WORLD-VOLUME EFFECTIVE ACTIONS OF EXOTIC FIVE-BRANES

arXiv:1404.5442 [hep-th] to appear in JHEP

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22 July 2014@YITP

INTRODUCTION

Exotic branes?

M-theory compactified on T^8

U-duality $E_{8(8)}(\mathbb{Z})$ multiplet (particles in 3d)

[Elitzur-Giveon-Kutasov-Rabinovici (1997),
Bau-O'Loughlin (1997), Obers-Pioline (1998)]

Particles multiplet in 3d

→ **"ordinary branes" wrapped on cycles**

- **waves**
- **F-strings**
- **D-branes**
- **NS5-branes**
- **KK-monopoles**

Other states : **Exotic branes (Q-branes) wrapped on cycles**

Exotic branes are found in a duality chain

Example

NS5-brane

↓ T-dual (transverse direction) R_1

KK-monopole

↓ T-dual (non Taub-NUT direction) R_2

5^2_2-brane ← an exotic brane

5 2 quadratic dependence on radii

2 string coupling

world-volume space dim

$$T \sim g_s^{-2}(R_1 R_2)^2$$

Exotic brane as 1/2 BPS state

5^2_2-brane is a 1/2 BPS solution in SUGRA
[de Boer-Shigemori (2010)]

$$ds^2_{5^2_2} = dx^2_{012345} + H(d\rho^2 + \rho^2(d\theta)^2) + \frac{H}{K}((dx^8)^2 + (dx^9)^2),$$

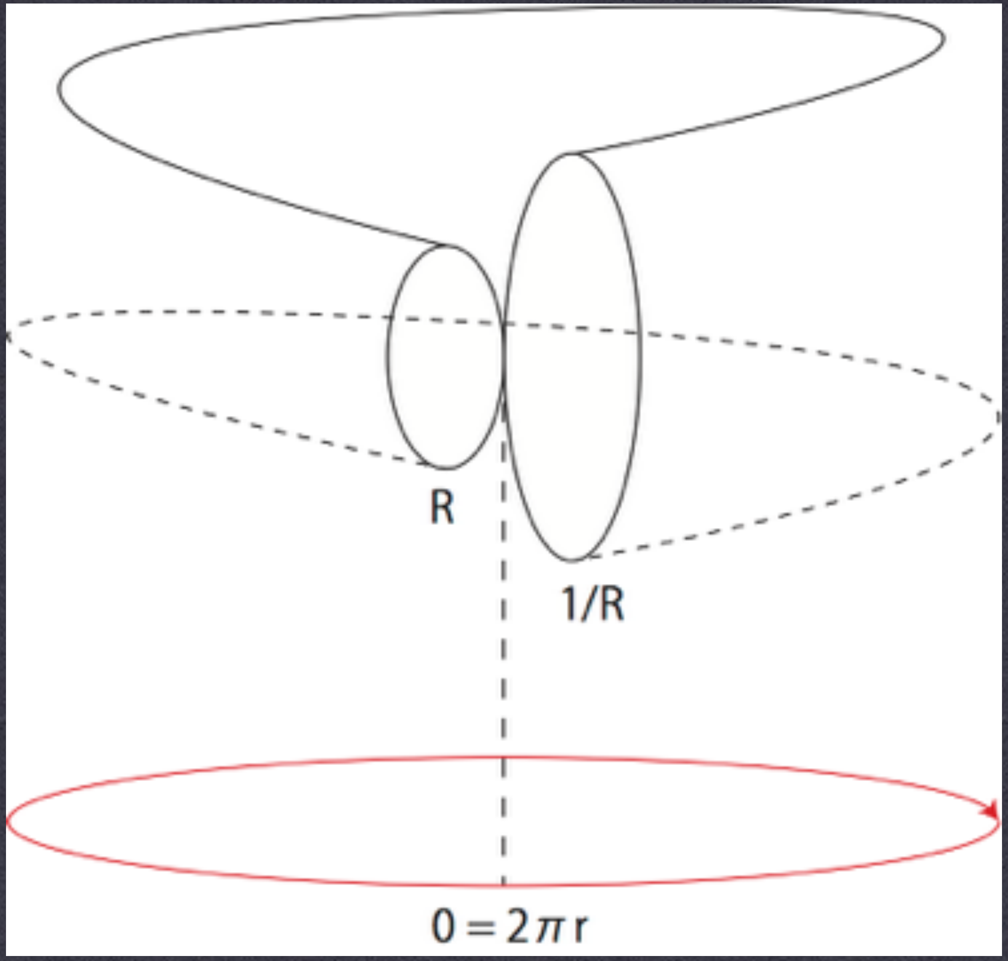
$$e^{2\phi} = \frac{H}{K}, \quad B = -\frac{\sigma\theta}{K} dx^8 \wedge dx^9,$$

