Soft graviton effects on Local matter dynamics in de Sitter space

Hiroyuki Kitamoto (SNU)
with Yoshihisa Kitazawa (KEK, Sokendai)

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Introduction
(dS symmetry breaking)

We focus on time dependent quantum effects in dS space

$$dS_4:\ ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2 = \frac{-d\tau^2 + d\mathbf{x}^2}{H^2\tau^2}$$

H: Hubble const.

$a = e^{Ht} = -1/H\tau$

It is crucial whether the dS symmetry is broken or not

$$\tau \to C\tau, \quad \mathbf{x} \to C\mathbf{x}$$
$$(\mathbf{p} \to C^{-1}\mathbf{p})$$

Quantum effects expressed in functions of physical momentum $P = p/a(\tau) = -Hp\tau$ respect the dS symmetry

Such effects are time independent as we observe phenomena at fixed physical momentum scales

Scalar field in dS space

$$\mathcal{L}_s = -\frac{1}{2}\int \sqrt{-g}d^D x \; [g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + (m^2+\xi R)\varphi^2]$$

B. D. vac.: $\phi_{\mathbf{p}}(x) = \frac{\sqrt{\pi}}{2} H^{\frac{D-2}{2}}(-\tau)^{\frac{D-1}{2}} H_\nu^{(1)}(-p\tau)e^{+i\mathbf{p}\cdot\mathbf{x}}$

$$\nu = \sqrt{(\frac{D-1}{2})^2 - \frac{m^2}{H^2} - \xi D(D-1)}$$

$$\sim H^{\frac{D-2}{2}}(-\tau)^{\frac{D-1}{2}} \times \frac{1}{(-p\tau)^\nu} e^{+i\mathbf{p}\cdot\mathbf{x}}$$

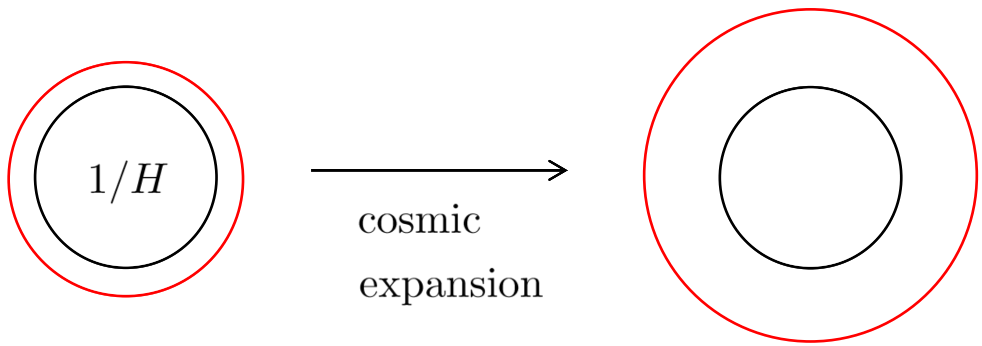
super-horizon
$P \ll H \;\; (-p\tau \ll 1)$

Massless, minimally coupled field ($m^2 = 0, \;\; \xi = 0$):

$$\langle\varphi(x)\varphi(x)\rangle \sim H^{D-2}\int d^{D-1}(-p\tau)\; \frac{1}{(-p\tau)^{D-1}}$$

scale invariant spectrum
$\Rightarrow$ IR log. divergence

We introduce an IR cut-off $p_{\min} = 1/L$: $\int_{\color{red}1/a(\tau)L}^{H} dP$



$a(\tau)L$:
size of universe

$$\langle \varphi^2(x) \rangle = (\text{UV const.}) + \frac{H^2}{4\pi^2} \int_{1/a(\tau)L}^{H} \frac{dP}{P}$$
$$= (\text{UV const.}) + \underline{\frac{H^2}{4\pi^2} \log HLa(\tau)}$$

