M5-branes and Wilson Surfaces in AdS₇/CFT₆ Correspondence

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Introduction and Conclusion

What we did : tests of AdS_7/CFT_6

N M5-branes

Near-horizon limit → M-theory on $AdS_7 \times S^4$

Low energy limit → 6D (2,0) SCFT

M-theory on $AdS_7 \times S^4$ ⇔ ? ⇔ 6D (2,0) SCFT

We compare: M-brane ⇔ ? ⇔ Wilson surface

New evidence for AdS_7/CFT_6

M5-brane		Wilson surface
wrapping $AdS_3 \times S^3 \subset AdS_7$	⇔	Symmetric rep. $\langle W_{S_k} \rangle$
wrapping $AdS_3 \times \tilde{S}^3 \subset AdS_7 \times S^4$	⇔	Anti-symmetric rep. $\langle W_{A_k} \rangle$

CFT side : 6D (2,0) theory

6D (2,0) theory on $S^1 \times S^5$ ⇔ 5D MSYM on S^5

■ We use the conjecture of the equivalence between 6D (2,0) theory on $S^1 \times S^5$ and 5D maximal SYM (MSYM) on S^5 under [Douglas '10] [Lambert-Papageorgakis-SchmidtSommerfeld '10]

$$R_6 = \frac{g_{\rm YM}^2}{8\pi^2} \qquad (R_6 : \text{ radius of compactified } S^1, \quad g_{\rm YM} : \text{ 5D gauge coupling})$$

■ Partition function of 5D MSYM on S^5 calculated by localization is Chern-Simons matrix model. [Kim-Kim '12] [Källén-Minahan-Nedelin-Zabzine '13]

$$\mathcal{Z} \sim \int \prod_{i=1}^{N} d\nu_i \exp\left[-\frac{N^2}{\beta} \sum_{i=1}^{N} \nu_i^2 + \sum_{\substack{i,j \\ i \neq j}} \ln \left| 2\sinh \frac{N(\nu_i - \nu_j)}{2} \right| \right]$$

$$\left(\beta = \frac{g_{\rm YM}^2}{2\pi r}, \qquad r : \text{ radius of } S^5 \right)$$

⇒ Large N limit (fixed β)

$$\mathcal{Z} \sim \int \prod_{i=1}^{N} d\nu_i \exp\left[-\frac{N^2}{\beta} \sum_{i=1}^{N} \nu_i^2 + \frac{N}{2} \sum_{\substack{i,j \\ i \neq j}} |\nu_i - \nu_j| \right]$$

Since both terms in the exponent are $O(N^3)$, we can evaluate this by saddle point method.

➢ Saddle point equations

$$0 = -\frac{2N^2}{\beta}\nu_i + N \sum_{j, i\neq j} \text{sign}(\nu_i - \nu_j)$$

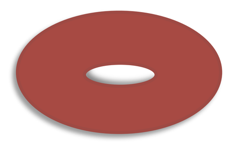
⇒

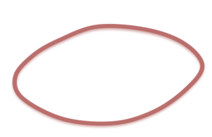
➢ Eigenvalue distribution

$$\nu_i = \frac{\beta}{2}\left(1 - \frac{2i}{N}\right) \qquad (\nu_1 > \nu_2 > \cdots > \nu_N)$$

Expectation values of Wilson surfaces

Wilson surfaces ($S^1 \times S^1$) ⇔ Wilson loops (S^1)



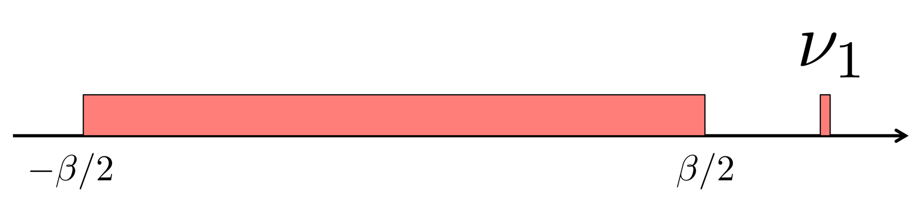


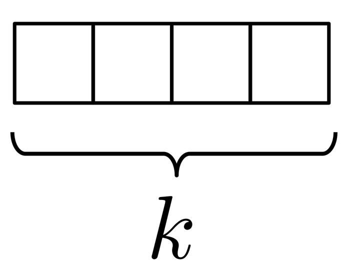
Wilson surfaces compactified on S^1 correspond to Wilson loops in 5D MSYM. Thus, We can compute the expectation values of Wilson surfaces in terms of Wilson loops.

