有限温度・密度系におけるカイラル対称性の自発的破れのくりこみ群による解析

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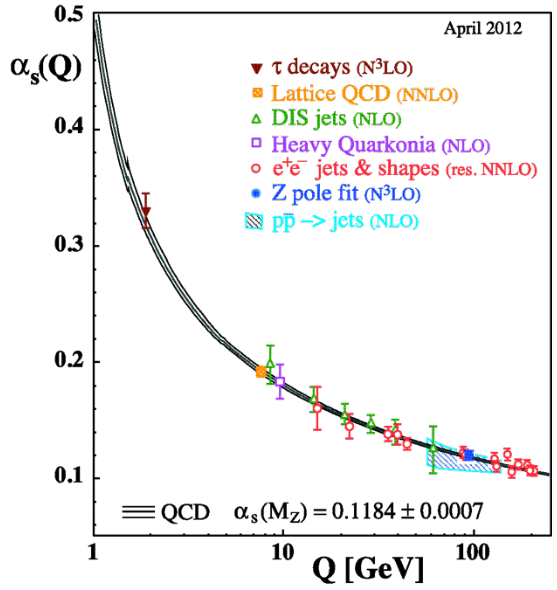
基研研究会「場の理論と弦理論」

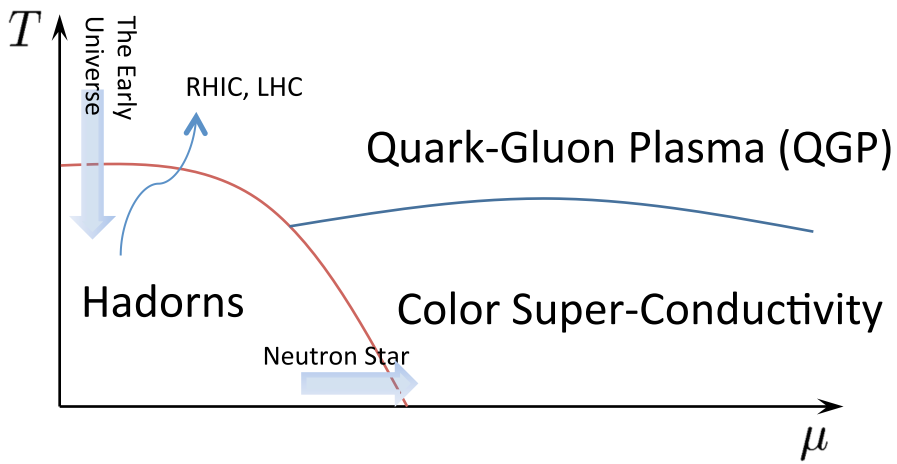
Introduction



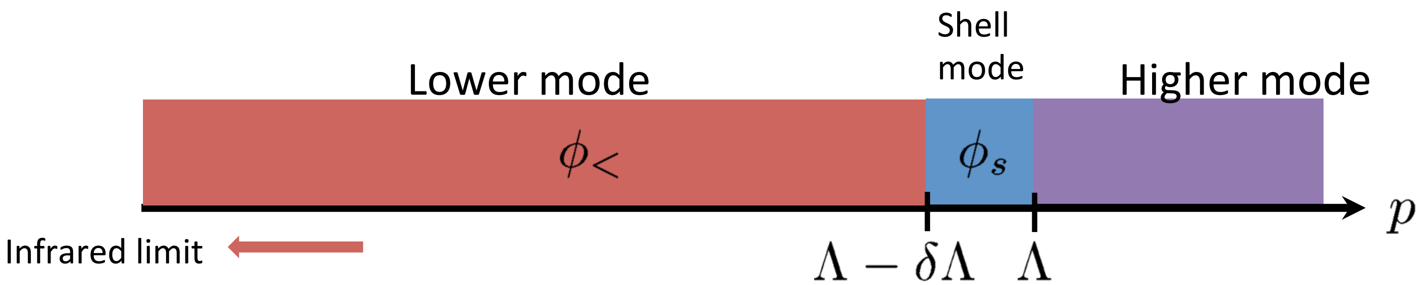
- Quantum Chromodynamics (QCD)
  - Strong Coupling at low energy scale $\alpha_s \gg 1$
  - Hadron mass $\mathcal{O}(10^3)$ MeV
    - ➡ Constituent quark mass 300 MeV
    - Current quark mass $\mathcal{O}(10)$ MeV

Dynamical Chiral Symmetry Breaking(DχSB) $\langle \bar{\psi}\psi \rangle \neq 0$

- QCD at finite temperature and density
  - Phase diagram



Introduction
Non-Perturbative Renormalization Group(NPRG)



$$Z = \int^{\Lambda_0} \mathcal{D}\phi \; e^{-S_0} = \int^{\Lambda} \mathcal{D}\phi_< \int_{\Lambda}^{\Lambda_0} \mathcal{D}\phi_s \; e^{-S_0[\phi_< + \phi_s]}$$

$$= \int^{\Lambda} \mathcal{D}\phi_< \; e^{-S_{\text{eff}}[\phi_<;\Lambda]}$$

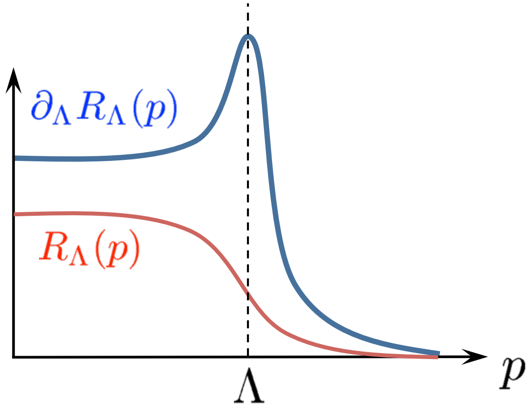
$$e^{-S_{\text{eff}}[\phi_<;\Lambda]} = \left(\int_{\Lambda}^{\Lambda_0} \mathcal{D}\phi_s \; e^{-S_0[\phi_s]} \right)$$

Wilsonian effective action

NPRG equation

$$\frac{dS_{\text{eff}}}{d\Lambda} = \beta$$

NPRG



- Legendre effective action with IR cutoff $\Gamma_\Lambda[\Phi]$
  - ➢ Propagator with regulator $R_\Lambda(p)$

$$G_{0,\Lambda} = \frac{1}{G_0^{-1} + R_\Lambda(p)} \sim \frac{1}{\not{p} + R_\Lambda(p)}$$

➡ The regulator suppresses the lower modes with $p < \Lambda$

➡ The higher modes with $\Lambda < p < \Lambda_0$ are integrated out.

Λ is regarded as an infrared cutoff scale.

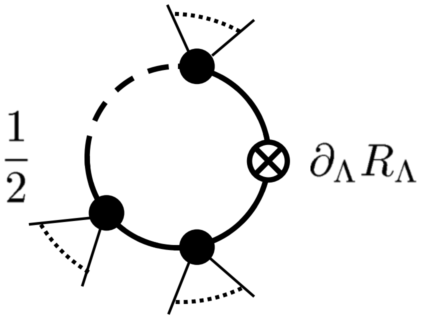
ex: Optimized cutoff function

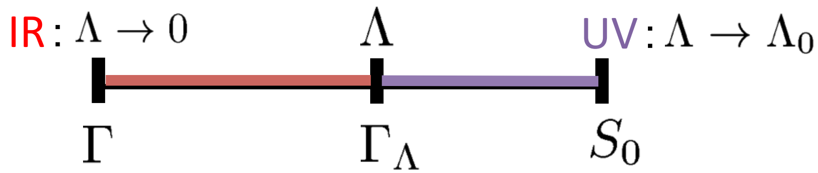
D.F.Litim Phys. Rev D64, 105007

$$R_\Lambda(p) = \not{p}\left(\frac{\Lambda}{|p|} - 1\right)\theta(1 - \frac{p^2}{\Lambda^2})$$

- Wetterich flow equation

$$\partial_\Lambda \Gamma_\Lambda[\Phi] = \frac{1}{2}\mathrm{STr}\left\{\left[\frac{\overrightarrow{\delta}}{\delta\Phi}\Gamma_\Lambda[\Phi]\frac{\overleftarrow{\delta}}{\delta\Phi} + R_\Lambda\right]^{-1} \cdot (\partial_\Lambda R_\Lambda)\right\} = \frac{1}{2}$$





Approximations for NPRG

- Approximation methods

Ex: ϕ^4 theory with Z_2 symmetry

  - Derivative expansion

$$\Gamma_{\Lambda_0}[\phi] = S_0[\phi] = \int d^4x \left[\frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2!}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4\right]$$

$$\Gamma_\Lambda[\phi] = \int d^4x \left[V_\Lambda(\phi) + \frac{1}{2}Z_\Lambda(\phi)(\partial_\mu\phi)^2 + \frac{1}{2}Y_\Lambda(\phi)\phi(\partial^2\phi) + \cdots\right]$$

Local Potential Approximation(LPA) $\phi(p) = (2\pi)^4\delta^4(p)\phi \quad Z_\Lambda = 1$

$$\Rightarrow \Gamma_\Lambda[\phi] = \int d^4x \left[V_\Lambda(\phi) + \frac{1}{2}(\partial_\mu\phi)^2\right]$$

  - Truncation

$$V_\Lambda(\phi) = \frac{1}{2!}m_\Lambda^2\phi^2 + \frac{1}{4!}\lambda_\Lambda\phi^4 + \cdots$$

The potential function is spanned by the polynomials of field. We need to truncate the expansion to some finite order.

