Anomaly-free Multiple Singularity Enhancement in F-theory

Theory Center, KEK
Shun'ya Mizoguchi
in collaboration with: Taro Tani

SM,Tani arXiv:1508.07423
SM JHEP 1407(2014) 018 arXiv:1403.7066

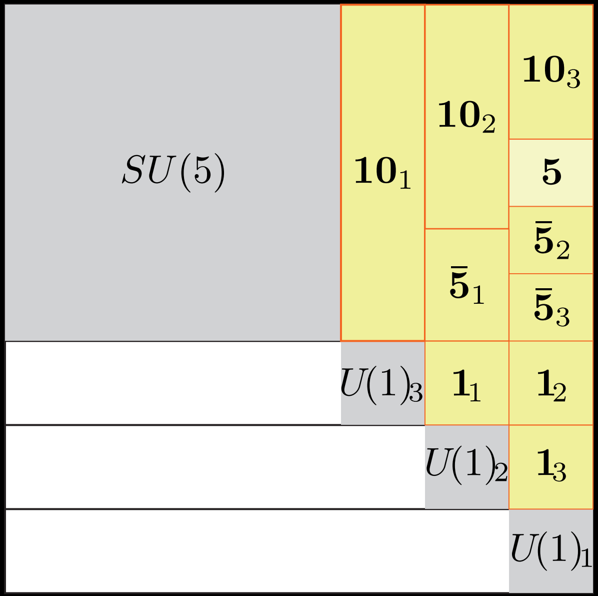
The Standard Model – Why is it as it is?

- One of the biggest challenges that we face in string theory is to explain why nature is as it is
- Why is the top quark so heavy? Why are the lepton-flavor mixing angles large? And in the first place, why are there three generations of quarks and leptons in nature?
- The conventional approaches to string compactification cannot answer to these questions

WHY 3??

"Family Unification"

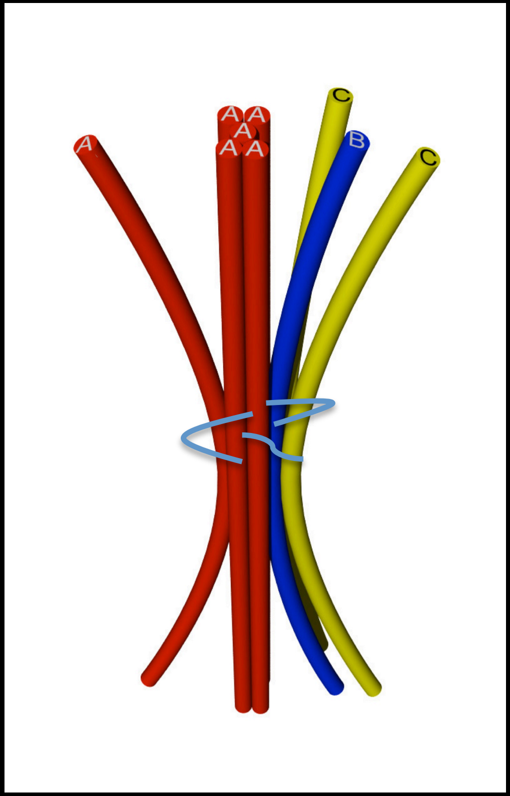
- Family unification is the idea that the quarks and leptons are the fermionic partners of the scalars of some coset supersymmetric non-linear sigma model
  Buchmuller,Peccei,Yanagida; Kugo,Yanagida; Irie,Yasui; Ong; Bando,Kuramoto,Maskawa,Uehara; Itoh,Kugo,Kunitomo…
- Remarkably, $E_7/(SU(5) \times U(1)^3)$ model automatically realizes precisely three non-universal generations of matter fields needed for the SU(5) GUT
  Kugo,Yanagida



$E7/(SU(5) \times U(1)^3)$

“F-theory” Family Unification SM

- Last year, in YITP workshop 2014, it was pointed out that such a coset spectrum may be realized by a set of localized matter multiplets near a “multiple” singulary on 7-branes in 6D F-theory
- The key observation was that, in 6D, the representation of chiral matter localized at an enhanced (split-type) singularity is labeled by some homogeneous Kähler manifold, corresponding to the space of string junctions near the singularity Tani



$E7/(SU(5) \times U(1)^3)$

Four essential aspects of F-theory

1. Instead of considering a configuration of the IIB complex scalar, one considers a configuration of a FICTITIOUS torus whose modulus equals τ Vafa

IIB complex scalar as a modulus

$$\sqrt{-g_4}R_4 \quad = \quad \sqrt{-g}\left(R - \frac{1}{2}g^{\mu\nu}\frac{\partial_\mu\tau\partial_\nu\tau}{\mathrm{Im}\tau^2} - \frac{1}{2}(\partial_\mu \log\rho)^2\right)$$

complex structure (shape)

