Five dimensional O(N)-symmetric CFTs and conformal bootstrap

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Motivation

Is non-renormalizable theory

- Renormalizable?
- Sensible?
- Predictive?

e.g. Einstein gravity in d=4, N=8 SUGRA in d=4, or maximally supersymmetric Yang-Mills in d=5

(c.f. when I was a student ~~long~~ sometime ago there was a ~~legendary~~ popular(?) thread "renormalization of non-renormaliable field theories" in 2ch)

Asymptotic safety?

Suppose your (non-renormalizable) theory has a (non-trivial) UV fixed point, then such a theory may be

- sensible
- predictive
- can appear in nature
- may replace string theory

But in reality, it is hard to find an example starting from non-renormalizable Lagrangian.

- If any, unitarity? Stability? Uniqueness? Question remains…

Example O(N) model in $d = 4 - \epsilon$

- Consider O(N) vector model in $d = 4 - \epsilon$, ϵ will be eventually negative

$$S = \int d^d x \partial^\mu \phi^i \partial_\mu \phi^i + \lambda (\phi^i \phi^i)^2$$

- 1-loop beta function

$$\beta_\lambda = -\epsilon\lambda + (N+8)\frac{\lambda^2}{8\pi^2} + \mathcal{O}(\lambda^3)$$

- (Conformal) fixed point

$$\lambda^* = \frac{\epsilon}{(N+8)8\pi^2}$$

- Seems to exist for both positive/negative ϵ
- For ϵ → 1, it should describe O(N) symmetric critical phenomena in d=3 (and agrees with experiment after careful resummation)

Example O(N) model in $d = 4 - \epsilon$

- In d = 5, it is a little bit suspicious
- Sign of coupling constant. Unstable?

$$\lambda^* = \frac{\epsilon}{(N+8)8\pi^2}$$

- In Wilsonian picture, we have to tune infinitely many UV parameters (non-renormalizablity)

$$\phi^6 \qquad (\partial_\mu \phi \partial^\mu \phi)^2$$

- Maybe can these terms stabilize the potential? Who knows?
- For larger (negative) ϵ, the unitary bound can be violated for small N

Conjecture by Fei, Giombi, Klebanov

- Despite these subtleties, Fei et al (1404.1094) conjectured that O(N) vector models in d=5 should have sensible unitary UV fixed points
- Dual to large N higher spin AdS6 theory
- Using large N method

$$\frac{C_T^{d=5}}{C_T^{\text{free},d=5}} = 1 - \frac{0.0905669}{N} + \cdots$$

$$\frac{C_J^{d=5}}{C_J^{\text{free},d=5}} = 1 - \frac{0.461124}{N} + \cdots$$

- Using $d = 6 - \epsilon$ expansion, they claim it may have an alternative description (as IR fixed point with the same universality)

$$\int d^d x \partial_\mu \phi^i \partial^\mu \phi^i + \partial_\mu \sigma \partial^\mu \sigma + g_1 \sigma^3 + g_2 \sigma (\phi_i \phi_i)$$

- Conjecture for the conformal window

$$N \geq 35 \quad (d = 5) \qquad N \geq 1039 \quad (d = 6 - \epsilon)$$

Conformal Bootstrap approach

Success of conformal bootstrap

- Idea of conformal bootstrap is revised in higher dimensional (d>2) CFTs with tremendous success

- Solved d=3 Ising model (c.f. El-Showk et al, 1203.6064 1403.4545 1406.4858)

- Solved QCD chiral phase transitions and frustrated magnets (c.f. Nakayama-Ohtsuki **arXiv:1407.6195**)

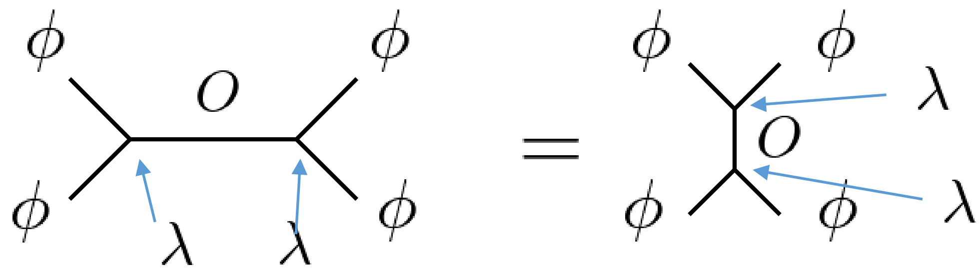
- Solve asymptotic safety ← Here!