Nambu—Jona-Lasinio Model

- 4-fermi interaction

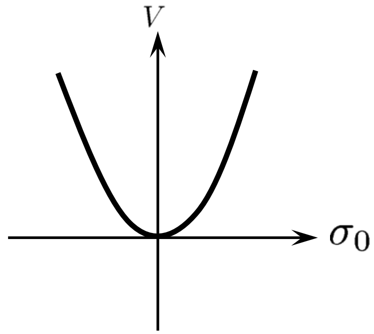
$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \frac{G}{2}\{(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2\}$$

- Invariant under Chiral global U(1) transformation

$$\psi(x) \to e^{i\gamma_5\theta}\psi(x), \quad \bar{\psi}(x) \to \bar{\psi}(x)e^{i\gamma_5\theta}$$

⇒ Prohibit the mass term $m\bar{\psi}\psi$

$$\bar{\psi}\psi \to \bar{\psi}e^{2i\gamma_5\theta}\psi \neq \bar{\psi}\psi$$

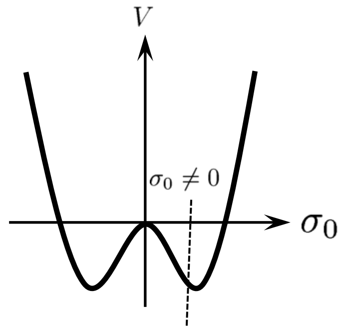


- Describe the DχSB of QCD

- 4-fermi coupling is fluctuation of chiral order parameter.

$\mathcal{L} + m_0\bar{\psi}\psi$

$$G \sim \langle(\bar{\psi}\psi)^2\rangle \sim \frac{\partial\langle\bar{\psi}\psi\rangle}{\partial m_0}$$



NPRG and NJL model at finite temperature and density

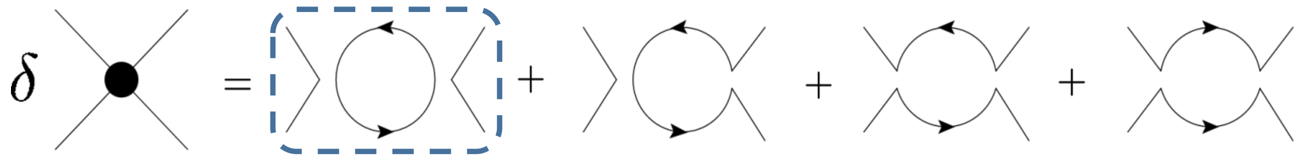
- Bare action

$$S_0 = \int d^4x \left[\bar{\psi} i \partial\!\!\!/ \psi + \mu \bar{\psi} \gamma_0 \psi - \frac{G_0}{2} \{ (\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \} \right]$$

- Effective action

$$\Gamma_\Lambda = \int d^4x \left[\bar{\psi} i \partial\!\!\!/ \psi + \mu \bar{\psi} \gamma_0 \psi - \frac{G_\Lambda}{2} \{ (\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \} \right]$$

- Generate the 4-fermi interaction



Large-N leading part

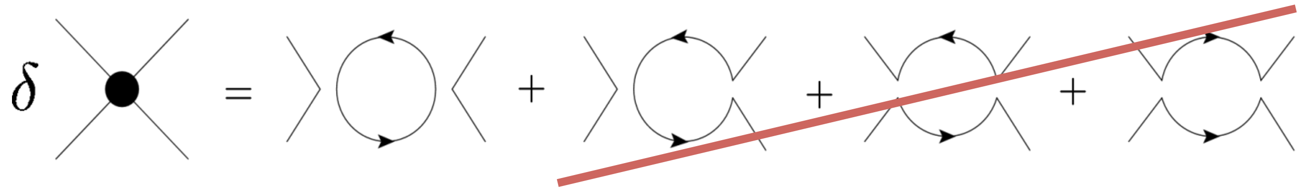
Large-N leading NPRG eq. ⇔ Mean field approximation

⇒ Large-N non-leading NPRG eq.

- 3d optimized cutoff function

$$R_\Lambda(\boldsymbol{p}) = \boldsymbol{p}\!\!\!/ \left(\frac{\Lambda}{|\boldsymbol{p}|} - 1 \right) \theta(1 - \frac{\boldsymbol{p}^2}{\Lambda^2}) = \boldsymbol{p}\!\!\!/ \; r(\boldsymbol{p}/\Lambda)$$

Large-N leading



$$\begin{cases} \partial_t g = -2g + \frac{4}{3}g^2 \left[\left(\frac{1}{2} - n_+ \right) + \left(\frac{1}{2} - n_- \right) + \frac{\partial}{\partial \omega}(n_+ + n_-) \right] \Big|_{\omega \to 1} \\ \partial_t T = T \\ \partial_t \mu = \mu \end{cases}$$

$$\partial_t = -\Lambda \frac{\partial}{\partial \Lambda}, \quad g = \frac{G\Lambda^2}{4\pi^2}, \quad n_\pm = \frac{1}{e^{\beta\epsilon_\pm} + 1}$$

$$\beta = \frac{1}{T}$$

$$\epsilon_\pm = \omega \pm \mu,$$

$$\omega = \boldsymbol{p}^2(1+r)$$

- $T \to 0 \ (\beta \to \infty)$ limit

$$T \equiv \frac{T}{\Lambda}, \quad \mu \equiv \frac{\mu}{\Lambda}$$

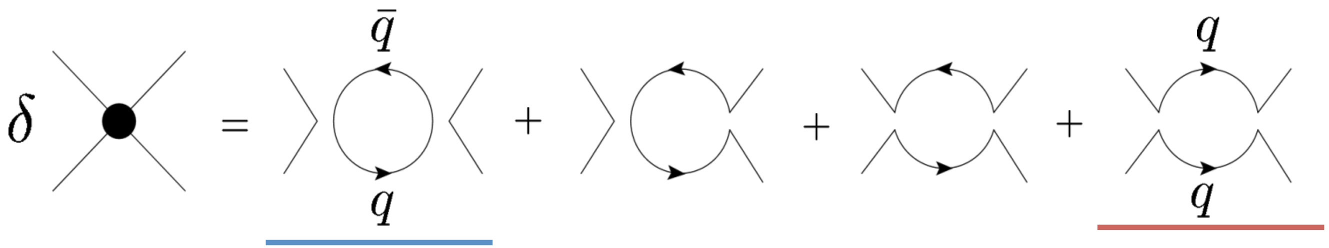
$$\partial_t g = -2g + \frac{4}{3}g^2 \left[1 - \theta(\mu - 1) + \delta(\mu - 1) \right]$$

$$= \begin{cases} -2g + \frac{4}{3}g^2 & (\mu < 1) \\ -2g + g^2 \left[\frac{1}{2} + \delta(\mu - 1) \right] & (\mu = 1) \\ -2g & (\mu > 1) \end{cases}$$

$$\frac{\partial}{\partial \omega} n_- \overset{\beta \to \infty}{=} \delta(\mu - 1)$$

$$\theta(\mu - 1) = \begin{cases} 0 & (\mu < 1) \\ \frac{1}{2} & (\mu = 1) \\ 1 & (\mu > 1) \end{cases}$$

Large-N non-leading



$$\begin{cases} \partial_t g = -2g + \frac{1}{3} g^2 (4I_0 - I_1) \\ \partial_t T = T \\ \partial_t \mu = \mu \end{cases}$$

$$\partial_t = -\Lambda \frac{\partial}{\partial \Lambda}, \quad g = \frac{G\Lambda^2}{4\pi^2}, \quad n_\pm = \frac{1}{e^{\beta\epsilon_\pm} + 1}$$

$$I_0 = \left[\left(\frac{1}{2} - n_+ \right) + \left(\frac{1}{2} - n_- \right) + \frac{\partial}{\partial \omega}(n_+ + n_-) \right] \Bigg|_{\omega \to 1}$$

$$I_1 = \left[\frac{1}{(1+\mu)^2} \left(\frac{1}{2} - n_+ \right) + \frac{1}{(1-\mu)^2} \left(\frac{1}{2} - n_- \right) + \frac{1}{1+\mu} \frac{\partial}{\partial \omega} n_+ + \frac{1}{1-\mu} \frac{\partial}{\partial \omega} n_- \right] \Bigg|_{\omega \to 1}$$

- $T \to 0 \ (\beta \to \infty)$ limit

$$I_0 = 1 - \theta(\mu - 1) + \delta(\mu - 1)$$

$$I_1 = \frac{1}{2(1+\mu)^2} + \frac{1}{(1-\mu)^2} \left(\frac{1}{2} - \theta(\mu - 1) \right) + \frac{1}{1-\mu} \delta(\mu - 1)$$

