4D N=1 gauge theories from M5 branes on A_k singularity with orientifold

Shoichi Kanno (Tokyo institute of Technology)

① introduction

6D N=(2,0) SCFT on Riemann surface

→ 4D N=2 theory

This procedure is very successful for understanding duality network of 4D N=2 theories.

We want to generalized to 4D N=1

6D N=(1,0) SCFT on Riemann surface

→ 4D N=1 theory

So many 6D N=(1,0) SCFT [Heckman et al.]

[Gaiotto Razamat] M5 on C^2/Z_k orbifold "class S_k "

In this talk, we further impose R^5/Z_2 orentifold action

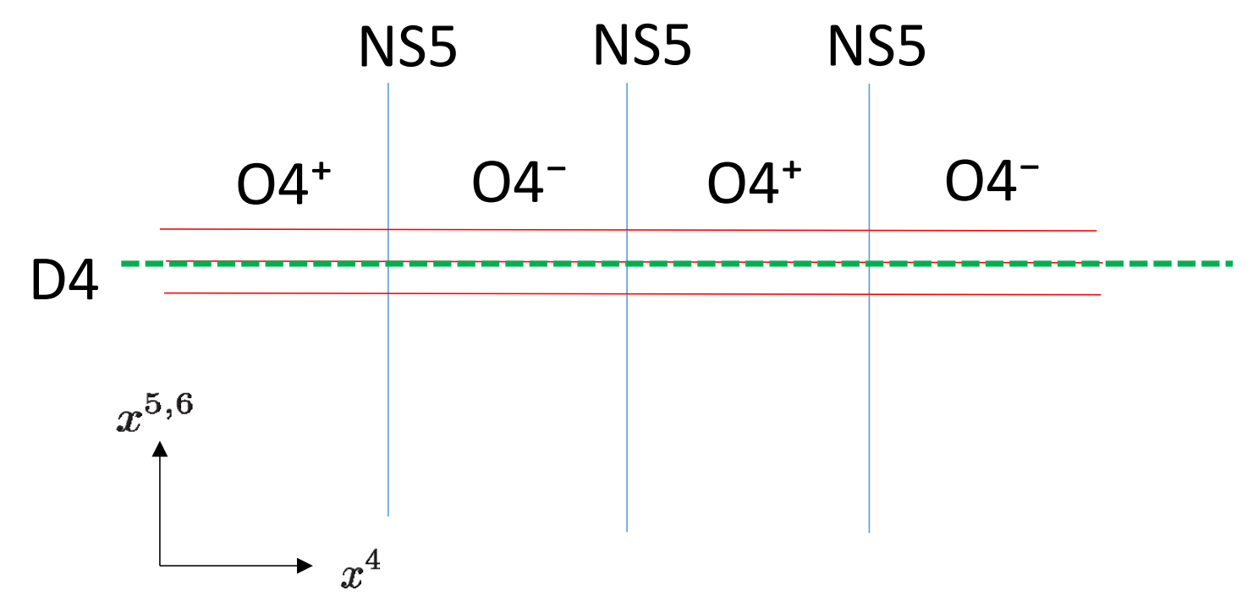
In addition, we restrict the case for odd k k=2m+1

②Type IIA brane system set up

orientifold

	0	1	2	3	4	5	6	7	8	9
N D4	○	○	○	○	○					
(n+1) NS5	○	○	○	○		○	○			

C^2/Z_k



orbifold action

$$Z_1 = x^5 + ix^6 \quad Z_2 = x^7 + ix^8$$

$$(z_1, z_2) \;\to\; (\xi z_1, \xi^{-1} z_2) \quad \xi = \exp(2\pi i/k)$$

orientifold action

$x^{5,6,7,8,9} \to -x^{5,6,7,8,9}$ + orientation change of strings

fixed plane $x^{5,6,7,8,9} = 0$ → O4-plane

two types of O4-plane distinguished by RR-charge: O4⁺ and O4⁻

across a NS5 → O4⁺ and O4⁻ interchange

③Orbifold and Orientifold projection

original theory : N=2 SU(N) conformal quiver

$N=2$ $\mathrm{SU}(N)_a$ vectormultiplet $(V^{(a)}, \Phi^{(a)})$ $(a = 1, \cdots, n)$

$\mathrm{SU}(N)_a \times \mathrm{SU}(N)_{a+1}$ bifundamental hypermultiplet $(Q^{(a)}, \tilde{Q}^{(a)})$ $(a = 1, \cdots, n-1)$