Kahler structure (size)

$$\mathcal{L}_{IIB} \quad = \quad \sqrt{-g}\left(R - \frac{1}{2}g^{\mu\nu}\frac{\partial_\mu\tau\partial_\nu\tau}{\mathrm{Im}\tau^2}\right)$$

At each point in 10d, one considers a 2-torus with its shape (modulus) varying from point to point

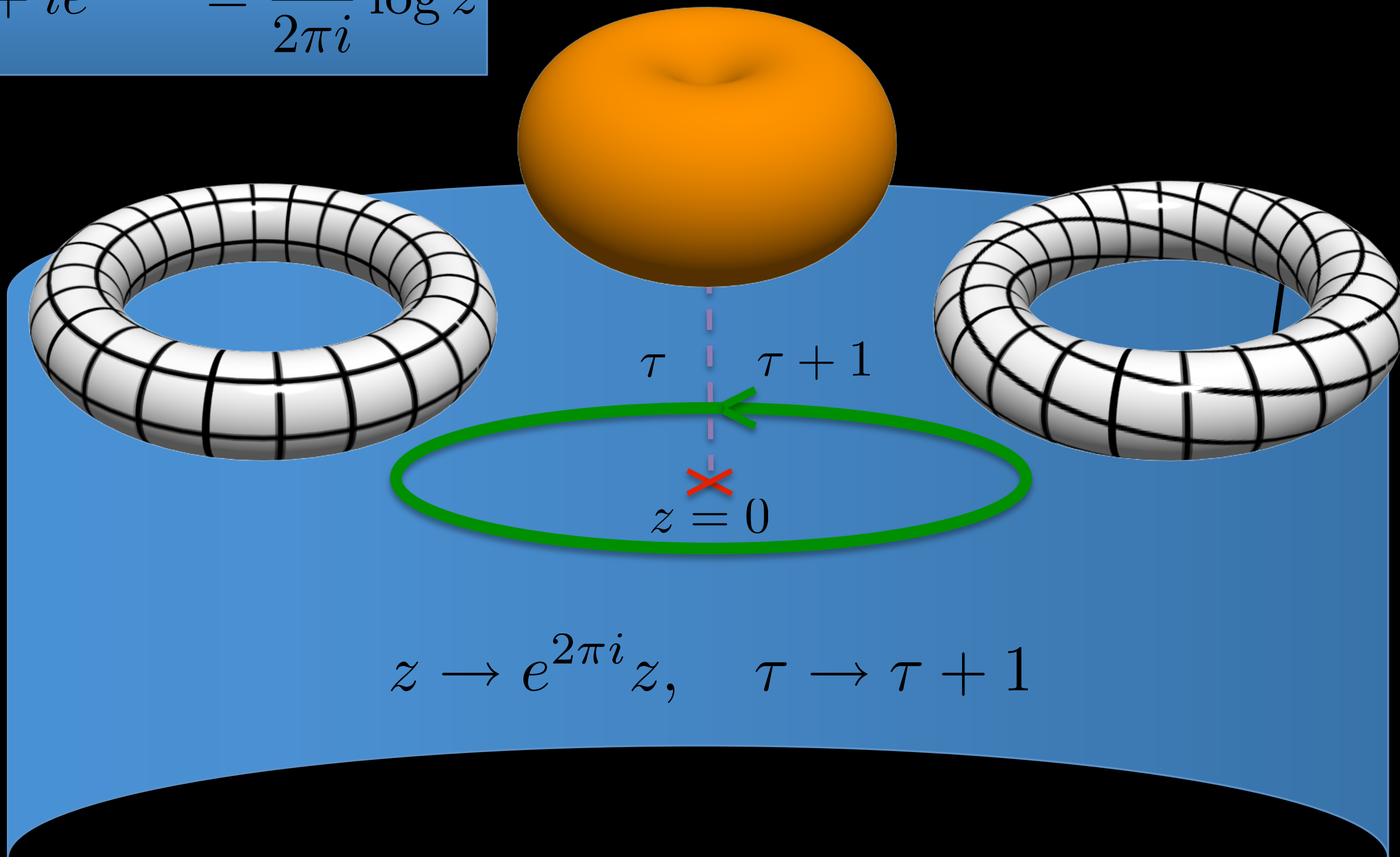
“elliptic fiberation”

Four essential aspects of F-theory

1. Instead of considering a configuration of the IIB complex scalar, one considers a configuration of a FICTITIOUS torus whose modulus equals τ Vafa
2. 7-branes are located where an elliptic (=torus) fiber degenerate and becomes singular