Analysis Method

$$\begin{cases} \partial_t g = -2g + \frac{4}{3} g^2 I_0 & \text{: leading} \\ \partial_t g = -2g + \frac{1}{3} g^2 (4I_0 - I_1) & \text{: non-leading} \end{cases}$$

$$\tilde{g} = \frac{1}{g}$$

⇒

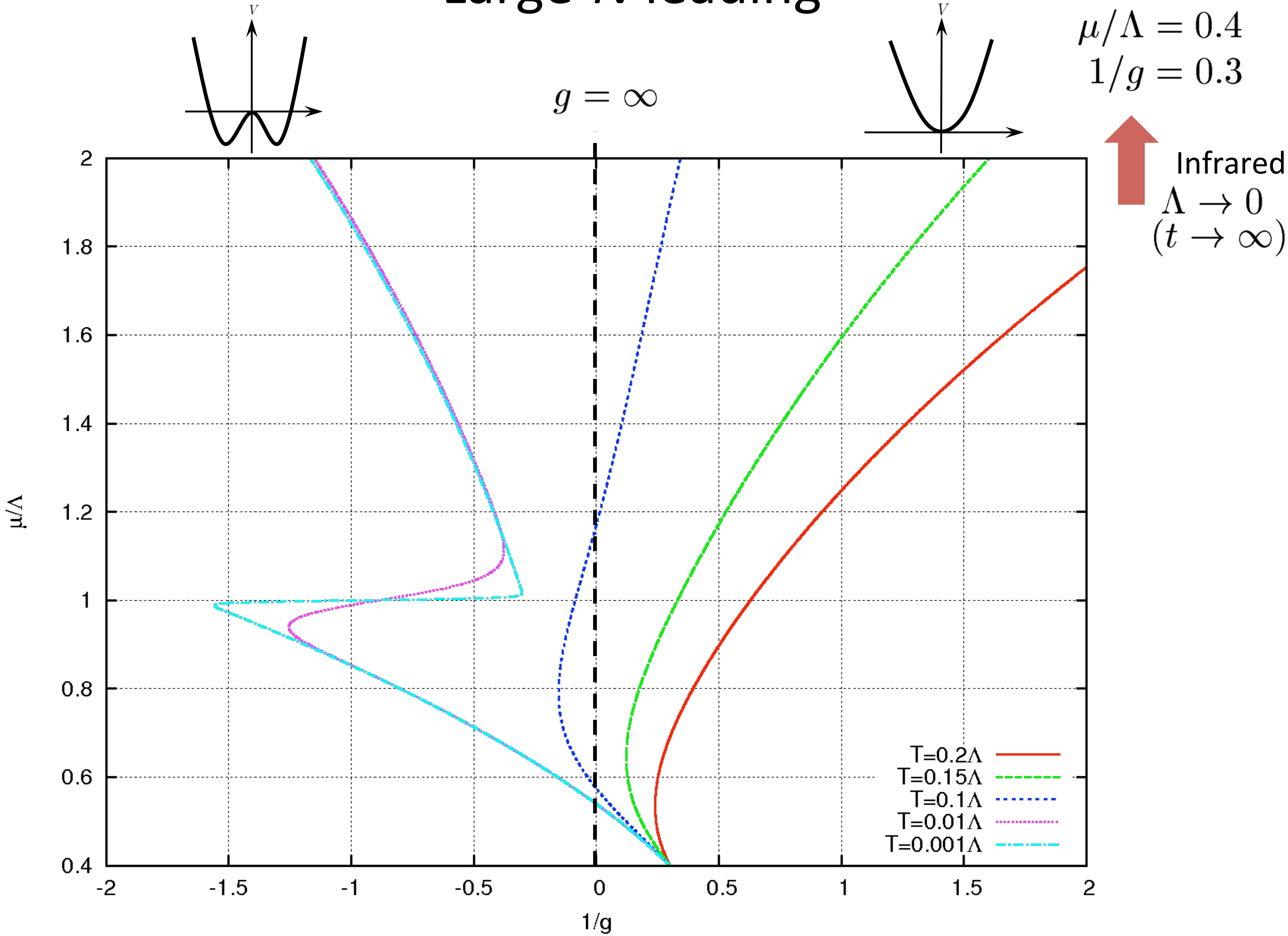
$$\begin{cases} \partial_t \tilde{g} = 2\tilde{g} - \frac{4}{3} I_0 & \text{: leading} \\ \partial_t \tilde{g} = 2\tilde{g} - \frac{1}{3} (4I_0 - I_1) & \text{: non-leading} \end{cases}$$

