Brane-Antibrane at Finite Temperature in the Framework of Thermo Field Dynamics

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Hagedorn Temparature

maximum temperature for perturbative string

The oscillation mode of a single energetic string captures most of the energy.

degeneracy of oscillation mode $\quad d_n \sim e^{2\pi\sqrt{2n}}$

density of state $\quad \Omega(E) \sim e^{\beta_H E}$

partition function $\quad Z(\beta) = \int_0^\infty dE\ \Omega(E)\ e^{-\beta E} = \mathrm{Tr}\ e^{-\beta H}$

Hagedorn temperature $\mathcal{T}_H$

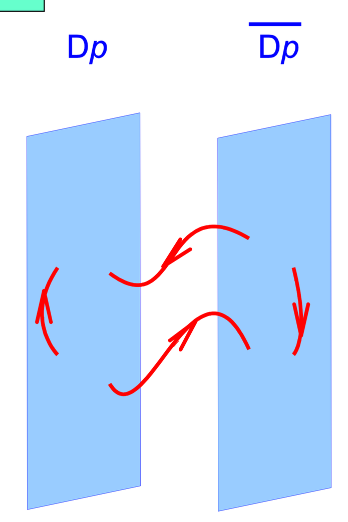
$$\beta_H \equiv \frac{1}{\mathcal{T}_H} = 2\pi\sqrt{2\alpha'} \qquad \beta = \frac{1}{\mathcal{T}}$$

The partition function diverges above $\mathcal{T}_H$

$$Z(\beta) \to \infty \quad \text{for} \quad \beta < \beta_H$$

Brane—antibrane in Matsubara Formalism (Hotta)

- Dp-$\overline{\text{Dp}}$ pair



unstable at zero temperature

open string tachyon ➞ tachyon potential

Sen's Conjecture $\quad$ potential hight = brane tention

- BSFT (Boudary String Field Theory)

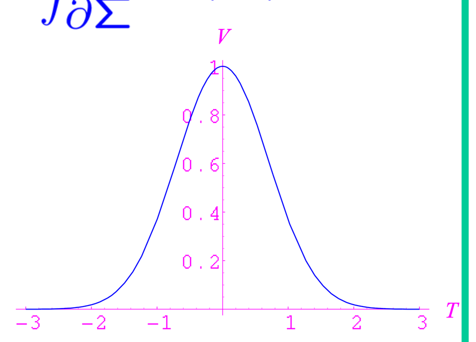
solution of classical master eq.

$$S_{eff} = Z$$

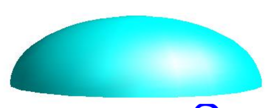
S_{eff} : 10-dim. effective action $\quad Z$: 2-dim. partition function

2-dim. action : $S_2 = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\ \partial_a X_\mu \partial^a X^\mu + \int_{\partial\Sigma} d\tau |T|^2 + \cdots$

- Tachyon potential of Dp-$\overline{\text{Dp}}$ based on BSFT



tree level (disk worldsheet)



$$V(T) = 2\tau_p \mathcal{V}_p \exp(-8|T|^2),$$

T : complex scalar field, τ_p : brane tension, $\mathcal{V}_p$: p-dim. volume

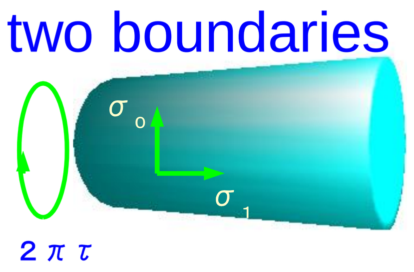
finite temperature case

Matsubara method ➞ 1-loop (cylinder worldsheet)

Conformal invariance is broken by the boundary terms.

➞ ambiguity in the choice of the Weyl factors of two boundaries

- Cylinder Boundary Action (Andreev-Oft)



$$S_b = \int_0^{2\pi\tau} d\sigma_0 \int_0^\pi d\sigma_1 [|T|^2\delta(\sigma_1) + |T|^2\delta(\pi - \sigma_1)].$$

Both sides of cylinder worldsheet are treated on an equal footing.

