Thermodynamics of black M-branes from SCFTs

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Based on a work with Takeshi Morita (KEK):

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Thermodynamics of blackholes

- Analogy of 2nd law
  - 2nd law: Entropy in isolated system always increases.
  - Area of horizon always increases (in classical level).
  - Bekenstein-Hawking entropy: $S = \frac{A}{4G_N}$
- Analogy of 1st law
  - 1st law: $dE = TdS$
  - Energy: mass of blackhole (ADM mass)
  - Temperature: Hawking temperature

    (It is derived from the spectrum of Hawking radiation.)

Black branes in supergravity

- Gravity solution in 10d N=IIA / IIB SUGRA
- Metric (in string frame) $i = 1, \ldots, p$

$$ds^2 = \alpha' \left(\frac{U^{\frac{7-p}{2}}}{\sqrt{a_p \lambda}} (-f dt^2 + dx^i dx^i) + \sqrt{a_p \lambda} \left(U^{-\frac{7-p}{2}} \frac{dU^2}{f} + U^{\frac{p-3}{2}} d\Omega^2_{(8-p)} \right) \right)$$

$$f(U) = 1 - \left(\frac{U_0}{U} \right)^{7-p}, \qquad a_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2}\right)$$

- Dilaton $$e^{\phi} = \frac{1}{N} (2\pi)^{2-p} (a_p)^{\frac{3-p}{4}} \left(\frac{U}{\lambda^{1/(3-p)}} \right)^{-\frac{(3-p)(7-p)}{4}}$$

- SUGRA description for superstring theory is valid for small curvature and small string coupling:

$$N^{-\frac{2}{7-p}} \ll \left(T \lambda^{-\frac{1}{3-p}}\right)^{\frac{3-p}{5-p}} \ll 1$$

Free energy of black brane

- Black Dp-branes in superstring theory

(Radius of horizon)

$$F \sim N^2 T^{\frac{2(7-p)}{5-p}} \lambda^{-\frac{3-p}{5-p}} V_p \qquad |\phi| \sim T^{\frac{2}{5-p}} \lambda^{\frac{1}{5-p}}$$

➢ We can similarly discuss the gravity solutions of 11d SUGRA (low-energy limit of M-theory).

$\beta = 1/T$

- Black M2-brane (on $AdS_4 \times S^7/Z_k$) [ABJM '08]

$$F \sim N^{\frac{3}{2}} \sqrt{k} T^3 V_2 \qquad |\phi| \sim \frac{r_0}{k^{\frac{1}{2}} l_p^{\frac{3}{2}}} \sim \frac{N^{\frac{1}{4}}}{k^{\frac{1}{4}} \beta^{\frac{1}{2}}}$$

Exotic N-dependence!

- Black M5-brane

$$F \sim N^3 T^6 V_5 \qquad |\phi| \sim \frac{r_0}{l_p^3} \sim \frac{N}{\beta^2}$$

Can a field theory reproduce it?

- Just as the D1-D5-P system successfully reproduced the Bekenstein-Hawking entropy…

➢ Dp-branes (Nontrivial dependence on T and λ)

- (p+1)-dim maximally supersymmetric Yang-Mills

➢ M2-branes (Nontrivial dependence on N)

- ABJM theory (3d Chern-Simons-matter theory)
- Free energy at T=0 is obtained via localization technique.

[Marino '10]

➢ M5-branes (Nontrivial dependence on N)

- 6d SCFT, but the details are not known.

Black D-branes:

Review of Smilga-Wiseman method

Effective theory on Dp-branes

- Maximally supersymmetric Yang-Mills theory

$$S_{\mathrm{D}p} = \frac{N}{\lambda} \int d\tau d^p x \, \mathrm{Tr} \left[\frac{1}{4} \mathcal{F}_{\mu\nu}^2 + \frac{1}{2} (D_\mu \Phi^I)^2 + \frac{i}{2} \bar{\Psi} \Gamma^\mu D_\mu \Psi - \frac{1}{4} [\Phi^I, \Phi^J]^2 - \frac{i}{2} \bar{\Psi} \Gamma^I [\Phi^I, \Psi] \right]$$

$$\boxed{\lambda = g_{YM}^2 N}$$

(Euclidean time with period β)

- Classical vacua are gauge equivalent to configurations:

$$A_{\mu,ab} = a_{\mu,a} \delta_{ab} \, , \quad \Phi^I_{ab} = \phi^I_a \delta_{ab} \, , \quad \Psi_{ab} = 0$$

- We assume only the scalar moduli play relevant roles.
- These moduli correspond to the positions of Dp-branes.