$g = \infty$ chiral symmetry braking

⇒ $\tilde{g} = 0$

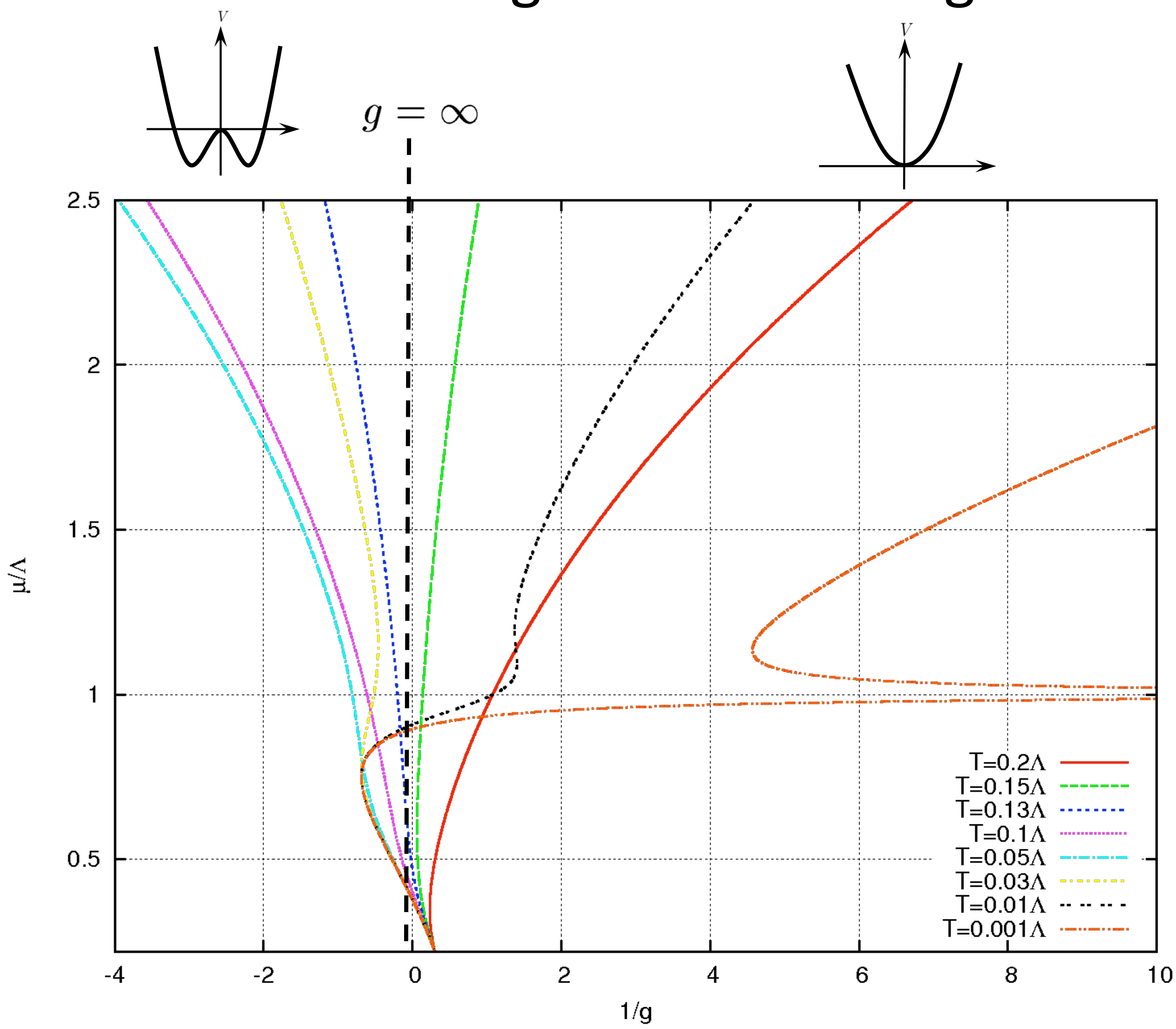
Large-*N* leading

$\mu/\Lambda = 0.4$
$1/g = 0.3$



Large-*N* non-leading

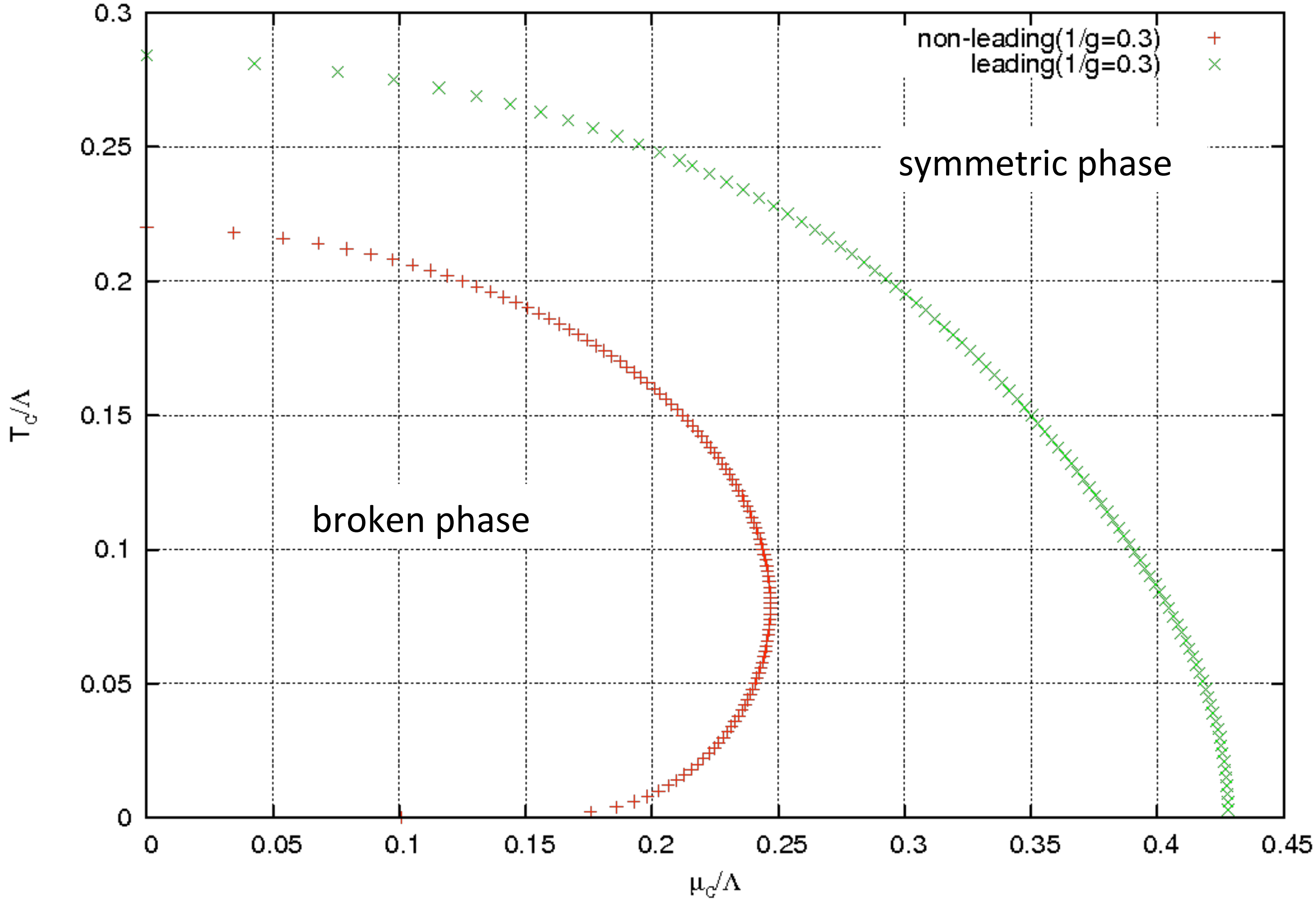
$\mu/\Lambda = 0.22$
$1/g = 0.3$



赤外
$\Lambda \to 0$
$(t \to \infty)$

Leading vs. non-leading

- Phase diagram



QM model and NPRG

B.-J Schaefer, J.Wambach Nucl. Phys. **A757** 479

- Effective action of Quark Meson model $N_c = 3 \quad N_f = 2$

$$\Gamma_\Lambda[\Phi] = \int d^4x \, \{ \bar{\psi}[\gamma^\mu \partial_\mu + h(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)]\psi + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + U_\Lambda(\sigma^2 + \vec{\pi}^2) \Big\}$$

- The mesonic effective potential U_Λ is a function of the O(4)-symmetric field $\vec{\phi}^2 = \sigma^2 + \vec{\pi}^2$.

- We use 3d optimized cutoff function.

$$R_\Lambda^{\mathrm{F}}(\boldsymbol{p}) = \not{\boldsymbol{p}} \left(\frac{\Lambda}{|\boldsymbol{p}|} - 1 \right) \theta(1 - \frac{\boldsymbol{p}^2}{\Lambda^2}) = \not{\boldsymbol{p}} \; r_{\mathrm{F}}(\boldsymbol{p}/\Lambda)$$

$$R_\Lambda^{\mathrm{B}}(\boldsymbol{p}) = \boldsymbol{p}^2 \left(\frac{\Lambda^2}{\boldsymbol{p}^2} - 1 \right) \theta(1 - \frac{\boldsymbol{p}^2}{\Lambda^2}) = \boldsymbol{p}^2 \; r_{\mathrm{B}}(\boldsymbol{p}/\Lambda)$$

QM model and NPRG

- NPRG eq. of the scale-dependent grand canonical thermodynamic potential $\Omega(T,\mu;\phi)$

$$-\Lambda\frac{\partial}{\partial\Lambda}\Omega_\Lambda(T,\mu;\phi) = -\frac{\Lambda^5}{12\pi^2}\left[\frac{3}{E_\pi}\coth\left(\frac{E_\pi}{2T}\right)+\frac{1}{E_\sigma}\coth\left(\frac{E_\sigma}{2T}\right)\right.$$
$$\left.-\frac{2N_\mathrm{c}N_\mathrm{f}}{E_q}\left\{\tanh\left(\frac{E_q-\mu}{2T}\right)+\tanh\left(\frac{E_q+\mu}{2T}\right)\right\}\right]$$

$$E_i = \sqrt{\Lambda^2+M_i^2} \qquad i = q,\sigma,\pi$$

$$M_q^2 = h^2\phi^2 \quad M_\sigma = 2\frac{\partial\Omega_\Lambda}{\partial\phi^2}+4\phi^2\frac{\partial^2\Omega_\Lambda}{\partial(\phi^2)^2} \quad M_\pi = 2\frac{\partial\Omega_\Lambda}{\partial\phi^2}$$

