Defect Holography and the Fractional Quantum Hall Effect

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Strings and Fields, YITP (26 July, 2014)

work in progress
with M. Fujita, R. Meyer, S. Sugimoto

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Today's focus:

Holography of domain walls in Yang-Mills–Chern-Simons theory.

Yang-Mills–Chern-Simons? (YMCS)

level

3D: $$S = -\frac{1}{4g^2}\int_{M_3} d^3x\,\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \frac{k}{4\pi}\int_{M_3}\omega_3(A)$$

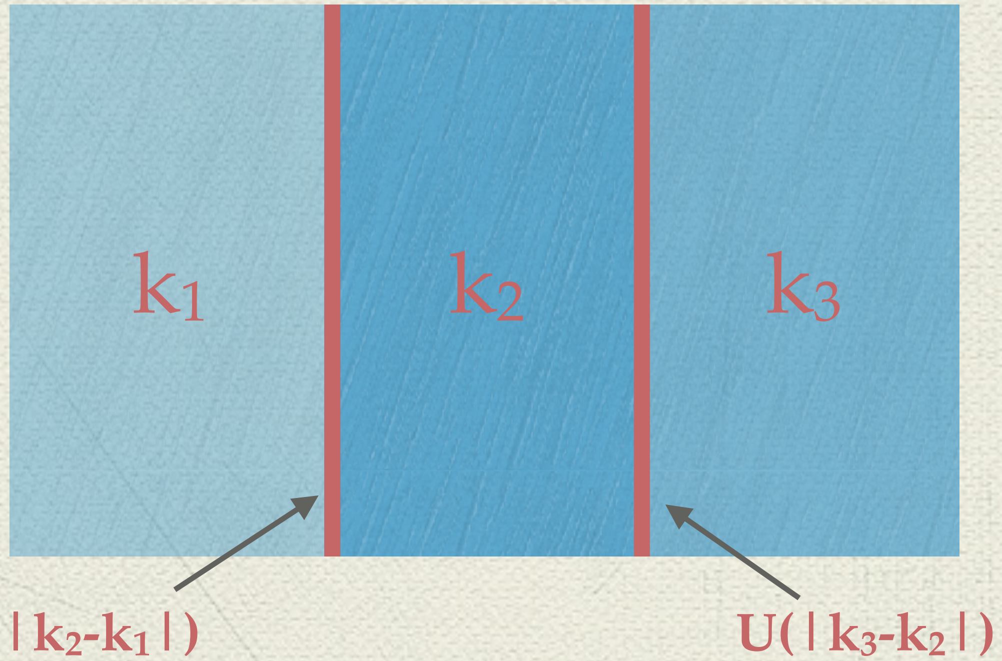
$$\omega_3(A) = \mathrm{Tr}(A\wedge dA + \frac{2}{3}A\wedge A\wedge A)$$ ⇦ **Chern-Simons term**

CS term only gauge invariant up to a total derivative: $\delta_\Lambda \omega_3(A) = d\,\mathrm{Tr}(\Lambda F)$

Consider domain walls separating phases at different levels **=> not gauge invariant!**

Must be additional degrees of freedom living on the defects to compensate

New degrees transform under a $U(|\Delta k|)$ global symmetry.



Holographic Embedding of YM~~CS~~

N=4 SYM ⟺ AdS5

Compactify N=4 SYM on a circle, impose anti-periodic boundary conditions on fermions ⇨ ~~SUSY~~