$$H = h_0 + \sigma \log\frac{\mu}{\rho}, \quad K = H^2 + (\sigma\theta)^2, \quad \sigma = \frac{R_8 R_9}{2\pi\alpha'}$$

(ρ, θ) : x^6-x^7plane

$$\begin{cases} \theta = 0 \;\; : \;\; g_{88} = g_{99} = H^{-1}, \\ \theta = 2\pi \;\; : \;\; g_{88} = g_{99} = \frac{H}{H^2 + (2\pi\sigma)^2} \end{cases}$$

Monodromy $\neq$ diffeo or gauge transformations
(multi-valued function)



non-geometric

Monodromy $= SO(2,2)$ T-duality

T-fold (U-fold)

Nature of exotic branes?

- **Supergravity viewpoint**

  [Lozano Tellechea-Ortin (2000), Hull (2004),
  de Boer-Shigemori (2010,2012), de Boer-Mayerson-Shigemori (2014), and more..]

- **World-sheet viewpoint**

  [Kikuchi-Okada-Sakatani (2012)]

  [Kimura-S.S, NPB876(2013)876, JHEP08(2013)126, JHEP03(2014)128]

  [cf. today's talk by Kimura and Poster by Yata (Friday)]

- **Double field theory viewpoint**

  [Andriot-Hohm-Larfors-Lust-Patalong (2012), Andriot-Betz (2013),
  Blumenhagen-Desr-Plauschinn-Rennecke-Schmid (2012,2013),
  Geissbuhler-Marques-Nunez-Penas (2013), and more]

- **World-volume viewpoint (This talk)**

  [Chatzistavrakidis-Gautason-Moutsopoulos-Zagermann (2014)]

  [Kimura-Yata-S.S, arXiv:1404.5442]