Monodromy around a singular torus

$$\tau = C_0 + ie^{-\phi} \quad = \frac{1}{2\pi i}\log z$$



SL(2,Z)Modular transformation ~ SL(2,Z)S-duality

Four essential aspects of F-theory

1. Instead of considering a configuration of the IIB complex scalar, one considers a configuration of a FICTITIOUS torus whose modulus equals τ Vafa
2. 7-branes are located where an elliptic (=torus) fiber degenerate and becomes singular
3. Singularities of elliptic fiberations were classified according to their types investigated by Kodaira Kodaira

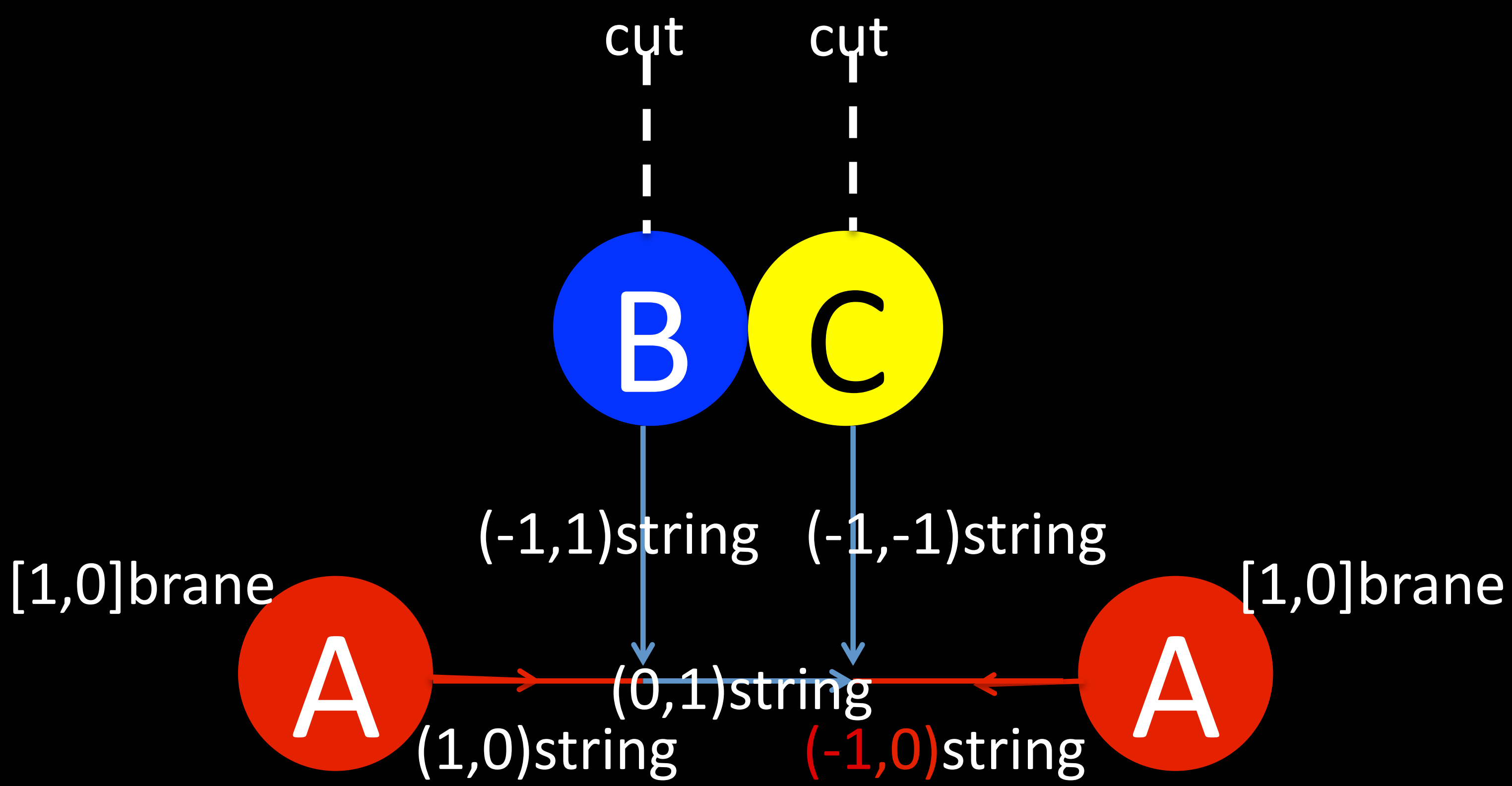
Collapsible set of 7-branes are classified: Kodaira's classification

Fiber type	Singularity type	7-branes	Brane type
I_n	A_{n-1}	A^n	A_{n-1}
II	A_0	AC	H_0
III	A_1	A^2C	H_1
IV	A_2	A^3C	H_2
I0*	D_4	A^4BC	D_4
I_n*	D_{n+4}	$A^{n+4}BC$	D_{n+4}
II*	E_8	A^7BC^2	E_8
III*	E_7	A^6BC^2	E_7
IV*	E_6	A^5BC^2	E_6

