Soliton Star as Confining Fermi Liquid

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Based on : Bhattacharya Ogawa Takayanagi TU
[arXiv:1201.0764] JHEP 1202 (2012) 137

Holographic model of fermi liquid

There are various attempts to construct horographic dual of realistic fermi liquid at zero temperature.

- Extremal Charged black hole ➡ Non fermi liquid but large entropy
  [Hong Liu et al]
- Electron star ➡ Lifshitz scaling at IR
  [Hartnoll et al]

Both solutions correspond to deconfining phase of dual field theory.
Various colorful excitations appears near fermi surface.

➡ Can we construct holographic dual of confining fermi liquid?

Today's Topic

Can we realize Confining Fermi Liquid holographically?

→ we consider Einstein fermion fluid Maxwell system and find AdS soliton like solution.

Plan of the talk

1. Brief summary of fermi liquid

2. Holographic set up (Einstein Maxwell fermionic fluid system)

3. Soliton star

4. Thermodynamical stability of the solution

Fermi Liquid

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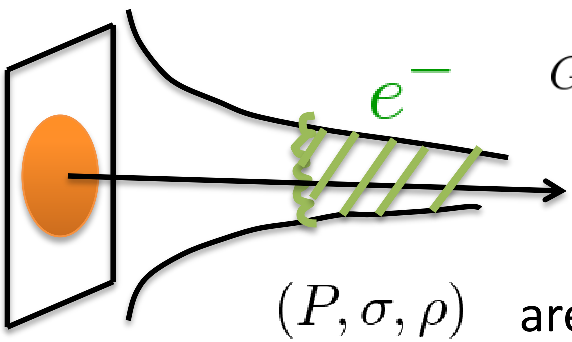
Two important properties of LFL: $C \sim T$ and Luttinger Relation. $\langle Q \rangle = A_F$
$\langle Q \rangle$:Total charge of fermi sea. A_F :volume enclosed by fermi surface.

Fermionic fluid system

We would like to consider a bulk system in which all U(1) charges are carried by only fermions.

Since bulk fermions don't backreact at classical level, we need to treat them at 1 loop level [Sachidev 11] . However one can treat it by effective fluid dynamic description of fermion .



$$G_{\mu\nu} - \Lambda g_{\mu\nu} = T\mu\nu \qquad T^f_{\mu\nu} = (P+\rho)u_\mu u_\nu + P g_{\mu\nu}$$

$$\nabla_\mu F^{\mu\nu} = J^\nu \qquad J_\mu = e\sigma u_\mu$$

(P, σ, ρ) are thermodynamical quantites of fluid that can be expressed by single bulk chemical potential $\mu(r)$.

$$\rho = \beta \int_m^{\mu(r)} \epsilon D(\epsilon) d\epsilon \qquad \sigma = \beta \int_m^{\mu(r)} D(\epsilon) d\epsilon \qquad \mu\sigma = P + \rho$$

Fermionic fluid system

We assume (P, σ, ρ) is given by simple fermion destribution function $D(\epsilon)$ in flat space and chemical potential at each point $\mu(r)$.

$$\rho = \beta \int_m^{\mu(r)} \epsilon D(\epsilon) d\epsilon \qquad \sigma = \beta \int_m^{\mu(r)} D(\epsilon) d\epsilon \qquad \mu\sigma = P + \rho$$

We also assume the configuration is static ,

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + k(r)d\theta^2 + r^2(dx^2 + dy^2) \qquad A_t = h(r)$$

then velocity vector field is,

$$u^2 = -1 \Longrightarrow u_t = \sqrt{f(r)}\ ,\quad \text{others} = 0$$

and bulk chemical potential is determined by conservation law of EM tensor.

$$\nabla_\mu T^{\mu\nu} = 0 \Longrightarrow \mu(r) = \frac{h(r)}{\sqrt{f(r)}}$$

Electron star solution [Hartnoll et.al]

Lifshitz geometry is an exact solution of the system:

$$ds^2 = -r^z dt^2 + \frac{dr^2}{r^2} + r^2(dx^2 + dy^2 + d\theta^2)$$

This is due to charge screening effect of fermion $\longrightarrow$ gauge field become massive.