For simplicity, $L = 1/H$

Not inv. under
$\tau \to C\tau,\ \mathbf{p} \to C^{-1}\mathbf{p}$

'82 A. Vilenkin, L. H. Ford,
A. D. Linde,
A. A. Starobinsky

Massless Dirac, Gauge fields are conformally coupled
and so do not induce the dS symmetry breaking

Gravitational field in dS space

$$g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu},\ \Omega = a(\tau)e^{\kappa w(x)},$$
$$\det\tilde{g}_{\mu\nu} = -1,\ \tilde{g}_{\mu\nu} = \eta_{\mu\rho}(e^{\kappa h(x)})^{\rho}{}_{\nu}$$

$$\kappa^2 = 16\pi G$$

GF term: $\mathcal{L}_{\rm GF} = -\frac{1}{2}a^2 F_\mu F^\mu,$

$$F_\mu = \partial_\rho h_\mu{}^\rho - 2\partial_\mu w + 2h_\mu{}^\rho \partial_\rho \log a + 4w\partial_\mu \log a$$

'94 N. C. Tsamis, R. P. Woodard

Conformally coupled modes

$$\langle h^{0i}(x)h^{0j}(x')\rangle = -\delta^{ij}\langle\phi(x)\phi(x')\rangle,$$
$$\langle Y(x)Y(x')\rangle = \langle\phi(x)\phi(x')\rangle,$$
$$\langle b^0(x)\bar{b}^0(x')\rangle = -\langle\phi(x)\phi(x')\rangle$$

$$\langle\phi(x)\phi(x')\rangle = \frac{H^2}{4\pi^2}\frac{1}{y}$$

Minimally coupled modes

$$\langle X(x)X(x')\rangle = -\langle\varphi(x)\varphi(x')\rangle,$$
$$\langle\tilde{h}^i{}_j(x)\tilde{h}^k{}_l(x')\rangle = (\delta^{ik}\delta_{jl} + \delta^i{}_l\delta_j{}^k - \frac{2}{3}\delta^i{}_j\delta^k{}_l)\langle\varphi(x)\varphi(x')\rangle,$$
$$\langle b^i(x)\bar{b}^j(x')\rangle = \delta^{ij}\langle\varphi(x)\varphi(x')\rangle$$

$$\langle\varphi(x)\varphi(x')\rangle = \frac{H^2}{4\pi^2}\{\frac{1}{y} - \frac{1}{2}\log y + \frac{1}{2}\underline{\log a(\tau)a(\tau')} + 1 - \gamma\}$$

$\tilde{h}^i{}_j$:traceless, $X \equiv 2\sqrt{3}w - h^{00}/\sqrt{3},\ Y \equiv h^{00} - 2w$

$y \equiv \Delta x^\mu \Delta x_\mu/\tau\tau'$: dS invariant distance^2

Motivation

- Gravitational field contains massless and minimally coupled modes without fine-tuning
- Gravitational effects seem to be suppressed by $GH^2 \ll 1$, but there exist enhancement factors: $(GH^2 \log a(\tau))^n$
- Although we cannot observe the super-horizon modes directly, it is possible that virtual gravitons at the super-horizon scale affect local dynamics of matter fields at the sub-horizon scale
- This investigation is up to the 1-loop level: $\log a(\tau) \gg 1,$
  $GH^2 \log a(\tau) \ll 1$

Free scalar field theory

$$\mathcal{L}_s = -\frac{1}{2}\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{R}{6}\phi^2\right] \rightarrow -\frac{1}{2}\left[\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{\tilde{R}}{6}\phi^2\right]$$

$g_{\mu\nu} = \Omega^2\tilde{g}_{\mu\nu}$, $\Omega\phi \rightarrow \phi$

- We focus on soft gravitons in internal momenta
- We set external momentum to be at the sub-horizon scale

Effective e. o. m.:

$$\frac{\delta\Gamma}{\delta\hat{\phi}} = \partial_\mu\partial^\mu\hat{\phi} + \kappa^2 H^2\{c_1 a^2 H^2 + c_2 aH\partial + (c_3\log a + c_4)\partial\partial$$

