Emission spectrum of soft massless states from heavy superstring

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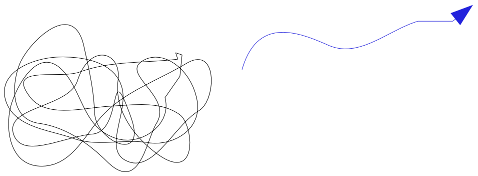
Why heavy (super)string?

Highly excited string is very small, heavy, and unstable



Decay modes, lifetime…?

(accessible by perturbation theory)

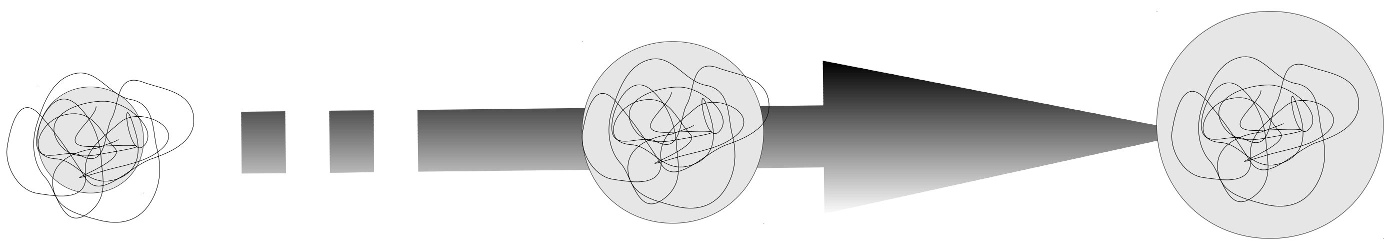
This decay may be seen as a stringy toy model of Hawking radiation.

e.g. Long-lived states and cosmological application

Blackhole/String correspondnce

Highly excited string has enormous entropy

May explain Bekenstein-Hawking entropy?



$\longleftrightarrow$

$\sim \sqrt{M}(\alpha')^{3/2}$

Typical size (random walk)

$g_s \simeq N^{-\frac{1}{4}}$

Corresponding point

$$S_{\text{string}} \sim S_{\text{BH}} \sim \sqrt{N}$$

$\longleftrightarrow$

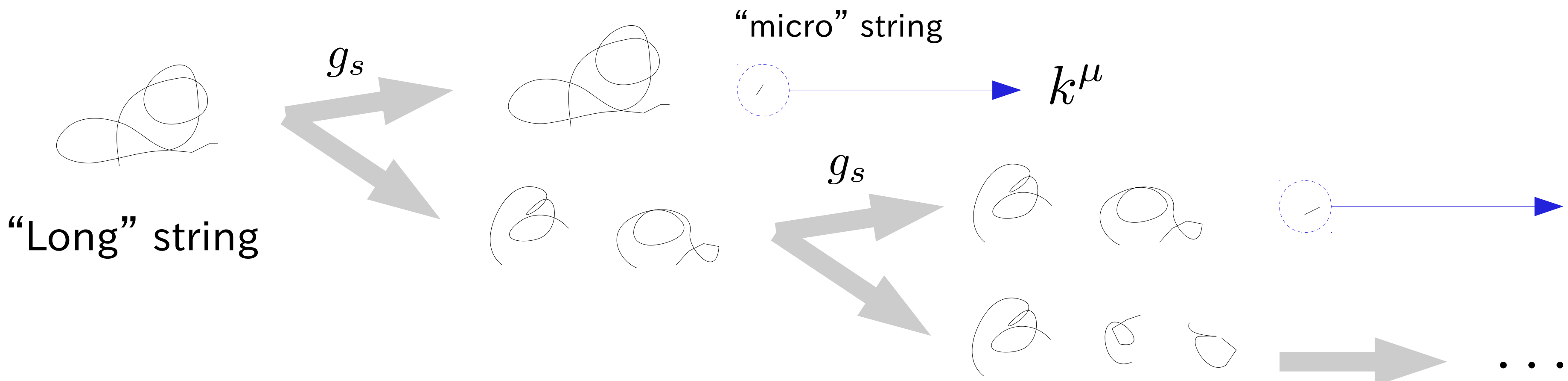
$\sim GM \simeq M g_s^2 \alpha'$

Horizon size

[Susskind ('93), Horowitz-Polchinski ('97)]

