LINEAR RESPONSES OF D0-BRANES VIA GAUGE/GRAVITY CORRESPONDENCE

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1. Introduction

What occurs in a black hole when we apply a time-dependent external field to the black hole?

Membrane paradigm

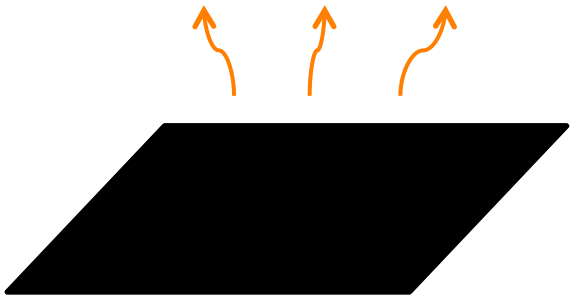
Damour (1978), Thorne, Price, MacDonald (1986)
Parikh, Wilczek (1998) etc.

- Predictions of classical general relativity
- Responses occur on the horizon
- The responses follow the hydrodynamic laws

What is the microscopic origin?

<u>Black hole in string theory</u>

Strominger, Vafa (1996)
Callan, Maldacena (1996), etc.



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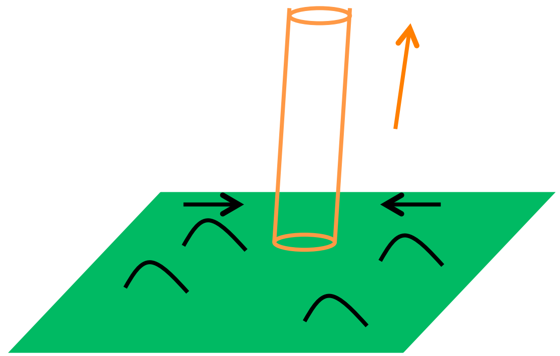

Bekenstein-Hawking entropy ⇔ Number of states on D-branes

Hawking radiation ⇔ open + open → closed

It will be possible to understand the membrane paradigm using string theory.

D0-brane black hole

Itzhaki, Maldacena, Sonnenshein, Yankielowicz (1998), etc.

Motivations

- D0-brane black hole is spherical.
- There is no spatial direction along the brane.

Strongly coupled Matrix theory ((0+1)-dimensional U(N) SYM) ⇔ Near horizon geometry of D0-brane black hole solution

By using gauge/gravity correspondence, we calculate the linear responses of D0-branes (Matrix theory).

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2. Gauge/gravity correspondence for Matrix theory

Sekino, Yoneya (1999),etc

Near extremal D0-brane black hole solution (near horizon limit)

$$ds_s^2 = e^{\frac{2}{7}\tilde{\phi}} ds_w^2,$$

$$ds_w^2 = \tilde{R}^2 \left[z^{-2}(-f dt^2 + f^{-1} dz^2) + \left(\frac{5}{2}\right)^2 d\Omega_8^2 \right],$$

$$f = 1 - \left(\frac{z}{z_0}\right)^{\frac{14}{5}},$$

$$e^{\tilde{\phi}} = \left(\frac{z}{\tilde{R}}\right)^{\frac{21}{10}},$$

$$A_0 = \frac{1}{g_s}\left(\frac{\tilde{R}}{z}\right)^{\frac{14}{5}}$$

String frame

AdS frame

($AdS_2 \times S^8$ if extremal)

$R =$ Radius of S^8

V_8 = Volume of S^8

$\sim R^8$

where $z \equiv \frac{2}{5} R^{\frac{7}{2}} r^{-\frac{5}{2}}$, $\tilde{R} \equiv \frac{2}{5} R$, $R = (60\pi^3)^{\frac{1}{7}} (g_s N)^{\frac{1}{7}} l_s$

$$T_H = \frac{7}{10\pi z_0} \quad , \quad S_{BH} = \frac{V_8}{4G}\left(\frac{\tilde{R}}{z_0}\right)^{\frac{9}{5}} \quad , \quad q = \frac{7 g_s}{16\pi G R} V_8$$

“GKPW relation” for Matrix theory

Gubser, Klebanov, Polyakov (1998), Witten (1998), Sekino, Yoneya (1999) Policastro, Son, Starinets (2002)

Boundary conditions of linear perturbations

- Ingoing boundary condition at the horizon $z = z_0$
- Dirichlet boundary condition at the cutoff surface $z = z_c$