Schematic conformal bootstrap equations

- Consider 4pt functions $\langle \phi^{i_1}(x_1)\phi^{i_2}(x_2)\phi^{i_3}(x_3)\phi^{i_4}(x_4)\rangle$
- OPE expansions

$$\phi^i \times \phi^i = \sum_{I \in \mathbf{R}\otimes\mathbf{R}, l:\mathrm{spin}} \lambda_{\phi\phi O} O^{I,l}$$

- I: S,T and A (S: Singlet, T: Traceless symmetric, A: Anti-symmetric)
- Crossing relations



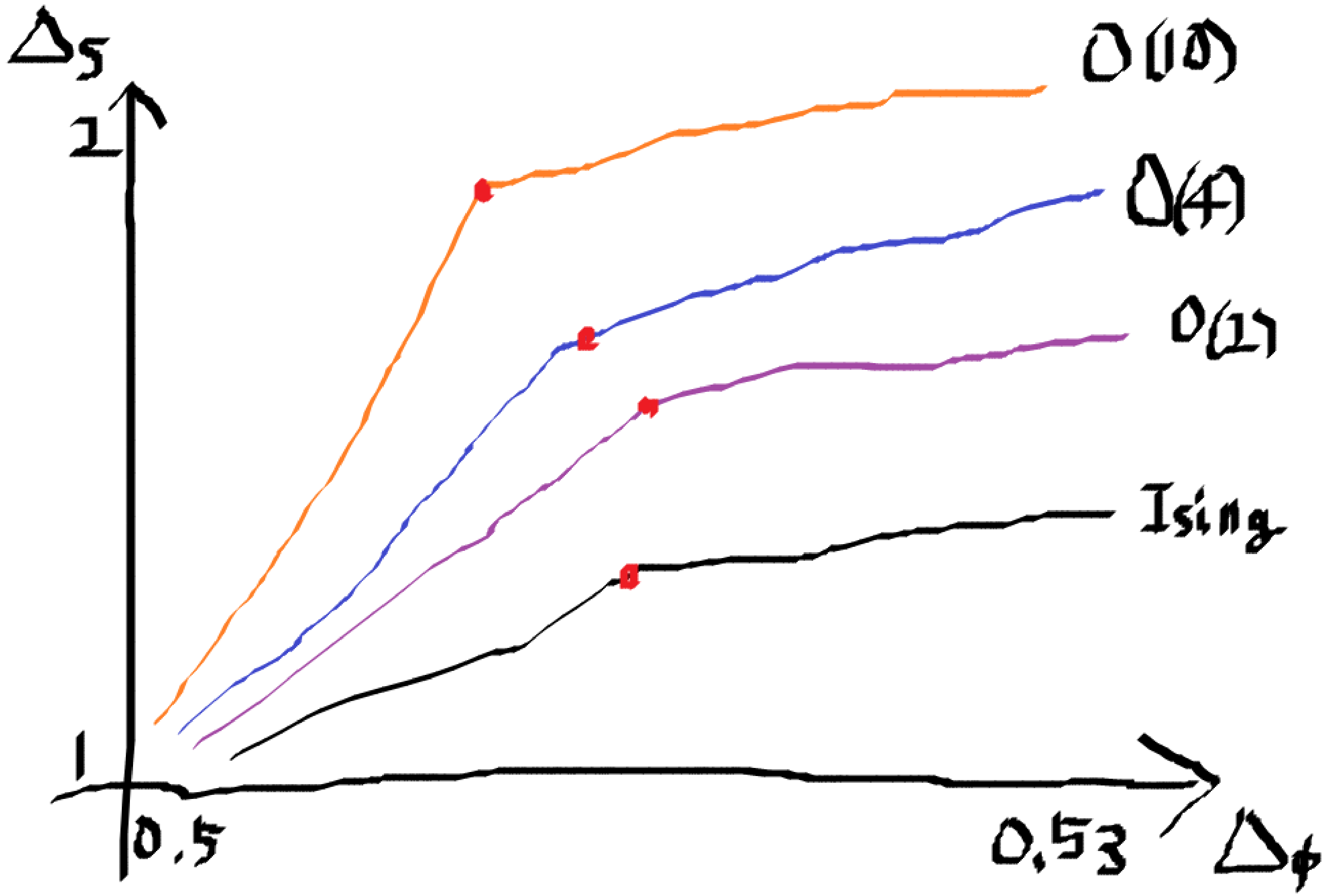
- Assume spectra (e.g. $\Delta_\phi = \delta$, $\Delta^{I,l} = \Delta_c^{I,l}(\delta)$)

to see if you can solve the crossing relations (non-trivial due to unitarity $\lambda^2_{\phi\phi O} > 0$)

$$1 = \sum_O \lambda_O^2 (f_O(z) - f_O(1-z))$$

→ convex optimization problem

Results in d=3 (Kos et al 1307.6856)