$\hat{\phi}$: classical field

including soft gravitons

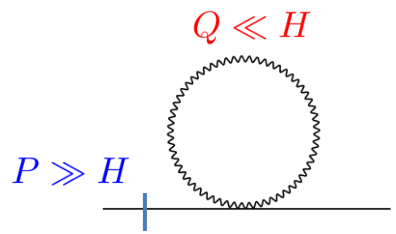
$$+c_5 a^{-1}H^{-1}\partial\partial\partial + c_6 a^{-2}H^{-2}\partial\partial\partial\partial\}\hat{\phi} = 0$$

$$\Downarrow \qquad P \gg H \Leftrightarrow \partial\hat{\phi} \gg aH\hat{\phi}$$

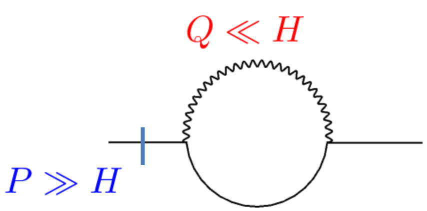
$$\partial_\mu\partial^\mu\hat{\phi} + \kappa^2 H^2 \log a\{d_1\partial_0^2 + d_2\partial_i^2\}\hat{\phi} \simeq 0$$

$d_1 = -d_2$?

Effective Lorentz inv.



$$\simeq \frac{\kappa^2 H^2}{4\pi^2} \log a(\tau) \{ \frac{3}{8}\partial_0^2 + \frac{13}{8}\partial_i^2 \} \hat{\phi}$$



$$\simeq \frac{\kappa^2 H^2}{4\pi^2} \log a(\tau) \{ -\frac{3}{4}\partial_0^2 - \frac{5}{4}\partial_i^2 \} \hat{\phi}$$

$P \gg H \Rightarrow \log a \partial \gg \partial \log a$

$$\{1 + \frac{3\kappa^2 H^2}{32\pi^2} \log a(\tau)\} \partial_\mu \partial^\mu \hat{\phi} \simeq 0$$

Lorentz inv.

↑

Normalizable by

$$\phi \to Z_\phi \phi, \;\; Z_\phi \simeq 1 - \frac{3\kappa^2 H^2}{64\pi^2} \log a(\tau)$$

Similary, $\{1 + \frac{3\kappa^2 H^2}{128\pi^2} \log a(\tau)\} i\gamma^\mu \partial_\mu \hat{\psi} \simeq 0$ $\quad \Omega^{\frac{3}{2}}\psi \to \psi$

Lorentz inv.

↑

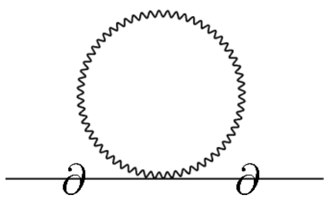
$$\psi \to Z_\psi \psi, \;\; Z_\psi \simeq 1 - \frac{3\kappa^2 H^2}{256\pi^2} \log a(\tau)$$

Soft gravitons do not contribute to these free field theories

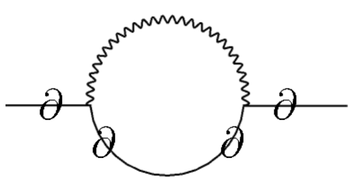
Local, Nonlocal IR singularities

Nonlocal IR singularities from soft or colinear particles are canceled between real and virtual processes

On the other hand, soft gravitons induce the dS symmetry breaking IR effects through the local terms:



$$\Sigma_{\text{4-pt}}(x,x') = \frac{\kappa^2}{2}\langle h^{\mu\rho}(x)h_\rho{}^\nu(x)\rangle\partial_\mu\partial_\nu\underline{\delta^{(4)}(x-x')}$$



$$\Sigma_{\text{3-pt}}(x,x') \to \kappa^2\langle h(x)h(x)\rangle\partial\partial\underline{\delta^{(4)}(x-x')}$$