+ fund.(antifund.) hyper at end nodes

◆Orbifold projection

(a) spatial rotation

spatial rotation charge = R-charge

→ Φ : 1 Q : -1/2 $\tilde{Q}$: -1/2 (charge of scalar component)

(b)action on gauge index

for charge (i,j) sector filed $\Psi_{ij} \to \xi^{i-j}\Psi_{ij}$

Q $\tilde{Q}$ have half-integer R-charge

→ assign (half-)integer charge at even(odd) gauge node

Thus, we found

$(V^{(a)}, \Phi^{(a)})$ → N=1 SU(N)^k necklace quiver $\mathcal{N}^{(a)}$

$(Q^{(a)}, \tilde{Q}^{(a)})$ → bifundamental chiral which zig-zag back and forth between $\mathcal{N}^{(a)}$ and $\mathcal{N}^{(a+1)}$

◆Orientifold projection

identify fields with their orientifold image

If image of a field is itself, then

gauge node : SU → SO (O4⁻) /Usp (O4⁺)

bifundamental Φ → antisymmetric rep.(O4⁻)
symmetric rep. (O4⁺)

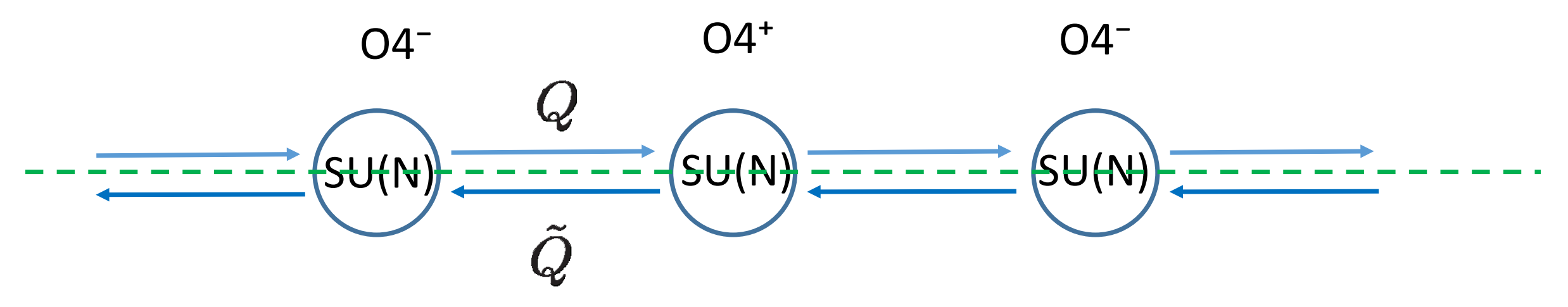
Since orientifold action causes $Q \leftrightarrow \tilde{Q}$

such situation does not occur for Q $\tilde{Q}$

For simplicity, we assign O4⁺ plane to gauge node with integer Z_k charge

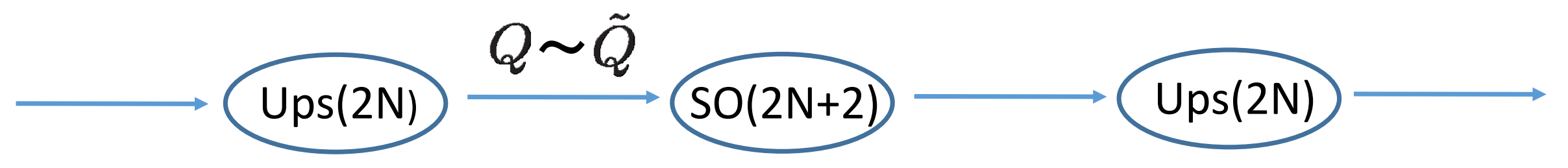
④Quiver diagram representation of the projection