Geometry: AdS soliton

Massive fermions induce scalar masses

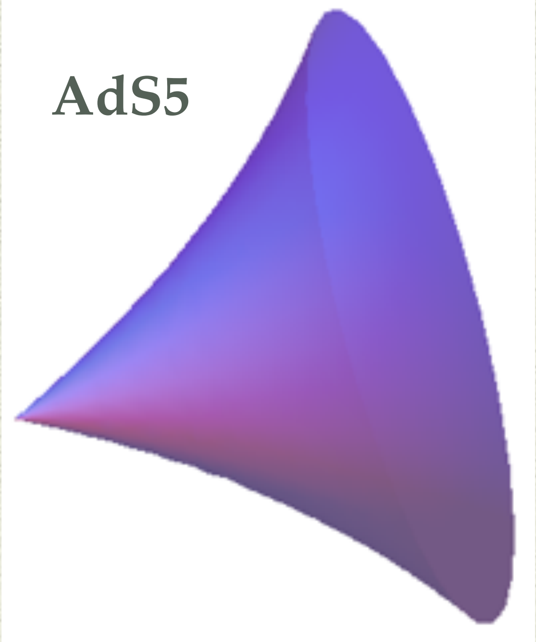
Gauge field confines

} => gapped theory

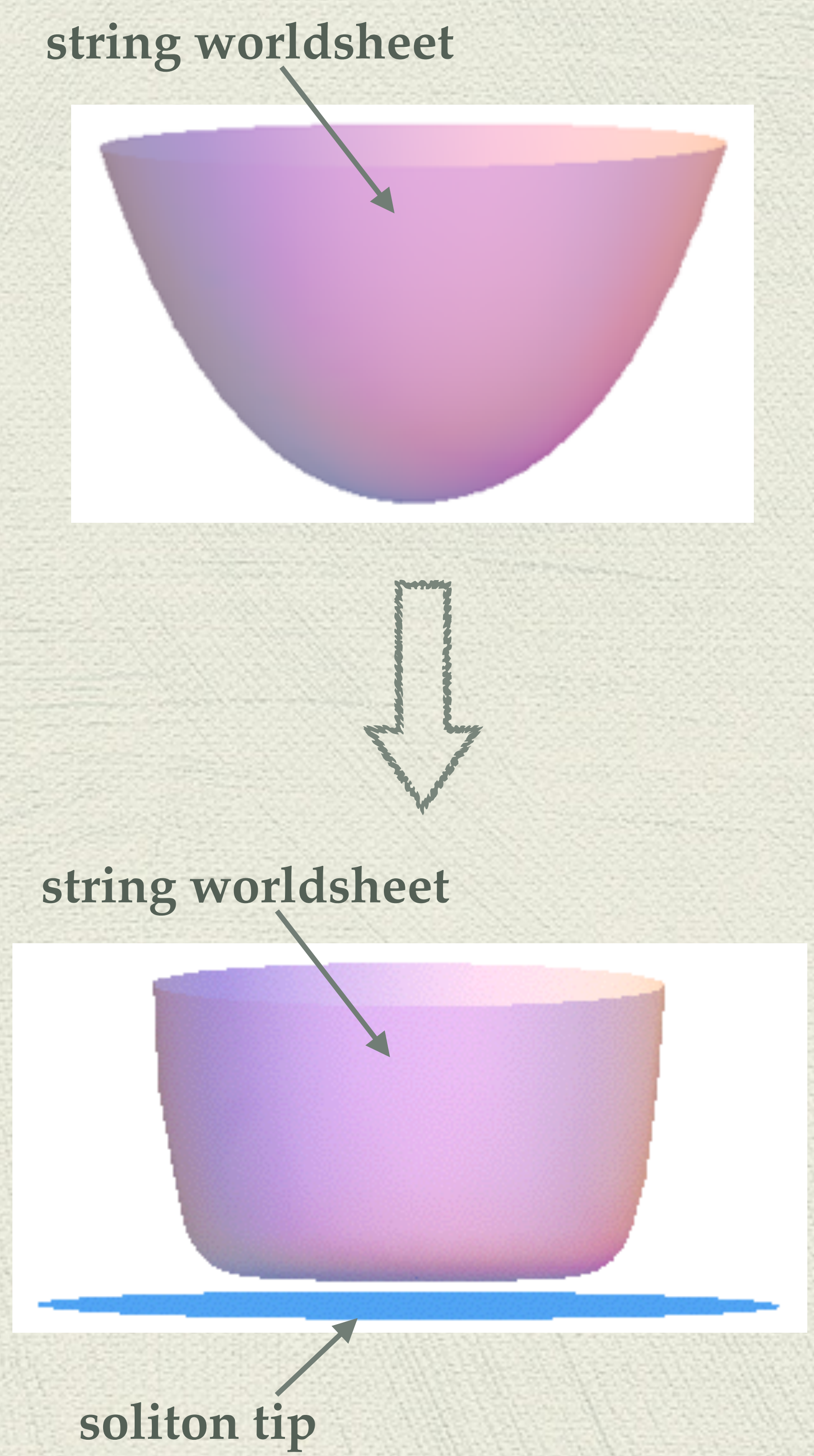
Dual geometry caps off at a scale determined by the compactification radius