Four essential aspects of F-theory

1. Instead of considering a configuration of the IIB complex scalar, one considers a configuration of a FICTITIOUS torus whose modulus equals τ Vafa
2. 7-branes are located where an elliptic (=torus) fiber degenerate and becomes singular
3. Singularities of elliptic fiberations were classified according to their types investigated by Kodaira Kodaira
4. The Kodaira singularities are described by joining/parting of 7-branes, which involves not only D-branes but general (p,q) branes DeWolfe,Hauer,Iqbal,Zwiebach

String junction: (p,q) analogue of open string



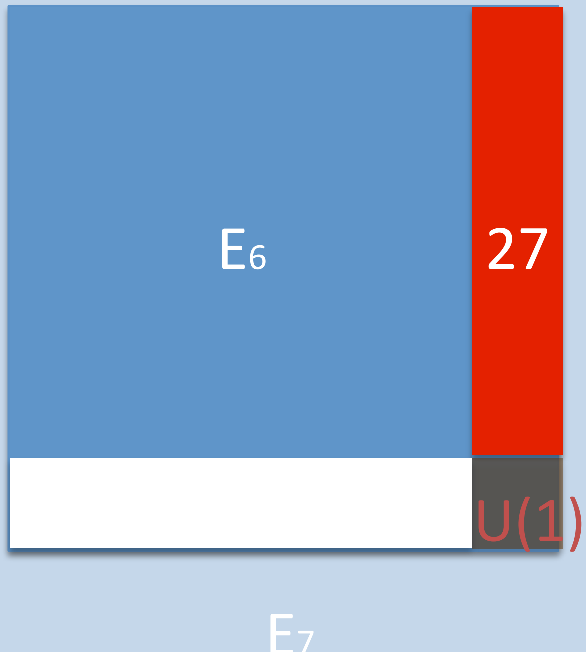
- (-1,1) and (-1,-1) strings are pulled out when the string crosses over the B and C branes