$$\langle W_R \rangle = \langle {\rm Tr}_R e^{N\nu} \rangle$$

➢ Symmetric representation S_k (assume k is order of N)

$${\rm Tr}_{S_k} e^{N\nu} = \sum_{1 \le i_1 \le \cdots \le i_k \le N} \exp\left[N \sum_{l=1}^{k} \nu_{i_l} \right] \xrightarrow{\text{① extract the leading term}} \exp[Nk\nu_1] \xrightarrow[\nu_1 \text{ changes}]{\text{② consider if it changes the eigenvalue distribution}} \nu_1 = \frac{\beta}{2}\left(1 + \frac{k}{N}\right)$$



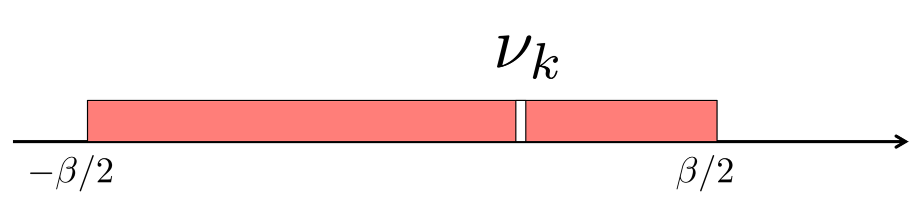


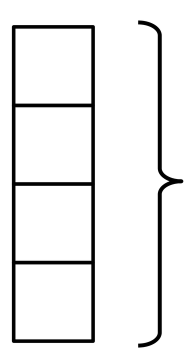
③ substitute eigenvalues into the matrix model →

$$\langle W_{S_k} \rangle \sim \exp\left[\frac{\beta N}{2} k \left(1 + \frac{k}{2N} \right) \right]$$

➢ Anti-symmetric representation A_k

$${\rm Tr}_{A_k} e^{N\nu} = \sum_{1 \le i_1 < \cdots < i_k \le N} \exp\left[N \sum_{l=1}^{k} \nu_{i_l} \right] \xrightarrow{\text{① extract the leading term}} \exp[N(\nu_1 + \cdots + \nu_k)] \xrightarrow{\text{② consider if it changes the eigenvalue distribution}} \nu_i \text{ unchanged}$$





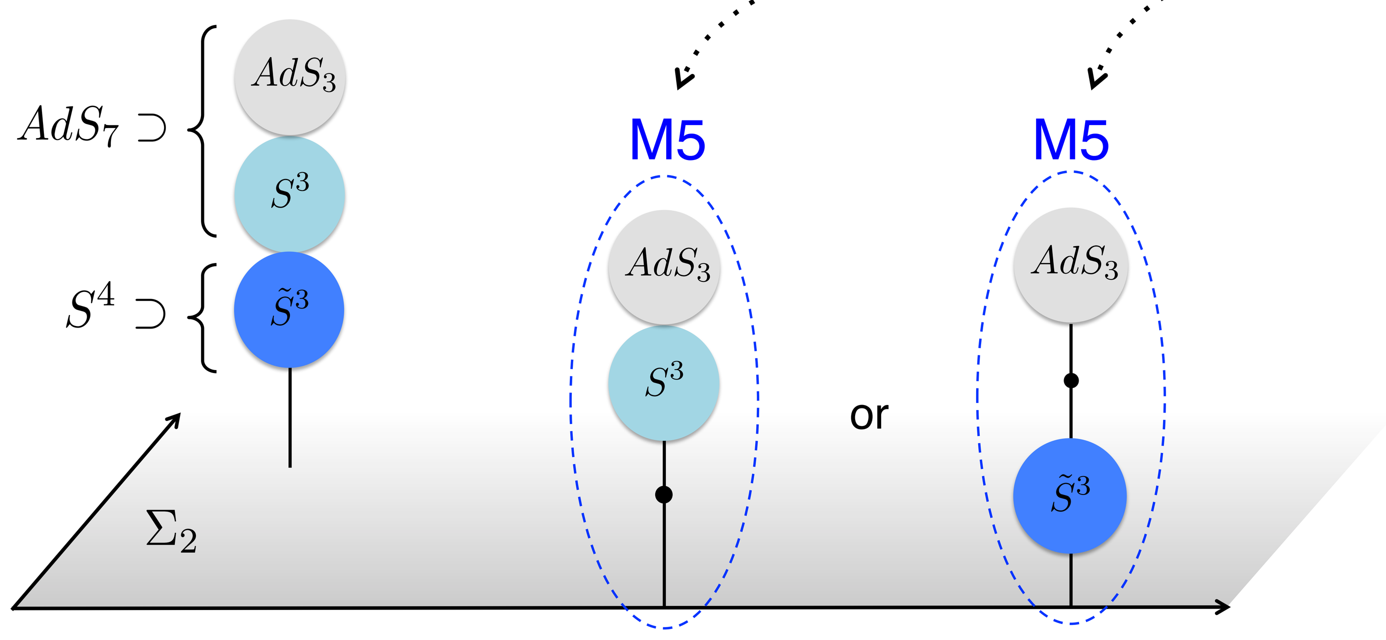
③ substitute eigenvalues into the matrix model →