First Results in d=5

- Bootstrapping O(N) models in S sector (or T sector) as in d=3

- No interesting behavior at all…
- No kink

- Expected because large N formula tells that they are below the generalized free curve

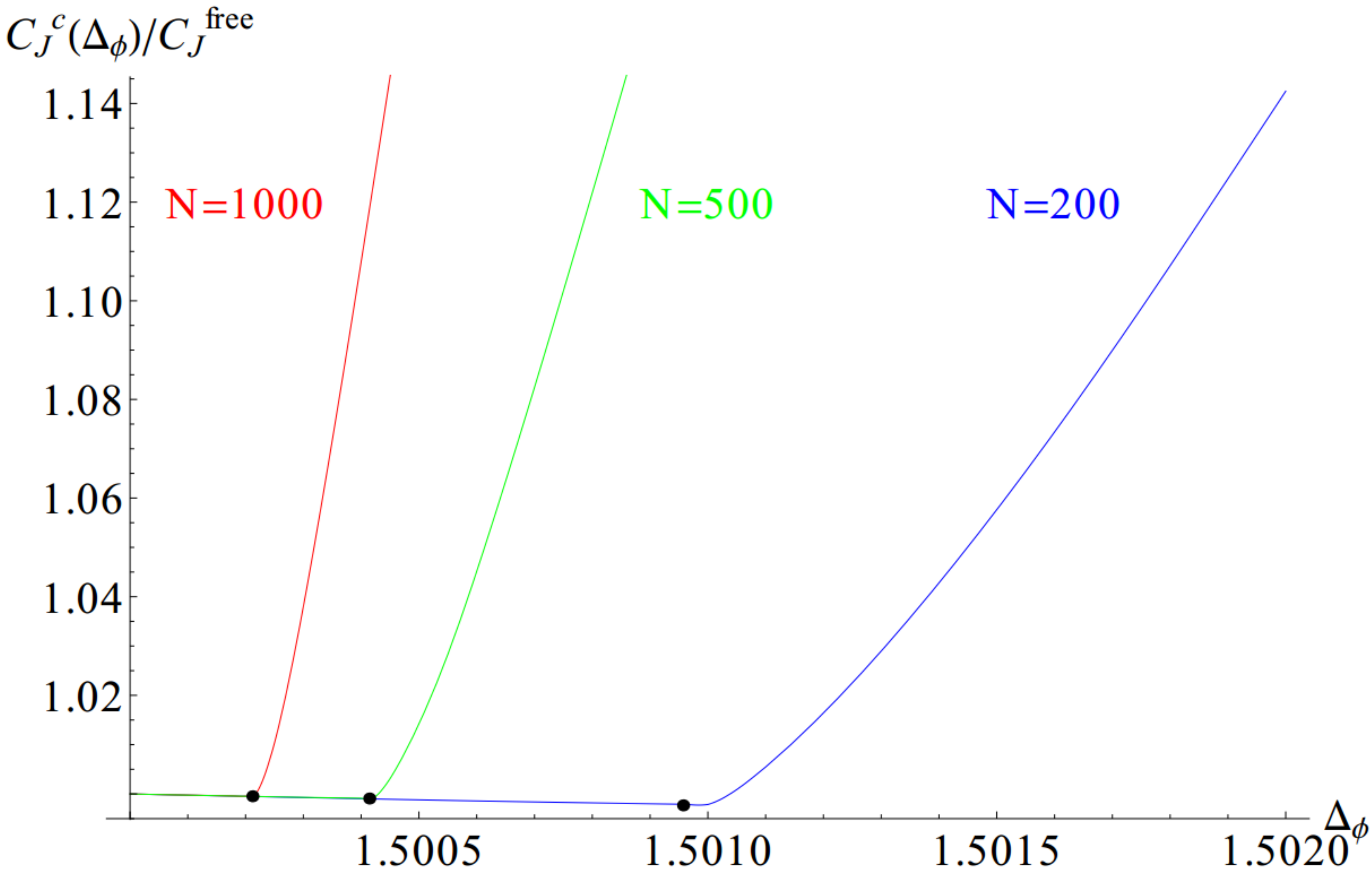
$$\Delta_\phi = \frac{3}{2} + O(1/N) \qquad \Delta_S = 2 + O(1/N)$$

- Generalized free theory (fake CFT)