WORLD-VOLUME THEORIES

World-volume effective theory

We focus on 1/2 BPS exotic five-branes in type IIB string theory

- Brane fluctuation modes
- World-volume gauge fields

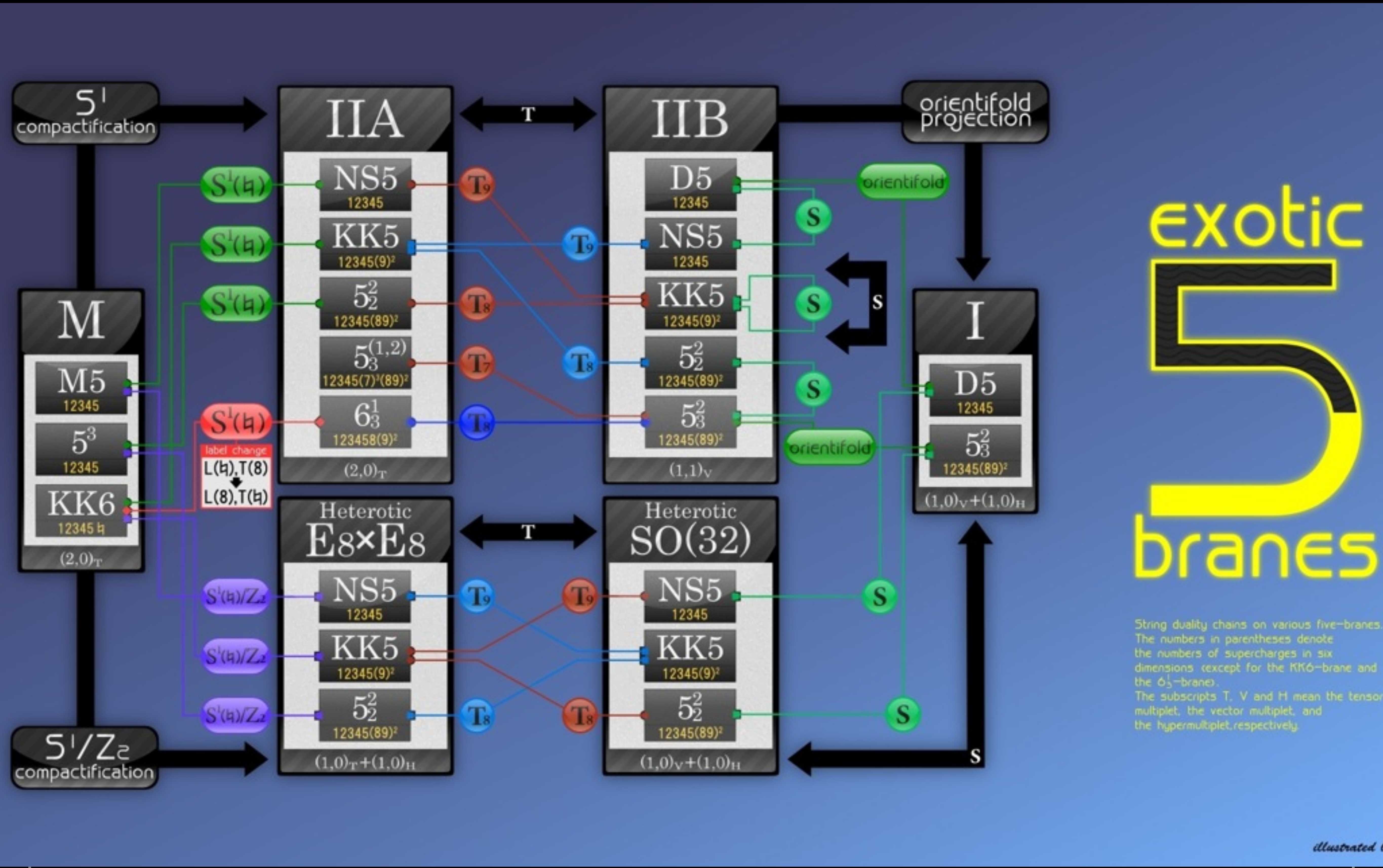
zero-modes associated with the geometry (solution)

should be embedded into the supermultiplets

16 SUSY in 6 dim

- N=(1,1) vector multiplet
- N=(2,0) tensor multiplet

Duality chain is helpful



String duality chains on various five-branes. The numbers in parentheses denote the numbers of supercharges in six dimensions (except for the KK6-brane and the 6^1_3-brane). The subscripts T, V and H mean the tensor multiplet, the vector multiplet, and the hypermultiplet, respectively.

image by M. Yata

IIB 5^2_2-brane

Duality chain

6d N=(1,1) vector multiplet

- $U(1)^2$ isometries $\longrightarrow$ $k_1^\mu,\ k_2^\mu$ Killing vectors

Space-time covariant expression of T-duality rule

Covariant Buscher rule

T-duality in NS-NS sector

$$g_{\mu\nu} \longrightarrow g'_{\mu\nu} = g_{\mu\nu} - \frac{(i_k g)_\mu (i_k g)_\nu - ((i_k B)_\mu - \lambda\partial_\mu\varphi)((i_k B)_\nu - \lambda\partial_\nu\varphi)}{k^2},$$

$$B_{\mu\nu} \longrightarrow B'_{\mu\nu} = B_{\mu\nu} - \frac{(i_k g)_\mu ((i_k B)_\nu - \lambda\partial_\nu\varphi) - ((i_k B)_\mu - \lambda\partial_\mu\varphi)(i_k g)_\nu}{k^2},$$

$$e^{2\phi} \longrightarrow e^{2\phi'} = \frac{1}{k^2} e^{2\phi}$$

winding zero-modes φ

Covariant Buscher rule

non-covariant Buscher rule in R-R sector [Meessen-Ortin (1998)]

↓ Covariantized

Effective rule of T-duality transformation

$$
\begin{aligned}
C'^{(n)} &= (-)^n i_k C^{(n)} - C^{(n-1)} \wedge (i_k B - \lambda d\varphi) \\
&\quad - (-)^{n-2} k^{-2} i_k C^{(n-1)} \wedge (i_k B - \lambda d\varphi) \wedge i_k g
\end{aligned}
$$

Available only inside the pull-back

After long calculations ….

