3-pt. functions in AdS_5/CFT_4 @ weak coupling from integrability

Takuya Nishimura

The University of Tokyo, Komaba

Based on arXiv:1304.5011 and work in progress with Y. Kazama, S. Komatsu

Introduction

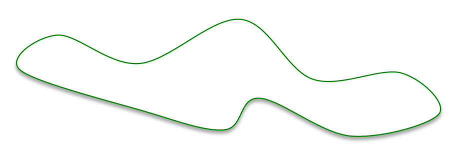
Correlation functions are fundamental observables in AdS/CFT.

$$\langle \mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)\rangle = \int \langle \mathcal{V}_1(z_1;x_1)\cdots\mathcal{V}_n(z_n;x_n)\rangle_{\text{WS}}$$

In particular,

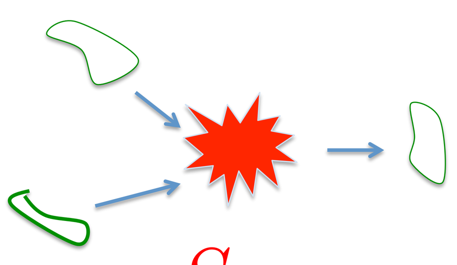
$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\rangle$ ⟷ Spectrum

$\Delta, J, S, \cdots$ ⟷ $E, J, S, \cdots$



$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle$ ⟷ Interaction

C_{ijk} ⟷ S_{ijk}



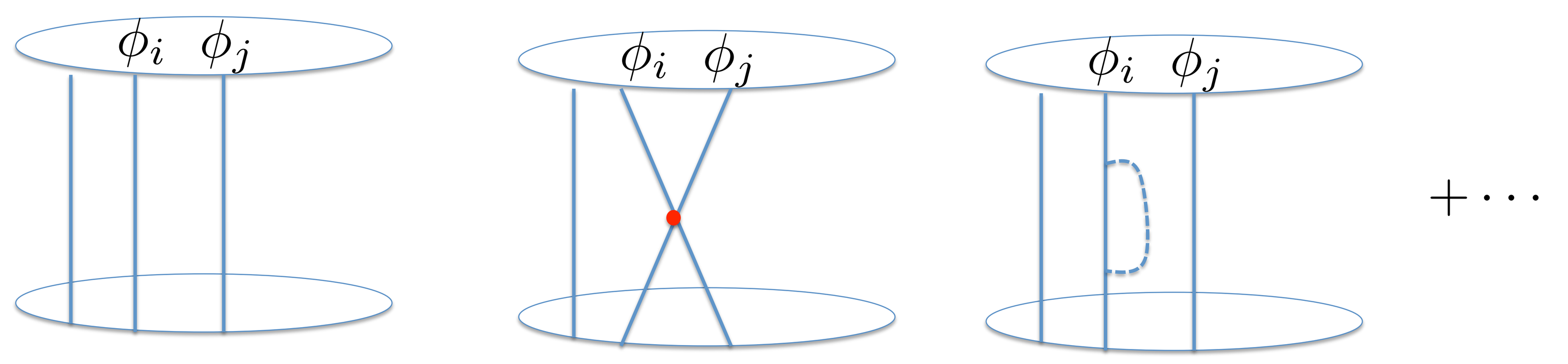
It would be quite important to study these fundamental observables in detail to reveal the underlying mechanism of AdS/CFT!!

The power of integrability

2pt. Function

The spectrum Δ can be obtained by diagonalizing the operator mixing, which turned out to be equivalent to diagonalize the integrable spin chain Hamiltonian.