- 1-loop Free Energy of Open Strings

$$F_o(T,\beta) = -\frac{16\pi^4 \mathcal{V}_p}{{\beta_H}^{p+1}} \int_0^\infty \frac{d\tau}{\tau}\ \tau^{-\frac{p+1}{2}} e^{-4\pi|T|^2\tau} \times \left[\left(\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4 \left\{\vartheta_3\left(0\left|\frac{i\beta^2}{8\pi^2\alpha'\tau}\right.\right) - 1\right\} - \left(\frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4 \left\{\vartheta_4\left(0\left|\frac{i\beta^2}{8\pi^2\alpha'\tau}\right.\right) - 1\right\}\right]$$

finite temperature effective potential

$$V(T,\beta) = V(T) + F_o(T,\beta)$$

- Brane-antibrane Pair Creation Transition
- N D9-$\overline{\text{D9}}$ pairs

$|T|^2$ term of $V(T,E)$ $\quad \left[-16N\tau_9\mathcal{V}_9 + \frac{8\pi N^2 \mathcal{V}_9}{{\beta_H}^{10}} \ln\left(\frac{\pi {\beta_H}^{10} E}{2N^2\mathcal{V}_9}\right)\right] |T|^2.$

critical temperature $\quad \mathcal{T}_c \simeq {\beta_H}^{-1}\left[1 + \exp\left(-\frac{{\beta_H}^{10}\tau_9}{\pi N}\right)\right]^{-1}.$

Above $\mathcal{T}_c$, $T = 0$ becomes the potential minimum.

➞ A phase transition occurs and D9-$\overline{\text{D9}}$ pairs become stable.

$\mathcal{T}_c$ is a decreasing function of N

➞ Multiple D9-$\overline{\text{D9}}$ pairs are created simultaneously.

- N Dp-$\overline{\text{Dp}}$ pairs with $p \le 8$

No phase transition occurs.

Thermo Field Dynamics (Takahashi-Umezawa)

- Canonical Ensemble

expectation value

$$\langle A \rangle = Z^{-1}(\beta) \sum_n \langle n|\hat{A}|n\rangle e^{-\beta E_n}$$

We can represent it as

$$\langle A \rangle = \left\langle 0(\beta) \middle| \hat{A} \middle| 0(\beta) \right\rangle$$

by introducing a fictitious copy of the system.

$$|0(\beta)\rangle = Z^{-\frac{1}{2}}(\beta) \sum_n e^{-\frac{\beta E_n}{2}} |n, \tilde{n}\rangle \qquad |n,\tilde{n}\rangle = |n\rangle \otimes |\tilde{n}\rangle$$

thermal vacuum state

We cannot represent it as

$$|0(\beta)\rangle = \sum_n |n\rangle\, f_n(\beta)$$

for ordinary number $f_n(\beta)$, since

$$f_n^*(\beta) f_m(\beta) = Z^{-1}(\beta) e^{-\beta E_n} \delta_{nm}$$

cannot be satisfied.

- Ensemble of Free Fermions (example)

Hamiltonian $\quad H = \omega a^\dagger a$

anti-commutation relation $\quad \left\{a, a^\dagger\right\} = 1$

We introduce fictitious system.

Hamiltonian $\quad \tilde{H} = \omega \tilde{a}^\dagger \tilde{a}$

anti-commutation relation $\quad \left\{\tilde{a}, \tilde{a}^\dagger\right\} = 1$

generator of Bogoliubov tr.

