Brane-Antibrane and Closed Superstrings at Finite Temperature in the Framework of Thermo Field Dynamics

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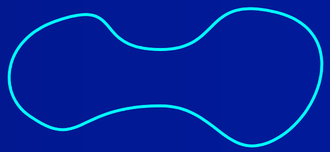
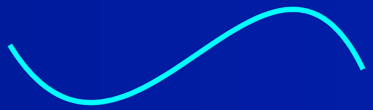
1. Introduction

- Hagedorn Temperature $\mathcal{T}_H$ (type II)

  maximum temperature for perturbative strings

  A single energetic string captures most of the energy.





$$d_n \sim e^{2\pi\sqrt{2n}}$$

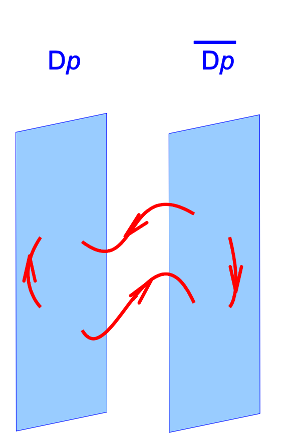
$$\Omega(E) \sim e^{\beta_H E}$$

$$Z(\beta) = \int_0^\infty dE\ \Omega(E)\ e^{-\beta E}$$

$$\beta_H \equiv \frac{1}{\mathcal{T}_H} = 2\pi\sqrt{2\alpha'}$$

$Z(\beta) \to \infty$ for $\beta < \beta_H$

■ Dp-$\overline{\mathrm{D}p}$ pairs (type II)



unstable at zero temperature

open string tachyon → tachyon potential

Sen's conjecture   potential height=brane tension

■ Brane-antibrane Pair Creation Transition Hotta

finite temperature system of Dp-$\overline{\mathrm{D}p}$ pairs

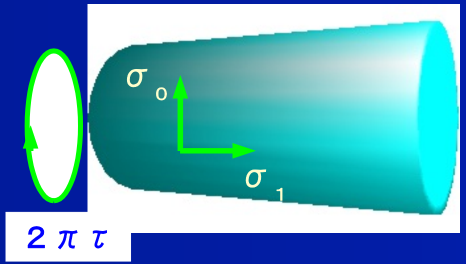
↓ based on Matsubara formalism and on BSFT

1-loop (cylinder world sheet)

Conformal invariance is broken by the boundary terms.

ambiguity in choosing the Weyl factors

→ cylinder boundary action Andreev-Oft



finite temperature effective potential

↓

D9-$\overline{\mathrm{D9}}$ pairs become stable near the Hagedorn temperature.

Thermo Field Dynamics (TFD) Takahashi-Umezawa

statistical average

$$\langle A \rangle = Z^{-1}(\beta) \sum_n \langle n|\hat{A}|n\rangle e^{-\beta E_n}$$

We can represent it as

$$\langle A \rangle = \left\langle 0(\beta) \left| \hat{A} \right| 0(\beta) \right\rangle$$

by introducing a fictitious copy of the system.

$$|0(\beta)\rangle = Z^{-\frac{1}{2}}(\beta) \sum_n e^{-\frac{\beta E_n}{2}} |n, \tilde{n}\rangle$$

thermal vacuum state

$$|n, \tilde{n}\rangle = |n\rangle \otimes |\tilde{n}\rangle$$

We cannot represent it as

$$|0(\beta)\rangle = \sum_n |n\rangle\, f_n(\beta)$$

for ordinary number $f_n(\beta)$, since

$$f_n^*(\beta) f_m(\beta) = Z^{-1}(\beta) e^{-\beta E_n} \delta_{nm}$$

cannot be satisfied.

Hawking-Unruh effect can be described by TFD.

It is expected that TFD is available to non-equilibrium system.

(real time formalism)

TFD has been applied to string theory

string field theory	Leblanc
D-brane	Vancea, Cantcheff, etc.
closed bosonic string	Abdalla-Gadelha-Nedel
AdS background	Grada-Vancea, etc.
pp-wave background	Nedel-Abdalla-Gadelha, etc.