$$ds^2 = w^2 dx_3^2 + f(w) w^2 d\theta^2 + \frac{R^2}{f(w)} \frac{dw^2}{w^2}$$

cap at $w = w_0$ $\quad f(w) = 1 - \frac{w_0^4}{w^4}$



Holographic Embedding of YM~~CS~~ -ish



Wilson line

Computed by attaching a string.

AdS => conformal => no dependence on size
(after renormalization)

AdS soliton => Wilson lines larger than the compactification radius hit the bottom

Dominated by area of worldsheet at bottom
=> area law *confinement

Infrared physics = confined phase of 3D YM

Holographic Embedding of YMCS-ish

Fujita, Li, Ryu, Takayanagi
0901.0924

How to get the CS in YM-CS?

Starting with D3 branes, introduce *k* units of C_0 flux on the circle.

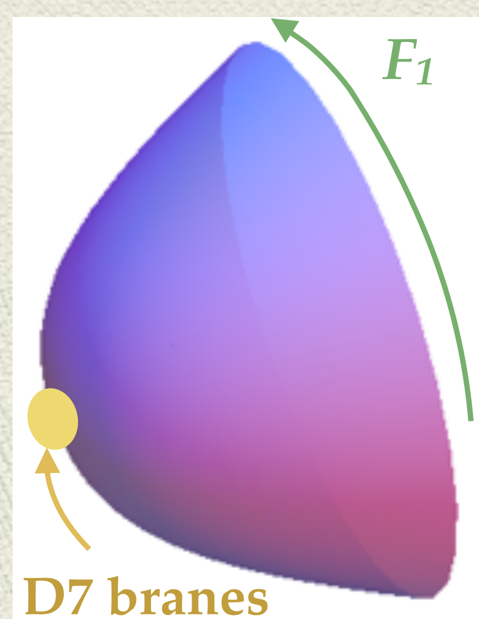
WZ part of action: $$\frac{1}{8\pi^2}\int_{M_4} \mathrm{Tr}(F \wedge F) C_0 = \frac{1}{8\pi^2}\int_{M_4} \omega_3(A) \wedge (k\, d\theta) = \frac{k}{4\pi}\int_{M_3} \omega_3(A)$$ **Chern-Simons**

=> With twisted fermion b.c., dual low-energy effective theory is YMCS, in the confined phase.