Smilga-Wiseman method (1)

[Wiseman '13]

- We assume the dominant configuration satisfies

$$\beta|\phi_a - \phi_b| \gg 1 \qquad |\phi_a - \phi_b| := \sqrt{\sum_I (\phi_a^I - \phi_b^I)^2}$$

- All the Dp-branes are separated by large distances.
- All the off-diagonal components have large mass.

- 1-loop effective action can be calculated as the quadratic fluctuations around classical vacua.
- Contribution from each field (scalar, gauge field, fermion, FP ghost) is of the form

$$\sum_n \sum_{a<b} \mathrm{Tr}' \left[\ln \left(-\partial^i \partial_i + \left(\frac{2\pi n}{\beta} \right)^2 + |\phi_a - \phi_b|^2 + \ldots \right) \right]$$

Smilga-Wiseman method (2)

- Classical and 1-loop effective actions

$$S_{\mathrm{D}p}^{\mathrm{classical}} = \frac{N}{\lambda}\int d\tau d^p x \sum_a \left(\frac{1}{2}\partial^\mu \phi_a^I \partial_\mu \phi_a^I\right)$$

starts from $(\partial\phi)^4$ due to SUSY

$$S_{\mathrm{D}p,T=0}^{\mathrm{one\text{-}loop}} = -\int d\tau d^p x \sum_{a<b} \frac{\Gamma\left(\frac{7-p}{2}\right)}{(4\pi)^{\frac{1+p}{2}}} \left(2\frac{\{\partial_\mu(\phi_a^I - \phi_b^I)\partial_\nu(\phi_a^I - \phi_b^I)\}^2}{|\phi_a - \phi_b|^{7-p}} - \frac{\{\partial_\mu(\phi_a^I - \phi_b^I)\partial^\mu(\phi_a^I - \phi_b^I)\}^2}{|\phi_a - \phi_b|^{7-p}}\right) + \dots$$

- Condition 1: Higher derivative terms are suppressed.

$$|(\partial\phi)^2/\phi^4| \ll 1$$

- Condition 2: Temperature dependent terms are suppressed. (They are proportional to $\exp(-\beta|\phi_a - \phi_b|)$.)

$$\beta|\phi_a - \phi_b| \gg 1$$

Smilga-Wiseman method (3)

- Assumption 1: moduli and their difference

$$\phi_a^I \sim \phi_a^I - \phi_b^I \sim \phi$$

- It means that all the branes are uniformly distributed, roughly speaking. (All distances are of the same order.)

- Assumption 2: derivative and temperature

$$\partial \phi_a^I \sim \partial(\phi_a^I - \phi_b^I) \sim \frac{1}{\beta}\phi$$

- The dependence on temperature in this discussion is determined only here. The meaning seems unclear.

Smilga-Wiseman method (4)

- Assumption 3: classical and 1-loop are balanced.

$$S_{\mathrm{D}p}^{\text{classical}} \sim S_{\mathrm{D}p,T=0}^{\text{one-loop}} \sim S_{\mathrm{D}p}$$

- It does not mean the higher loop contributions can be neglected.
- According to SUGRA analysis, it may mean that all loop contributions are of the same order.
- It seems reasonable, since we here consider the strong-coupling region in the dual field theory.

Reproduction of SUGRA results

- Estimation $\int d\tau \sim \beta\,,\ \sum_a \sim N$ and $\sum_{a<b} \sim N^2.$

$$S_{\mathrm{D}p}^{\mathrm{classical}} \sim \int d^p x \frac{N^2}{\beta\lambda}\phi^2\,, \qquad S_{\mathrm{D}p,T=0}^{\text{one-loop}} \sim \int d^p x \frac{N^2}{\beta^3\phi^{3-p}}$$

- Results

$$\phi \sim T^{\frac{2}{5-p}}\lambda^{\frac{1}{5-p}}$$

$$F_{\mathrm{D}p} \sim S_{\mathrm{D}p}/\beta \sim N^2 T^{\frac{2(7-p)}{5-p}}\lambda^{-\frac{3-p}{5-p}}V_p$$

- Wiseman discussed only p<3 cases, but this discussion can be done for all Dp-brane cases. $\beta\phi \gg 1$
- It is because the temperature condition is equivalent to the small curvature condition, interestingly.