$\Nearrow$

$$\partial_\rho\partial_\sigma\langle T\phi(x)\phi(x')\rangle$$
$$\to -i\delta_\rho{}^0\delta_\sigma{}^0\underline{\delta^{(4)}(x-x')}$$

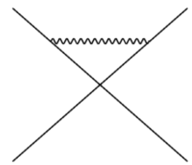
The coefficients are IR divergent
$\langle h(x)h(x)\rangle \to H^2\log a(\tau)$

Interacting field theories

$$\mathcal{L}_{\rm int} = -\frac{\lambda_4}{4!}\phi^4 - \lambda_Y \phi\bar{\psi}\psi \;\rightarrow\; -\frac{\lambda_4}{4!}Z_\phi^4\phi^4 - \lambda_Y Z_\phi Z_\psi^2 \phi\bar{\psi}\psi$$

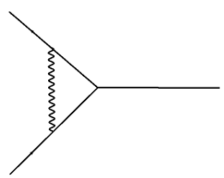
$\sqrt{-g}$ is absorbed by $\Omega\phi \rightarrow \phi,\ \Omega^{\frac{3}{2}}\psi \rightarrow \psi$

$$-\frac{\lambda_4}{4!}Z_\phi^4\hat{\phi}^4 \quad +$$

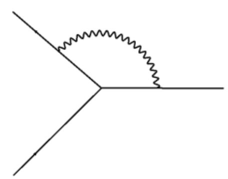


$$\simeq\; -\frac{\lambda_4}{4!}\left\{1 - \frac{21\kappa^2 H^2}{16\pi^2}\log a(\tau)\right\}\hat{\phi}^4$$

$$-\lambda_Y Z_\phi Z_\psi^2 \hat{\phi}\hat{\bar{\psi}}\hat{\psi} \quad +$$



$$+$$



$$\simeq\; -\lambda_Y\left\{1 - \frac{39\kappa^2 H^2}{128\pi^2}\log a(\tau)\right\}\hat{\phi}\hat{\bar{\psi}}\hat{\psi}$$

Both couplings decrease with cosmic evolution

$$(\lambda_4)_{\rm eff} \simeq \lambda_4\left\{1 - \frac{21\kappa^2 H^2}{16\pi^2}\log a(\tau)\right\}, \quad (\lambda_Y)_{\rm eff} \simeq \lambda_Y\left\{1 - \frac{39\kappa^2 H^2}{128\pi^2}\log a(\tau)\right\}$$

Gauge field theories

Effective action:

$$-\frac{1}{4g^2}\{1+\frac{3\kappa^2H^2}{8\pi^2}\log a(\tau)\}(\hat{F}_{\mu\nu})^a(\hat{F}^{\mu\nu})^a$$

$$gA_\mu^a \to A_\mu^a$$

$$+\{1+\frac{3\kappa^2H^2}{128\pi^2}\log a(\tau)\}\hat{\bar{\psi}}\gamma^\mu\hat{A}_\mu^a t^a\hat{\psi}$$

Nomalized after $\psi \to Z_\psi \psi$

$$\Downarrow$$

$$g_{\mathrm{eff}} \simeq g\{1-\frac{3\kappa^2H^2}{16\pi^2}\log a(\tau)\}$$

g_{eff} decreases with cosmic evolution

The behavior is independent of a gauge group

Gauge dependence

We introduce a gauge parameter β as

$$F_\mu = \beta\partial_\rho h_\mu{}^\rho - 2\beta\partial_\mu w + \frac{2}{\beta}h_\mu{}^\rho\partial_\rho \log a + \frac{4}{\beta}w\partial_\mu \log a$$

original gauge is at $\beta = 1$

For a continuous β ($|(\beta^2-1)| \ll 1$), the effective Lorentz inv. is respected

$$Z_\phi^{-2}(\beta,\tau)\partial_\mu\partial^\mu\hat{\phi} \simeq 0, \quad Z_\psi^{-2}(\beta,\tau)i\gamma^\mu\partial_\mu\hat{\psi} \simeq 0$$

$$Z_\phi(\beta,\tau) \simeq 1 - \left(1-(\beta^2-1)^1\right)\frac{3\kappa^2H^2}{64\pi^2}\log a(\tau),$$
$$Z_\psi(\beta,\tau) \simeq 1 - \left(1-(\beta^2-1)^1\right)\frac{3\kappa^2H^2}{256\pi^2}\log a(\tau)$$