***k* units of C_0 flux on the circle**
⇨ *k* D7 branes must lie at tip

probe branes wrapping internal S^5

=> 3D surface in AdS soliton

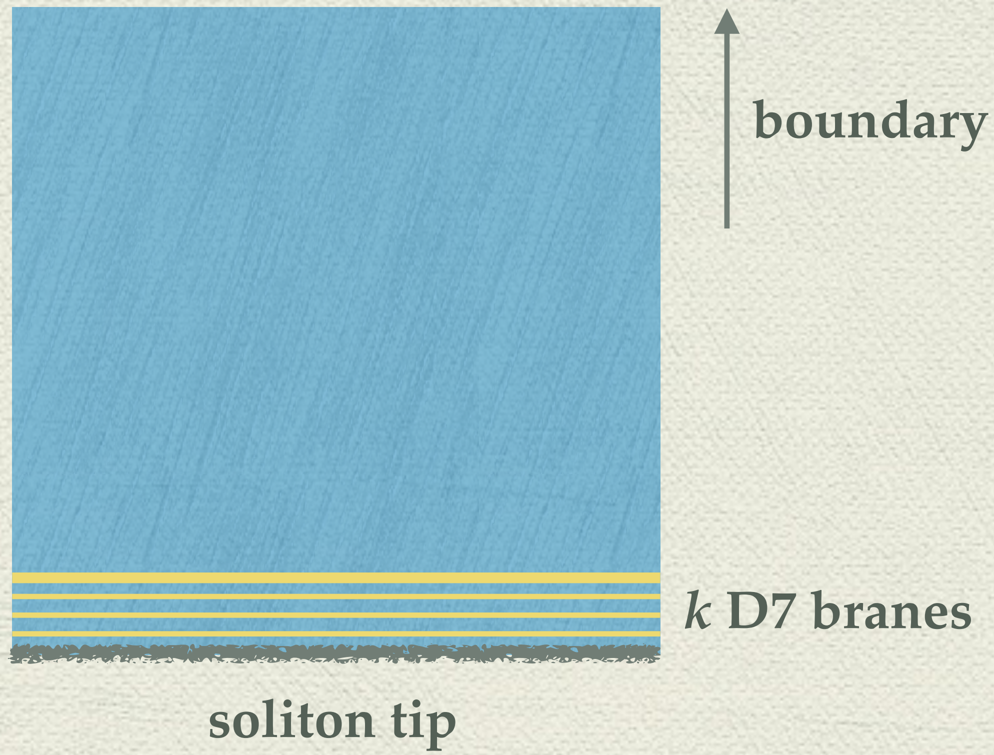


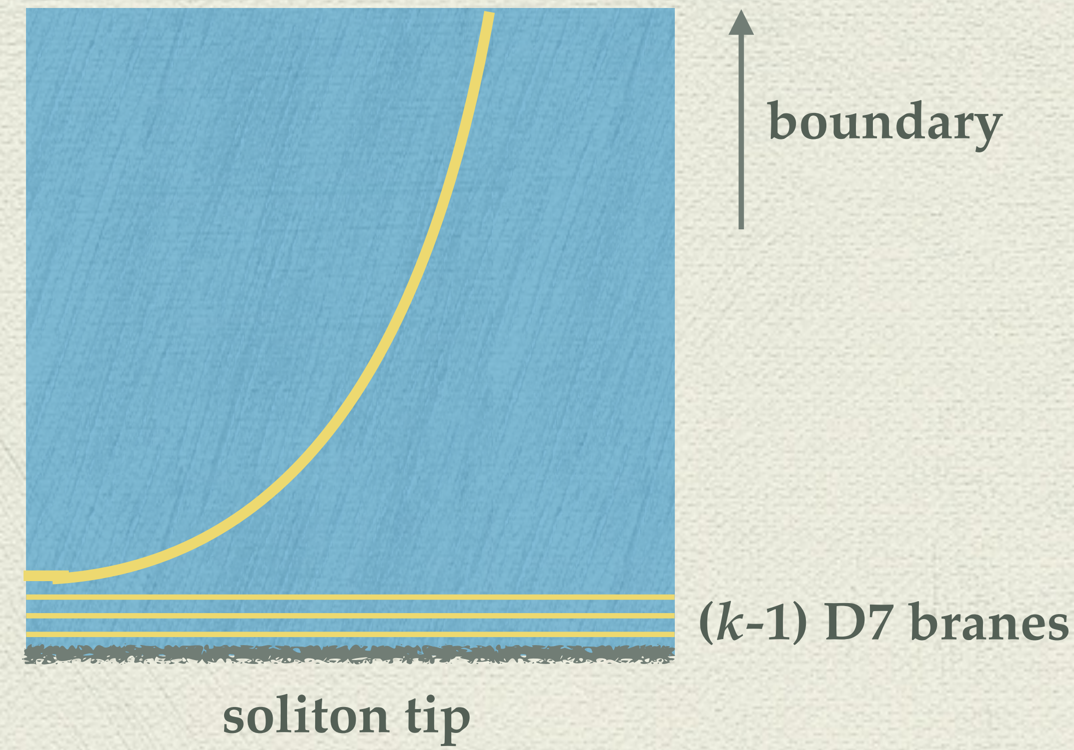
String worldsheet can attach to D7 probe branes:

Dominant contribution to on-shell action from Wilson loop on D7 brane => level-rank duality