Matter from string junction

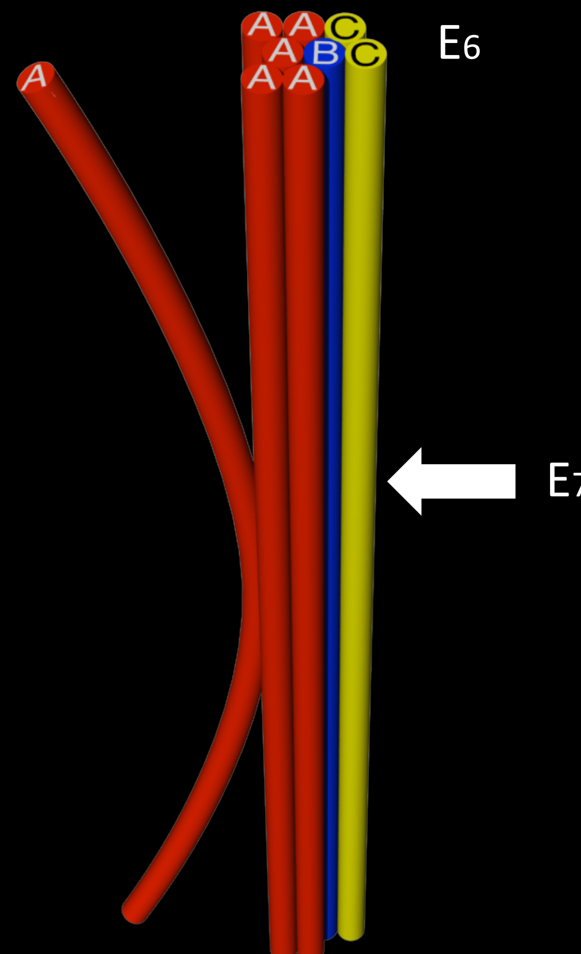
T.Tani, Nucl.Phys.B602(2001)434

- in F
the
n 7-

- s
erms
e and



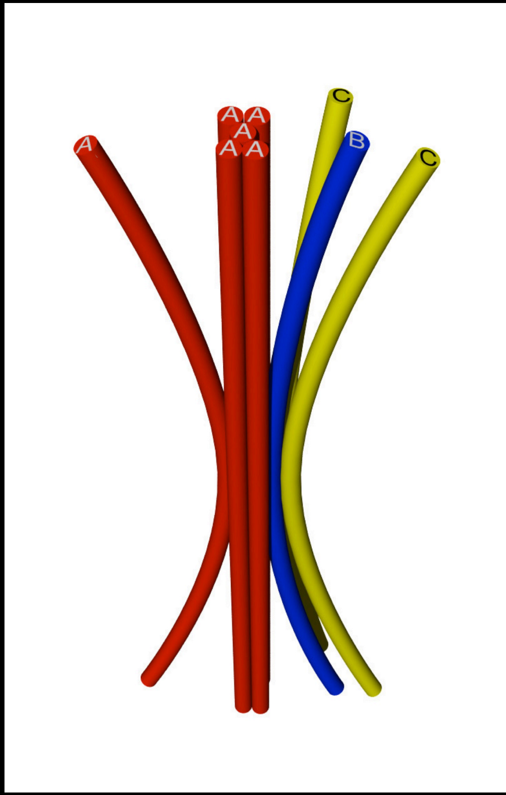
Matter corresponds to a Kahler coset



Gives a perfectly consistent picture

Kugo-Yanagida model via F-theory Family Unification SM, JHEP 1407(2014) 018, arXiv:1403.7066 [hep-th]

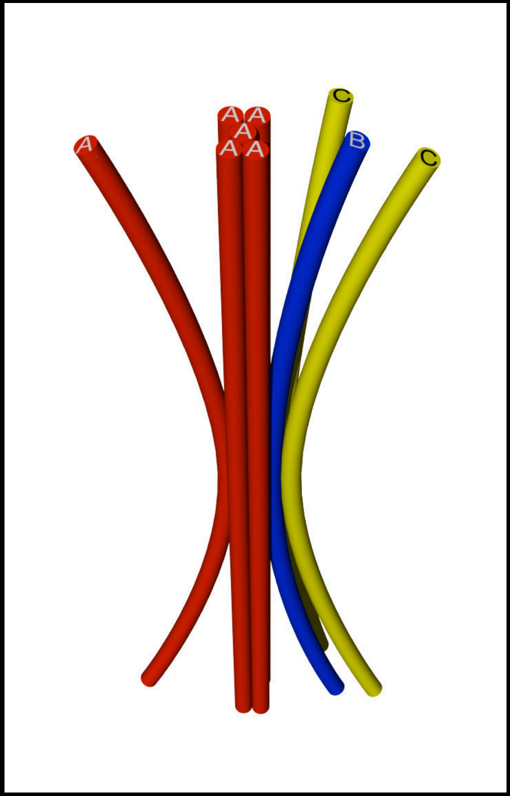
- Implementing this mechanism, it was argued that the E_7/ $(SU(5)xU(1)^3)$ Kugo-Yanagida coset appears at a multiple singularity enhancement from SU(5) to E7



$E7/(SU(5) \times U(1)^3)$

The aim of this talk

- is to prove that this is correct by an anomaly consideration
- We clarify whether one can realize a Kahler coset of the form $G/(H \times U(1)^r)$ with $r \geqq 2$ as a local matter spectrum without conflicting anomaly cancellation
- We will show that such a coset spectrum can indeed be realized at certain points in the moduli space of a 6D F-theory compactification on an elliptic CY3 over a Hirzebruch surface
  SM,Tani arXiv:1508.07423