$$\langle W_{A_k} \rangle \sim \exp\left[\frac{\beta N}{2} k \left(1 - \frac{k}{N} \right) \right]$$

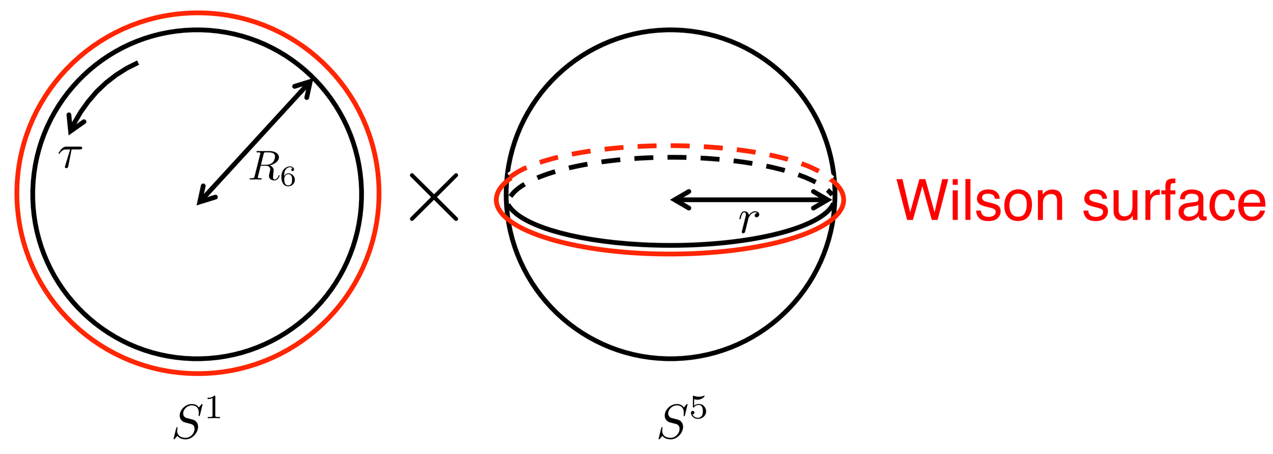
Gravity side : M-theory

Supergravity on $AdS_7 \times S^4$

■ We consider a probe M5-brane wrapping $AdS_3 \times S^3$ or $AdS_3 \times \tilde{S}^3$.



■ We take $S^1 \times S^5$ as the boundary of AdS_7 in global coordinates.

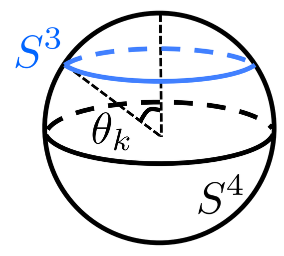


$$ds^2 = L^2\left(\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\Omega_5^2\right) + \frac{L^2}{4}\left(d\theta^2 + \sin^2\theta d\tilde{\Omega}_3^2\right)$$

Identification : $\tau \sim \tau + 2\pi \frac{R_6}{r}$

■ There exists the *flux quantization condition* associated with the coupling of a M2-brane to the M5-brane wrapping S^3. [Camino-Paredes-Ramallo '01]

(ex) Flux through S^3 in S^4



Flux

$$F_3 = \frac{k}{2N}L^3\omega_3 \quad \Rightarrow \quad \cos\theta_k = 1 - \frac{2k}{N}$$

$$\begin{cases} k : \text{integer} \\ \omega_3 : \text{volume form of unit } S^3 \end{cases}$$