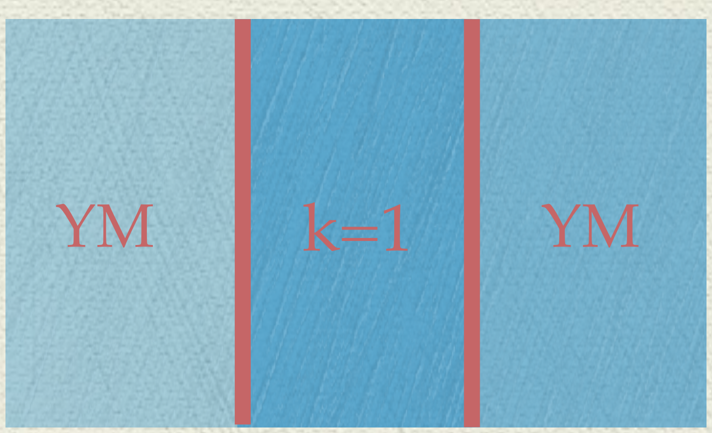
Holographic Embedding of Domain Walls

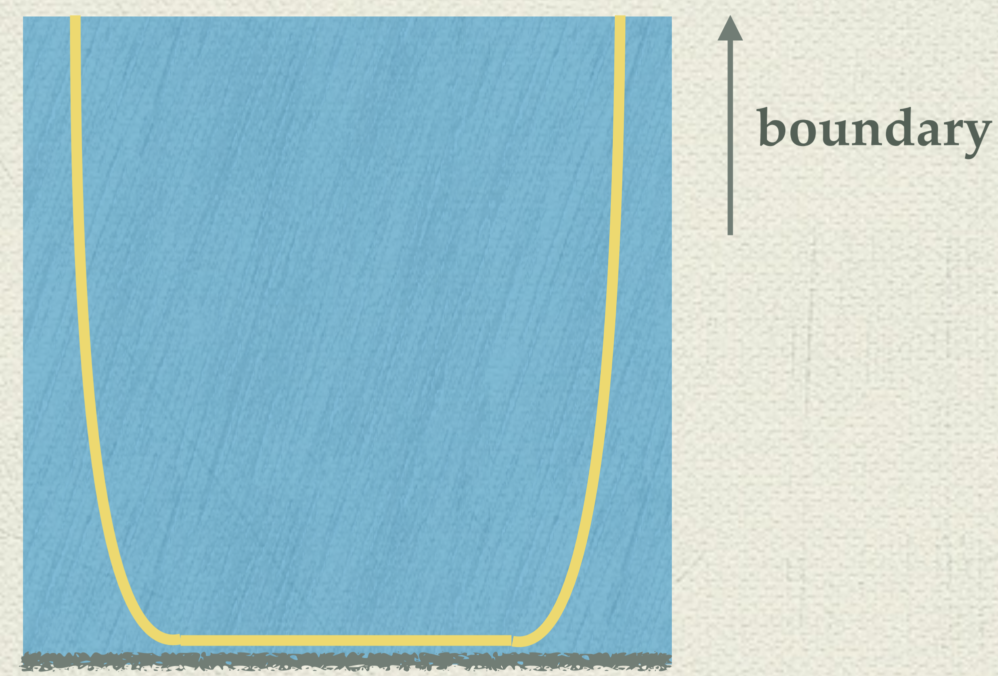
Interested in defects changing the level k ⇨ need to change the number of branes





Will mostly consider interval of finite length with level $k = 1$; $k = 0$ elsewhere.