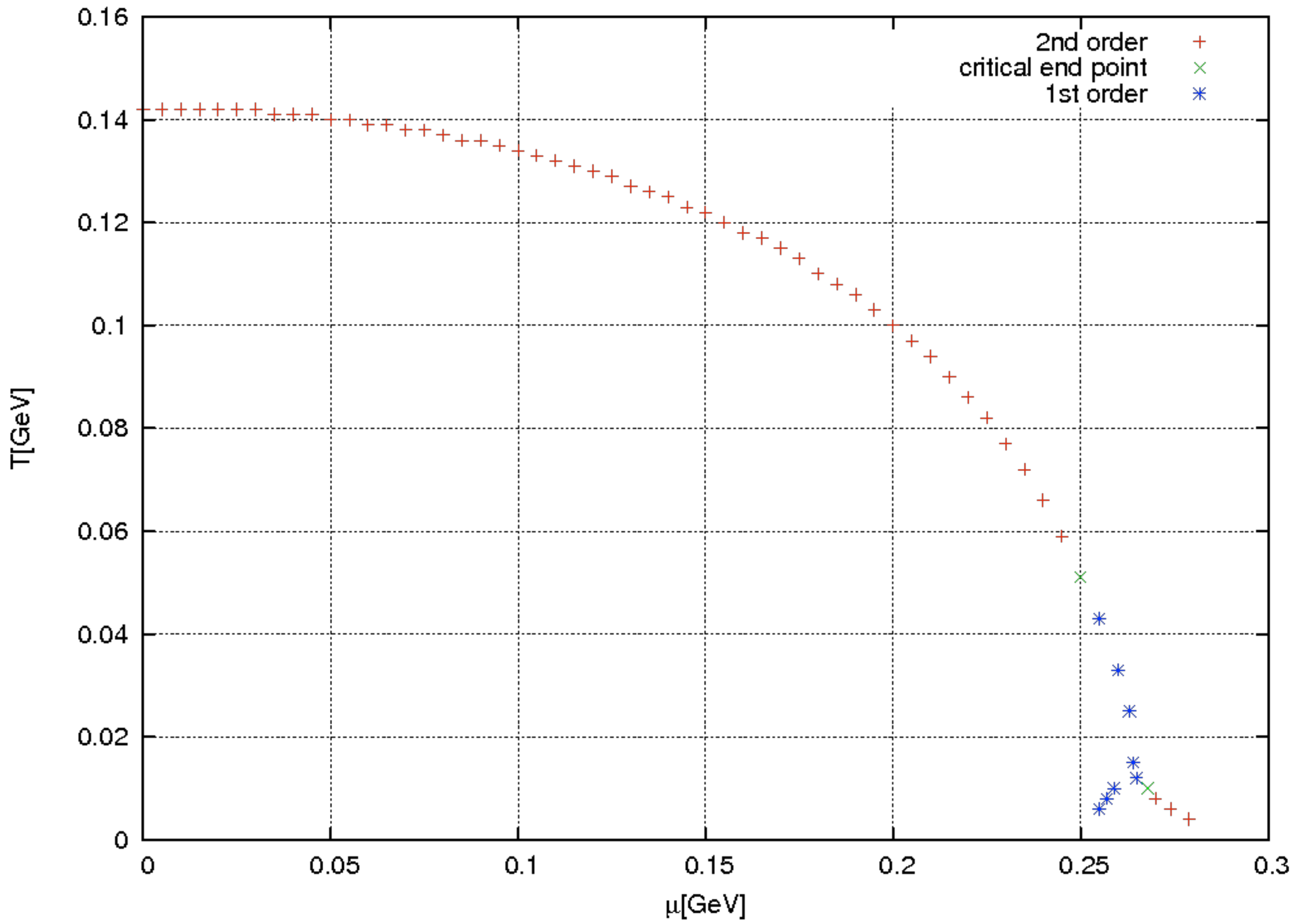
- Initial conditions

$$U_{\Lambda_0}(\phi^2) = \frac{\lambda}{4}(\phi^2)^2$$

$$\Lambda_0 = 500\ \mathrm{MeV} \qquad \lambda = 10 \qquad h = 3.2$$

Numerical results

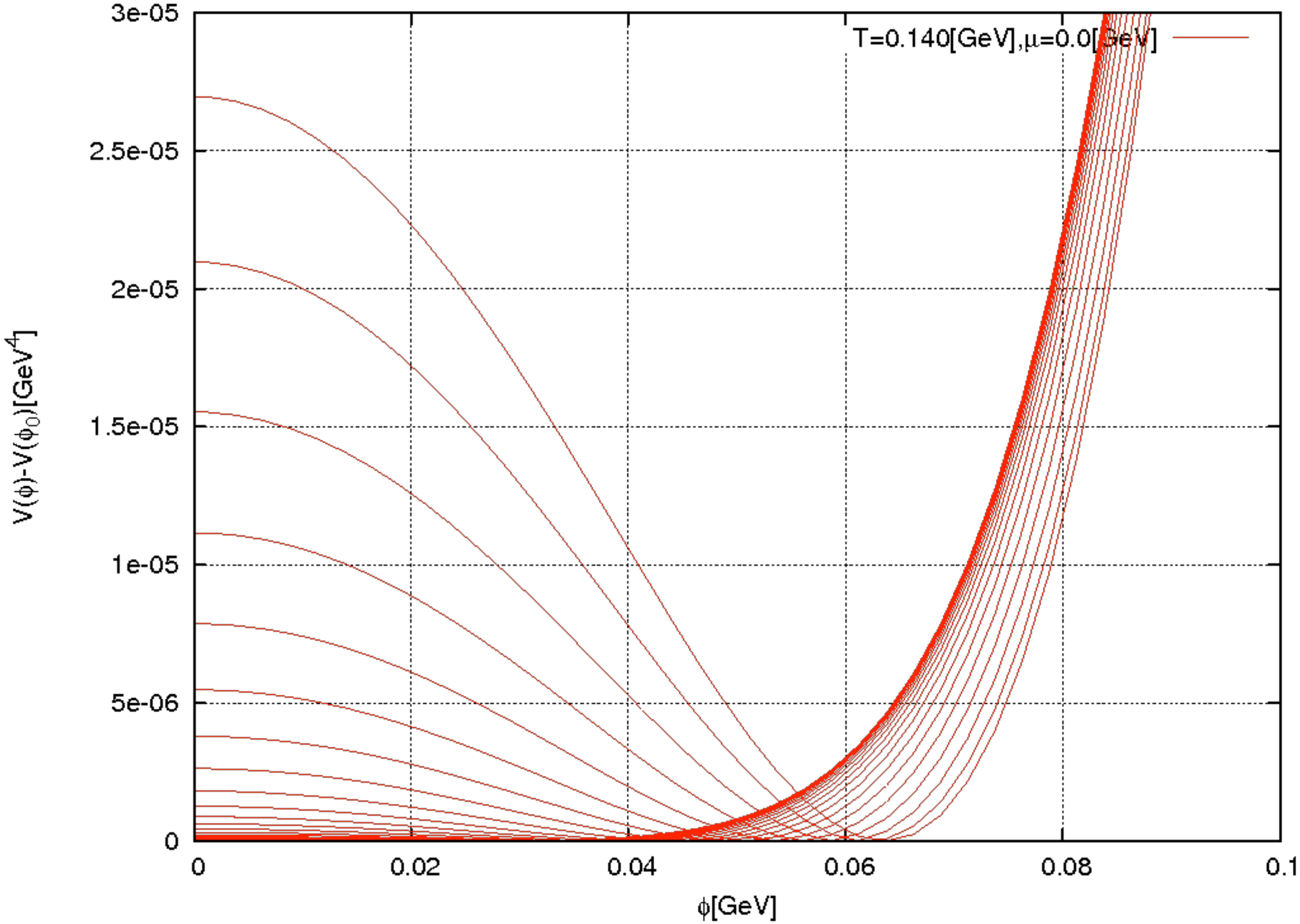
- Phase diagram



Numerical results

- 2nd order transition

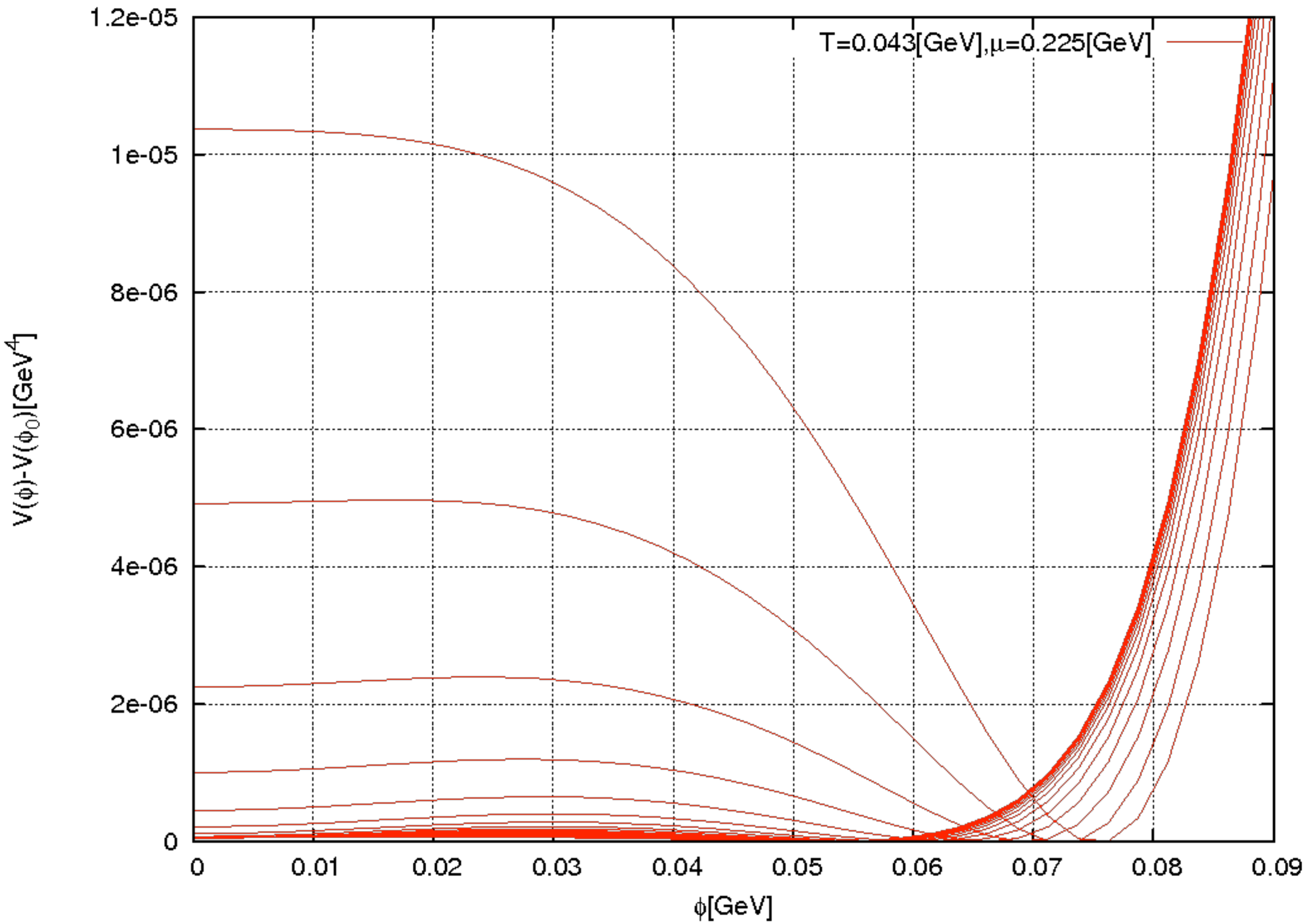
$T = 140\ \mathrm{MeV},\ \mu = 0\ \mathrm{MeV}$



Numerical results

- 1st order transition

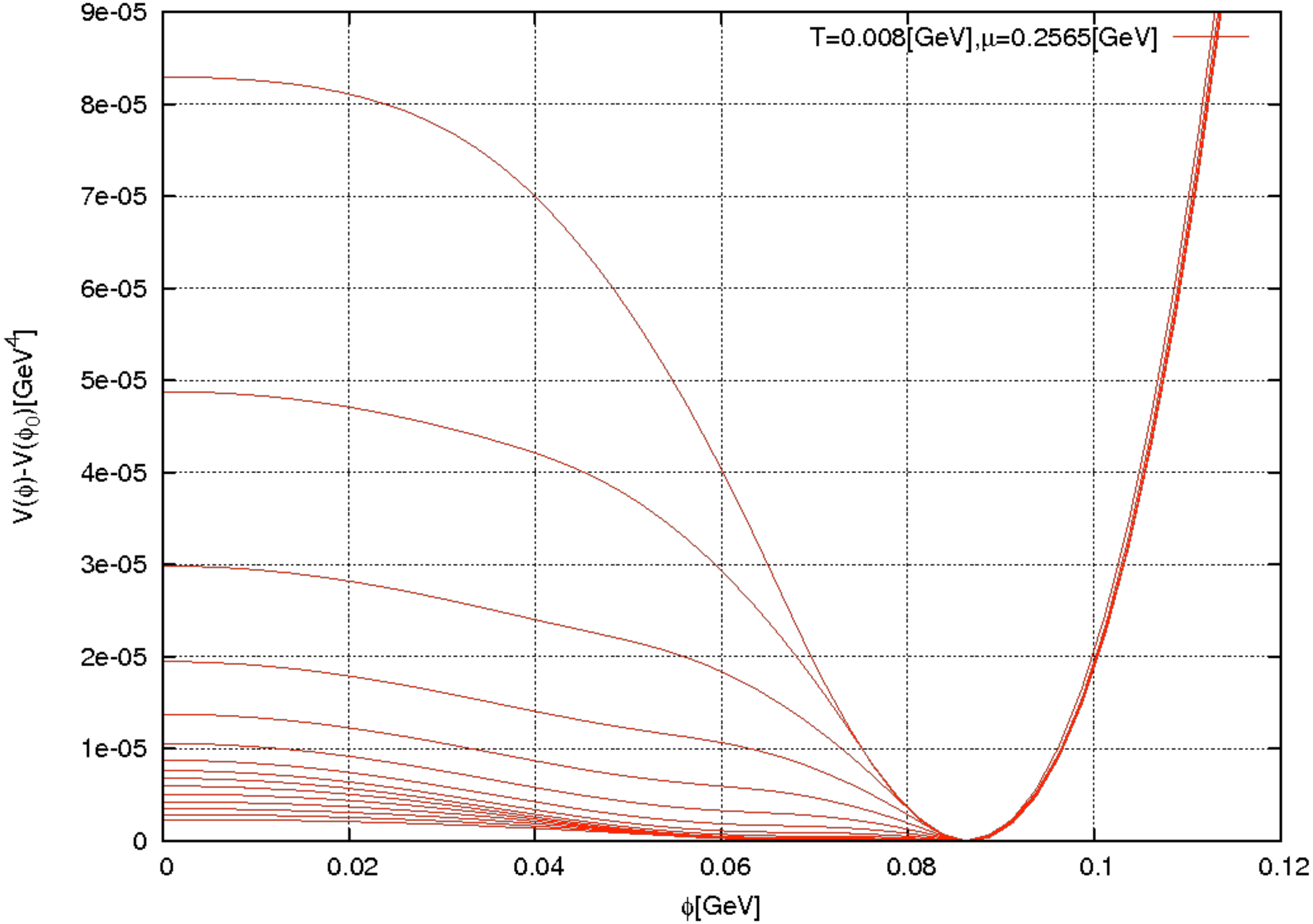
$T = 43\ \mathrm{MeV},\ \mu = 225\ \mathrm{MeV}$