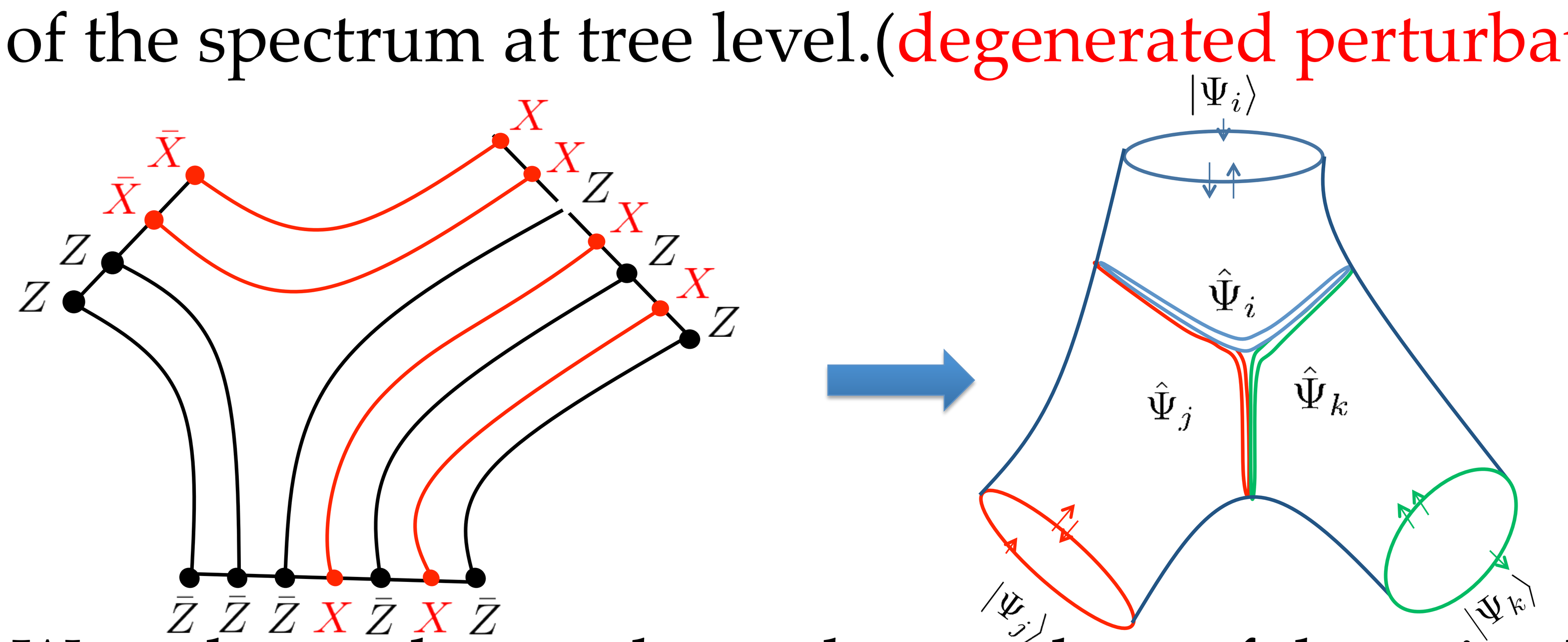
['02 Minahan, Zarembo], ['03 Beisert], ['03 Beisert, Kristjansen, Staudacher]



This can be solved by Bethe ansatz. And continued to S-matrix and ABA, finite size correction via TBA, Y-system and T-system, Quantum spectral curve… See details for ['10 Beisert et al]

3pt. Function

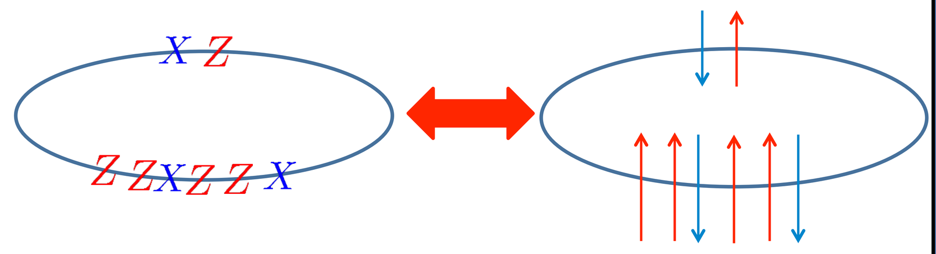
At tree level, 3pt. Functions can be calculated by summing up all possible planar Wick contractions. However, even at tree level, we need to prepare 1-loop eigenstates of the dilatation due to the huge degeneracy of the spectrum at tree level.(degenerated perturbation)



We only need to evaluate the overlaps of the spin chain wave functions! ['04 Okuyama, Tsen] ['04 Roiban, Volvich] ['05 Alday, et al] ['10 Escobedo, Gromov, Sever, Vieira] ['11 Foda]

Algebraic Bethe ansatz

The dilatation op. for the SU(2) sector is mapped to the XXX spin chain Hamiltonian.



$$Z=\phi^1+i\phi^2,\ X=\phi^3+i\phi^4 \quad H_{XXX}\propto\sum_n (S_n^xS_{n+1}^x+S_n^yS_{n+1}^y+S_n^zS_{n+1}^z)$$