Electron star is a domain wall solution which connects IR Lifshitz geometry and UV AdS5 geometry. In the solution, fermion condensates between $0 \leq r \leq r_s$ where $\mu(r_s) = m$ out side the star $r_s \leq r$ is described by AdS-Reissner metric.

To be more realistic, we would like to consider confining solution.

(In LFL, There are no gapless colorful excitation.)

AdS soliton metric [Witten 98] [Horowitz Meyers 98]

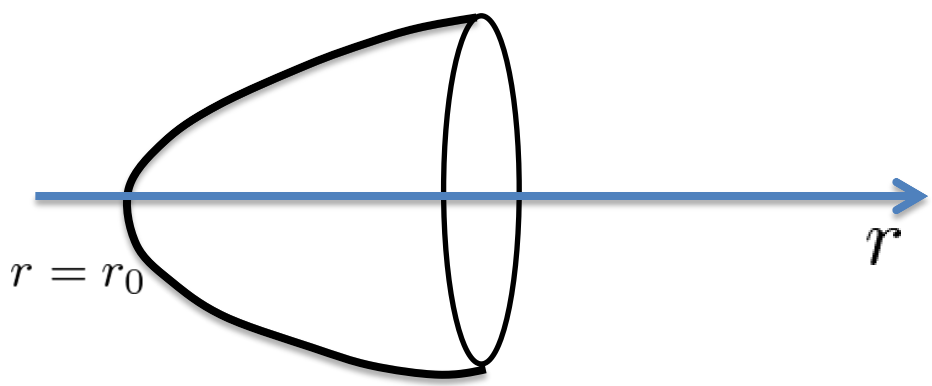
Consider the metric :

$$ds^2 = f(r)d\theta^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2 - dt^2) \qquad f(r) = r^2 - \frac{r_0^4}{r^2}$$

No Singularity at the tip $r = r_0$ $\longrightarrow$ $\theta \sim \theta + \beta_\theta$

Area law for holographic wilson loop

$\longrightarrow$ The metric describes confining phase of the dual theory.



Today's Topic

Can we realize Confining Fermi Liquid holographically?

Only fermion carry charges in FL

→ we consider Einstein fermion fluid Maxwell system and find AdS Soliton like solutions.

No gapless colorful excitations in real FL

Soliton star solution

Back to our bulk fermion system, we would like to find soliton star solutions with fermion charge which interpolate IR AdS Soliton geometry and UV AdS5 geometry. We take an ansatz

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + k(r)d\theta^2 + r^2(dx^2 + dy^2) \qquad A_t = h(r)$$

and demand

$$g(r) \to \frac{g_0}{(r-r_0)} + g_1 + g_2(r-r_0) + \cdots \qquad h(r) \to h_0 + h_1(r-r_0) + h_2(r-r_0)^2 + \cdots$$

$$k(r) \to k_0(r-r_0) + k_1(r-r_0)^2 + \cdots \qquad f(r) \to f_0 + f_1(r-r_0) + f_2(r-r_0)^2 + \cdots$$

at the tip $r = r_0$. We solve the equations numerically, and find 1 parameter family of solutions which are labeled by charge Q.

