Self-duality of tensor gauge fields on topologically nontrivial manifolds

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based on arXiv [hep-th] 1406.6023 [by Hiroshi Isono]

Consider an abelian tensor gauge field whose field strength is self-dual

Conventional derivation of self-duality (schemeatically)

EoM : d(F-*F)=0 ⟶ F-*F=d(something) ⟶ F-*F=0 by gauge fixing

This method makes sense on topologically trivial manifolds (Poincare's lemma)

Here we give the derivation of self-duality on topologically nontrivial manifold

motivation : application of worldvolume action [PST] to single M5 wrapped on $\mathrm{AdS}_3 \times S^3$ [Mori-Yamaguchi 1404.0930]

to check the $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence for the Wilson surface operators of 6 dimensional (2,0) A_n superconformal field theory (or its S^1-reduction to 5 dimensional maximally supersymmetric Yang-Mills theory). This work obtained the nice correspondence.

But the self-duality does not seem to be derived on such manifolds (see above for WHY)

Can we derive self-duality ? ————— My answer : YES

Consider 2n-form gauge field on (4n+2)D spacetime : n=1 —> M5, n=2 —> RR 5-form in IIB SUGRA

covariant action $S = \int d^D x \frac{\sqrt{-g}}{(\partial a)^2} \partial^a a (M - *M)_{a a_1 \cdots a_{2n}} (*M)^{a_1 \cdots a_{2n} b} \partial_b a = \int e_{\hat{v}} \iota_v (M - *M) \wedge M$ [Pasti-Sorokin-Tonin]

3 gauge symmetries

$\delta_1 a = 0, \quad \delta_1 A = d\Lambda$ ordinary tensor gauge symmetry

$\delta_2 a = 0, \quad \delta_2 A = da \wedge \Phi$ used for deriving self-duality (main topic)

$\delta_3 a = \varphi, \quad \delta_3 A = \frac{\varphi}{\sqrt{(\partial a)^2}} \iota_v (M - *M)$ used for eliminating the field a

notations $M_{2n+1} = dA_{2n}$ $\quad \hat{v} := \frac{da}{\sqrt{(\partial a)^2}} \quad v := \frac{\partial^a a}{\sqrt{(\partial a)^2}} \partial_a \quad e_\theta \omega := \theta \wedge \omega$

equations of motion

$$\delta A : \; d[\hat{v} \wedge \iota_v (M - *M)] = 0$$

it suffices to consider only this EoM

$$\delta a : \; d\left[\frac{1}{\sqrt{(\partial a)^2}} \hat{v} \wedge \iota_v (M - *M) \wedge \iota_v (M - *M)\right] = 0$$

this can be derived from the 1st EoM

derivation of self-duality

exists when the spacetime (worldvolume) is topologically nontrivial

1. solve the EoM $\quad \hat{v} \wedge \iota_v (M - *M) = da \wedge (2n\text{-form}) \to d(2n\text{-form}) = 0 \to (2n\text{-form}) = d\eta + \omega$

2. gauge transform the solution $\quad \hat{v} \wedge \iota_v (M - *M) = da \wedge (d\eta + \omega)$, $\quad \delta_2[\hat{v} \wedge \iota_v (M - *M)] = da \wedge d\Phi$ ⟶ we can eliminate $da \wedge d\eta$; we can NOT eliminate $da \wedge \omega$

3. new gauge transformation [1406.6023 HI]

the action is invariant under $\quad \delta_2' a = 0, \quad \delta_2' A = a\xi \quad$ ξ is just a closed form

this transformation allows us to eliminate both $da \wedge d\eta$ and $da \wedge \omega$ since $\delta_2'[\hat{v} \wedge \iota_v (M - *M)] = -da \wedge \xi$

4. self-duality

gauge-fixed solution $\hat{v} \wedge \iota_v (M - *M)] = 0$ $\xrightarrow{\hat{v}\iota_v * = *\iota_v \hat{v}}$ $\iota_v \hat{v} \wedge (M - *M) = 0$ $\xrightarrow{\iota_v \hat{v} + \hat{v} \iota_v = 1}$ $M - *M = 0$ self-duality

generalisations

1. DBI-like extension : the 2nd gauge transformation is the same even with the self-interactions
2. noncovariant actions : Perry-Schwarz is a partially gauge fixed version of PST