At the lowest order we do not use one-loop amplitude.

→ There is no problem of the choice of Weyl factors.

finite temperature system of D*p*-D*p* and closed superstring based on TFD?

Contents

2. Brane-antibrane Pair in TFD

- Light-Cone Momentum

We consider a single first quantized string.

light-cone momentum

$$\begin{aligned} p^+ &= p^0 + p^1 \\ p^- &= p^0 - p^1 \end{aligned}$$

$$p^0 = \frac{1}{2}(p^+ + p^-)$$

$$p^+ p^- - |\boldsymbol{p}|^2 - M^2 = 0$$

$$p^- = \frac{|\boldsymbol{p}|^2 + M^2}{p^+}$$

partition function for a single string

$$\begin{aligned} Z_1(\beta) &= \mathrm{Tr}\ \exp\left(-\beta p^0\right) = \mathrm{Tr}\ \exp\left[-\frac{1}{2}\beta(p^+ + p^-)\right] \\ &= \mathrm{Tr}\ \exp\left[-\frac{1}{2}\beta\left(p^+ + \frac{|\boldsymbol{p}|^2 + M^2}{p^+}\right)\right] \end{aligned}$$

- **BSFT** (Boundary String Field Theory) (BV formalism)

  solution of classical master eq. (superstring)

$$S_{eff} = Z$$

S_{eff} : effective action $\quad Z$: 2-dim. partition function

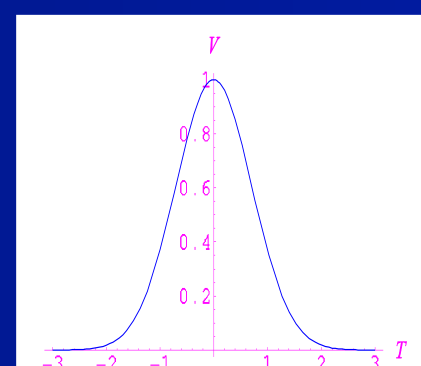
$$S_2 = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \ \partial_a X_\mu \partial^a X^\mu + \int_{\partial\Sigma} d\tau |T|^2 + \cdots$$

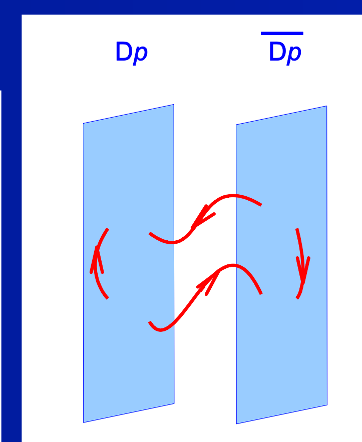
- **Disk** (tree level tachyon potential)

$$V(T) = 2\tau_p v_p \exp(-8|T|^2), \qquad \tau_p = \frac{1}{(2\pi)^p \alpha'^{\frac{p+1}{2}} g_s}$$

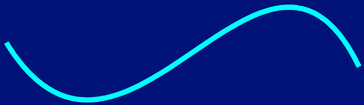
T: complex scalar field $\quad \tau_p$: tension of Dp-brane

$g_s = e^\phi$: coupling of strings $\quad v_p$: p-dim. volume





Mass Spectrum



We consider an open string

on a Brane-antibrane pair.

mass spectrum

$$M_{NS}{}^2 = \frac{1}{\alpha'}\left(N_B + N_{NS} + 2|T|^2 - \frac{1}{2}\right)$$ space time boson

$$M_R{}^2 = \frac{1}{\alpha'}\left(N_B + N_R + 2|T|^2\right)$$ space time fermion

number ops.

$$N_B = \sum_{l=1}^{\infty}\sum_{I=1}^{8} \alpha_{-l}^I \alpha_l^I$$ oscillation mode of world sheet boson

$$N_{NS} = \sum_{r=\frac{1}{2}}^{\infty}\sum_{I=1}^{8} r b_{-r}^I b_r^I$$ oscillation mode of world sheet fermion (NS b. c)

$$N_R = \sum_{m=1}^{\infty}\sum_{I=1}^{8} m d_{-m}^I d_m^I$$ oscillation mode of world sheet fermion (R b. c)

We will show only the NS mode case.