We plot as function of Q

Result

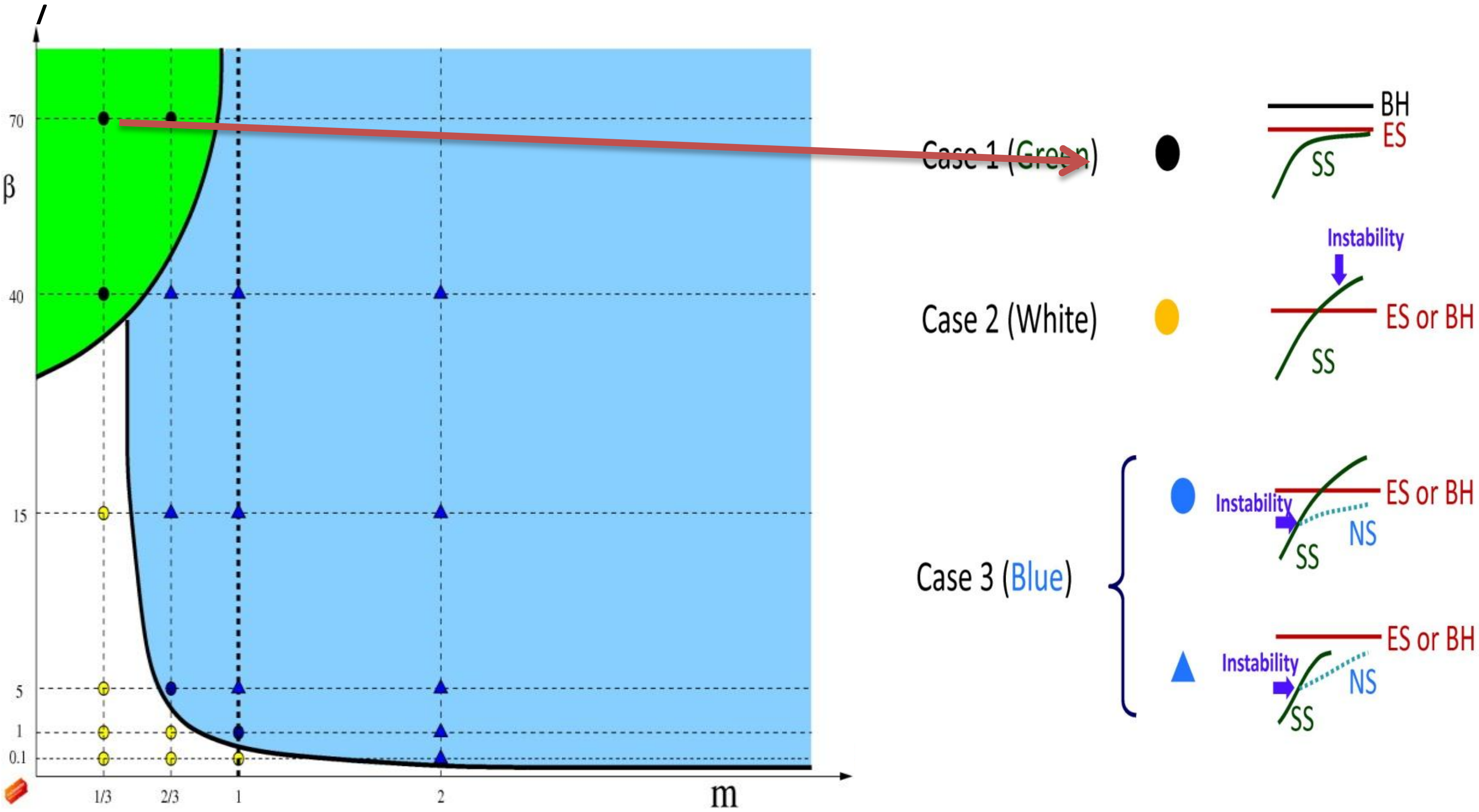
At zero temperature, one can consider three different solutions of the system: Namely, extreamally charged black hole and electron star and soliton star.

For all the geometries, one boundary direction is compactified with period R

We calculate energies of the solutions $\frac{\langle T_{tt} \rangle^3}{Q^4}$ and decide most stable solution for fixed charge $QR^{\frac{3}{2}}$

When Q is large, soliton star become unstable. We verify various value of (m, β) ,and find that there are 3 types of instabilities.

