathbf{1}_n \quad \dots (*)$$

$$\text{cf.) } \frac{\mathcal{B}}{\mathcal{L}} Q + Q \frac{\mathcal{B}}{\mathcal{L}} = 1, \quad \Delta_g Q + Q \Delta_{g-1} = 1.$$

In bosonic SFT, the above relations are **crucial** for the proof of the **gauge independence of on-shell amplitudes**. [Asano-Kato]

$\implies$  (\*) will play the same role in SSFT.

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*Let's thank the organizers!*