$$G_F = -i\theta(\beta)\left(\tilde{a}a - a^\dagger\tilde{a}^\dagger\right)$$

$$\sin\theta(\beta) = \left(1 + e^{\beta\omega}\right)^{-\frac{1}{2}}$$

$$\cos\theta(\beta) = \left(1 + e^{-\beta\omega}\right)^{-\frac{1}{2}}$$

$$\tan\theta(\beta) = e^{-\frac{\beta\omega}{2}}$$

thermal vacuum state

$$|0(\beta)\rangle = e^{-iG_F}|0\rangle = \left\{\cos\theta(\beta) + \sin\theta(\beta) a^\dagger \tilde{a}^\dagger\right\}|0\rangle = \cos\theta(\beta) \exp\left[\tan\theta(\beta) a^\dagger \tilde{a}^\dagger\right]|0\rangle$$

Bogoliubov tr. of annihilation ops.

$$a(\beta) = e^{-iG_F} a e^{iG_F} = \cos\theta(\beta) a - \sin\theta(\beta)\tilde{a}^\dagger$$

$$\tilde{a}(\beta) = e^{-iG_F} \tilde{a} e^{iG_F} = \cos\theta(\beta)\tilde{a} + \sin\theta(\beta) a^\dagger$$

Thermal vacuum state satisfies

$$a(\beta)\,|0(\beta)\rangle = \tilde{a}(\beta)\,|0(\beta)\rangle = 0$$

Fermi distribution

$$\left\langle 0(\beta) \middle| a^\dagger a \middle| 0(\beta) \right\rangle = \sin^2\theta(\beta) = \frac{e^{-\beta\omega}}{1 + e^{-\beta\omega}}$$

fictitious system as `holes'

$$\frac{1}{\cos\theta(\beta)}\, a^\dagger\,|0(\beta)\rangle = -\frac{1}{\sin\theta(\beta)}\, \tilde{a}\,|0(\beta)\rangle$$

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- Ensemble of Free Bosons (example)

Hamiltonian $H = \omega a^\dagger a$

commutation relation $\left[a, a^\dagger\right] = 1$

We introduce fictitious system.

Hamiltonian $\tilde{H} = \omega \tilde{a}^\dagger \tilde{a}$

commutation relation $\left[\tilde{a}, \tilde{a}^\dagger\right] = 1$

generator of Bogoliubov tr.

$$G_B = -i\theta(\beta)\left(\tilde{a}a - a^\dagger \tilde{a}^\dagger\right)$$

$$\begin{aligned} \sinh\theta(\beta) &= \left(e^{\beta\omega} - 1\right)^{-\frac{1}{2}} \\ \cosh\theta(\beta) &= \left(1 - e^{-\beta\omega}\right)^{-\frac{1}{2}} \\ \tanh\theta(\beta) &= e^{-\frac{\beta\omega}{2}} \end{aligned}$$

thermal vacuum state

$$\begin{aligned} |0(\beta)\rangle &= e^{-iG_B}|0\rangle \\ &= \frac{1}{\cosh\theta(\beta)} \exp\left[\tanh(\beta) a^\dagger \tilde{a}^\dagger\right] |0\rangle \end{aligned}$$

Bogoliubov tr. of annihilation ops.

$$\begin{aligned} a(\beta) &= e^{-iG_B} a e^{iG_B} = \cosh\theta(\beta) a - \sinh\theta(\beta)\tilde{a}^\dagger \\ \tilde{a}(\beta) &= e^{-iG_B} \tilde{a} e^{iG_B} = \cosh\theta(\beta)\tilde{a} - \sinh\theta(\beta) a^\dagger \end{aligned}$$

Thermal vacuum state satisfies

$$a(\beta)\,|0(\beta)\rangle = \tilde{a}(\beta)\,|0(\beta)\rangle = 0$$

Bose distribution

$$\left\langle 0(\beta) \left| a^\dagger a \right| 0(\beta) \right\rangle = \sinh^2\theta(\beta) = \frac{e^{-\beta\omega}}{1 - e^{-\beta\omega}}$$

fictitious system as `holes'

$$\frac{1}{\cosh\theta(\beta)}\, a^\dagger\, |0(\beta)\rangle = \frac{1}{\sinh\theta(\beta)}\, \tilde{a}\, |0(\beta)\rangle$$

Brane—antibrane in TFD

- Light-Cone

We consider a single first quantized string.