Plan

1. Introduction
2. Anomaly analysis
3. Conclusions and discussion

2. ANOMALY ANALYSIS

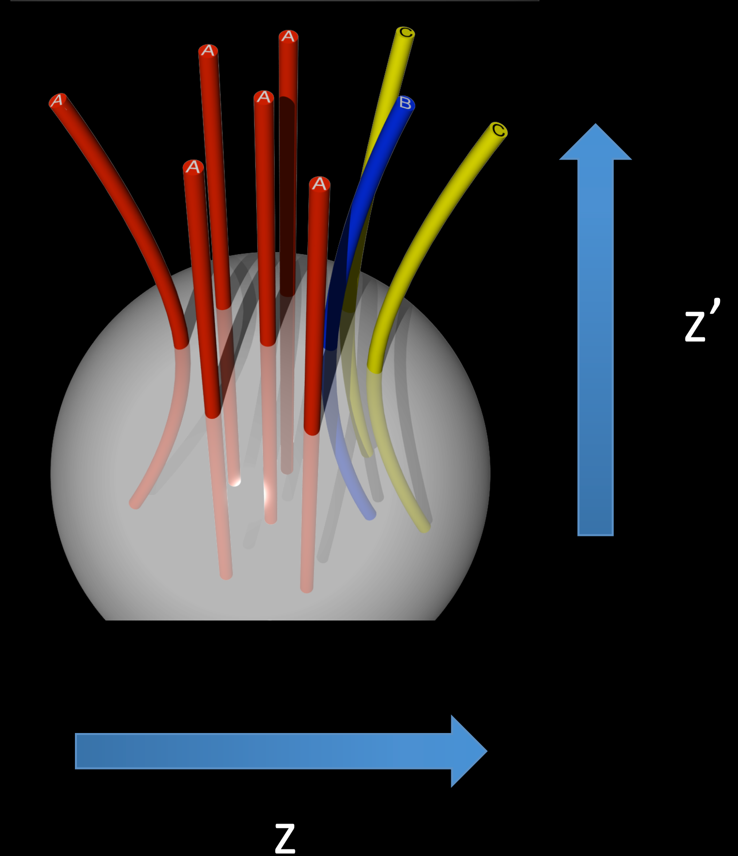
F-theory on an elliptic CY3 over Fn

F theory on an elliptic fibration over Fn

Morrison,Vafa

$$y^2 = x^3 + x \sum_{i=0}^{8} z^i f_{8+(4-i)n}(z') + \sum_{i=0}^{12} z^i g_{12+(6-i)n}(z')$$

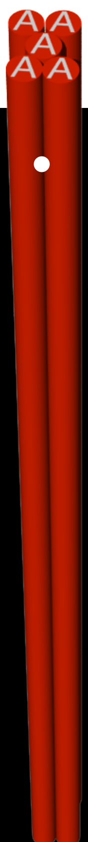
- Dual to heterotic on K3 BIKMVS
- 12+n of 24 instantons embedded in one of E8



Unbroken SU(5) curve

$$y^2 = x^3 + x\sum_{i=0}^{8} z^i f_{8+(4-i)n}(z') + \sum_{i=0}^{12} z^i g_{12+(6-i)n}(z')$$

- We take the coefficient functions f's and g's to be of the particular form



- They are so arranged that the discriminant starts with z^5

SU(5) singularity

$$\begin{aligned}
f_{4n+8} &= -3h_{n+2}^4,\\
f_{3n+8} &= 12h_{n+2}^2 H_{n+4},\\
f_{2n+8} &= 12\left(h_{n+2}q_{n+6} - H_{n+4}^2\right),\\
g_{6n+12} &= 2h_{n+2}^6,\\
g_{5n+12} &= -12h_{n+2}^4 H_{n+4},\\
g_{4n+12} &= 12h_{n+2}^2(2H_{n+4}^2 - h_{n+2}q_{n+6}),\\
g_{3n+12} &= -f_{n+8}h_{n+2}^2 + 24h_{n+2}H_{n+4}q_{n+6} - 16H_{n+4}^3,\\
g_{2n+12} &= -f_8 h_{n+2}^2 + 2f_{n+8}H_{n+4} + 12q_{n+6}^2
\end{aligned}$$

Independent polynomials

$$
\begin{aligned}
f_{4n+8} &= -3h_{n+2}^4, \\
f_{3n+8} &= 12h_{n+2}^2 H_{n+4}, \\
f_{2n+8} &= 12\left(h_{n+2}q_{n+6} - H_{n+4}^2\right), \\
g_{6n+12} &= 2h_{n+2}^6, \\
g_{5n+12} &= -12h_{n+2}^4 H_{n+4}, \\
g_{4n+12} &= 12h_{n+2}^2(2H_{n+4}^2 - h_{n+2}q_{n+6}), \\
g_{3n+12} &= -f_{n+8}h_{n+2}^2 + 24h_{n+2}H_{n+4}q_{n+6} - 16H_{n+4}^3, \\
g_{2n+12} &= -f_8 h_{n+2}^2 + 2f_{n+8}H_{n+4} + 12q_{n+6}^2
\end{aligned}
$$

- They are parameterized by the five functions

h_{n+2}, H_{n+4}, q_{n+6}, f_{n+8} and g_{n+12}

- The total degrees of freedom is

$$(n+3)+(n+5)+(n+7)+(n+9)+(n+13)-1 = 5n+36,$$

→ complex structure moduli

Discriminant

$$
\begin{aligned}
f_{4n+8} &= -3h_{n+2}^4, \\
f_{3n+8} &= 12h_{n+2}^2 H_{n+4}, \\
f_{2n+8} &= 12\left(h_{n+2}q_{n+6} - H_{n+4}^2\right), \\
g_{6n+12} &= 2h_{n+2}^6, \\
g_{5n+12} &= -12h_{n+2}^4 H_{n+4}, \\
g_{4n+12} &= 12h_{n+2}^2(2H_{n+4}^2 - h_{n+2}q_{n+6}), \\
g_{3n+12} &= -f_{n+8}h_{n+2}^2 + 24h_{n+2}H_{n+4}q_{n+6} - 16H_{n+4}^3, \\
g_{2n+12} &= -f_8 h_{n+2}^2 + 2f_{n+8}H_{n+4} + 12q_{n+6}^2
\end{aligned}
$$

- The singularity gets enhanced wherever either of h_{n+2} and P_{3n+16} vanishes