Black M2-brane:

ABJM theory

Effective theory on M2-branes

- ABJM theory (dual to M-theory on $AdS_4 \times S^7/Z_k$)

$$S_{\rm ABJM} = \frac{k}{2\pi}\int d\tau d^2x \left(\mathrm{Tr}\left[(D_\mu\Phi^\dagger_A)(D^\mu\Phi^A) + i\Psi^{\dagger A}\gamma^\mu D_\mu\Psi_A\right] + \mathcal{L}^{(1)}_{\rm CS} - \mathcal{L}^{(2)}_{\rm CS} - V_B - V_F\right),$$

where $A, B, C, D = 1, \ldots, 4$

$$\mathcal{L}^{(i)}_{\rm CS} = \frac{1}{2}\epsilon^{\mu\nu\rho}\,\mathrm{Tr}\left[A^{(i)}_\mu\partial_\nu A^{(i)}_\rho + \frac{2}{3}A^{(i)}_\mu A^{(i)}_\nu A^{(i)}_\rho\right],$$

$$V_B = \frac{1}{3}\mathrm{Tr}\Big[\Phi^\dagger_A\Phi^A\Phi^\dagger_B\Phi^B\Phi^\dagger_C\Phi^C + \Phi^A\Phi^\dagger_A\Phi^B\Phi^\dagger_B\Phi^C\Phi^\dagger_C + 4\Phi^A\Phi^\dagger_B\Phi^C\Phi^\dagger_A\Phi^B\Phi^\dagger_C - 6\Phi^A\Phi^\dagger_B\Phi^B\Phi^\dagger_A\Phi^C\Phi^\dagger_C\Big],$$

$$V_F = i\mathrm{Tr}\Big[\Phi^\dagger_A\Phi^A\Psi^{\dagger B}\Psi_B - \Phi^A\Phi^\dagger_A\Psi_B\Psi^{\dagger B} - 2\Phi^\dagger_A\Phi^B\Psi^{\dagger A}\Psi_B + 2\Phi^A\Phi^\dagger_B\Psi_A\Psi^{\dagger B} - \epsilon^{ABCD}\Phi^\dagger_A\Psi_B\Phi^\dagger_C\Psi_D + \epsilon_{ABCD}\Phi^A\Psi^{\dagger B}\Phi^C\Psi^{\dagger D}\Big],$$

Effective action

- Classical moduli: $\Phi^A_{ab} = \frac{1}{\sqrt{2}}(\phi^A_a + i\phi^{A+4}_a)\delta_{ab}\,, \quad \Psi_{ab} = 0$
- Dominant configuration: $\beta|\phi_a - \phi_b|^2 \gg 1$

  (The exponent is changed, since mass dims of scalars are changed.)
- Classical action

$$S^{\text{classical}}_{\text{ABJM}} = \frac{k}{2\pi}\int d\tau d^2x \sum_a \left(\frac{1}{2}\partial^\mu \phi^I_a \partial_\mu \phi^I_a\right)$$

- 1-loop effective action at T=0

starts from $(\partial\phi)^4$

$$S^{\text{one-loop}}_{\text{ABJM},T=0} \sim -\int d\tau d^2x \sum_{a<b}\left(\frac{\{\partial(\phi_a-\phi_b)\partial(\phi_a-\phi_b)\}^2}{|\phi_a-\phi_b|^6}\right) + \dots$$

[Baek, Hyun, Jang, Yi '08]

Reproduction of SUGRA results

- Conditions: They are almost the same, but changed.

  (since, again, the scalars have different mass dims.) e.g. $\beta\phi^2 \gg 1$

- Assumptions: They remain the same.

- Estimation: $S_{\rm ABJM}^{\rm classical} \sim S_{{\rm ABJM},T=0}^{\rm one\text{-}loop} \sim S_{\rm ABJM}$

$$S_{\rm ABJM}^{\rm classical} \sim \int d^2x \frac{kN}{\beta}\phi^2 \,, \quad S_{{\rm ABJM},T=0}^{\rm one\text{-}loop} \sim \int d^2x \frac{N^2}{\beta^3\phi^2}$$

- Results

$$F_{\rm ABJM} \sim S_{\rm ABJM}/\beta \sim N^{\frac{3}{2}}\sqrt{k}T^3V_2 \qquad \phi \sim \frac{N^{\frac{1}{4}}}{k^{\frac{1}{4}}\beta^{\frac{1}{2}}}$$

Black M5-brane:

6d SCFT with some assumptions

Effective theory on M5-branes

- 6d N=(2,0) SCFT
- The details have been not known yet.