N=2 SU(N) conformal quiver



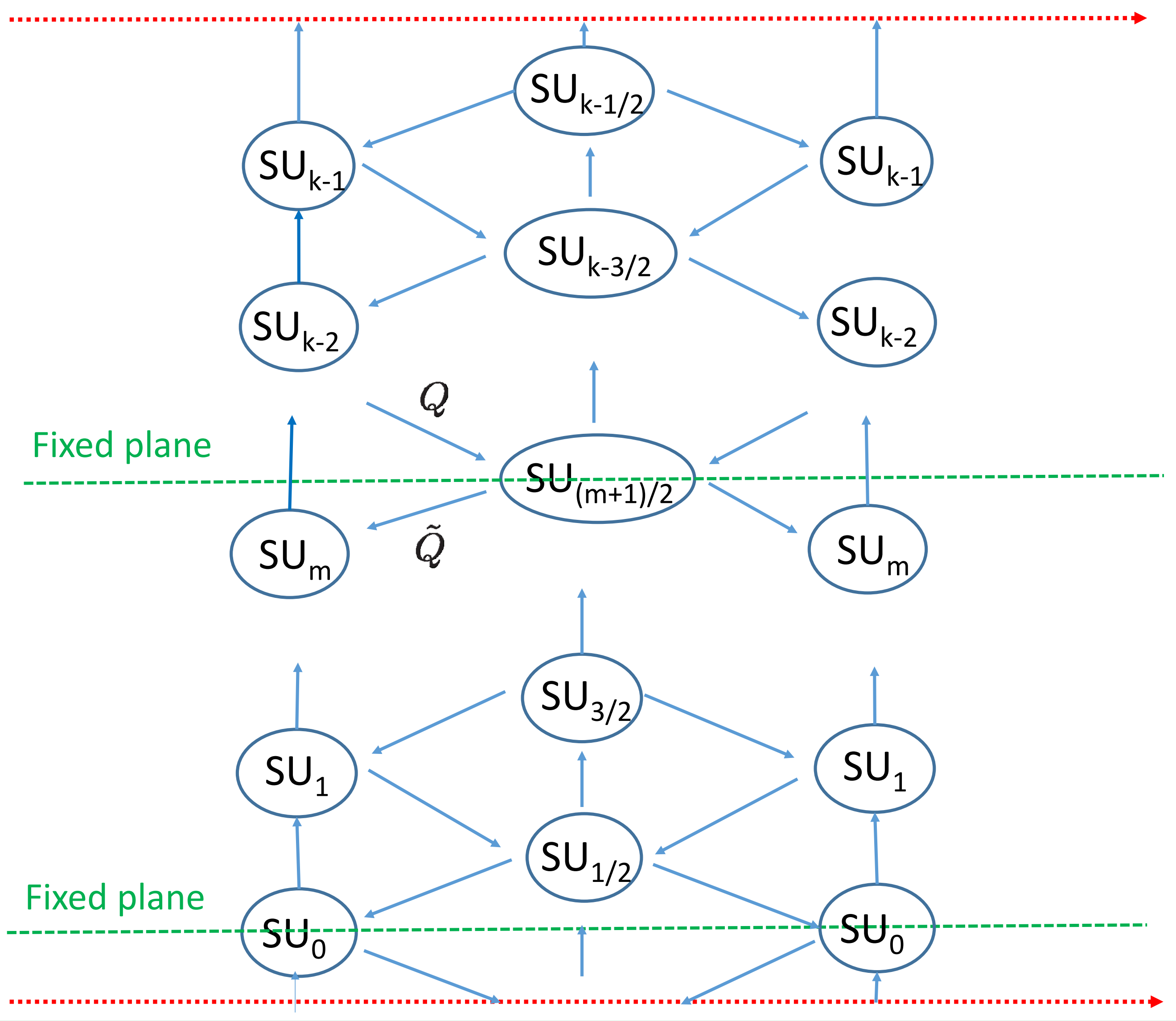
→ orientifold

N=2 Usp-SO conformal quiver



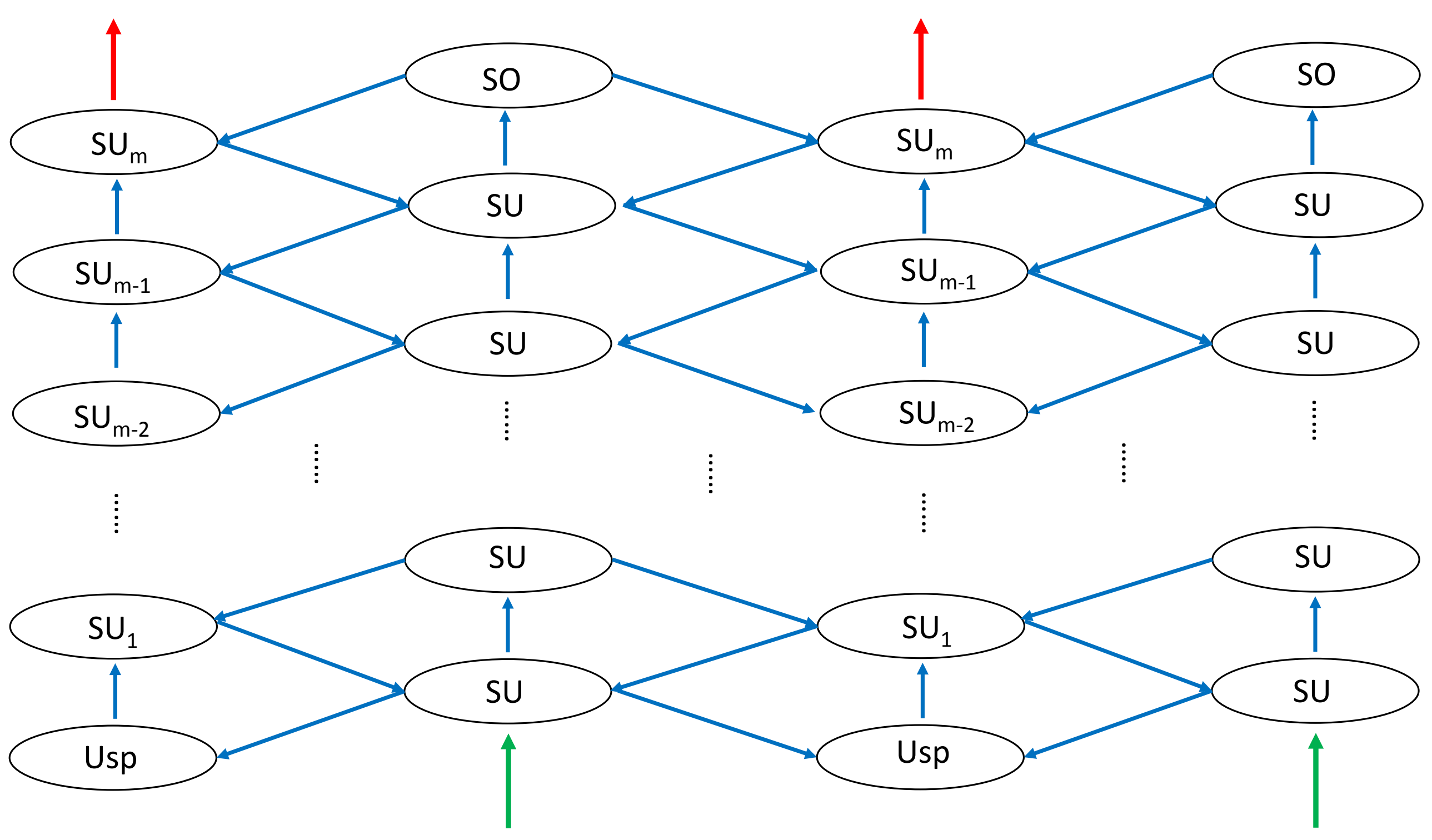
↓ orbifold

core theory of class S_k



→ orientifold

4D N=1 quiver for the brane system



🟩→ antisymmetric rep. chiral

🟥→ symmetric rep. chiral

⑤Anomaly free theory

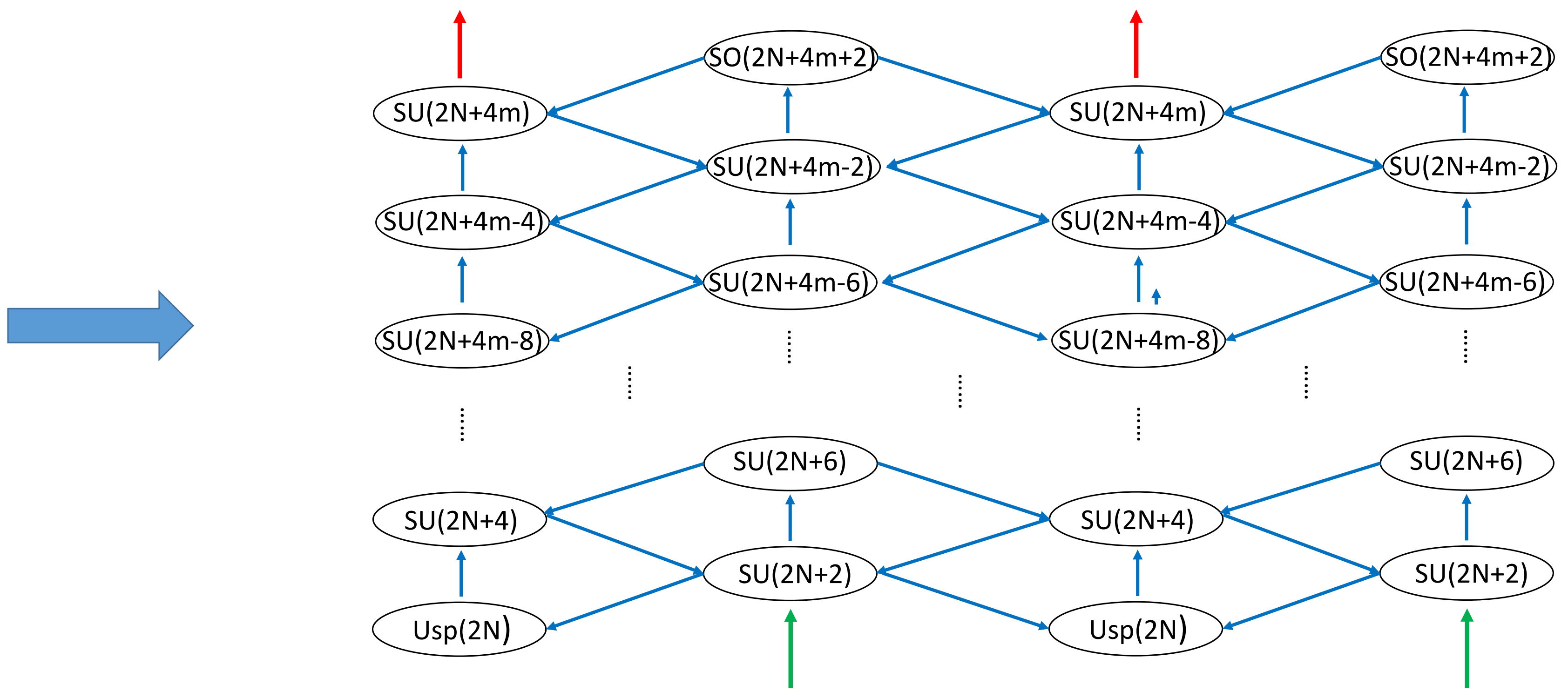
orientifoled theory is chiral

→ tune the rank of gauge groups to cancel gauge anomaly

We impose two conditions:

① anomaly free