Numerical results

- 1st order transition at low temperature and high density

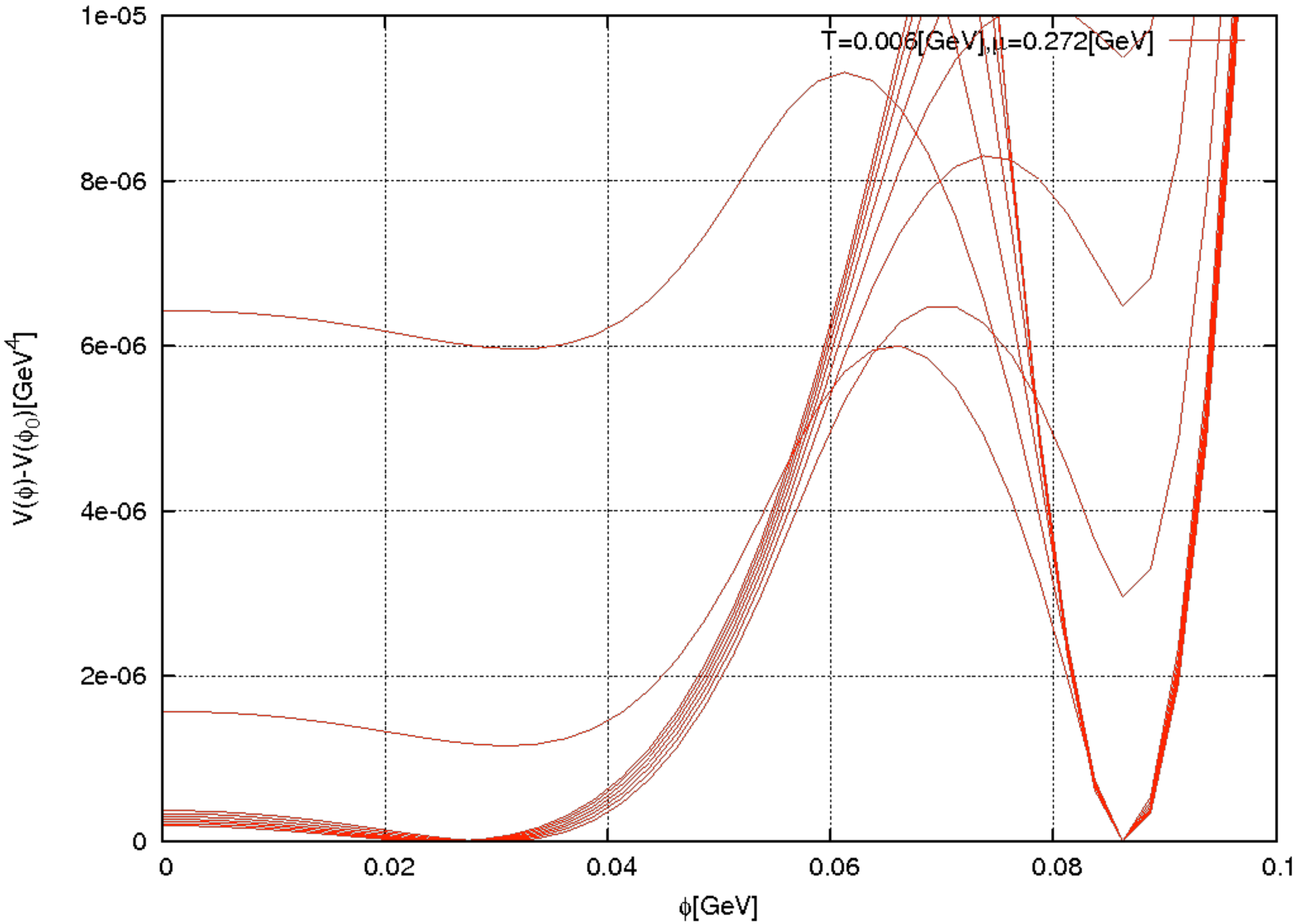
$$T = 8\ \mathrm{MeV},\ \mu = 256.5\ \mathrm{MeV}$$



Numerical results

- 2^{nd} order transition at low temperature and high density

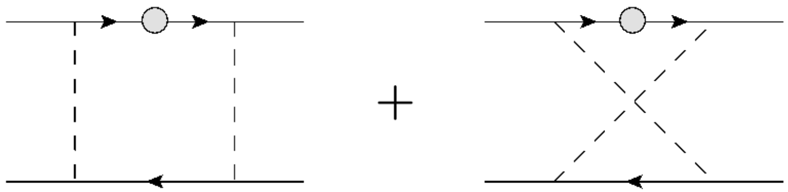
$$T = 6 \text{ MeV},\ \mu = 272 \text{ MeV}$$



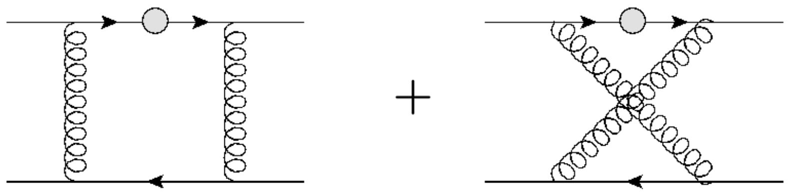
Summary

- We study effective models of QCD at finite temperature and density beyond mean field approximation using Non-Perturbative Renormalization Group.
- Running Yukawa coupling
- Bosonized 4-fermi interaction

$$\Gamma_\Lambda[\Phi] = \int d^4x \left\{ \bar{\psi}[\gamma^\mu \partial_\mu + h(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)]\psi + \frac{G}{2}\{(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\} + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + U_\Lambda(\sigma^2 + \vec{\pi}^2) \right\}$$