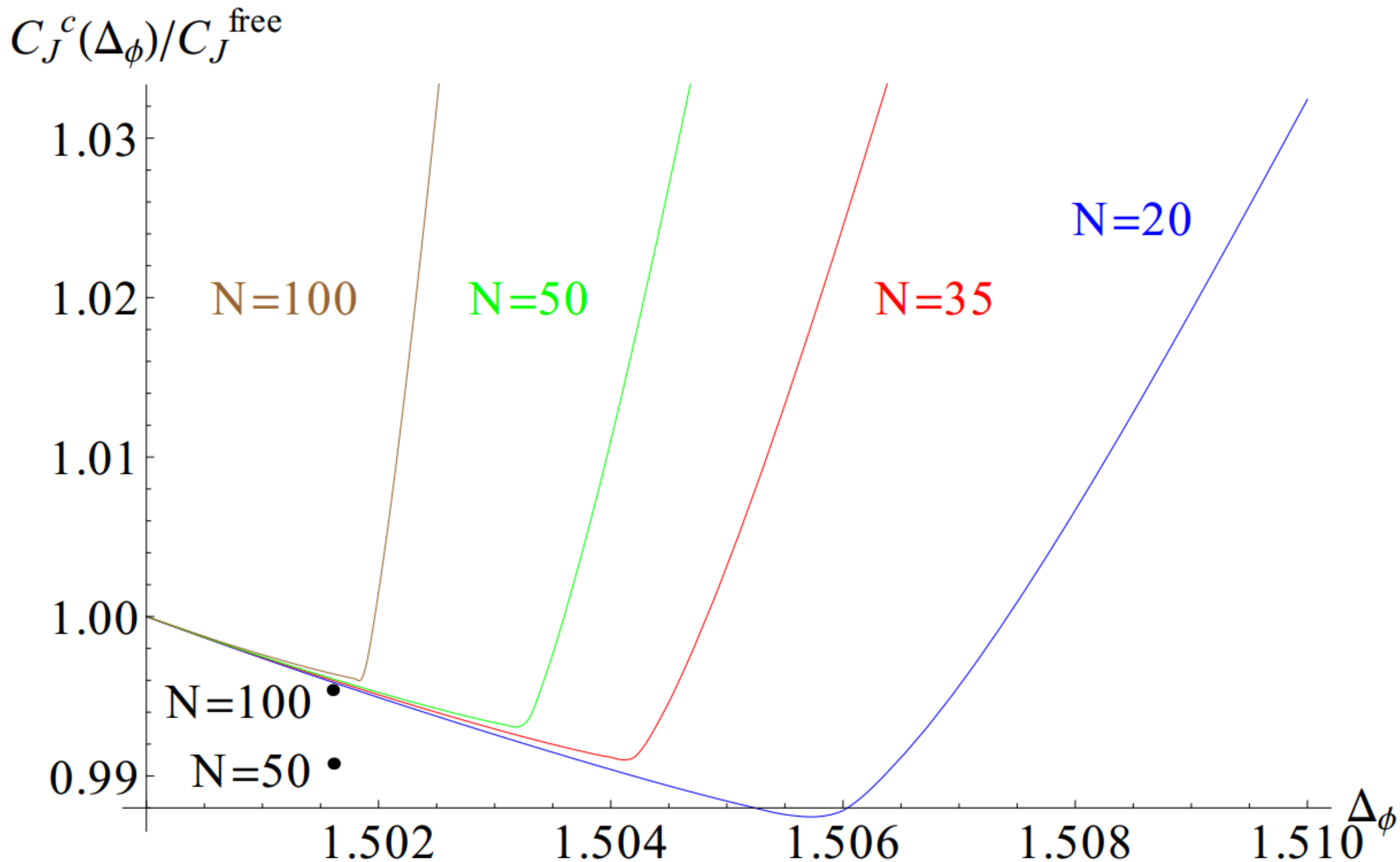
$$\Delta_S = 2\Delta_\phi \qquad \langle\phi\phi\phi\phi\rangle = \langle\phi\phi\rangle\langle\phi\phi\rangle + \mathrm{perm}$$

- Since they are always consistent, the non-trivial CFT below this curve would not show up
- Study central charges instead!