- The discriminant becomes $\Delta = 108z^5 h_{n+2}^4 P_{3n+16} + \cdots,$

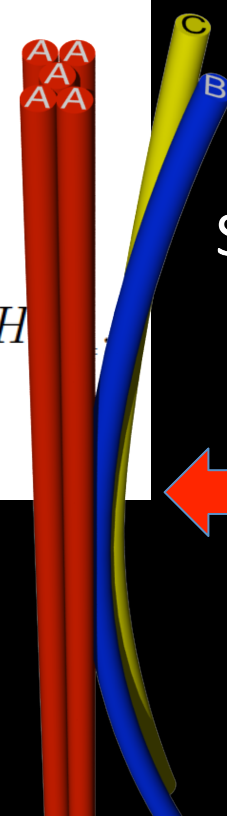
$$P_{3n+16} \equiv -2f_8 h_{n+2}^2 H_{n+4} - 2f_{n+8}h_{n+2}q_{n+6} + f_{8-n}h_{n+2}^4 + g_{n+12}h_{n+2}^2 - 24H_{n+4}q_{n+6}^2$$

Locus of h_{n+2} : 10 representation

$$
\begin{aligned}
\cancel{f_{4n+8}} &= -3h_{n+2}^4, \\
\cancel{f_{3n+8}} &= 12h_{n+2}^2 H_{n+4}, \\
f_{2n+8} &= 12\left(h_{n+2}q_{n+6} - H_{n+4}^2\right), \\
\cancel{g_{6n+12}} &= 2h_{n+2}^6, \\
\cancel{g_{5n+12}} &= -12h_{n+2}^4 H_{n+4}, \\
\cancel{g_{4n+12}} &= 12h_{n+2}^2(2H_{n+4}^2 - h_{n+2}q_{n+6}), \\
g_{3n+12} &= -f_{n+8}h_{n+2}^2 + 24h_{n+2}H_{n+4}q_{n+6} - 16H \\
g_{2n+12} &= -f_8 h_{n+2}^2 + 2f_{n+8}H_{n+4} + 12q_{n+6}^2
\end{aligned}
$$

- It turns out that the order of the discriminant becomes 7

SO(10) singularity



10

- The matter localized here is

  SO(10)/(SU(5) × U(1))

  = 10 representation

Locus of P_{3n+16} : 5 representation

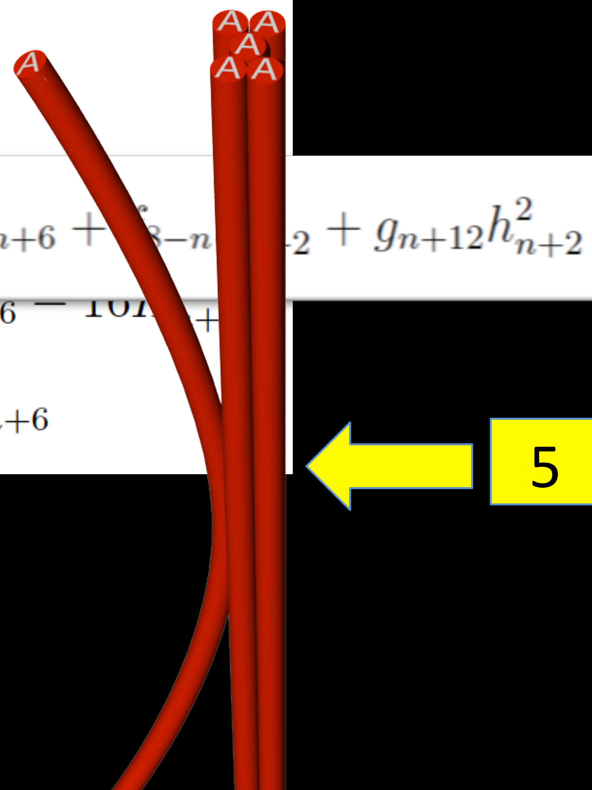
$$f_{4n+8} = -3h^4_{n+2},$$
$$f_{3n+8} = 12h^2_{n+2}H_{n+4},$$
$$f_{2n+8} = 12\left(h_{n+2}q_{n+6} - H^2_{n+4}\right),$$
$$g_{6n+12} = 2h^6_{n+2},$$
$$g_{5n+12} = -12h^4_{n+2}H_{n+4},$$

$$P_{3n+16} \equiv -2f_8h^2_{n+2}H_{n+4} - 2f_{n+8}h_{n+2}q_{n+6} + f_{8-n} \ldots {}_{2} + g_{n+12}h^2_{n+2} - 24H_{n+4}q^2_{n+6}$$

$$g_{3n+12} = -f_{n+8}h^4_{n+2} + 24h_{n+2}H_{n+4}q_{n+6} - 16 \ldots$$
$$g_{2n+12} = -f_8h^2_{n+2} + 2f_{n+8}H_{n+4} + 12q^2_{n+6}$$