Instability of soliton stars



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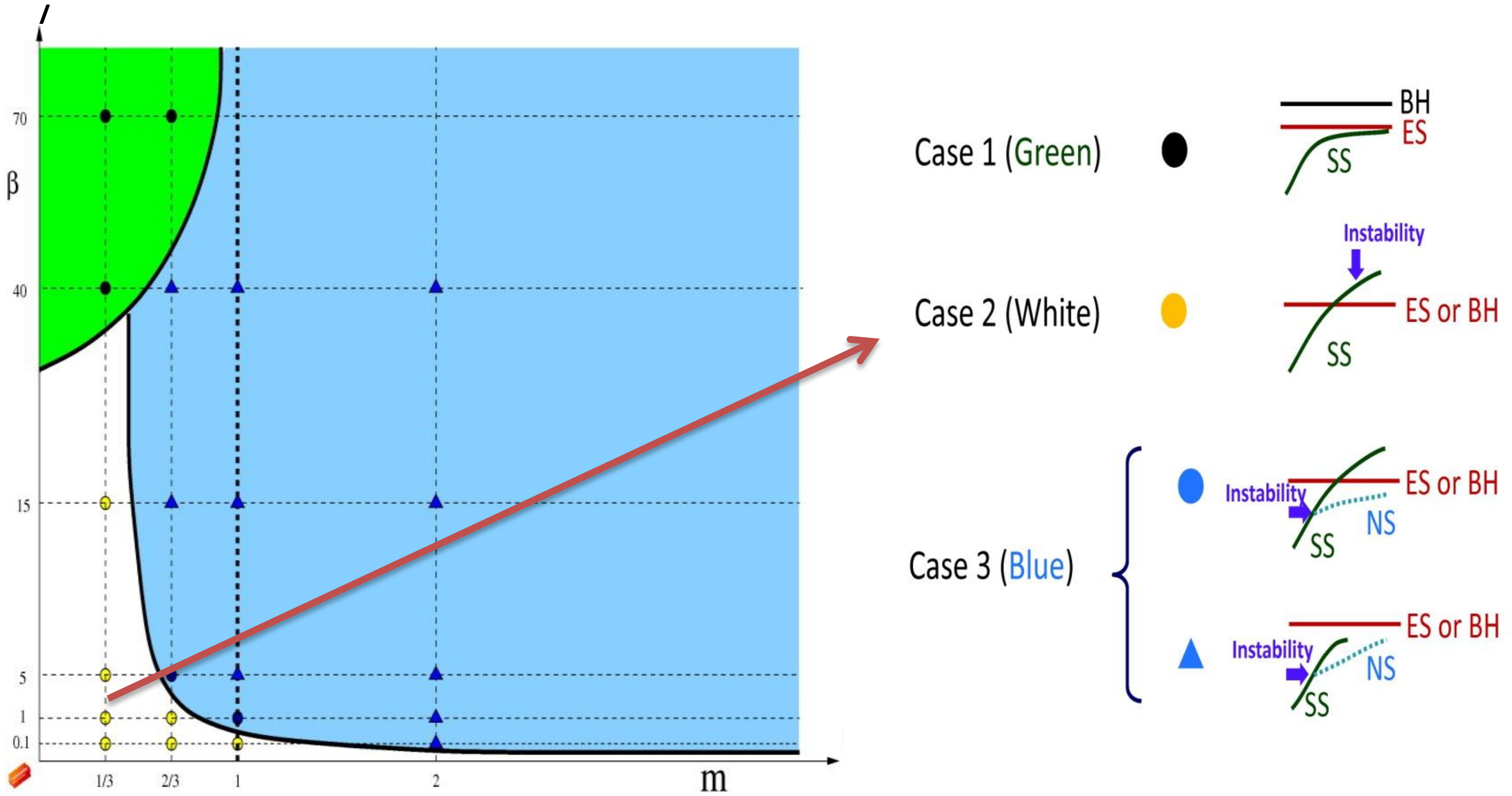