Decay process

[Dai-Polchinski ('89)]



Leading order in g_s : Splitting only once

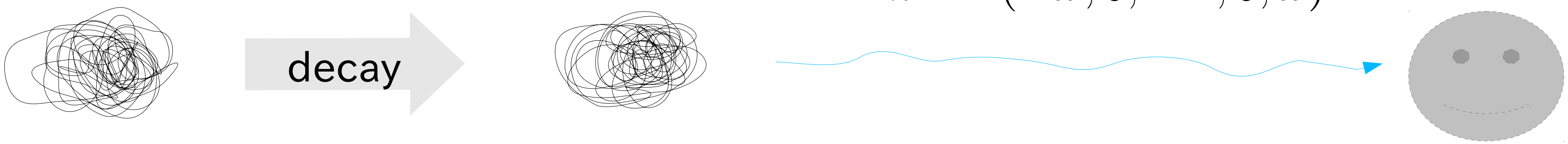
Not so large momentum transfer: microscopic strings may fly away (soft emission)

$\longrightarrow$ Turns out that massless state emission is dominant

Setup

Rest frame

$$k^\mu = (-\omega, 0, \cdots, 0, \omega)$$



$$P^\mu_{\rm ini} = (\sqrt{2N}, 0, \cdots, 0) \qquad P^\mu_{\rm fin} = (-\sqrt{\omega^2 + 2N'}, 0, \cdots, 0, -\omega)$$

$(\alpha' = 1/2)$

- Observe a heavy string from far away → Averaged initial state
- Not observe final states → sum over the possible states
- Initial and final states are “heavy”

$$N,\ N' \gg \omega \longrightarrow \boxed{N - N' = \sqrt{2N}\omega}$$

Semi-inclusive decay

Decay rate: $\Gamma = \frac{\omega^{D-2} d\omega}{M^2} P\big(\Phi(N) \to V(k) + \Phi(N')\big)$

$(D = 9)$

$$\underbrace{\frac{1}{\mathcal{G}(N)} \sum_{\Phi(N)}}_{\text{averaging}} \underbrace{\sum_{\Phi(N')} \sum_{\zeta}}_{\text{sum over}} \left| \langle \Phi(N') | V(\zeta, k) | \Phi(N) \rangle \right|^2$$

A state at level N

Exponentially many states at level $N \to$ difficult to sum over

(Open superstring) density of states: $\mathcal{G}(N) \simeq N^{-\frac{11}{4}} e^{\pi\sqrt{8N}}$

[Amati-Russo ('99)]

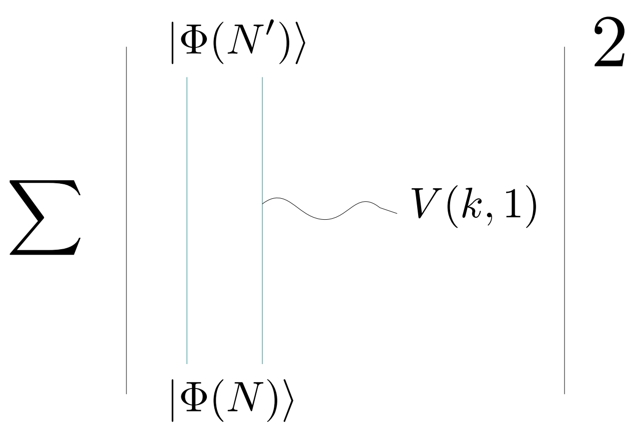
$$\hat{P}_N = \oint \frac{dw}{2\pi i w} w^{\hat{N}-N} \longrightarrow \sum_{\Phi(N)} |\Phi(N)\rangle = \sum_{\phi=\mathrm{All}} \hat{P}_N |\phi\rangle$$