IIB 5^2_2 -brane

$$S^{\rm DBI}_{5^2_2} = -T_{5^2_2} \int d^6\xi \; e^{-2\phi}(\det h_{IJ})\sqrt{1 + \frac{e^{2\phi}(i_{k_1} i_{k_2}(C^{(2)} + C^{(0)} B^{(2)}))^2}{\det h_{IJ}}}$$

$$\times \sqrt{-\det(\Pi^2_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + \frac{K^{(1)}_a K^{(1)}_b}{(k_2)^2} - \frac{(k_2)^2(K^{(2)}_a K^{(2)}_b - K^{(3)}_a K^{(3)}_b)}{\det h_{IJ}} + \lambda F_{ab})}$$

$$h_{IJ} = k^\mu_I k^\nu_J (g_{\mu\nu} + B_{\mu\nu}), \quad \Pi^2_{\mu\nu} = g_{\mu\nu} - \frac{1}{(k_2)^2}(i_{k_2} g)_\mu (i_{k_2} g)_\nu,$$

$$K^{(1)}_\mu = (i_{k_2} B - \lambda d\varphi')_\mu,$$

$$K^{(2)}_\mu = \left(i_{k_1} g - \frac{k_1 \cdot k_2}{(k_2)^2}(i_{k_2} g) + \frac{1}{(k_2)^2}(i_{k_1} i_{k_2} B)(i_{k_2} B - \lambda d\varphi') \right)_\mu ,$$

$$K^{(3)}_\mu = \left((i_{k_1} B - \lambda d\varphi) - \frac{k_1 \cdot k_2}{(k_2)^2}(i_{k_2} B - \lambda d\varphi') + \frac{1}{(k_2)^2}(i_{k_1} i_{k_2} B) i_{k_2} g \right)_\mu$$

5^2_2 Four transverse directions? $X, \ X', \ X^1, \ X^2$ → NO

- Two geometric modes X, X' are projected out

$$-\det(\Pi_{\mu\nu}\partial_a X^\mu \partial_b X^\nu \cdots)$$

$$\Pi_{\mu\nu} = g_{\mu\nu} - h^{IJ} g_{\mu\rho} g_{\nu\sigma} k_I^\rho k_J^\sigma$$ projection operator

- Two winding zero-modes φ, φ' appear

$$(i_{k_1} B - d\varphi), \quad (i_{k_2} B - d\varphi')$$

$$\delta B = d\Lambda^{(1)}, \quad \delta\varphi = -i_{k_1}\Lambda^{(1)}, \quad \delta\varphi' = -i_{k_2}\Lambda^{(1)}$$

- **Appropriate tension**

$$T \sim e^{-2\phi} \det h_{IJ} \sim e^{-2\phi} R_1^2 R_2^2 \qquad 5^2_2$$

solitonic

volume of fibered torus

- **World-volume gauge symmetry**

$$\delta A_a = \partial_a \chi$$

- **Space-time gauge symmetry**

$$\begin{aligned}
&\delta B = d\Lambda^{(1)}, \\
&\delta C^{(0)} = 0, \quad \delta C^{(2)} = d\lambda^{(1)}, \quad \delta C^{(4)} = d\lambda^{(3)} - B \wedge d\lambda^{(1)}, \\
&\delta(d\tilde{A}^{(1)}) = P[\delta \tilde{C}^{(2)}], \quad \delta\varphi = -i_{k_1}\Lambda^{(1)}, \quad \delta\varphi' = -i_{k_2}\Lambda^{(1)}.
\end{aligned}$$

Wess-Zumino term

Wess-Zumino term — formal T-duality of $B^{(6)}$

A mixed-symmetry tensor needs to be introduced

[Bergshoeff-Riccioni (2010), Bergshoeff-Ortin-Riccioni (2011)]

$$B^{(8,2)}_{\hat{\mu}_1 \cdots \hat{\mu}_6, mn} \qquad \hat{\mu}_i \neq \text{isometry directions}$$

$$S^{WZ}_{5^2_2} = -\mu_5 \int_{\mathcal{M}_6} P[B^{(8,2)}] + \cdots$$

Hodge dual of $k_1^\mu k_2^\nu B^{(2)}_{\mu\nu}$

cf. dual graviton for KKM

No scalar modes associated with isometries

[Kimura-Yata-S.S. (2014)]