Bogoliubov Transformation

generator of Bogoliubov tr.

$$G_{1NS} = \mathcal{G}_B + \mathcal{G}_{NS}$$

$$\mathcal{G}_B = i\sum_{l=1}^{\infty} \frac{1}{l}\, \theta_l \left(\alpha_{-l}\cdot\tilde{\alpha}_{-l} - \tilde{\alpha}_l\cdot\alpha_l\right)$$

$$\mathcal{G}_{NS} = i\sum_{r=\frac{1}{2}}^{\infty} \theta_r \left(b_{-r}\cdot\tilde{b}_{-r} - \tilde{b}_r\cdot b_r\right)$$

$$\tanh\theta_l = \exp\left(-\frac{\beta l}{4\alpha' p^+}\right)$$

$$\tan\theta_r = \exp\left(-\frac{\beta r}{4\alpha' p^+}\right)$$

Thermal Vacuum State

thermal vacuum state for a single string

$$\begin{aligned} |0_{1NS}(\theta)\rangle &\equiv e^{-iG_{1NS}}|0\rangle\rangle \left|p^+\right\rangle |\boldsymbol{p}\rangle \\ &= \prod_{l=1}^{\infty}\left\{\left(\frac{1}{\cosh(\theta_l)}\right)^8 \exp\left[\frac{1}{l}\ \tanh(\theta_l)\alpha_{-l}\cdot\tilde{\alpha}_{-l}\right]\right\} \\ &\quad \times \prod_{r=\frac{1}{2}}^{\infty}\left\{(\cos(\theta_r))^8 \exp\left[\ \tan(\theta_r) b_{-r}\cdot\tilde{b}_{-r}\right]\right\}|0\rangle\rangle \left|p^+\right\rangle |\boldsymbol{p}\rangle \end{aligned}$$

$$\alpha_l|0\rangle\rangle = b_r|0\rangle\rangle = \tilde{\alpha}_l|0\rangle\rangle = \tilde{b}_r|0\rangle\rangle = 0 \qquad \text{for positive } l, r$$

Free Energy for a Single String

$$F_{1NS}(\theta) = \left\langle 0_{1NS}(\theta) \right| \left(H_{1NS} - \frac{1}{\beta} K_{1NS} \right) \left| 0_{1NS}(\theta) \right\rangle$$

Hamiltonian

$$H_{1NS} = \frac{1}{2}\left(p^+ + \frac{|\boldsymbol{p}|^2 + M_{NS}^2}{p^+}\right)$$

entropy

$$K_{1NS} = -\sum_{l=1}^{\infty} \frac{1}{l}\left\{\alpha_{-l}\cdot\alpha_l \ln \sinh^2\theta_l - \alpha_l\cdot\alpha_{-l}\ln\cosh^2\theta_l\right\} - \sum_{r=\frac{1}{2}}^{\infty}\left\{b_{-r}\cdot b_r \ln\sin^2\theta_r + b_r\cdot b_{-r}\ln\cos^2\theta_r\right\}$$

$$F_{1NS}(\beta) = \frac{1}{2}\left(p^+ + \frac{|\boldsymbol{p}|^2}{p^+}\right) + \frac{|T|^2}{\alpha' p^+} + \frac{8}{\beta}\sum_{l=1}^{\infty}\ln\left[1 - \exp\left(-\frac{\beta l}{2\alpha' p^+}\right)\right] - \frac{1}{4\alpha' p^+} - \frac{8}{\beta}\sum_{r=\frac{1}{2}}^{\infty}\ln\left[1 + \exp\left(-\frac{\beta r}{2\alpha' p^+}\right)\right]$$

This is not useful for analysis of thermodynamical system of strings.