$\because$ At the coincident point,
$\langle h^{\mu\nu}(x)h^{\rho\sigma}(x)\rangle \to \left(1-(\beta^2-1)^1\right)\langle h^{\mu\nu}(x)h^{\rho\sigma}(x)\rangle$

Each coeffcient of the screening of coulings is gauge dependent:

$$\left.\begin{aligned} \delta\lambda_4 &\simeq -\lambda_4\big(1-(\beta^2-1)^1\big)\frac{21\kappa^2H^2}{16\pi^2}\log a(\tau), \\ \delta\lambda_Y &\simeq -\lambda_Y\big(1-(\beta^2-1)^1\big)\frac{39\kappa^2H^2}{128\pi^2}\log a(\tau), \\ \delta g &\simeq -g\big(1-(\beta^2-1)^1\big)\frac{3\kappa^2H^2}{16\pi^2}\log a(\tau) \end{aligned}\right\} \quad \delta\lambda_j \sim \beta_j t$$

$$\because \ \log a(\tau) = Ht$$

because there is no unique way to specify the time as it depends on an observer

Our proposal is to pick a particular coupling as a physical time such as $\lambda_i \sim t$:

$$\delta\lambda_j \sim \frac{\beta_j}{\beta_i}t$$

The relative scaling exponents β_j/β_i are gauge invariant

’90 H. Kawai, M. Ninomiya
’93 H. Kawai, Y Kitazawa, M. Ninomiya

Parametrization dependence

Lorentz inv. is broken in a general choice of parametrization of the metric

$$\begin{aligned} g_{\mu\nu} &= a^2 e^{2\kappa w} \eta_{\mu\rho} (e^{\kappa h})^\rho{}_\nu \qquad &, h^\mu{}_\mu = 0 \\ &= a^2 (\eta_{\mu\nu} + 2\kappa\Phi\eta_{\mu\nu} + \kappa\Psi_{\mu\nu}) \qquad &, \Psi^\mu{}_\mu = 0 \end{aligned}$$

The difference between them starts at the non-linear level and contributes only to tadpole diagrams at 1-loop level

$$\begin{aligned} \kappa w &= \kappa\Phi + \mathcal{O}(\kappa^2\Phi^2, \kappa^2\Psi^2), \\ \kappa h_{\mu\nu} &= \kappa\Psi_{\mu\nu} + \mathcal{O}(\kappa^2\Phi\Psi, \kappa^2\Psi\Psi) \end{aligned} \quad \Rightarrow \quad \begin{aligned} \kappa^2\langle h_{\mu\nu}(x) h_{\rho\sigma}(x')\rangle &= \kappa^2\langle \Psi_{\mu\nu}(x)\Psi_{\rho\sigma}(x')\rangle, \\ \kappa\langle h_{\mu\nu}(x)\rangle &\neq 0 \end{aligned}$$

Such effects can be eliminated by shifting the background metric

$$h_{\mu\nu} \to h_{\mu\nu} + v_{\mu\nu}$$

Soft graviton effects at 1-loop on Local matter dynamics

- Soft gravitons contribute to the dS symmetry breaking only through the local terms
- The IR effects on kinetic terms respect Lorentz invariance and can be absorbed by wave function renormalizations
- Soft gravitons screen dimensionless couplings whose relative scaling exponents are invariant against a slight gauge deformation
- Parametrization dependence of the metric can be eliminated by shifting the background metric