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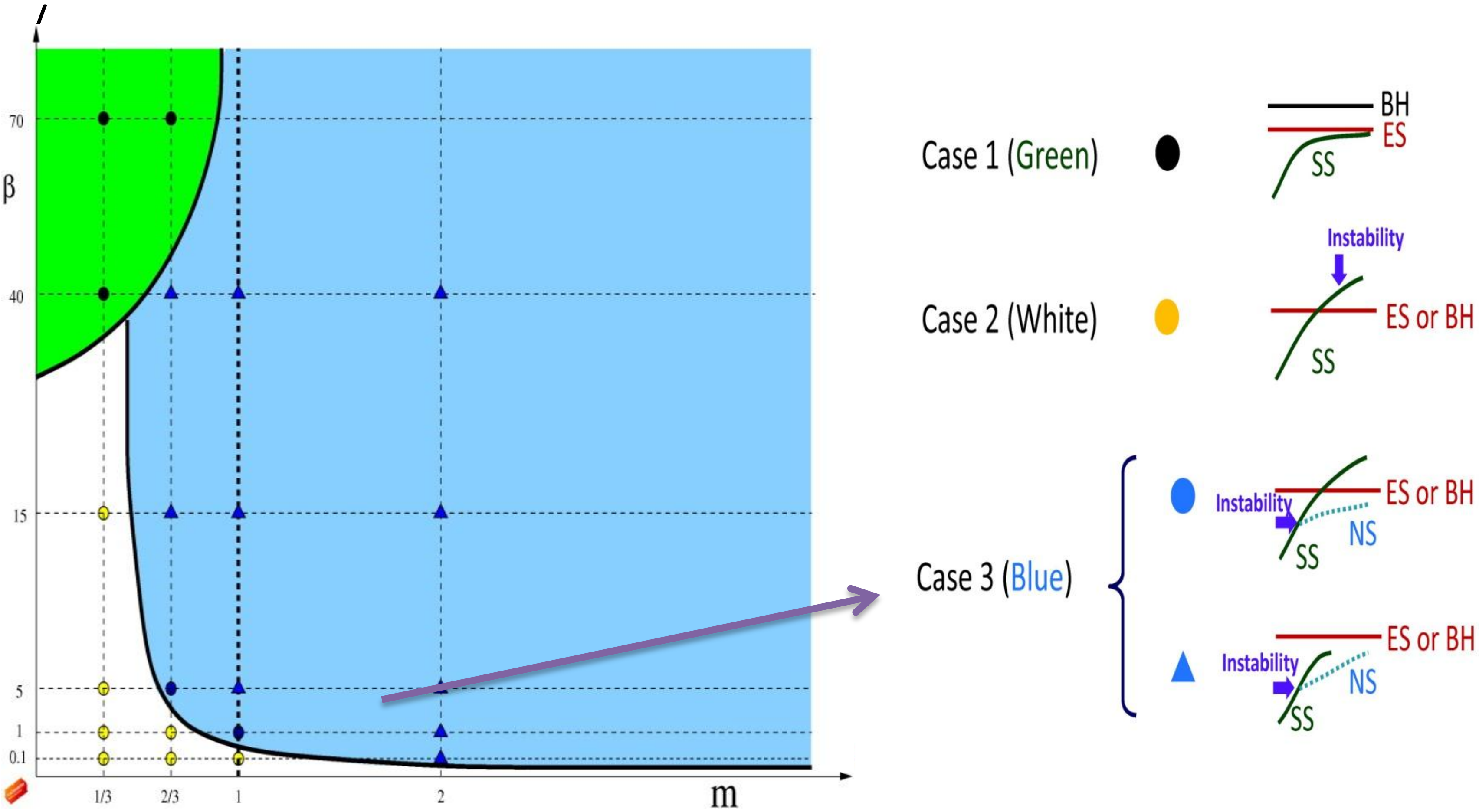

Conclusions

We find solutions in Einstein -Maxwell – fermionic fluid system which are dual to confining fermi liquid.

We find various instabilities of soliton star as we change (m, β) when Q is large. There are 3 types of instabilities:

(i) Soliton Star approaches to electron star

(ii) first order phase transition to electron star (or extremal black hole).

(iii) Thermodynamical instability appear $\frac{\partial \mu}{\partial Q} < 0$.

We confirm existence of fermi surface and Luttinger relation.