free energy for a single string

→ partition function for a single string → free energy for multiple strings (string gas)

- **Partition Function for a Single String**

$$Z_{1NS}(\beta) = \frac{v_p}{(2\pi)^p}\int_0^\infty dp^+ \int_{-\infty}^{\infty} d^p p \ \exp\left(-\beta F_{1NS}\right)$$

$$\tau \equiv \frac{2\pi\beta}{{\beta_H}^2 p^+} = \frac{\beta}{4\pi\alpha' p^+}, \quad \beta_H = 2\pi\sqrt{2\alpha'}$$

$$Z_{1NS}(\beta) = \frac{16\pi^4\beta v_p}{{\beta_H}^{p+1}}\int_0^\infty \frac{d\tau}{\tau}\, \tau^{-\frac{p+1}{2}} e^{-4\pi|T|^2}\left\{\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right\}^4 \exp\left(-\frac{\pi\beta^2}{{\beta_H}^2\tau}\right)$$

- **Free Energy for Multiple Strings**

Free energy for multiple strings can be obtained from the following eq.

$$F(\beta) = -\sum_{w=1}^{\infty}\frac{1}{\beta w}\left\{Z_{1NS}(\beta w) - (-1)^w Z_{1R}(\beta w)\right\}$$

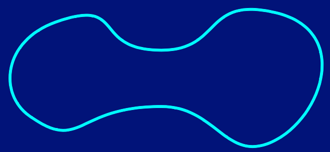
$$F(\beta) = -\frac{16\pi^4 v_p}{{\beta_H}^{p+1}}\int_0^\infty \frac{d\tau}{\tau}\, \tau^{-\frac{p+1}{2}} e^{-4\pi|T|^2\tau} \times\left[\left(\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4\left\{\vartheta_3\left(0\left|\frac{i\beta^2}{{\beta_H}^2\tau}\right.\right)-1\right\} - \left(\frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4\left\{\vartheta_4\left(0\left|\frac{i\beta^2}{{\beta_H}^2\tau}\right.\right)-1\right\}\right]$$

This equals to the free energy based on Matsubara formalism.

This implies that our choice of Weyl factors

in the case of Matsubara formalism is quite natural.

3. Closed Superstring in TFD



- Mass Spectrum

$$
\begin{aligned}
{M_{NSNS}}^2 &= \frac{2}{\alpha'}\left(N_B + N_{NS} + \overline{N}_B + \overline{N}_{NS} - 1\right) \\
{M_{RR}}^2 &= \frac{2}{\alpha'}\left(N_B + N_R + \overline{N}_B + \overline{N}_R\right)
\end{aligned}
$$

space time boson

$$
\begin{aligned}
{M_{NSR}}^2 &= \frac{2}{\alpha'}\left(N_B + N_{NS} + \overline{N}_B + \overline{N}_R - \frac{1}{2}\right) \\
{M_{RNS}}^2 &= \frac{2}{\alpha'}\left(N_B + N_R + \overline{N}_B + \overline{N}_{NS} - \frac{1}{2}\right)
\end{aligned}
$$

space time fermion

We show only the NS-NS mode case.

GSO projection

$$\text{left-moving modes}: \ \frac{1}{2}(1+G)$$

$$\text{right-moving modes}: \ \frac{1}{2}\left(1+\overline{G}\right)$$

$$G = -(-1)^{\sum_{r=\frac{1}{2}}^{\infty} b_{-r} b_r}$$

level-matching condition

$$N_B + N_{NS} - \overline{N}_B - \overline{N}_{NS} = 0$$

$$\delta_{n,n'} = \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \ \exp\left[2\pi i \tau_1 \left(n - n'\right)\right]$$

Thermal Vacuum State

generator of Bogoliubov tr.