Spectrum of operators

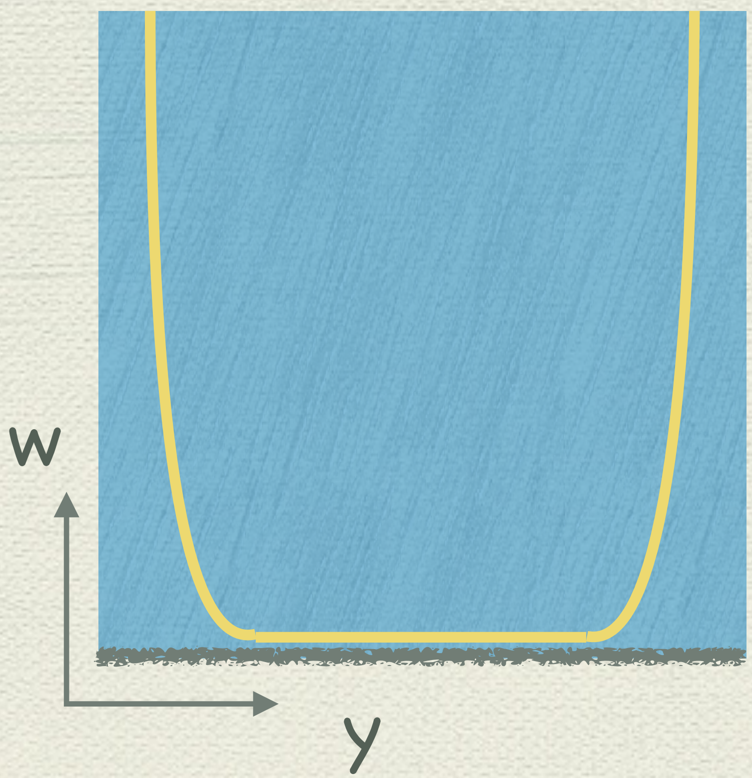
Consider only bosonic operators uncharged under SO(6) or SO(2) global symmetries.

The brane is extended infinitely in the t, x directions.

WZ term + F_5 flux ⇨ CS term on brane worldvolume

Level = N, number of D3 branes ⇨
Probe brane theory = DBI + CS at level N

Fields (near right-hand boundary; left-hand boundary fields have opposite chirality):



Fields	Asymptotic form
y	$y_0 - \frac{Rw_*^3}{4w^4} + \cdots$
A	$a_-^{(4)} w^4 + \cdots + (\text{flat})$
A	$(a_-^{(4)}\ \text{terms}) + (\text{flat}) + a_+^{(-4)} w^{-4} + \cdots$

Spectrum of operators

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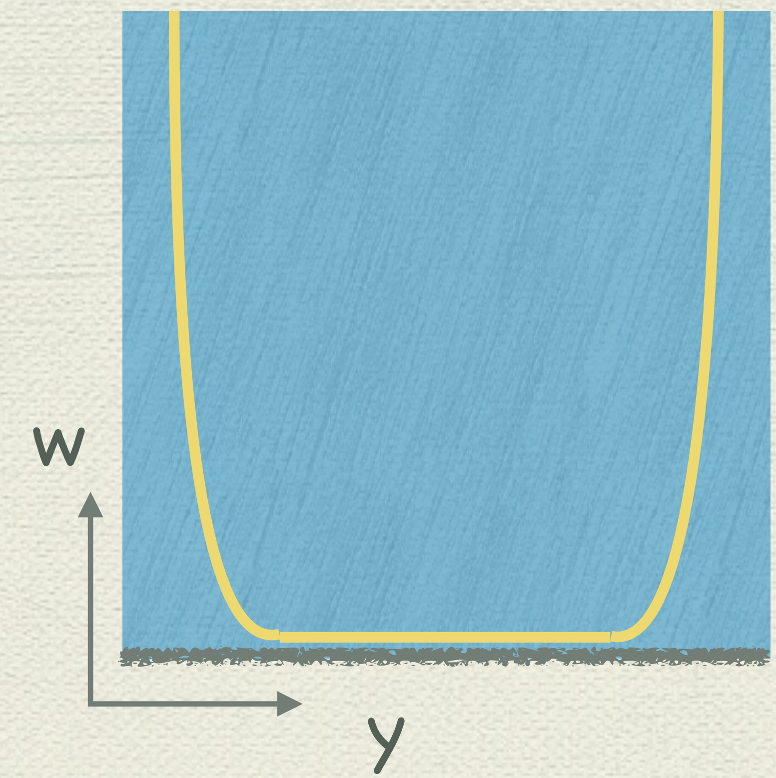
Source	Response	Dimension
y		3
a	a	5
a	a	1

← irrelevant chiral operator (dimension 5 row)

← chiral current => U(k) symmetry (dimension 1 row)



Spectrum of operators

Harvey, Roysten
0804.2854

Source	Response	Dimension
y		3
a	a	5
a	a	1

c.f.