- Geometric zero-modes X^1, X^2
- Winding modes φ, φ'
- Gauge 1-form A_a

embedded into 6d N=(1,1) vector multiplet

- World-volume and space-time gauge invariance

IIB 5^2_3-brane

$$5^2_2 \xrightarrow[\text{S-dual}]{} 5^2_3$$

After long calculations ….

IIB 5^2_3-brane

$$S^{\mathrm{DBI}}_{5^2_3} = -T_{5^2_3}\int d^6\xi\ |\tau| e^{-2\phi}(\det l_{IJ})\sqrt{1+\frac{e^{2\phi}}{\det l_{IJ}}\{i_{k_1}i_{k_2}(B+|\tau|^{-2}C^{(0)}C^{(2)})\}}$$

$$\times\sqrt{-\det\left(\Pi^2_{\mu\nu}\partial_a X^\mu\partial_b X^\nu+\frac{L_a^{(1)}L_b^{(1)}}{|\tau|^2k_2^2}-\frac{k_2^2(L_a^{(2)}L_b^{(2)}-|\tau|^{-2}L_a^{(3)}L_b^{(3)})}{\det l_{IJ}}+\lambda F_{ab}\right)}$$

$$\tau = C^{(0)} + ie^{-\phi},$$

$$l_{IJ} = k_I^\mu k_J^\nu(g_{\mu\nu} - |\tau|^{-1}C^{(2)}_{\mu\nu}),$$

$$L^{(1)}_\mu = (i_{k_1}C^{(2)} + \lambda d\tilde{\varphi}')_\mu,$$

$$L^{(2)}_\mu = \left(i_{k_1}g - \frac{k_1\cdot k_2}{k_2^2}(i_{k_2}C^{(2)} + \lambda d\tilde{\varphi}) + \frac{1}{k_2^2}(i_{k_1}i_{k_2}C^{(2)})i_{k_2}g\right)_\mu,$$

$$L^{(3)}_\mu = \left((i_{k_1}C^{(2)} + \lambda d\tilde{\varphi}) - \frac{k_1\cdot k_2}{k_2^2}(i_{k_2}C^{(2)} + \lambda d\tilde{\varphi}) + \frac{1}{k_2^2}(i_{k_1}i_{k_2}C^{(2)}i_{k_2}g\right)$$

$$F_{ab} = \partial_a A_b - \partial_b A_a + (\mathrm{RR\ forms})$$

- **Two geometric modes X, X' are projected out**

$$-\det(\Pi_{\mu\nu}\partial_a X^\mu \partial_b X^\nu \cdots)$$

$$\Pi_{\mu\nu} = g_{\mu\nu} - h^{IJ} g_{\mu\rho} g_{\nu\sigma} k_I^\rho k_J^\sigma$$

- **Appropriate tension**

$$T \sim |\tau| e^{-2\phi} \det l_{IJ} \sim e^{-3\phi} R_1^2 R_2^2 \qquad 5_3^2$$

- **Mixed-symmetry tensor in R-R sector**

$$S_{5_2^2}^{WZ} = -\mu_5 \int_{\mathcal{M}_6} P[C^{(8,2)}] + \cdots$$

S-dual of $B^{(8,2)}$

- **Scalar gauge transformation associated with R-R form**

$$\delta C^{(2)} = d\lambda^{(1)}, \quad \delta\tilde{\varphi} = -i_{k_1}\lambda^{(1)} \quad \delta\tilde{\varphi}' = -i_{k_2}\lambda^{(1)}$$

SUMMARY AND FUTURE WORKS

Summary

- We construct world-volume effective actions of type IIB 5^2_2-brane and 5^2_3-brane
- World-volume fields are embedded into the 6d N=(1,1) vector multiplet

$$X^1, \quad X^2, \quad \varphi, \quad \varphi', \quad A_a$$

- Source of mixed-symmetry tensors (non-geometric flux)

$$B^{(8,2)}, \qquad C^{(8,2)}$$

- Type IIA 5^2_2-brane — 6d N=(2,0) tensor multiplet
- Type I 5^2_3-brane
- 5^2_2-brane in $SO(32), E_8 \times E_8$ heterotic theories

Future directions

- Physics from the effective theories
- Supertube effect on world-volume of exotic branes?
- Monodromy ?
- Intersecting configurations of exotic branes?
- World-volume action of M-theory exotic branes?
- Treatment in the double field theory (DFT) or generalized geometry?
- R-branes?