On-shell action

$$S_{\mathrm{on-shell}} \sim \int \frac{d\omega}{2\pi} \bar{h}_I^s(-\omega)\mathcal{F}_I(\omega, z)\bar{h}_I^{s'}(\omega)\Big|_{z=z_c}$$

Linear response of Matrix operator $\mathcal{O}^s(\omega)$

$$\delta\langle \mathcal{O}^s(\omega)\rangle = -G_R^{ss'}(\omega)\bar{h}^{s'}(\omega),$$

$$G_R^{ss'}(\omega) = \begin{cases} -2\mathcal{F}(\omega, z)|_{z=z_c}, & (\text{for } s = s') \\ -\mathcal{F}(\omega, z)|_{z=z_c}, & (\text{for } s \neq s') \end{cases}$$

3. Linear responses of D0-branes

Matsuo, Y.S., Sekino (2013)

IIA action in AdS frame

$$S_{IIA} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-\frac{6}{7}\phi} \left(R + \frac{16}{49} \partial_\mu \phi \partial^\mu \phi - \frac{g_s^2}{4} e^{\frac{12}{7}\phi} F_{\mu\nu} F^{\mu\nu} \right)$$

$$(\mu, \nu = 0, \cdots, 9)$$

Consider tensor mode and vector modes.

With some gauge conditions,

Tensor mode $$h_{ij}(x^\mu) - \frac{1}{8} g_{ij} h^k{}_k(x^\mu) = \sum_I b^I(t, z) Y^I_{ij}(x^i)$$

Vector modes $$h^0_i(x^\mu) = \sum_I b^0_I(t, z) Y^I_i(x^i), \qquad h^z_i(x^\mu) = \sum_I b^z_I(t, z) Y^I_i(x^i),$$

$$\hat{A}_i(x^\mu) = \sum_I a_I(t, z) Y^I_i(x^i),$$ (Only 2 physical d.o.f.)

$$(i, j = 1, \cdots, 8)$$

Tensor mode

Equation of motion

$$0 = f^{-1}(fb')' + \tilde{\omega}^2 u^{-\frac{9}{7}} f^{-2} b - \frac{l(l+7)}{49} u^{-2} f^{-1} b \qquad (l \geq 2)$$

where $u = \left(\frac{z}{z_0}\right)^{\frac{14}{5}}$, $f = 1 - u$, $\tilde{\omega} = \frac{\omega}{4\pi T_H}$.

Solution for $\tilde{\omega} \ll 1$

$$b(\tilde{\omega}, u) = \bar{b}(\tilde{\omega}) \frac{u^{-\frac{l}{7}} F(u) - X(\tilde{\omega}) u^{1+\frac{l}{7}} \tilde{F}(u)}{u_c^{-\frac{l}{7}} F(u_c) - X(\tilde{\omega}) u_c^{1+\frac{l}{7}} \tilde{F}(u_c)},$$

where

$$F \equiv {}_2F_1\left(-\frac{l}{7}, -\frac{l}{7}; -\frac{2l}{7}; u\right), \qquad \tilde{F} \equiv {}_2F_1\left(1+\frac{l}{7}, 1+\frac{l}{7}; 2+\frac{2l}{7}; u\right)$$

$$X \equiv \frac{\Gamma(-\frac{2l}{7})}{\Gamma(-\frac{l}{7})^2} \frac{\Gamma(1+\frac{l}{7})^2}{\Gamma(2+\frac{2l}{7})} \left[1 + 2\pi i \tilde{\omega} \cot\left(\frac{l\pi}{7}\right)\right]$$

Inserting the solution into the action,

$$2\kappa^2 S_{\text{on-shell}} = \frac{7}{5}\tilde{R}^{\frac{9}{5}} z_0^{-\frac{14}{5}} D_2 \int_{u=u_c} dt[fbb' \text{ + contact terms]}$$

$$= \frac{7}{5}\tilde{R}^{\frac{9}{5}} z_0^{-\frac{14}{5}} D_2 \int \frac{d\omega}{2\pi}(1-u_c)\bar{b}(-\omega)\bar{b}(\omega)$$

$$\cdot \frac{-\frac{l}{7}u_c^{-1}F(u_c) + F'(u_c) - X u_c^{\frac{2l}{7}}((1+\frac{l}{7})\tilde{F}(u_c) + u_c\tilde{F}'(u_c))}{F(u_c) - X u_c^{1+\frac{2l}{7}}\tilde{F}(u_c)}$$