- Assumptions (to apply Smilga-Wiseman method)
- Kinetic term of scalar fields is $\sim \partial\Phi^I_{ab}\partial\Phi^I_{ba}$

  The NxN matrices representing the collective motion of M5-branes in the transverse directions.

  (There is no evidence to deny the scalars with matrix rep.)

- Classical vacua have scalar moduli ϕ^I_a

  The diagonal components of matrices of scalar fields.

  This breaks the original gauge symmetry into $U(1)^N$.

Effective theory on M5-branes

- Besides, 1-loop effective potential for the scalar moduli should start from $(\partial\phi)^4$ for a small $\partial\phi$ and a long distance $|\phi_a - \phi_b|$.

✓ It seems a natural assumption: due to supersymmetry, the quadratic terms should not receive quantum corrections.

✓ The contribution of each field in 1-loop effective action comes from quadratic fluctuation and should be of the form

$$\sum_n \sum_{a<b} \mathrm{Tr}' \left[\ln \left(-\partial^i \partial_i + \left(\frac{2\pi n}{\beta} \right)^2 + |\phi_a - \phi_b|^1 + \dots \right) \right]$$

Does it mean the three-scalar interactions on M5-branes…?

Effective action

- Dominant configuration: $\beta\sqrt{|\phi_a - \phi_b|} \gg 1$
- Classical action

$$S_{\rm M5}^{\rm classical} \sim \int d\tau d^5x \sum_a \left(\frac{1}{2}\partial^\mu \phi_a^I \partial_\mu \phi_a^I\right)$$

- 1-loop effective action at T=0

$$S_{{\rm M5},T=0}^{\rm one\text{-}loop} \sim -\int d\tau d^5x \sum_{a<b} \left(\frac{\{\partial(\phi_a - \phi_b)\partial(\phi_a - \phi_b)\}^2}{|\phi_a - \phi_b|^3}\right) + \ldots$$

- Again, it starts from $(\partial\phi)^4$ terms due to supersymmetry.
- The exponent in the denominator can be determined by dimensional analysis.

Reproduction of SUGRA results

- Conditions and assumptions
  - We impose them in a similar manner. e.g. $\beta\sqrt{\phi} \gg 1$
- Estimation: $S_{\mathrm{M5}}^{\mathrm{classical}} \sim S_{\mathrm{M5},T=0}^{\mathrm{one\text{-}loop}}$

$$S_{\mathrm{M5}}^{\mathrm{classical}} \sim \int d^5x \frac{N}{\beta}\phi^2 \,, \quad S_{\mathrm{M5},T=0}^{\mathrm{one\text{-}loop}} \sim \int d^5x \frac{N^2\phi}{\beta^3}$$

- Results

$$F_{\mathrm{M5}} \sim S_{\mathrm{M5}}/\beta \sim N^3 T^6 V_5 \qquad \phi \sim \frac{N}{\beta^2}$$

Conclusion and discussions

- All the results agree with SUGRA predictions through AdS/CFT correspondence.
  - Nontrivial dependences on T and λ for D-branes and on N for M-branes are perfectly reproduced.
- Then we can believe Smilga-Wiseman method captures the dynamics of branes.
- The assumptions in this method should be justified in the future works. Especially…
  - Relation of temperature and derivative: $\partial\phi \sim T\phi$
  - Higher loop contributions
  - Interaction terms in M5-brane theory

Comments on $O(N^2)$ d.o.f.s

- In D-brane theory, the free energy should be $O(N^2)$ both at weak and strong couplings.
  - According to Smilga-Wiseman method, the dynamics of N classical moduli dominates at strong coupling region.
  - The 't Hooft limit seems to ensure $O(N^2)$ free energy.

- In M-brane theory, 't Hooft limit cannot be taken.
  - Smilga-Wiseman method can be applied to M-brane systems, and reproduce the exotic N-dependences.
  - This may mean that M-theory dynamics is not special, but the 't Hooft limit in superstrings is special.