$\partial_t G \sim$  $\sim h^4$

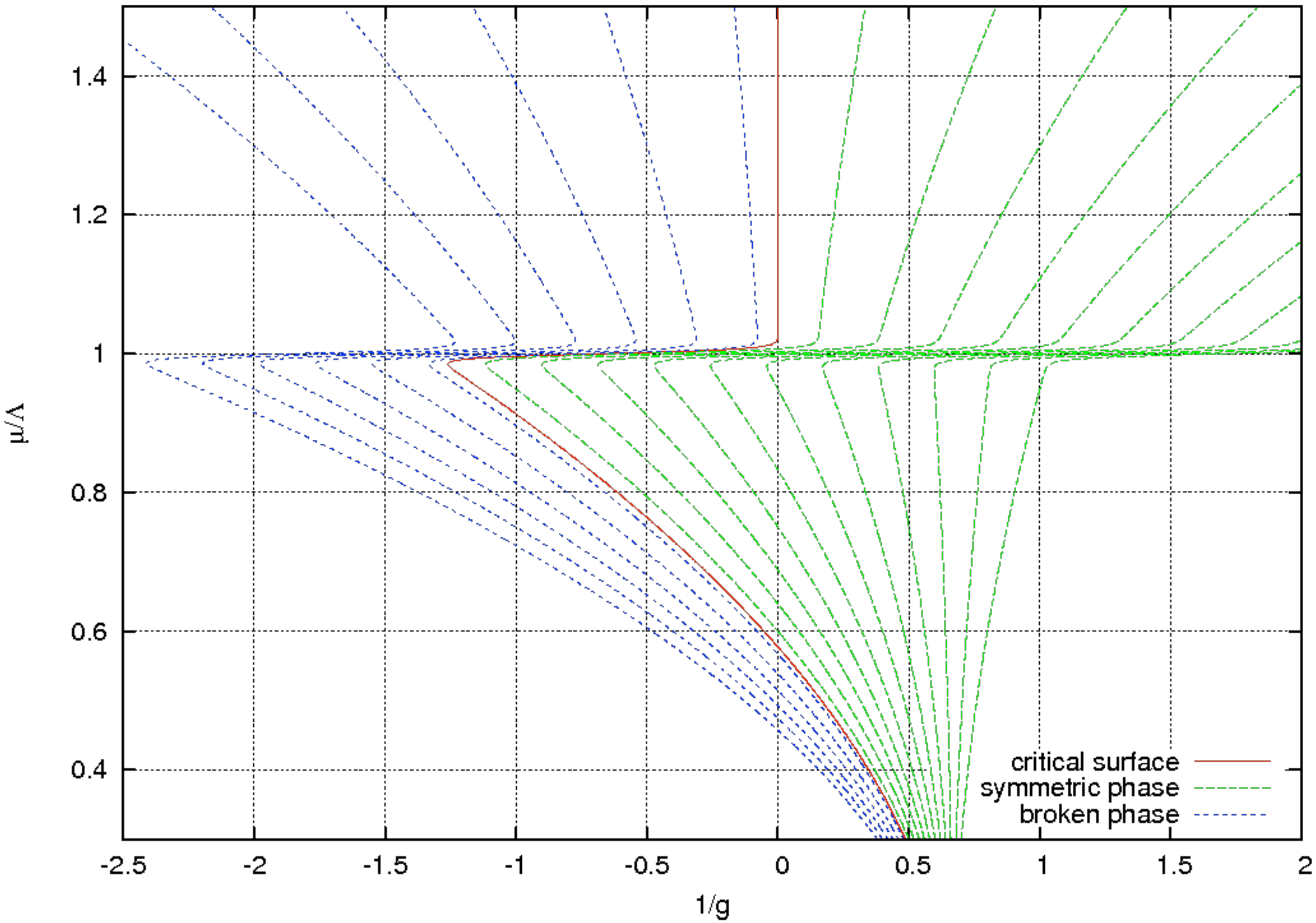
- QCD

$\partial_t G \sim$  $\sim g^4$

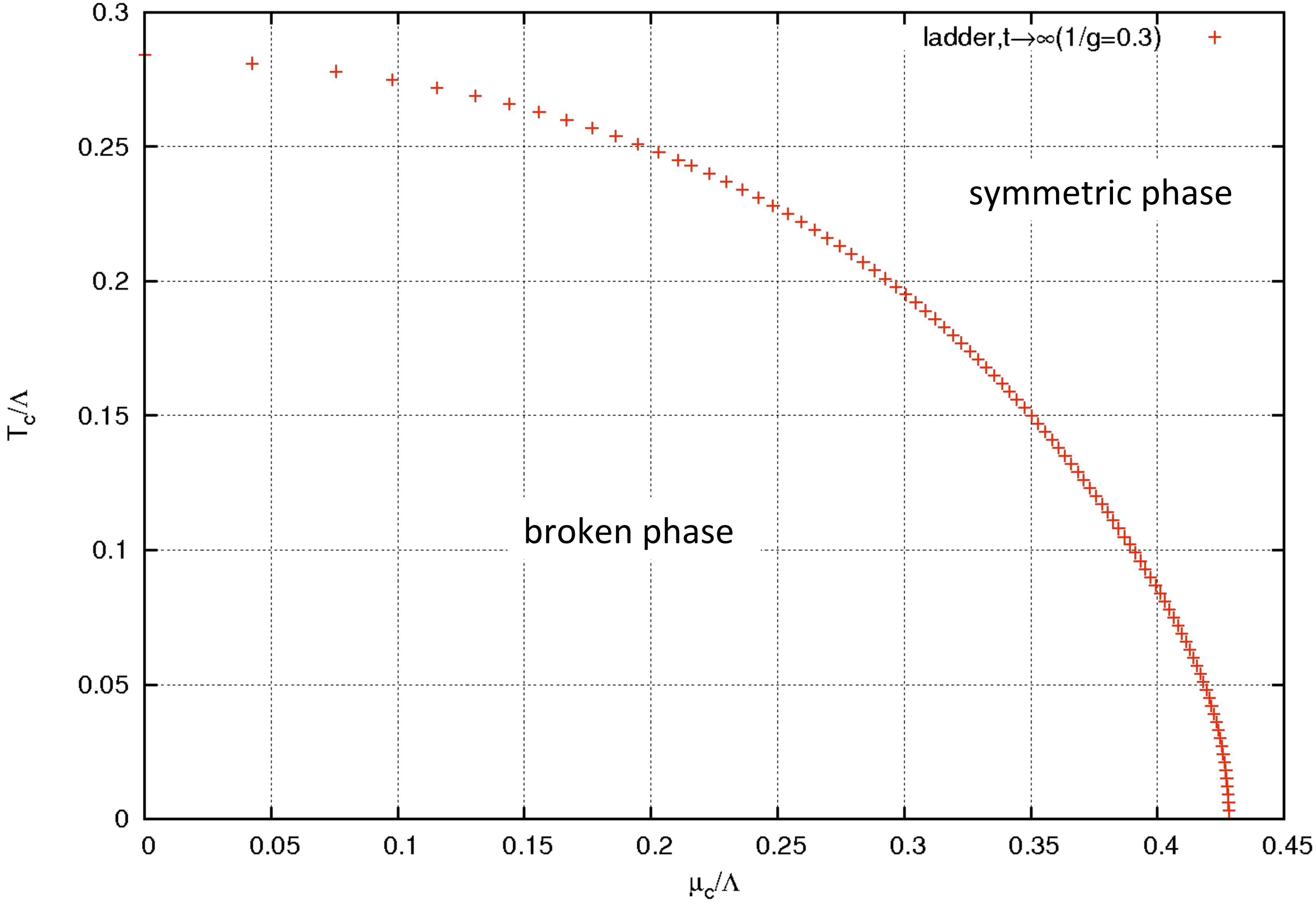
Appendix

Large-N leading

$\mu/\Lambda = 0.30,\ T/\Lambda = 10^{-4}$

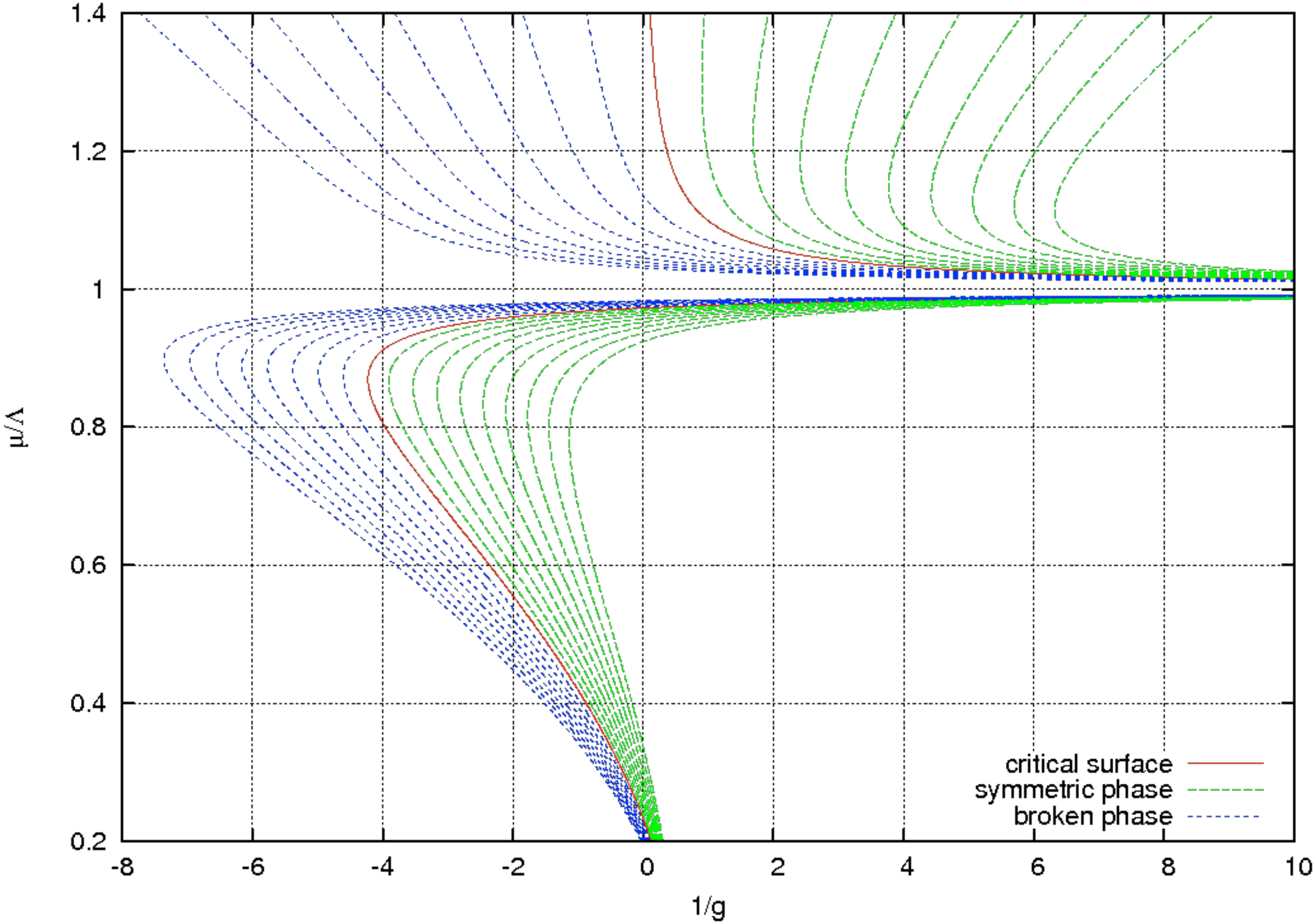


Large-N leading

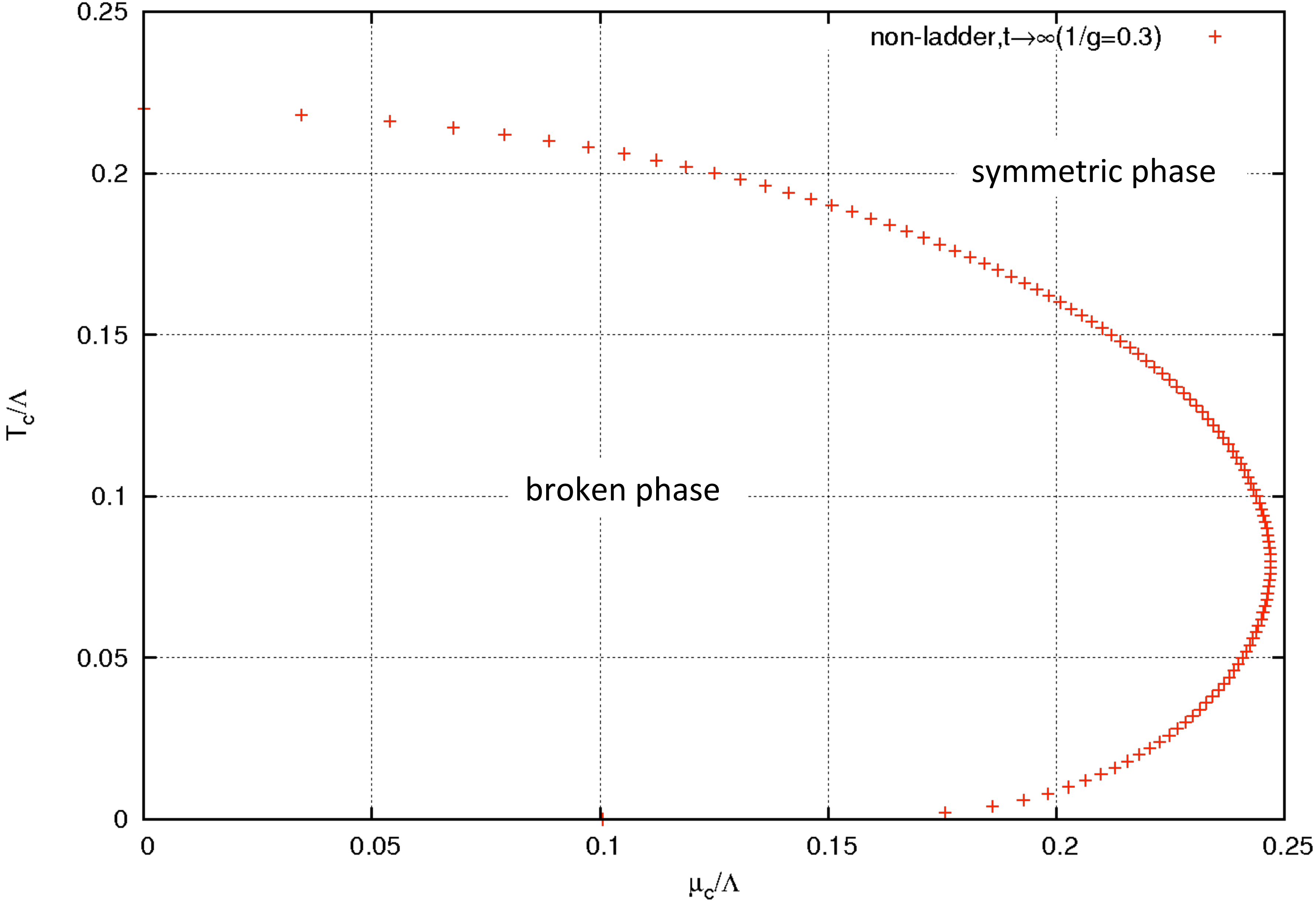


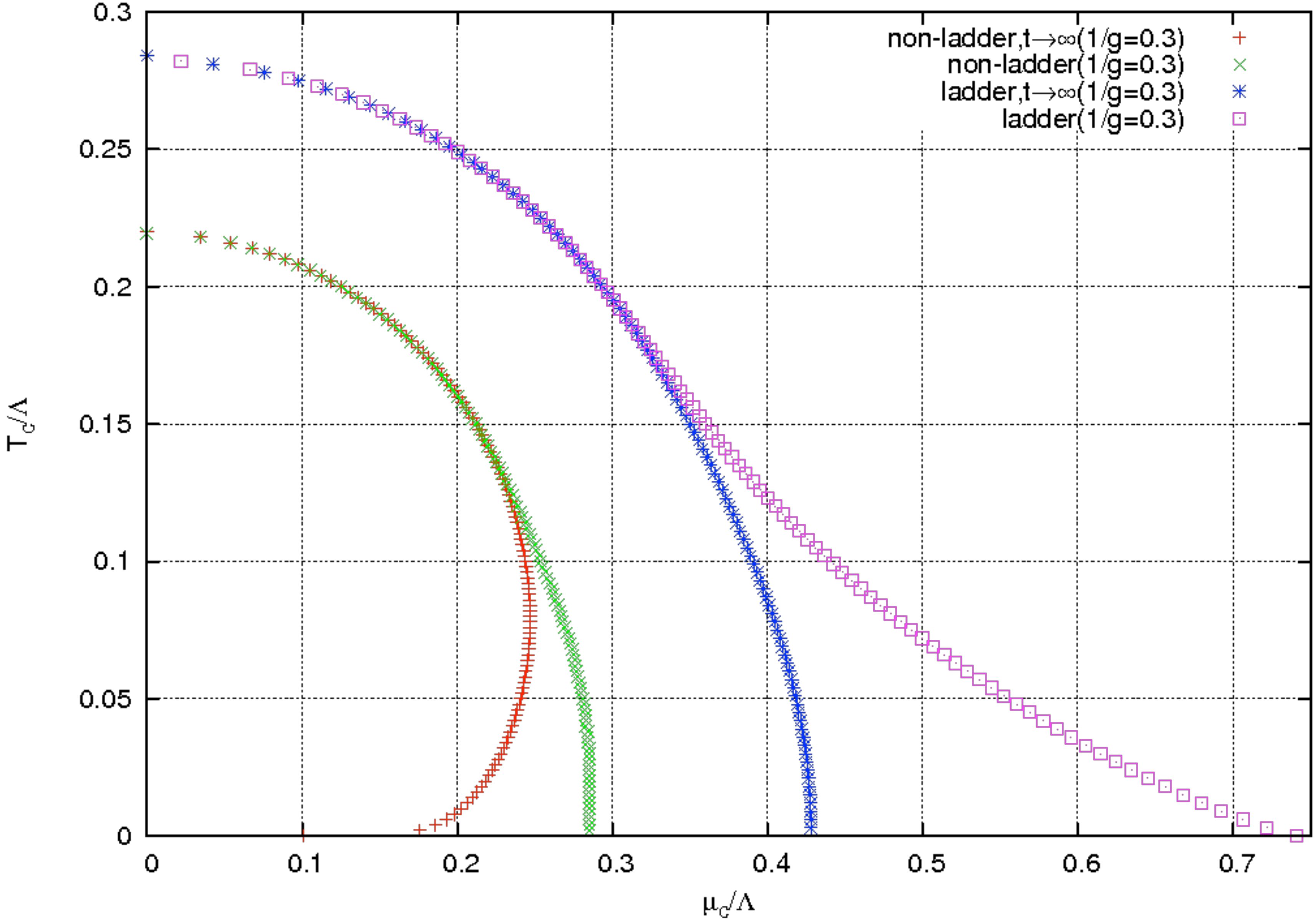
Non-ladder extended NPRG eq.

$\mu/\Lambda = 0.20,\ T/\Lambda = 10^{-4}$



Large-N non-leading





non-ladder,t→∞(1/g=0.3)
non-ladder(1/g=0.3)
ladder,t→∞(1/g=0.3)
ladder(1/g=0.3)
T_c/Λ
μ_c/Λ
0
0.05
0.1
0.15
0.2
0.25
0.3
0.4
0.5
0.6
0.7

Appendix

- Effective interaction : Not Lorentz invariant

$$V_{\text{eff}} = -\frac{G}{4}\{(\bar{\psi}\gamma_\mu\psi)^2 - (\bar{\psi}\gamma_5\gamma_\mu\psi)^2\}$$
$$\equiv -\frac{g_0}{4}\{(\bar{\psi}\gamma_0\psi)^2 - (\bar{\psi}\gamma_5\gamma_0\psi)^2\} - \frac{g_1}{4}\{(\bar{\psi}\gamma_i\psi)^2 - (\bar{\psi}\gamma_5\gamma_i\psi)^2\}$$

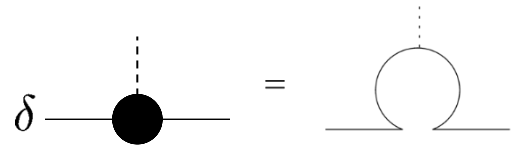
- Non-ladder extended NPRG eq.

$$\partial_t g_0 = -2g_0 + \frac{1}{6}g_0^2(I_0 - I_2) + \frac{1}{2}g_0 g_1(I_1 - I_3) + \frac{1}{2}g_1^2(I_0 - I_2) + \frac{1}{3}g_1^2(I_1 + I_3)$$