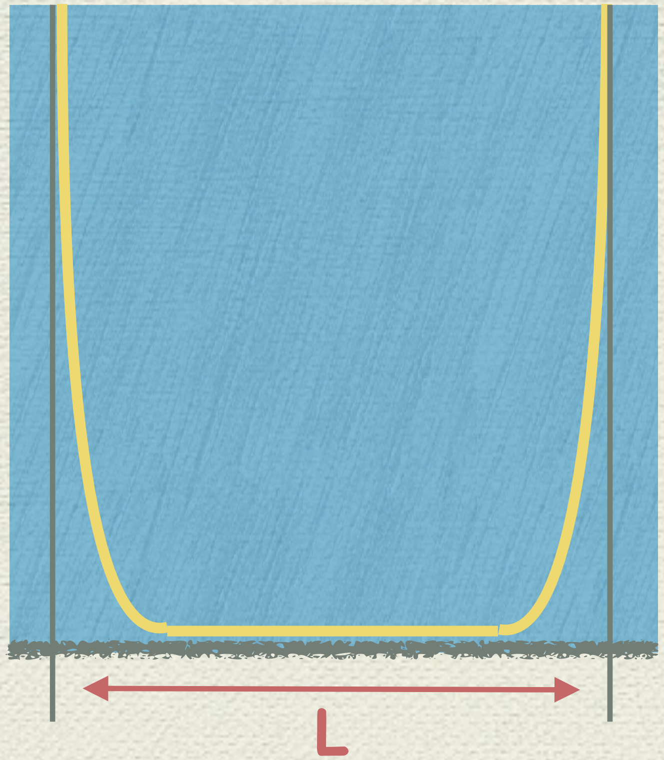
Operator
$\Phi^{(3)} \sim \bar{q}\gamma_+ F_{-y} q$
$V^{(5)} \sim \bar{q}\gamma_+ F_{-y} F_{-y} q$
$J_+ \sim \bar{q}\gamma_+ q$

D3-D7 brane intersection

System Properties

Metric: $ds^2 = w^2 dx_3^2 + f(w) w^2 d\theta^2 + \frac{R^2}{f(w)} \frac{dw^2}{w^2}$

Embedding: $y(w) = \int_{w_*}^{w} \frac{R\, w^4\, dw}{\sqrt{(w^6 - w_*^6)(w^4 - w_0^4)}}$



Free energy: $F = 2R\left(\int_{w_*}^{\Lambda} \frac{w^6\, dw}{\sqrt{(w^6 - w_*^6)(w^4 - w_0^4)}} - \frac{1}{2}\Lambda^2\right)$

For *L* large, *F* grows linearly with length.

Behavior due to the bulk geometry ending; if the effective background were AdS_4, there would be no *L* dependence.

Interpretation as order parameter for confinement?

System Properties

c.f.: Yee, Zahed 1103.6286

Edge mode [right-hand side chiralities]

Conserved current sector given by boundary values of flat connections: $A = d\varphi$

Source: $A_+^{(\text{flat})}$ = external gauge field $\mathcal{A}_+$ VeV: $A_-^{(\text{flat})}$ = current j_-

Flatness condition: $\partial_+ A_- = \partial_- A_+$

When source is turned off, flatness ⇨ $\partial_+ j_- = 0$ so current is chiral.

c.f.: Jensen 1012.4831
Yee, Zahed 1103.6286

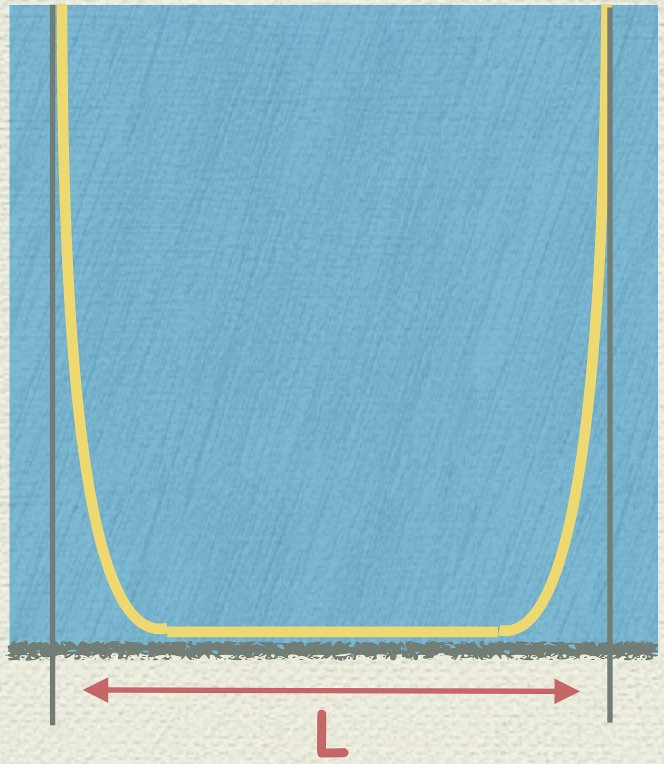
Anomaly [right-hand side chiralities]