② obtain N=2 Usp-SO conformal quiver for m=0



⑥Field content, superpetential

Gauge group

$$\prod_{a\in even}\left\{\mathrm{Usp}^{(a)}(2N)\times\prod_{i=1}^{m}\mathrm{SU}^{(a)}(2N+4i)\right\}\times\prod_{a\in odd}\left\{\prod_{i=0}^{m-1}\mathrm{SU}^{(a)}(2N+4i+2)\times\mathrm{SO}^{(a)}(2N+4m+2)\right\}$$

Chiral fields for even a

$\Phi^{(a,0)}$ $(\square,\overline{\square})$ of $\mathrm{Usp}^{(a)}(2N)\times\mathrm{SU}^{(a)}(2N+4)$

$\Phi^{(a,i)}$ $(\square,\overline{\square})$ of $\mathrm{SU}^{(a)}(2N+4i)\times\mathrm{SU}^{(a)}(2N+4i+4)\quad(i=1,2,\cdots,m-1)$

$X^{(a)}$ $\square\!\square$ of $\mathrm{SU}^{(a)}(2N+4m)$

$Q^{(a,i)}$ $(\square,\overline{\square})$ of $\mathrm{SU}^{(a)}(2N+4i)\times\mathrm{SU}^{(a+1)}(2N+4i-2)\quad(i=1,2,\cdots,m)$

$\tilde{Q}^{(a,0)}$ $(\overline{\square},\square)$ of $\mathrm{Usp}^{(a)}(2N)\times\mathrm{SU}^{(a+1)}(2N+2)$

$\tilde{Q}^{(a,i)}$ $(\overline{\square},\square)$ of $\mathrm{SU}^{(a)}(2N+4i)\times\mathrm{SU}^{(a+1)}(2N+4i+2)\quad(i=1,2,\cdots,m-1)$