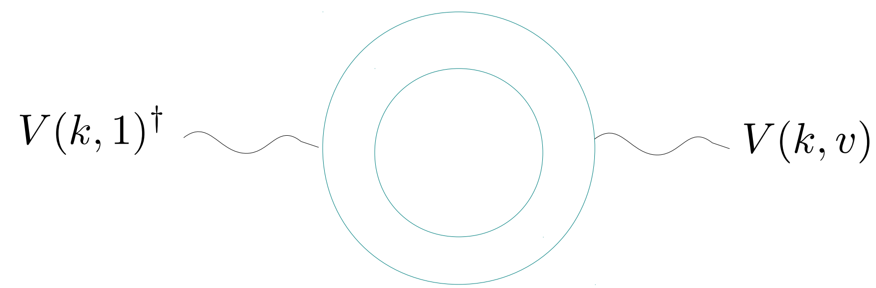
projection operator onto level N

$$\sum_{\Phi(N)} \sum_{\Phi(N')} |\langle \Phi(N')|V(k)|\Phi(N)\rangle|^2 = \sum_{\phi,\phi'} \left|\langle \phi'|\hat{P}_{N'} V(k) \hat{P}_N|\phi\rangle\right|^2$$

$$= \oint \frac{dv}{2\pi i v} v^{N-N'} \oint \frac{dw}{2\pi i w} w^{-N} \mathrm{tr}\left[V^\dagger(k,1)V(k,v)w^{\hat{N}}\right]$$

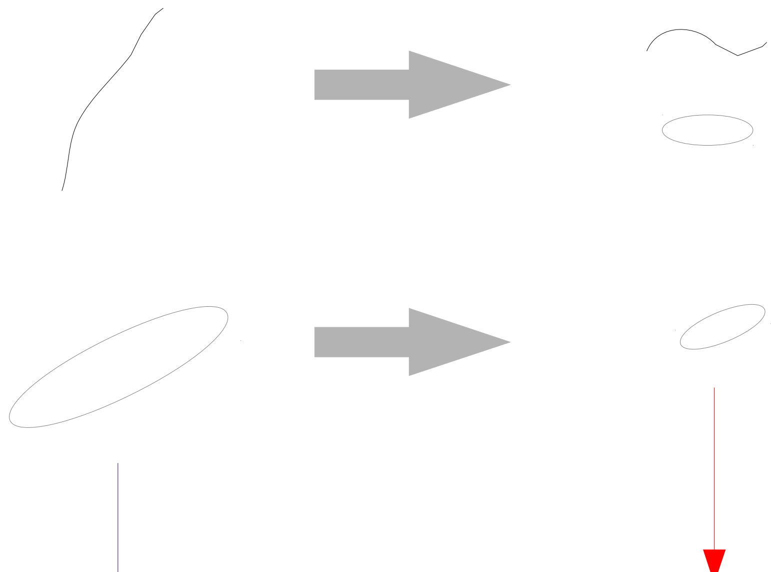
$$v^{\hat{N}} V(k,1) v^{-\hat{N}} = V(k,v)$$





Oscillator part is given by 1-loop calc.

Consider open/closed superstring massless state emission from heavy open/closed superstring.



Boson/fermion massless states

Boson/fermion massless states

$$\mathrm{tr}\left[V^\dagger(k,1)V(k,v)w^{\hat{N}}\right]$$

Evalueate this trace by use of Green-Schwarz superstring in light-cone gauge

Note: This tr is not supertrace.

Fermion emission is discussed to be subleading effect.

[Iengo-Russo, Chen-Li-She]

Vertex Operators

[Green-Schwarz]

Light-cone vertex operator: $\boxed{k^+ = 0}$

$$e^{-ik^+X^-}$$

$$\alpha_n^- \sim \sum : \alpha_{n-m}^i \alpha_m^i : + \cdots$$

SUSY

$$V_B(\zeta, k) = \left(\zeta^i(k) B^i - \zeta^-(k) p^+\right) e^{ik\cdot X}$$

Massless boson emission

$$B^i = \dot{X}^i - R^{ij} k^j$$

$$V_F(u, k) = \left(u^a(k) F^a + u^{\dot{a}}(k) F^{\dot{a}}\right) e^{ik\cdot X}$$