$$G_{NSNS} = \mathcal{G}_B + \mathcal{G}_{NS} + \overline{\mathcal{G}}_B + \overline{\mathcal{G}}_{NS}$$

$$\mathcal{G}_B = i \sum_{l=1}^{\infty} \frac{1}{l} \theta_l \left(\alpha_{-l} \cdot \tilde{\alpha}_{-l} - \tilde{\alpha}_l \cdot \alpha_l\right)$$

$$\mathcal{G}_{NS} = i \sum_{r=\frac{1}{2}}^{\infty} \theta_r \left(b_{-r} \cdot \tilde{b}_{-r} - \tilde{b}_r \cdot b_r\right)$$

thermal vacuum state for a single string

$$\begin{aligned} |0_{1NSNS}(\theta)\rangle &\equiv e^{-iG_{1NSNS}} |0\rangle\rangle \left|p^+\right\rangle |\boldsymbol{p}\rangle \\ &= |0_B(\theta)\rangle |0_{NS}(\theta)\rangle \left|\overline{0}_B\left(\overline{\theta}\right)\right\rangle \left|\overline{0}_{NS}\left(\overline{\theta}\right)\right\rangle \left|p^+\right\rangle |\boldsymbol{p}\rangle \end{aligned}$$

$$|0_B(\theta)\rangle = \prod_{l=1}^{\infty} \left\{ \left(\frac{1}{\cosh(\theta_l)}\right)^8 \exp\left[\frac{1}{l} \tanh(\theta_l) \alpha_{-l} \cdot \tilde{\alpha}_{-l}\right] \right\} |0\rangle\rangle$$

$$|0_{NS}(\theta)\rangle = \prod_{r=\frac{1}{2}}^{\infty} (\cos(\theta_r))^8 \exp\left[\tan(\theta_r) b_{-r} \cdot \tilde{b}_{-r}\right] |0\rangle\rangle$$

Free Energy for a Single String

$$F^{IJ}_{1NSNS}(\theta) = \left\langle 0_{1NSNS}(\theta) \middle| \left\{ H_{1NSNS} - \frac{1}{\beta}\left(K_{1NSNS} + C_{NSNS} + P^I_{NS} + \overline{P}^J_{NS}\right)\right\} \middle| 0_{1NSNS}(\theta) \right\rangle$$

$$I, J = + \quad \longrightarrow \quad \frac{1}{2} \text{ part of GSO projection}$$

$$I, J = - \quad \longrightarrow \quad \frac{1}{2}\, G \text{ part of GSO projection}$$

Hamiltonian

$$H_{1NSNS} = \frac{1}{2}\left(p^+ + \frac{|\boldsymbol{p}|^2 + M^2_{NSNS}}{p^+}\right)$$

entropy

$$\begin{aligned} K_{1NSNS} \;=\; & -\sum_{l=1}^{\infty} \frac{1}{l}\left\{\alpha_{-l}\cdot\alpha_l \ln\sinh^2\theta_l - \alpha_l\cdot\alpha_{-l}\ln\cosh^2\theta_l\right\} \\ & -\sum_{r=\frac{1}{2}}^{\infty}\left\{b_{-r}\cdot b_r \ln\sin^2\theta_r + b_r\cdot b_{-r}\ln\cos^2\theta_r\right\} \\ & +(\text{right-movers}) \end{aligned}$$

level-matching condition

$$C_{NSNS} = 2\pi i \tau_1 \left(\sum_{l=1}^{\infty}\alpha_{-l}\cdot\alpha_l + \sum_{r=\frac{1}{2}}^{\infty} r b_{-r}\cdot b_r - \sum_{l'=1}^{\infty}\overline{\alpha}_{-l'}\cdot\overline{\alpha}_{l'} - \sum_{r'=\frac{1}{2}}^{\infty} r'\overline{b}_{-r'}\cdot\overline{b}_{r'}\right)$$

GSO projection

$$P^+_{NS} = \overline{P}^+_{NS} = 0$$

$$P^-_{NS} = \pi i\left(\sum_{r=\frac{1}{2}}^{\infty} b_{-r}\cdot b_r + 1\right)$$

$$\overline{P}^-_{NS} = \pi i\left(\sum_{r=\frac{1}{2}}^{\infty} \overline{b}_{-r}\cdot \overline{b}_r + 1\right)$$