$\tilde{Q}^{(a,m)}$ $(\overline{\square},\square)$ of $\mathrm{SU}^{(a)}(2N+4m)\times\mathrm{SO}^{(a+1)}(2N+4m+2)$

Chiral fields for odd a

$\Phi^{(a,i)}$ $(\square,\overline{\square})$ of $\mathrm{SU}^{(a)}(2N+4i+2)\times\mathrm{SU}^{(a)}(2N+4i+2)\quad(i=0,1,\cdots,m-1)$

$\Phi^{(a,m)}$ $(\square,\overline{\square})$ of $\mathrm{SU}^{(a)}(2N+4m-2)\times\mathrm{SO}^{(a)}(2N+4m+2)$

$Y^{(a)}$ $\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ of $\mathrm{SU}^{(a)}(2N+2)$

$\tilde{Q}^{(a,i)}$ $(\overline{\square},\square)$ of $\mathrm{SU}^{(a)}(2N+4i-2)\times\mathrm{SU}^{(a+1)}(2N+4i)\quad(i=1,2,\cdots,m)$

$Q^{(a,0)}$ $(\square,\overline{\square})$ of $\mathrm{SU}^{(a)}(2N+2)\times\mathrm{Usp}^{(a+1)}(2N)$

$Q^{(a,i)}$ $(\square,\overline{\square})$ of $\mathrm{SU}^{(a)}(2N+4i+2)\times\mathrm{SU}^{(a+1)}(2N+4i)\quad(i=1,2,\cdots,m-1)$

$Q^{(a,m)}$ $(\square,\overline{\square})$ of $\mathrm{SO}^{(a)}(2N+4m+2)\times\mathrm{SU}^{(a+1)}(2N+4m)$

Superpotential coupling

closed triangle path in quiver = cubic superpotential

for even a

$$W^{(a,i)}=\mathrm{Tr}\tilde{Q}^{(a,i)}\Phi^{(a,i)}Q^{(a,i+1)}\qquad\qquad(i=0,1,\cdots,m-1)$$
$$\hat{W}^{(a,i)}=\mathrm{Tr}Q^{(a-1,i)}\Phi^{(a,i)}\tilde{Q}^{(a-1,i+1)}$$
$$W^{(a,m)}=\mathrm{Tr}\tilde{Q}^{(a,m)}X^{(a)}\tilde{Q}^{(a,m)}\qquad\hat{W}^{(a,m)}=\mathrm{Tr}Q^{(a,m)}X^{(a)}Q^{(a,m)}$$

for odd a

$$V^{(a,i)}=\mathrm{Tr}\tilde{Q}^{(a,i)}\Phi^{(a,i-1)}Q^{(a,i)}\qquad\qquad(i=1,2,\cdots,m)$$
$$\hat{V}^{(a,i)}=\mathrm{Tr}Q^{(a-1,i)}\Phi^{(a,i-1)}\tilde{Q}^{(a-1,i)}$$
$$V^{(a,0)}=\mathrm{Tr}Q^{(a,0)}Y^{(a)}Q^{(a,0)}\qquad\hat{V}^{(a,0)}=\mathrm{Tr}\tilde{Q}^{(a-1,0)}Y^{(a)}\tilde{Q}^{(a-1,0)}$$

⑦ global symmetry

intrinsic symmetry $\boxed{\{U(1)^k/U(1)\}}\times\boxed{U(1)_t}$ ← Φ , X , Y : -1
Q : 1/2 $\tilde{Q}$: 1/2