Thank you

BACKUP

IIA 5^2_2-brane

Duality chain

M5

NS5

KKM

5^2_2

Direct dimensional reduction

T-dual

T-dual

N=(2,0) tensor multiplet

6d N=(2,0) tensor multiplet

- Geometric zero-modes X^1, X^2
- Winding modes φ, φ'
- Scalar associated with M-circle Y
- Self-dual tensor field A_{ab}
- Auxiliary field (non-dynamical) a

DBI and WZ terms are given in the gauge invariant form
[Kimura-Yata-S.S (2014)]

M5-brane action

M5-brane action [Pasti-Sorokin-Tonin (1997)]

$$S_{\mathrm{M5}} \; = \; -T_{\mathrm{M5}} \int d^6\xi \, [\sqrt{-\det(P[\hat{g}]_{ab} + i\hat{H}^*_{ab})} + \frac{\sqrt{-\hat{g}}}{4g^{ef}\partial_e a \partial_f a} \hat{H}^{*abc} \hat{H}_{bcd} (\partial_a a \partial^d)]$$

$$+ T_{\mathrm{M5}} \int_{\mathcal{M}_6} \left(P[\hat{C}^{(6)}] - \frac{1}{2} F^{(3)} \wedge P[\hat{C}^{(3)}] \right)$$

Self-dual 2-form $F^{(3)} = dA^{(2)}$

$$\hat{H}^{(3)} = F^{(3)} + P[\hat{C}^{(3)}] \qquad H^{*(3)} = *_6 H^{(3)}$$

An auxiliary field a

— non dynamical (can be gauged away)

- Dimensional reduction along M-circle

  A scalar mode Y

- Introduce two isometries (Killing vectors) along transverse directions

$$k_1^\mu, \quad k_2^\mu$$

- T-duality transformations by the covariant Buscher rule

IIA 5^2_2-brane

$$S^{DBI}_{5^2_2} = -T_{5^2_2} \int d^6\xi e^{-2\phi} (\det h_{IJ})$$

$$\times \sqrt{- \det(\Pi^2_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + \frac{K^{(1)}_a K^{(1)}_b}{(k_2)^2} - \frac{(k_2)^2 (K^{(2)}_a K^{(2)}_a - K^{(3)}_a K^{(3)}_a)}{\det h_{IJ}} + \frac{\lambda^2 e^{2\phi}}{\det h_{IJ}} \tilde{F}_a \tilde{F}_b)}$$

$$\times \sqrt{\det(\delta_a{}^b + \frac{i e^{i\phi}}{3! \tilde{\mathcal{N}} \sqrt{\det h_{IJ} (\tilde{a})^2}} Z_a{}^b)}$$

$$- \frac{\lambda^2}{4} T_{5^2_2} \int d^6\xi \; \frac{\varepsilon^{abgd'e'f'} \tilde{H}_{d'e'f'} \tilde{H}_{abc} (\partial_g a \partial_d a)}{3! \tilde{\mathcal{N}}^2 \widetilde{\partial a}^2} \left[\tilde{g}^{cd} - \frac{\lambda^2 e^{2\phi} \tilde{g}^{ce} \tilde{g}^{df} \tilde{F}^{(1)}_e \tilde{F}^{(1)}_f}{\det h_{IJ} + \lambda^2 e^{2\phi} \tilde{g}^{a'b'} \tilde{F}^{(1)}_{a'} \tilde{F}^{(1)}_{b'}} \right]$$

$$\tilde{H}_{abc} = F^{(3)}_{abc} + (\text{RR-forms})$$

$$\tilde{F}^{(1)}_a = \partial_a Y + (\text{RR-forms})$$