Massless fermion emission

$$F^a = \sqrt{p^+} S^a$$

$$F^{\dot{a}} = \frac{1}{\sqrt{p^+}} \left((\gamma \cdot \dot{X} S)^{\dot{a}} + \frac{1}{3} : (\gamma^i S)^{\dot{a}} R^{ij} : k^j \right)$$

$$R^{ij} = \frac{1}{2} (S \gamma^{ij} S)(\tau)$$: generator of rotation

Basic traces

$$\mathrm{tr}\left(V_B(\zeta,1)^\dagger V_B(k,v)w^{\hat{N}}\right) = \left(\zeta^{*i}\zeta^i\Omega(v,w) + \zeta^{*-}\zeta^-(p^+)^2\right)\theta_4(0|\tau)^{-8}$$

$$\mathrm{tr}\left(V_F(u,1)^\dagger V_F(u,v)w^{\hat{N}}\right)$$
$$= 4\left[p^+u^{a*}u^a + u^{\dot{a}*}\gamma^i_{b\dot{a}}u^b p^i + u^{a*}\gamma^i_{a\dot{b}}u^{\dot{b}}p^i + \frac{u^{\dot{a}*}(k)u^{\dot{a}}(k)}{p^+}\left((p^i)^2 + \Omega(v,w)\right)\right]\Xi(v,w)\theta_4(0|\tau)^{-8}$$

1. $\oint \frac{dv}{2\pi i v} v^{N-N'}$ integral is easy to carry out.

$\longrightarrow$ $v^{-(N-N')}$ term survives $(N > N')$

2. $p^+ \to \sqrt{N}$: large-N factor

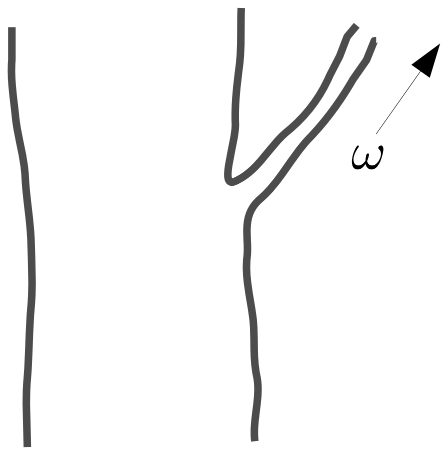
$$\Omega(v,w) = \sum_{n=1}^{\infty} n \frac{v^n + (w/v)^n}{1-w^n}$$

$$\Xi(v,w) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{v^n + (w/v)^n}{1+w^n}$$

$$\theta_4(0|\tau) = \prod_{n=1}^{\infty}\left(\frac{1-w^n}{1+w^n}\right)$$

$w = e^{i\pi\tau}$

Open string state from Open string



$$P_{\text{boson, open}} = \sum_{\zeta} |\zeta|^2 \frac{N-N'}{\mathcal{G}(N)} \oint \frac{dw}{2\pi i w} \frac{w^{-N'}\theta_4^{-8}}{1-w^{N-N'}}$$

$$P_{\text{fermion, open}} \simeq 4 \sum_{u} |u|^2 \frac{\sqrt{N}}{\mathcal{G}(N)} \oint \frac{dw}{2\pi i w} \frac{w^{-N'}\theta_4^{-8}}{1+w^{N-N'}}$$

Evaluate w-integral by saddle point method $(N' \, : \, \text{large})$

$$P_{\text{boson, open}} \sim \frac{N-N'}{\mathcal{G}(N)} \frac{e^{\pi\sqrt{8N'}} N'^{-11/4}}{1-e^{-\sqrt{2}\pi \frac{N-N'}{\sqrt{N'}}}}$$

$$P_{\text{fermion, open}} \sim \frac{\sqrt{N}}{\mathcal{G}(N)} \frac{e^{\pi\sqrt{8N'}} N'^{-11/4}}{1+e^{-\sqrt{2}\pi \frac{N-N'}{\sqrt{N'}}}}$$