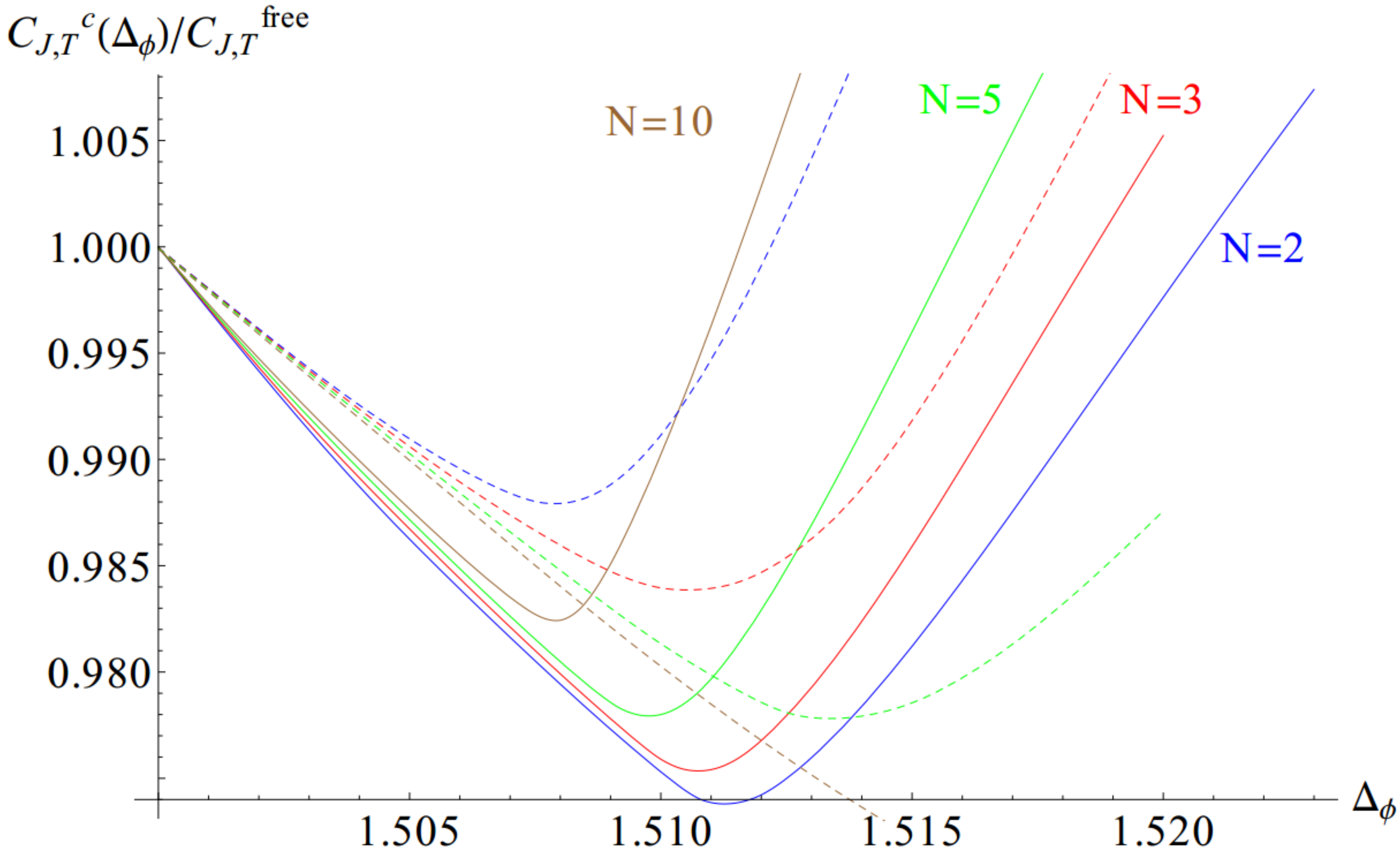
Results in d=5 (current central charges)



More results in d=5



More results in d=5 (current and EM tensor central charges)



Summaries in d=5

- Bootstrapping O(N) models in current/EM tensor central charges work
- We do see kinks/minima
- For large N, minima of current central charges agree with 1/N expansions (confirmation of Fei et al?)

- For smaller N, they deviate (1/N expansion is bad, however)
- Moreover the minima of EM central charge appear but the locations are different

- No (other) indications of conformal window?

Discussions

- O(N) symmetric unitary CFTs seem to exist in d=5
- Would be examples of asymptotic safety

- Really stable?
- Interpretations of different minima between current central charges and EM tensor central charges?
  - Proposed other fixed points with $1/\sqrt{N}$ expansion
- Mixed bootstrap to pin-point the fixed point

Legend of bootstrap

- Baron Munchhausen (famous for tall tales, ほら吹き男爵) told us he escaped from the swampland by pulling him up by his bootstrap (which means no string is needed to avoid swampland)
- Asymptotic safety is not a tall tale any longer
- How about gravity?