$$\partial_t g_1 = -2g_1 + \frac{1}{18}g_0^2(I_1 - I_3) + \frac{1}{3}g_0 g_1(I_0 - I_2) + \frac{2}{9}g_1^2(I_1 + I_3) + \frac{1}{3}g_1^2(I_0 + I_2) + \frac{7}{18}g_1^2(I_1 - I_3)$$

$$\partial_t \mu = \mu - \frac{2}{3}g_0 \; \frac{\partial}{\partial\omega}[n_+ - n_-]\Big|_{\omega\to 1}$$

$$\partial_t T = T$$



$$I_0 = \left[\left(\frac{1}{2} - n_+\right) + \left(\frac{1}{2} - n_-\right) + \frac{\partial}{\partial\omega}(n_+ + n_-) + \frac{\partial^2}{\partial\omega^2}(n_+ + n_-)\right]\Big|_{\omega\to 1}$$

$$I_1 = \left[\left(\frac{1}{2} - n_+\right) + \left(\frac{1}{2} - n_-\right) + \frac{\partial}{\partial\omega}(n_+ + n_-) - \frac{\partial^2}{\partial\omega^2}(n_+ + n_-)\right]\Big|_{\omega\to 1}$$

$$I_2 = \left[\frac{1}{(1+\mu)^2}\left(\frac{1}{2} - n_+\right) + \frac{1}{(1-\mu)^2}\left(\frac{1}{2} - n_-\right) + \frac{2\mu+1}{\mu(1+\mu)}\frac{\partial}{\partial\omega}n_+ + \frac{2\mu-1}{\mu(1-\mu)}\frac{\partial}{\partial\omega}n_-\right]\Big|_{\omega\to 1}$$

$$I_3 = \left[\frac{1}{(1+\mu)^2}\left(\frac{1}{2} - n_+\right) + \frac{1}{(1-\mu)^2}\left(\frac{1}{2} - n_-\right) - \frac{1}{\mu(1+\mu)}\frac{\partial}{\partial\omega}n_+ + \frac{1}{\mu(1-\mu)}\frac{\partial}{\partial\omega}n_-\right]\Big|_{\omega\to 1}$$