light-cone momentum

$$p^0 = \frac{1}{2}(p^+ + p^-)$$

$$p^+ p^- - |\boldsymbol{p}|^2 - m^2 = 0$$

$$p^- = \frac{|\boldsymbol{p}|^2 + m^2}{p^+}$$

partition function

$$\begin{aligned} Z(\beta) &= \mathrm{Tr}\ \exp\left(-\beta p^0\right) = \mathrm{Tr}\ \exp\left[-\frac{1}{2}\beta(p^+ + p^-)\right] \\ &= \mathrm{Tr}\ \exp\left[-\frac{1}{2}\beta\left(p^+ + \frac{|\boldsymbol{p}|^2 + m^2}{p^+}\right)\right] \\ &= \mathrm{Tr}\ \exp\left[-\frac{1}{2}\beta\left(p^+ + \frac{H}{p^+}\right)\right] \end{aligned}$$

light-cone Hamiltonian

$$H = |\boldsymbol{p}|^2 + m^2$$

- Mass Spectrum

We consider an open string on a Brane-antibrane pair with $T = 0$.

mass spectrum

$$\begin{aligned} {M_{NS}}^2 &= \frac{1}{\alpha'}\left(N_B + N_{NS} - \frac{1}{2}\right) \\ {M_R}^2 &= \frac{1}{\alpha'}\left(N_B + N_R\right) \end{aligned}$$

number ops.

$$\begin{aligned} N_B &= \sum_{l=1}^{\infty}\sum_{I=1}^{8} \alpha_{-l}^I \alpha_l^I \\ N_{NS} &= \sum_{r=\frac{1}{2}}^{\infty}\sum_{I=1}^{8} b_{-r}^I b_r^I \\ N_R &= \sum_{m=1}^{\infty}\sum_{I=1}^{8} d_{-m}^I d_m^I \end{aligned}$$

- Thermal Vacuum States

cf) D-brane in bosonic string theory
Vancea et al., Cantcheff

generator of Bogoliubov tr.

$$\begin{aligned} G_b &= G_B + G_{NS} \\ G_f &= G_B + G_R \end{aligned}$$

$$\begin{aligned} \tanh(\theta_l) &= \exp\left(-\frac{\beta l}{4p^+}\right) \\ \tan(\theta_r) &= \exp\left(-\frac{\beta r}{4p^+}\right) \\ \tan(\theta_m) &= \exp\left(-\frac{\beta m}{4p^+}\right) \end{aligned}$$

$$\begin{aligned} G_B &= i\sum_{l=1}^{\infty}\frac{1}{l}\,\theta_l\left(\alpha_{-l}\cdot\tilde{\alpha}_{-l} - \tilde{\alpha}_l\cdot\alpha_l\right) \\ G_{NS} &= i\sum_{r=\frac{1}{2}}^{\infty}\theta_r\left(b_{-r}\cdot\tilde{b}_{-r} - \tilde{b}_r b_r\cdot\right) \\ G_R &= i\sum_{m=1}^{\infty}\theta_m\left(d_{-m}\cdot\tilde{d}_{-m} - \tilde{d}_m\cdot d_m\right) \end{aligned}$$

oscillator part of thermal vacuum state

$$\begin{aligned} |0_{osc}(\beta)\rangle\rangle = \prod_{l=1}^{\infty}&\left\{\left(\frac{1}{\cosh(\theta_l)}\right)^8 \exp\left[\frac{1}{l}\ \tanh(\theta_l)\alpha_{-l}\cdot\tilde{\alpha}_{-l}\right]\right\} \\ &\times\left\{\prod_{r=\frac{1}{2}}^{\infty}(\cos(\theta_r))^8 \exp\left[\ \tan(\theta_r) b_{-r}\cdot\tilde{b}_{-r}\right]\right. \\ &\left. + \prod_{m=1}^{\infty}(\cos(\theta_m))^8 \exp\left[\ \tan(\theta_m) d_{-m}\cdot\tilde{d}_{-m}\right]\right\}|0\rangle\rangle \end{aligned}$$