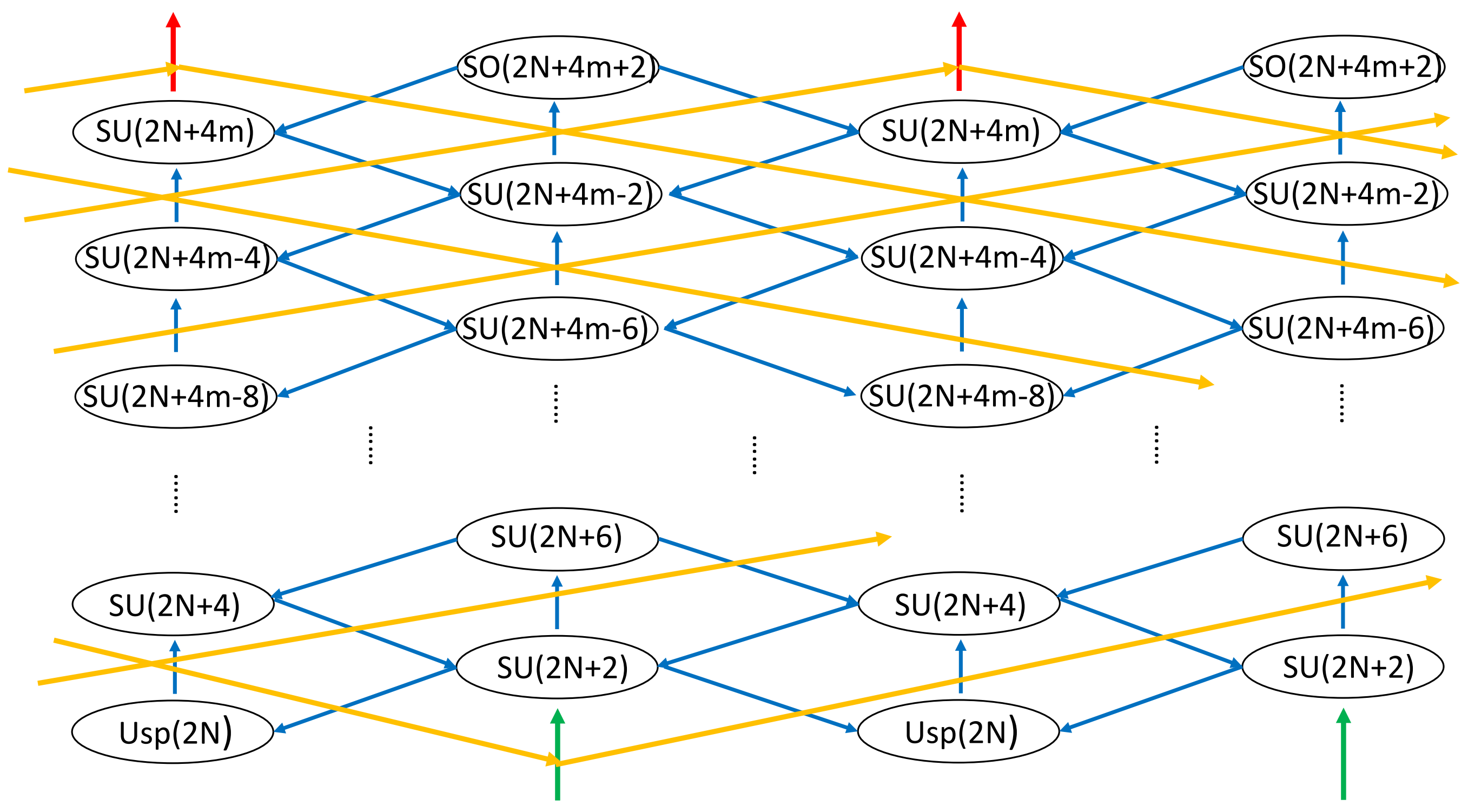
↑

abelian remnant of global symmetry in 6D theory

U(1) rotation of fields crossing with each orange arrow (k-1 independent ones)

(anti)symmetric rep. fields X (Y) have charge 2 and others have charge 1

Sign of charge determined by orientation



Flavor symmetry

$U(1)_{\alpha_a}$ $Q^{(a,i)}$:+1 $\tilde{Q}^{(a,i)}$:-1 $a=1,2,\ldots,n-1$

+ symmetry associated with both ends of the quiver

Superpotential is invariant in these global symmetry

⑧ Exactly marginal deformation

To identify the 4D theory with compactification of 6D theory on Riemann surface, it must be hold

# of exact marginal deformation parameter = # of complex structure moduli parameter of Riemann suface

We check it according to Leigh-Strassler argument

scaling coefficients for even a $\gamma(\Phi)$: anomalous dimension of Φ

$$A(\mathrm{Usp}^{(a)}(2N))=-1+(N+1)\gamma(\tilde{Q}^{(a,0)})+(N+1)\gamma(Q^{(a-1,0)})+(N+2)\gamma(\Phi^{(a,0)})$$

$$\begin{aligned}A(\mathrm{SU}^{(a)}(2N+4i))=&(N+2i+1)\gamma(\tilde{Q}^{(a,i)})+(N+2i+1)\gamma(Q^{(a-1,i)})\\&+(N+2i-1)\gamma(Q^{(a,i)})+(N+2i-1)\gamma(\tilde{Q}^{(a-1,i)})\\&+(N+2i+2)\gamma(\Phi^{(a,i)})+(N+2i-2)\gamma(\Phi^{(a,i-1)})\end{aligned}$$