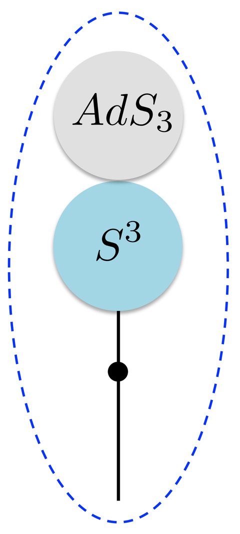
■ We use the so-called PST action including the Wess-Zumino term on a single M5-brane. [Pasti-Sorokin-Tonin '97]

$$S_{\mathrm{M5}} = T_5 \int d^6x\sqrt{-g}\left[\mathcal{L} + \frac{1}{4}\tilde{H}^{mn}H_{mn}\right] + T_5\int\left(C_6 - \frac{1}{2}C_3 \wedge H_3\right)$$

$$\left[\begin{array}{llll} \mathcal{L} = \sqrt{\det\left(\delta_m^{\ n} + i\tilde{H}_m^{\ n}\right)} & H_3 = F_3 - C_3 & g : \text{induced metric} & \\ & H_{mn} = H_{mnp}v^p & & T_5 = \frac{1}{(2\pi)^5\ell_p^6} \\ v_p = \frac{\partial_p a}{\sqrt{-g^{mn}\partial_m a\partial_n a}} & \tilde{H}^{mn} = (*_6H)^{mnp}v_p & a : \text{auxiliary field} & \end{array}\right]$$

Probe M5-branes

■ We must carefully determine the boundary term S_{bdy} to regularize S_{M5} .



Let's consider a M5-brane wrapping $AdS_3 \times S^3$ in Poincare coordinates: $ds^2 = \frac{L^2}{y^2}\left(dy^2 + dr_1^2 + r_1^2 d\phi^2 + dr_2^2 + r_2^2 d\Omega_3^2\right) + \frac{L^2}{4}d\Omega_4^2$

Ansatz : $r_2 = \kappa y$

Flux quantization : $\kappa = \sqrt{\frac{k}{2N}}$

$$\Rightarrow \quad S_{\mathrm{M5}} = \frac{k}{4N}\mathcal{K}z_0, \qquad \mathcal{K} = T_5 \times (\text{volume of the M5-brane on the boundary})$$

$(z_0 : \text{cutoff})$

We expect $S_{\mathrm{M5}} + S_{\mathrm{bdy}} = 0$ because the Wilson surface in Poincare coordinates preserves a half of supersymmetry.

$\longrightarrow$ $S_{\mathrm{bdy}} \propto$ (volume of the M5-brane on the boundary)

coordinate transformation

➢ M5-brane wrapping $AdS_3 \times S^3$ in global coordinates: $ds^2 = L^2\left[\cosh^2 u\left(\cosh^2 w d\tau^2 + dw^2 + \sinh^2 w d\phi^2\right) + du^2 + \sinh^2 u d\Omega_3^2\right] + \frac{L^2}{4}d\Omega_4^2$

Flux quantization : $\sinh u_k = \sqrt{\frac{k}{2N}}$

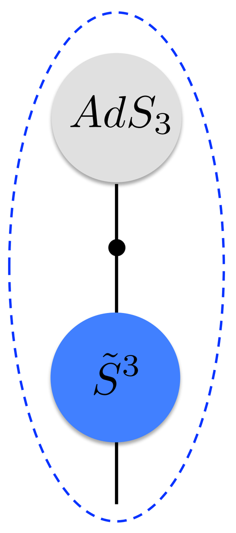
$$\Rightarrow \quad S_{\mathrm{M5}} = \frac{4\pi R_6}{r}Nk\left(1 + \frac{k}{2N}\right)\sinh^2 w_0 \qquad (w_0 : \text{cutoff})$$

$\Downarrow$ Adding the boundary term

$S_{\mathrm{bdy}} \propto \sinh w_0 \cosh w_0$

Agreement

$$\exp\left[-S_{\mathrm{M5}}\right] = \exp\left[\frac{\beta N}{2}k\left(1 + \frac{k}{2N}\right)\right]$$



➢ M5-brane wrapping $AdS_3 \times \tilde{S}^3$ in global coordinates: $ds^2 = L^2\left(\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\Omega_5^2\right) + \frac{L^2}{4}\left(d\theta^2 + \sin^2\theta d\tilde{\Omega}_3^2\right)$

Flux quantization : $\cos\theta_k = 1 - \frac{2k}{N}$

$$\Rightarrow \quad S_{\mathrm{M5}} = \frac{4\pi R_6}{r}Nk\left(1 - \frac{k}{N}\right)\sinh^2\rho_0 \qquad (\rho_0 : \text{cutoff})$$

$\Downarrow$ Adding the boundary term

$S_{\mathrm{bdy}} \propto \sinh\rho_0 \cosh\rho_0$

Agreement

$$\exp\left[-S_{\mathrm{M5}}\right] = \exp\left[\frac{\beta N}{2}k\left(1 - \frac{k}{N}\right)\right]$$