where $D_2 = \frac{1}{2}\int d^8x\sqrt{g_8}Y_{ij}Y^{ij}.$

3.1 Hydrodynamic stress tensor and current on S^8

$$\delta T^{ij}(\omega, x^i) = \sum_I T^I(\omega) Y_I^{ij}(x^i)$$
$$\delta T^{0i}(\omega, x^i) = \sum_I T_I^0(\omega) Y_I^i(x^i),$$
$$\delta J^i(\omega, x^i) = \sum_I J^I(\omega) Y_I^i(x^i),$$

where

$$T^I(\omega) = i\omega\eta b^I(\omega),$$

$$T_I^0(\omega) = \eta \frac{\frac{(l+8)(l-1)}{R^2}}{i\omega - D\frac{(l+8)(l-1)}{R^2}} b_I^0(\omega) - \bar{n}\frac{i\omega}{i\omega - D\frac{(l+8)(l-1)}{R^2}} a^I(\omega),$$

$$J^I(\omega) = \left(i\omega\sigma - \frac{\bar{n}^2}{\bar{\epsilon}+\bar{p}}\frac{i\omega}{i\omega - D\frac{(l+8)(l-1)}{R^2}}\right) a^I(\omega) + \bar{n}\frac{i\omega}{i\omega - D\frac{(l+8)(l-1)}{R^2}} b_I^0(\omega).$$

3.2 The case of $u_c \simeq 1$ horizon)

Tensor mode

$$S_{\rm on-shell} = \frac{1}{16\pi G}\frac{1}{2}\left(\frac{\tilde{R}}{z_0}\right)^{\frac{9}{5}} D_2 \int_{u_c\simeq 1} \frac{d\omega}{2\pi}\sqrt{-g_{00}}\, i\mathfrak{w}\bar{b}(-\omega)\bar{b}(\omega)$$

where $\mathfrak{w} = \dfrac{\omega}{\sqrt{-g_{00}}}$: Proper frequency

Stress tensor

$$\mathcal{T}(\omega) = \frac{1}{16\pi G}\left(\frac{r_0}{R}\right)^{\frac{9}{2}} i\mathfrak{w}\bar{b}(\omega)$$

This is the same as the hydrodynamic stress tensor on S^8

with

$$\eta = \frac{1}{16\pi G}\left(\frac{r_0}{R}\right)^{\frac{9}{2}}.$$

$$s = \frac{S_{BH}}{V_8} = \frac{1}{4G}\left(\frac{r_0}{R}\right)^{\frac{9}{2}} \Rightarrow \frac{\eta}{s} = \frac{1}{4\pi}$$

Vector modes

Stress tensor

$$\mathcal{T}^{\tilde{0}} = \frac{1}{16\pi G}\left(\frac{r_0}{R}\right)^{\frac{9}{2}} \frac{\frac{(l+8)(l-1)}{R^2}}{i\mathfrak{w} - \mathcal{D}\left(\frac{(l+8)(l-1)}{R^2} - \frac{14(2l^2+14l-7)}{(2l+7)S_l R^2}\right)} \bar{b}^{\tilde{0}} - \frac{7g_s}{16\pi G R} \frac{i\mathfrak{w} + \mathcal{D}\frac{14(l^2+7l+1)}{(2l+7)S_l R^2}}{i\mathfrak{w} - \mathcal{D}\left(\frac{(l+8)(l-1)}{R^2} - \frac{14(2l^2+14l-7)}{(2l+7)S_l R^2}\right)} \bar{a},$$

R-R 1-form current

$$\mathcal{J} = \frac{g_s^2}{16\pi G}\left(\frac{R}{r_0}\right)^{\frac{9}{2}} i\mathfrak{w}\bar{a} - \frac{49\mathcal{D}g_s^2}{16\pi G R^2}\left(\frac{R}{r_0}\right)^{\frac{9}{2}} \frac{i\mathfrak{w} - V}{i\mathfrak{w} - \mathcal{D}\left(\frac{(l+8)(l-1)}{R^2} - \frac{14(2l^2+14l-7)}{(2l+7)S_l R^2}\right)} \bar{a} + \frac{7g_s}{16\pi G R} \frac{i\mathfrak{w} + \mathcal{D}\frac{14(l^2+7l+1)}{(2l+7)S_l R^2}}{i\mathfrak{w} - \mathcal{D}\left(\frac{(l+8)(l-1)}{R^2} - \frac{14(2l^2+14l-7)}{(2l+7)S_l R^2}\right)} \bar{b}^{\tilde{0}},$$

where $\mathcal{D} \equiv \frac{1}{4\pi T} = \frac{\sqrt{-g_{00}}}{4\pi T_H}$, $S_l \equiv \frac{\pi \sin(\frac{2l}{7}\pi)}{\sin(\frac{l-1}{7}\pi)\sin(\frac{l+1}{7}\pi)}$.