Relation between β and θ .

$$\frac{\partial}{\partial \theta_l} F^{IJ}_{1NSNS}(\theta) = 0$$

$$\tanh \theta_l = \exp\left(- \frac{\beta l}{4\alpha' p^+} + \pi i \tau_1 l \right)$$

For $I = +$

$$\frac{\partial}{\partial \theta_r} F^{+J}_{1NSNS}(\theta) = 0$$

$$\tan \theta_r = \exp\left(- \frac{\beta r}{4\alpha' p^+} + \pi i \tau_1 r \right)$$

For $I = -$

$$\frac{\partial}{\partial \theta_r} F^{-J}_{1NSNS}(\theta) = 0$$

$$\tan \theta_r = \exp\left(- \frac{\beta r}{4\alpha' p^+} + \pi i \tau_1 r + \frac{\pi i}{2} \right)$$

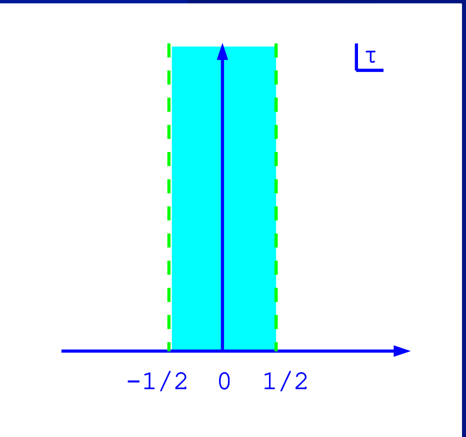
$$\begin{aligned} F^{++}_{1NSNS}(\beta) \;=\; & \frac{1}{2}\left(p^+ + \frac{|\boldsymbol{p}|^2}{p^+} \right) + \frac{8}{\beta} \sum_{l=1}^{\infty} \ln\left[1 - \exp\left(- \frac{\beta l}{\alpha' p^+} + 2\pi i \tau_1 l \right) \right] \\ & - \frac{1}{2\alpha' p^+} + \frac{\pi i \tau_1}{\beta} - \frac{8}{\beta} \sum_{r=\frac{1}{2}}^{\infty} \ln\left[1 + \exp\left(- \frac{\beta r}{\alpha' p^+} + 2\pi i \tau_1 r \right) \right] \\ & + \frac{8}{\beta} \sum_{l'=1}^{\infty} \ln\left[1 - \exp\left(- \frac{\beta l'}{\alpha' p^+} - 2\pi i \tau_1 l' \right) \right] \\ & - \frac{1}{2\alpha' p^+} - \frac{\pi i \tau_1}{\beta} - \frac{8}{\beta} \sum_{r'=\frac{1}{2}}^{\infty} \ln\left[1 + \exp\left(- \frac{\beta r'}{\alpha' p^+} - 2\pi i \tau_1 r' \right) \right] \end{aligned}$$

■ Partition Function for a Single String

$$Z_{1NSNS}(\beta) = \frac{v_9}{4(2\pi)^9}\int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \int_0^\infty dp^+ \int_{-\infty}^{\infty} d^8p \\ \times\left\{\exp\left(-\beta F^{++}_{1NSNS}\right) + \exp\left(-\beta F^{+-}_{1NSNS}\right) + \exp\left(-\beta F^{-+}_{1NSNS}\right) + \exp\left(-\beta F^{--}_{1NSNS}\right)\right\}$$

$$\tau_2 \equiv \frac{4\pi\beta}{\beta_H{}^2 p^+} \qquad \tau \equiv \tau_1 + i\tau_2 \qquad \beta_H = 2\pi\sqrt{2\alpha'}$$

$$Z_{1NSNS}(\beta) = \frac{8(2\pi)^8 \beta v_9}{\beta_H{}^{10}} \int_S \frac{d^2\tau}{\tau_2^6} \frac{1}{|\vartheta_1{}'(0|\tau)|^8} \\ \left\{\left(\vartheta_3^4 - \vartheta_4^4\right)\left(\overline{\vartheta}_3^4 - \overline{\vartheta}_4^4\right)\right\}(0|\tau) \exp\left(-\frac{2\pi\beta^2}{\beta_H{}^2\tau_2}\right)$$