Decay rate for open string emission

Using, $\mathcal{G}(N) \simeq N^{-\frac{11}{4}} e^{\pi\sqrt{8N}}$ $N - N' = \omega\sqrt{2N}$ and $\Gamma = \frac{\omega^7 d\omega}{M^2} P_{\text{boson or fermion}}$

$$\Gamma_{\text{boson, open}} \sim \frac{\omega^8 d\omega}{M^2} \frac{\sqrt{N}}{e^{2\pi\omega} - 1}$$

$$\Gamma_{\text{fermion, open}} \sim \frac{\omega^7 d\omega}{M^2} \frac{\sqrt{N}}{e^{2\pi\omega} + 1}$$

Thermal distribution of Hagdorn temperature $T_H = \frac{1}{2\pi}$

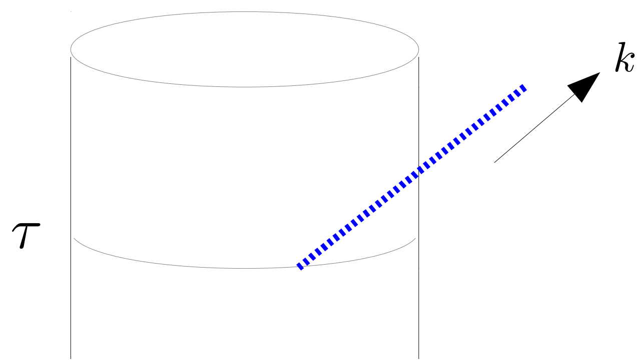
Closed string emission

Closed string vertex operator: open $\times$ open

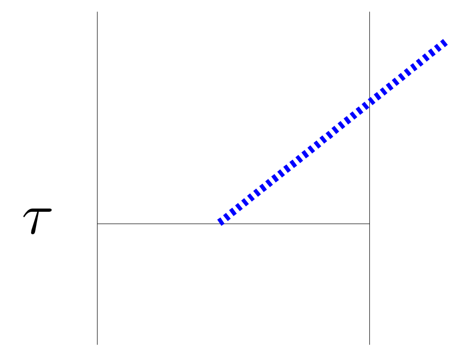
$$V_{\zeta\bar{\zeta}}(k,\tau) = \int_0^\pi \frac{d\sigma}{\pi} V_\zeta(k/2;\tau+\sigma) V_{\bar{\zeta}}(k/2;\tau-\sigma)$$

$\tilde{\alpha}_n^i, \tilde{S}_n^a$ from closed

α_n^i, S_n^a from open



from closed string



from open string

From Heavy closed string

Calculation is factorized $\left(\mathcal{G}^{\mathrm{cl}}(N) = (\mathcal{G}(N))^2\right)$

$$\frac{1}{\mathcal{G}^{\mathrm{cl}}(N)} \sum_{\Phi(N),\Phi'(N)} \left|\langle \Phi(N') | V_{\zeta\bar{\zeta}} | \Phi(N) \rangle \right|^2 = \frac{1}{\mathcal{G}(N)} \oint \frac{dv}{2\pi i v} v^{N-N'} \oint \frac{dw}{2\pi i w} w^{-N} \mathrm{tr}\left(V_\zeta(1)^\dagger V_\zeta(v) w^{\hat{N}}\right)$$
$$\times \frac{1}{\mathcal{G}(N)} \oint \frac{d\tilde{v}}{2\pi i \tilde{v}} \tilde{v}^{N-N'} \oint \frac{d\tilde{w}}{2\pi i \tilde{w}} \tilde{w}^{-N} \mathrm{tr}\left(V_{\bar{\zeta}}(1)^\dagger V_{\bar{\zeta}}(\tilde{v}) \tilde{w}^{\hat{N}}\right)$$

Product of the open result $(\alpha' = 2)$

For example, $$P^{ij}_{\mathrm{closed}} = \frac{\omega\sqrt{N}}{e^{2\pi\omega}-1} \cdot \frac{\omega\sqrt{N}}{e^{2\pi\omega}-1} = \frac{\omega^2 N (e^{4\pi\omega}-1)}{(e^{2\pi\omega}-1)^2} \cdot \frac{1}{e^{4\pi\omega}-1}$$