Although there are some differences, the thermodynamic quantities and transport coefficients agree with those of hydrodynamics on S^8.

Diffusion const.

$$D = \frac{\eta}{\bar{\epsilon} + \bar{p}} = \frac{\eta}{s} \cdot \frac{1}{T} = \frac{1}{4\pi T} = \mathcal{D}$$

Charge density

$$\bar{n} = \frac{q}{V_8} = \frac{7 g_s}{16\pi G R}$$

$$\frac{\bar{n}^2}{\bar{\epsilon} + \bar{p}} = \left(\frac{7 g_s}{16\pi G R}\right)^2 \frac{1}{T s} = \frac{49 \mathcal{D} g_s^2}{16\pi G R^2} \left(\frac{R}{r_0}\right)^{\frac{9}{2}}.$$

Conductivity

$$\sigma = \frac{g_s^2}{16\pi G} \left(\frac{R}{r_0}\right)^{\frac{9}{2}}$$

If we take l as large with l/R fixed, the extra terms are decoupled.

Rindler limit → Exactly agree with hydrodynamics

Bredberg, Keeler, Lysov, Strominger (2010)
Matsuo, Natsuume, Ohta, Okamura (2012)

3.3 The case of $u_c \simeq 0$ undary)

Vector modes

$$\mathcal{T}^0 = \frac{1}{16\pi G}\left(\frac{r_0}{R}\right)^{\frac{9}{2}} u_c^{-\frac{9}{7}-\frac{2l}{7}} \frac{2l+7}{18l(l-1)B^2} \cdot \frac{\frac{49}{R^2}(2l^2+23l-7) + \frac{5z_0}{R^2} l(2l+7)(l-1)B^2 u_c^{\frac{2l}{7}} i\omega}{i\omega - \frac{98(2l+7)}{45z_0(l-1)B^2} u_c^{-1-\frac{2l}{7}}} \bar{b}^0$$

$$- \frac{7g_s}{16\pi GR} \frac{(2l+7)^2}{63u_c} \frac{i\omega - \frac{343}{5z_0 l(2l+7)(l-1)B^2} u_c^{-\frac{2l}{7}}}{i\omega - \frac{98(2l+7)}{45z_0(l-1)B^2} u_c^{-1-\frac{2l}{7}}} \bar{a},$$

$$\mathcal{J} = \frac{g_s^2}{16\pi G}\left(\frac{R}{r_0}\right)^{\frac{9}{2}} u_c^{-\frac{5}{7}} \frac{2(2l+7)^2}{45} \frac{\frac{49(l+1)}{5l(l-1)z_0^2 B^2} u_c^{-\frac{2l}{7}} - \frac{i\omega}{z_0}}{i\omega - \frac{98(2l+7)}{45z_0(l-1)B^2} u_c^{-1-\frac{2l}{7}}} \bar{a}$$

$$+ \frac{7g_s}{16\pi GR} \frac{(2l+7)^2}{63u_c} \frac{i\omega - \frac{343}{5z_0 l(2l+7)(l-1)B^2} u_c^{-\frac{2l}{7}}}{i\omega - \frac{98(2l+7)}{45z_0(l-1)B^2} u_c^{-1-\frac{2l}{7}}} \bar{b}^0.$$

$$B \equiv \frac{\Gamma(-\frac{1}{7}+\frac{l}{7})\Gamma(\frac{8}{7}+\frac{l}{7})}{\Gamma(1+\frac{2l}{7})}.$$

→ Different behavior from the hydrodynamics!

4. Summary

- We have studied the linear responses of the near extremal D0-branes in low-frequency region by using gauge/gravity correspondence.

- When the cutoff surface is close to the horizon, the linear responses of the stress tensor and R-R 1-form current take forms similar to the hydrodynamics on S^8

- When the cutoff surface is far from the horizon, the linear responses do not correspond to the hydrodynamic stress tensor and current.

- How is the case of $\tilde{\omega} \gg 1$?

- Connection with fast scrambler ? Sekino, Susskind (2008)