domain of integration *S*

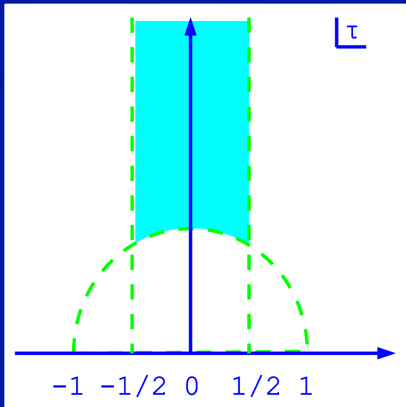
Free Energy for Multiple Strings

Free energy for multiple strings can be obtained from the following eq.

$$F(\beta) = -\sum_{w=1}^{\infty} \frac{1}{\beta w} \left[\{ Z_{1NSNS}(\beta w) + Z_{1RR}(\beta w) \} - (-1)^w \{ Z_{1NSR}(\beta w) + Z_{1RNS}(\beta w) \} \right]$$

$$\begin{aligned} F(\beta) = & - \frac{8(2\pi)^8 v_9}{{\beta_H}^{10}} \int_S \frac{d^2\tau}{\tau_2^6} \frac{1}{|\vartheta_1{}'(0|\tau)|^8} \\ & \times \Bigg[\left\{ \left(\vartheta_3^4 - \vartheta_4^4 \right) \left(\bar{\vartheta}_3^4 - \bar{\vartheta}_4^4 \right) + \vartheta_2^4 \bar{\vartheta}_2^4 \right\} (0|\tau) \sum_{w=1}^{\infty} \exp\left(- \frac{2\pi w^2 \beta^2}{{\beta_H}^2 \tau_2} \right) \\ & - \left\{ \left(\vartheta_3^4 - \vartheta_4^4 \right) \bar{\vartheta}_2^4 + \vartheta_2^4 \left(\bar{\vartheta}_3^4 - \bar{\vartheta}_4^4 \right) \right\} (0|\tau) \sum_{w=1}^{\infty} (-1)^w \exp\left(- \frac{2\pi w^2 \beta^2}{{\beta_H}^2 \tau_2} \right) \Bigg] \end{aligned}$$

This equals to the free energy in the S-representation based on Matsubara formalism.



We can transform this to the F-representation or the Dual-representation by using modular transformation.

4. Conclusion and Discussion

- Brane-antibrane in TFD

  We computed thermal vacuum state and partition function of a single string on a Brane-antibrane pair based on TFD.

  The free energy for multiple strings agrees with that based on the Matsubara formalism.

  There are no problem of the choice of the Weyl factors.

- Closed Superstring Gas in TFD

  We computed thermal vacuum state and partition function of a single closed superstring based on TFD.

  The free energy for multiple strings agrees with that based on the Matsubara formalism.

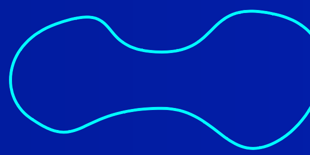
- String Field Theory

  We need to use second quantized string field theory in order to obtain the thermal vacuum state for multiple strings.

- D-brane boundary state of closed string   cf) Cantcheff

  The thermal vacuum state is reminiscent of the D-brane boundary state of a closed string.

$$|B9_{mat},\eta\rangle_{NSNS} = \exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\,\alpha_{-n}\cdot\tilde{\alpha}_{-n} + i\eta\sum_{u>0}\psi_{-u}\cdot\tilde{\psi}_{-u}\right]|B9_{mat},\eta\rangle^{(0)}_{NSNS}$$



- Hawking-Unruh Effect

  closed strings in curved spacetime

  black hole firewall   Almheiri-Marolf-Polchinski-Sully

  Planck solid model   Hotta