Including momentum part, thermal vacuum state is given by

$$\begin{aligned} |0_1(\beta)\rangle\rangle &= \mathcal{N}\int dp^+ \int d^{p-1}p\ \exp\left(-\frac{\beta p^+}{4}\right)\left|p^+\right\rangle \\ &\quad\times\exp\left(-\frac{\beta|\boldsymbol{p}|^2}{4p^+}\right)|\boldsymbol{p}\rangle\ |0_{osc}(\beta)\rangle\rangle \end{aligned}$$

- Free Energy for a Single String

$$F_1(\beta) = \left\langle\left\langle 0_1(\beta)\left|\left(\mathcal{H} - \frac{1}{\beta}K\right)\right|0_1(\beta)\right\rangle\right\rangle$$

$$\mathcal{H} = \frac{1}{2}\left(p^+ + \frac{|\boldsymbol{p}|^2 + m^2}{p^+}\right)$$

$$\begin{aligned} K = &-\sum_l\left\{\frac{1}{l}\ \alpha_{-l}\cdot\alpha_l \ln\sinh^2\theta_l - \frac{1}{l}\ \alpha_l\cdot\alpha_{-l}\ln\cosh^2\theta_l\right\} \\ &-\sum_r\left\{b_{-r}\cdot b_r\ln\sin^2\theta_r - b_r\cdot b_{-r}\ln\cos^2\theta_r\right\} \\ &-\sum_m\left\{d_{-m}\cdot d_m\ln\sin^2\theta_m - d_m\cdot d_{-m}\ln\cos^2\theta_m\right\} \end{aligned}$$

free energy for a single string

$$\begin{aligned} F_1(\beta) = &-\frac{16\pi^4\mathcal{V}_p}{{\beta_H}^{p+1}}\int_0^\infty\frac{d\tau}{\tau}\,\tau^{-\frac{p+1}{2}} \\ &\times\left[\left(\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4\exp\left(-\frac{\pi\beta^2}{{\beta_H}^2\tau}\right) + \left(\frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4\exp\left(-\frac{\pi\beta^2}{{\beta_H}^2\tau}\right)\right] \end{aligned}$$

Free energy of many strings can be obtained from the following eq.

$$F(\beta) = \frac{1}{2}\sum_{r=1}^{\infty}\frac{1}{r}\left\{F_{1b}(\beta r) - (-1)^r F_{1f}(\beta r)\right\}$$

$$\begin{aligned} F(\beta) = &-\frac{16\pi^4\mathcal{V}_p}{{\beta_H}^{p+1}}\int_0^\infty\frac{d\tau}{\tau}\,\tau^{-\frac{p+1}{2}} \\ &\times\left[\left(\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4\left\{\vartheta_3\left(0\left|\frac{i\beta^2}{{\beta_H}^2\tau}\right.\right) - 1\right\} - \left(\frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4\left\{\vartheta_4\left(0\left|\frac{i\beta^2}{{\beta_H}^2\tau}\right.\right) - 1\right\}\right] \end{aligned}$$

This equals to the free energy with $T = 0$ in Matsubara formalism.

Conclusion and Discussion

We have computed thermal vacuum state and free energy of a single string on a Brane-antibrane pair in the framework of TFD.

This thermal vacuum state is reminiscent of the D-brane boundary state of a closed string.

$$\left|D9 - \overline{D9}\right\rangle = \exp(-S_b)\left(|B9,+\rangle_{NSNS} - |B9,-\rangle_{NSNS}\right)$$

$$|B9_{mat},\eta\rangle_{NSNS} = \exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\ \alpha_{-n}\cdot\tilde{\alpha}_{-n} + i\eta\sum_{u>0}\psi_{-u}\cdot\tilde{\psi}_{-u}\right]|B9_{mat},\eta\rangle_{NSNS}^{(0)}$$

We need to use string field theory in order to obtain the thermal vacuum state for many strings.

We need to introduce open string tachyon.