The Bethe states are generated by the action of magnon creation operator.

$$|\boldsymbol{u}\rangle := B(u_1)\cdots B(u_M)|\uparrow^L\rangle \qquad u\text{: spectral parameter}$$

$$\Omega(u)=L_1(u)\cdots L_L(u)=\begin{pmatrix} A(u) & B(u)\\ C(u) & D(u)\end{pmatrix} \quad L_n(u)=\begin{pmatrix} u+iS_n^z & iS_n^-\\ iS_n^+ & u-iS_n^z\end{pmatrix}$$

The Bethe states are eigenstates of the Hamiltonian if and only if their rapidities satisfy the BA eq.

$$\left(\frac{u_k+i/2}{u_k-i/2}\right)^L=\prod_{l\neq k}\frac{u_k-u_l+i}{u_k-u_l-i} \quad \Longleftrightarrow \quad \text{On-shell or periodicity condition}$$

New analytic expression

The scalar products of our interest are given by

$$\langle\boldsymbol{u}|\boldsymbol{v}\rangle=\langle\uparrow^L|\prod_{i=1}^M C(u_i)\prod_{j=1}^M B(u_j)|\uparrow^L\rangle \quad \boldsymbol{u}\text{ or }\boldsymbol{v}\text{ are on-shell}$$

It is known that there exist M×M, L×L, 2M×2M determinant expressions. ['89 Slavnov], ['11 Foda, Wheler], ['12 Kostov, Matsuo]

We derive the following new expression. ['13 Kazama, Komatsu, T.N.]

$$\prod_{n=1}^{L-1}\oint_{\mathcal{C}_{\text{all}}}\frac{dx_n}{2\pi i}\underbrace{\prod_{k<l}(x_k-x_l)(e^{2\pi x_k}-e^{2\pi x_l})}_{\text{measure}}\prod_{m=1}^{L-1}\frac{Q_{\boldsymbol{u}}(x_m)Q_{\boldsymbol{v}}(x_m)e^{2\pi x_m}}{\underbrace{(x_m^2+1/4)^L}_{\text{spectrum}}}$$

Wave function

$$Q_{\boldsymbol{u}}(u)=\prod_{i=1}^M(u-u_i)$$

Q-function: The wave function of the spin chain in separation of variable (SoV) basis

Basic idea of derivation

1. Rewrite $\langle\boldsymbol{u}|\boldsymbol{v}\rangle\propto\langle\downarrow^L|(S^-)^{L-2M}\prod_{i=1}^M B(u_i)B(v_i)|\uparrow^L\rangle$ ['12 Kostov, Matsuo]
2. Diagonalize the magnon creation operator $B(u)$. ←Analogue of the coherent state!

Notice: We need to introduce twisting: $\Omega(u)\to\tilde{\Omega}(u)=K\Omega(u)$ $K=\begin{pmatrix}1 & \epsilon\\ -\epsilon & 1\end{pmatrix}$

3. Find the spectrum of $\hat{x}_k$, $\tilde{B}(u)=\prod_k(u-\hat{x}_k)$, and construct the completeness relation $\mathbf{1}=\sum_{\text{spec}}\mu(x)|\boldsymbol{x}\rangle\langle\boldsymbol{x}|$
4. Insert $\mathbf{1}=\sum_{\text{spec}}\mu(x)|\boldsymbol{x}\rangle\langle|\boldsymbol{x}|$ into the scalar product.

Summary and prospects

- The integral expression is quite simple and in particular, the wave functions are completely factorized.
- Semi-classical limit from this expression. ←Each variable is expected to correspond to the ``mode'' of string. ['06 Dorey, Vicedo]
  Angle variable representation seems to be more suitable.
  [Kazama, Konatsu, T.N. in progress]
- Semi-classical 3pt. Functions from Landau-Lifshitz model and monodromy condition: $\Omega_{cl}^1(u)\Omega_{cl}^2(u)\Omega_{cl}^3(u)=\mathbf{1}_{2\times2}$
  [Kazama, Konatsu, T.N. in progress]
- What is the quantum analogue of this condition?
  $(\hat{\Omega}^1\hat{\Omega}^2\hat{\Omega}^3-\hat{\mathbf{1}})C^{123}=0$ (Schwinger-Dyson eq.?)