$\xrightarrow{\text{interpret}}$ Thermal distribution of Hagedorn temp. $T_H = \dfrac{1}{4\pi}$

Closed states from open string

"Left" and "right" parts of closed vertex act on the same Fock space.

$$P_{\text{closed from open}} = \frac{1}{\mathcal{G}(N)} \sum_{\Phi(N),\Phi'(N)} \left|\langle\Phi(N')|V_{\zeta\bar{\zeta}}|\Phi(N)\rangle\right|^2$$

$$= \frac{1}{\mathcal{G}(N)} \int_0^\pi \frac{d\sigma}{\pi} \int_0^\pi \frac{d\tilde{\sigma}}{\pi} \oint \frac{dv}{2\pi i v} v^{N-N'} \oint \frac{dw}{2\pi i w} w^{-N} \mathrm{tr}\Big(\underbrace{(V_\zeta(e^{i\tilde{\sigma}})V_{\bar{\zeta}}(e^{-i\tilde{\sigma}}))^\dagger (V_\zeta(ve^{i\sigma})V_{\bar{\zeta}}(ve^{-i\sigma}))}_{\text{4 vertex insertion}} w^{\hat{N}}\Big)$$

Leading to a bit complicated result...

Example: Boson – Boson case

$$L = N - N' = \sqrt{2N}\omega + \mathcal{O}(1)$$

After v-integral,

$$P = \frac{1}{\mathcal{G}(N)} \oint \frac{dw}{2\pi i w} w^{-N} \left(\zeta^{ij}(\zeta^{ij*} + \zeta^{ji*}) P_1(w) \frac{1+(-1)^L}{2} + \zeta^{ij}(\zeta^{ij*} - \zeta^{ji*}) \frac{2}{\pi^2} P_2(w) \frac{1-(-1)^L}{2} \right) \theta_4^{-8}$$

$$P_1(w) = \left(\frac{L}{2}\right)^2 \frac{w^L}{\left(1 - w^{\frac{L}{2}}\right)^2}$$

$$P_2(w) = 2\sum_{n=1}^{\infty} \frac{n(n+L)}{(2n+L)^2} \frac{w^{L+n}}{(1-w^n)(1-w^{n+L})} + \sum_{n=1}^{L-1} \frac{n(L-n)}{(2n-L)^2} \frac{w^L}{(1-w^n)(1-w^{L-n})}$$

In this sum, $n = (L+1)/2 + \mathcal{O}(1)$ part gives the dominant contribution.

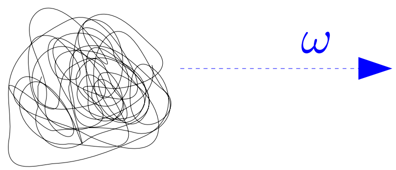
➡ Leading order part is the same as that from closed string!!

(ω dependence)

Emission rates (summary)

Emission rates:

$M \simeq \sqrt{N}$



$$\Gamma \sim \frac{\omega^8 d\omega}{M^2} \frac{\sigma(\omega)}{e^{\beta_H \omega} \mp 1} \qquad \beta_H = \pi\sqrt{8\alpha'}$$

Open from Open:

$$\sigma_{\rm boson} = g^2\sqrt{N} \cdot 1$$

$$\sigma_{\rm fermion} = g^2\sqrt{N} \cdot \omega^{-1}$$

Closed from Open/Closed:

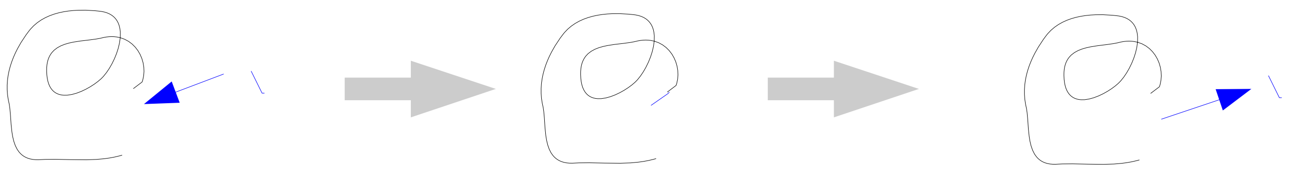
$$\sigma_{BB} = g^4 N \cdot \frac{\omega(e^{\beta_H \omega} - 1)}{(e^{\beta_H \omega/2} - 1)^2}$$