Flatness condition: $\partial_+ A_- = \partial_- A_+$ ⇨ $\partial_\mu j^\mu = -\partial_+ j_- \propto \partial_- \mathcal{A}_+ = E_{\text{external}}$

anomaly in 2 dimensions

Consistent with expectation from brane construction.

System Properties



Irrelevant vector operator:

$A_-|_{\text{right}} \sim a_- w^4 + \cdots$ **at the right-hand boundary**

leads to $A_-|_{\text{left}} \sim c\, a_- w^{-4} + \cdots$ **at the left boundary,**

and the opposite for A_+

Thus, $V_+|_{\text{right}} \sim A_+|_{\text{left}}$ **and** $V_-|_{\text{left}} \sim A_-|_{\text{right}}$ **;**

the field varied to obtain the VeV does not determine the VeV.

$\langle V_-|_{\text{left}} \; V_+|_{\text{right}} \rangle \sim e^{-2\pi L/\ell}$ **with** l **the circle compactification length.**

Implies mediation by field with mass on the KK scale.

Meson spectrum, response to bulk fields, general correlation functions, multiples types of phase transitions, …

Surprisingly rich.

Fractional Quantum Hall Effect

Fujita, Li, Ryu, Takayanagi
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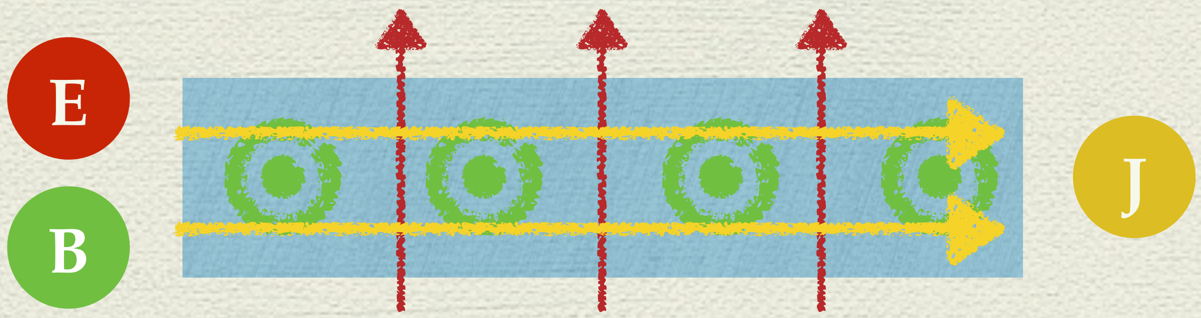
Quantum Hall Effect:



Apply a strong magnetic field normal to a conducting film

When an electric field is applied, a perpendicular current flows: E ⊥ J

For strong magnetic fields, this conductivity becomes quantized.

Effective field theory description by Chern-Simons action.

Gapped, exhibits edge currents, relies crucially on interactions (unlike IQHE)

Response to external EM field (encoded in *B* field) reproduces response in edge currents (probably)

Some peculiar subtleties in holographic renormalization…

Examples of Hall behavior in holographic systems may shed light on FQHE.

Summary

Holography for level-changing defects in YM-CS

Few degrees of freedom

Surprisingly rich structure:

anomalies, affine Lie symmetries, unusual correlation properties, relationship with confinement, (large *N*) symmetry breaking, phase transitions…

Relationship to FQH

終わり