$$A(\mathrm{SU}^{(a)}(2N+4m))=1+(N+2m+1)\left\{\gamma(\tilde{Q}^{(a,m)})+\gamma(Q^{(a-1,m)})+\gamma(X^{(a)})\right\}$$

$$2A(W^{(a,i)})=\gamma(\tilde{Q}^{(a,i)})+\gamma(\Phi^{(a,i)})+\gamma(Q^{(a,i+1)})$$

$$2A(\hat{W}^{(a,i)})=\gamma(\tilde{Q}^{(a-1,i)})+\gamma(\Phi^{(a,i)})+\gamma(\tilde{Q}^{(a-1,i+1)})$$

$$2A(W^{(a,m)})=2\gamma(\tilde{Q}^{(a,m)}+\gamma(X^{(a)})\qquad 2A(\hat{W}^{(a,m)})=2\gamma(Q^{(a-1,m)}+\gamma(X^{(a)})$$

We found the following relation

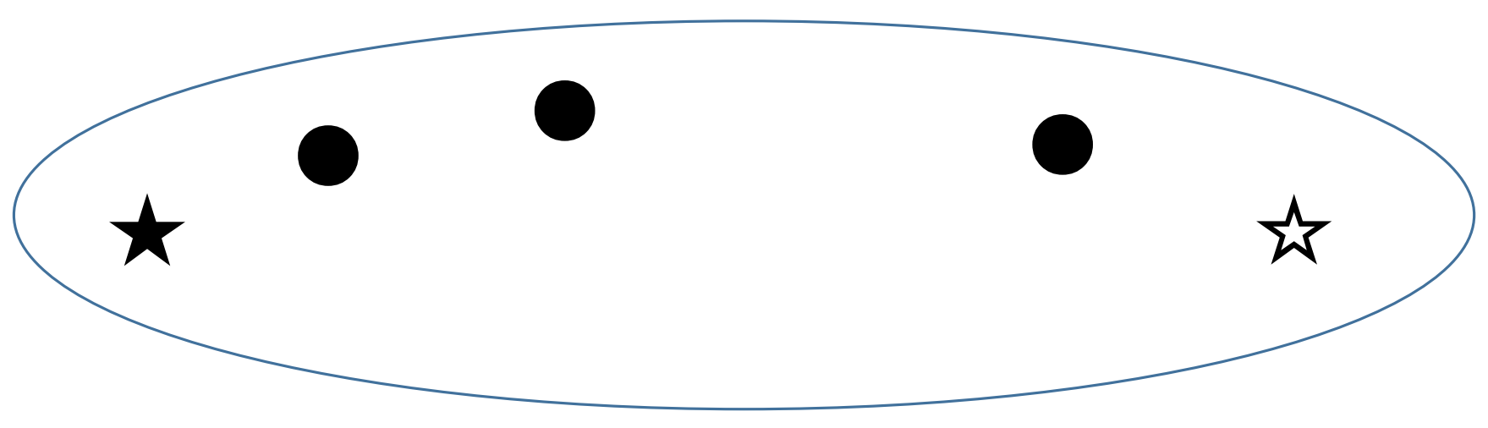
$$\boxed{\begin{aligned}A(\mathrm{Usp}^{(a)}(2N))+\sum_{i=1}^{m}A(\mathrm{SU}^{(a)}(2N+4i))=&\sum_{i=0}^{m-1}2(N+2i+1)\left\{A(W^{(a,i)})+A(\hat{W}^{(a,i)})\right\}\\&+(N+2m+1)\left\{A(W^{(a,m)})+A(\hat{W}^{(a,m)})\right\}\end{aligned}}$$

Similar relation holds for odd a. Thus we have n relations

→ **n exactly marginal deformations**

⑨associated Riemann surface

We identify the quiver theory with a sphere with (n+1) punctures



(n-1) puncture associated with $U(1)_{\alpha_a}$

two punctures associated with ends of the quiver

We can glue punctures (gauging the flavor symmetry) and obtain more general theories in the same way as for N=2 class S theories.

Conjecture

exchange of same type of punctures = duality of the theory

Future work

Check the conjecture in terms of index

Closing puncture (giving VEV to meson or baryon op.)

How about even k (need additional flavor for anomaly cancellation)