$$\sigma_{BF} = g^4 N \cdot \frac{e^{\beta_H \omega} + 1}{(e^{\beta_H \omega/2} - 1)(e^{\beta_H \omega/2} + 1)}$$

$$\sigma_{FF} = g^4 N \cdot \frac{\omega^{-1}(e^{\beta_H \omega} - 1)}{(e^{\beta_H \omega/2} + 1)^2}$$

Observations I

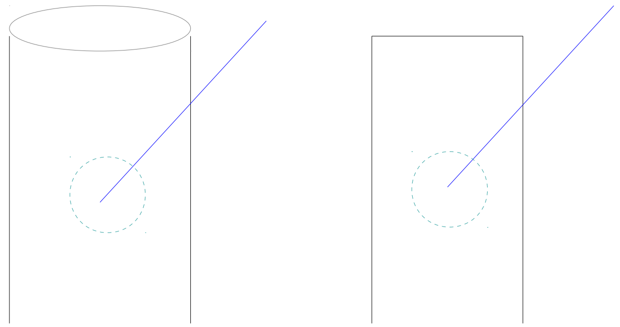
Open string is like a blackbody.



$$\sim \frac{1}{N}$$

Behaves like a cavity? (Once absorbed, hardly emitted)

Closed string emission from open/closed string takes the same form.



Locality of the interaction

Blackhole greybody factors

Greybody factors: $$\Gamma = \frac{\sigma_{js}(\omega)\omega^8 d\omega}{e^{\beta\omega} \mp 1}$$

s: spin, j: total ang.mon.

$j \geq s,\ \omega \ll 1$

Spherical BH in an asymptotically flat space

[Harmark-Natario-Schiappa ('07), Kanti-March-Russel(02)]

$\sigma_{j0} \propto \omega^{2j}$: scalar (s=0) $\longrightarrow \omega^0$

$\sigma_{j\frac{1}{2}} \propto \omega^{2j-1}$: Dirac fermion $\longrightarrow \omega^0$

blackbody

$\sigma_{j1} \propto \omega^{2j}$: vector $\longrightarrow \omega^2$

Dominant j=s modes

Heavy string

ω^{-1}

ω^0

(j=0 ?)

$d^9k \to \omega^8 d\omega$

Blackhole greybody factors

5D D5-D1-KK near extremal BH

$$\sigma_{s=0} \propto \frac{\omega(e^{\beta_{BH}\omega}-1)}{(e^{\beta_{BH}\omega/2}-1)^2}$$

[Das-Mathur ('96), Maldacena-Strominger(97)]

$$\sigma_{s=1/2} \propto \frac{\omega e^{\beta_{BH}\omega}+1}{(e^{\beta_{BH}\omega/2}-1)(e^{\beta_{BH}\omega/2}+1)}$$

[Hosomichi ('97)]

Closed string emission from
Heavy superstring

Bosons

$$\sigma \propto \frac{\omega(e^{\beta_H\omega}-1)}{(e^{\beta_H\omega/2}-1)^2}$$

$$\sigma \propto \frac{\omega^{-1}(e^{\beta_H\omega}-1)}{(e^{\beta_H\omega/2}+1)^2}$$

Fermions

$$\sigma \propto \frac{e^{\beta_H\omega}+1}{(e^{\beta_H\omega/2}-1)(e^{\beta_H\omega/2}+1)}$$

Why these kinds of black holes?

Summary

- Calculate open/closed massless state emission from heavy open/closed superstring
- Open string state emission: blackbody like
- Closed string state emission: same for open/closed string
- Greybody factors are somehow blackhole like
    (Our setup is non-BPS)

- Numerical coefficient?
- Next order? Counpling constant vs. large-N