- The order of the discriminant = 6

SU(6) singularity



SU(6)/(SU(5) × U(1))
= 5 representation

Matter for a generic SU(5) curve

$$(n+2)\mathbf{10}, \quad (3n+16)\mathbf{5}, \quad (5n+36)\mathbf{1}.$$

- Dual to K3 compactication of $E_8 x E_8$ heterotic string with instanton numbers (12 − n, 12 + n)
- Anomaly free

What happens when $h_{n+2} = P_{3n+16} = 0$?

$$\Delta = 108 z^5 h_{n+2}^4 P_{3n+16} + \cdots,$$

$$P_{3n+16} \equiv -2 f_8 h_{n+2}^2 H_{n+4} - 2 f_{n+8} h_{n+2} q_{n+6} + f_{8-n} h_{n+2}^4 + g_{n+12} h_{n+2}^2 - 24 H_{n+4} q_{n+6}^2$$

H_{n+4} or q_{n+6} has a common zero with h_{n+2}

- H_{n+4} has a common zero → E6
- q_{n+6} has a common zero → D6 = SO(12)

Common locus of h_{n+2} and q_{n+6}

$$
\begin{aligned}
f_{4n+8} &= -3h_{n+2}^4, \\
f_{3n+8} &= 12h_{n+2}^2 H_{n+4}, \\
f_{2n+8} &= 12\left(h_{n+2}q_{n+6} - H_{n+4}^2\right), \\
g_{6n+12} &= 2h_{n+2}^6, \\
g_{5n+12} &= -12h_{n+2}^4 H_{n+4}, \\
g_{4n+12} &= 12h_{n+2}^2(2H_{n+4}^2 - h_{n+2}q_{n+6}), \\
g_{3n+12} &= -f_{n+8}h_{n+2}^2 + 24h_{n+2}H_{n+4}q_{n+6} - 16H_{n+4}^3, \\
g_{2n+12} &= -f_8 h_{n+2}^2 + 2f_{n+8}H_{n+4} + 12q_{n+6}^2
\end{aligned}
$$

- The orders of f and g do not change
- The order of discriminant = 8

SO(12) singularity

- The localized matter will be

SO(12)/(SU(5) × U(1)^2)

= 10(SO(10))+10(SU(5))= 10+5+5 plus 1 from Cartan

Note that previously $\Delta = 108z^5h_{n+2}^4P_{3n+16} + \cdots,$

$$P_{3n+16} \equiv -2f_8h_{n+2}^2H_{n+4} - 2f_{n+8}h_{n+2}q_{n+6} + f_{8-n}h_{n+2}^4 + g_{n+12}h_{n+2}^2 - 24H_{n+4}q_{n+6}^2$$

- The localized matter will be

SO(12)/(SU(5) × U(1)^2)

= 10(SO(10))+10(SU(5))= 10+5+5 plus 1 from Cartan

→ n+2 SO(10), 3n+16 SU(6)

- Let us suppose (maximally degenerate case)

$$q_{n+6} = h_{n+2}q_4$$

for some q_4

- In this case the discriminant becomes

$$\Delta = 108z^5h_{n+2}^6P_{n+12} + \cdots,$$

$$P_{n+12} \equiv -2q_4f_{n+8} + g_{n+12} - 24q_4^2H_{n+4}$$

→ n+2 SO(12) singularities, n+12 SU(6) singularities

Common locus of h_{n+2} and q_{n+6}

- The localized matter will be

SO(12)/(SU(5) × U(1)^2)

= 10(SO(10))+10(SU(5))= 10+5+5 plus 1 from Cartan

- Maximally degenerate case

$$q_{n+6} = h_{n+2}q_4$$

⇨ n+2 SO(12) singularities, n+12 SU(6) singularities

- Independent polynomials

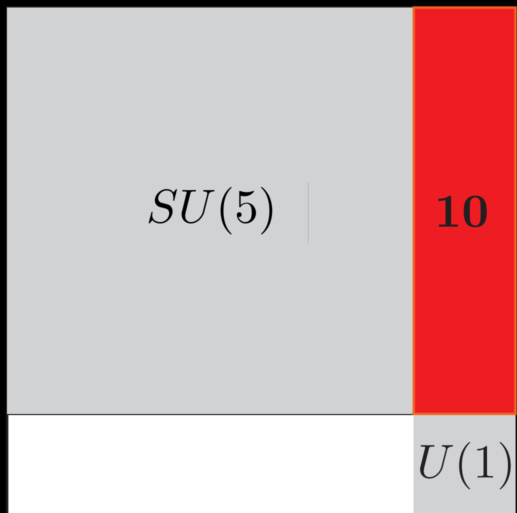
$$h_{n+2},\ H_{n+4},\ q_4,\ f_{n+8} \text{ and } g_{n+12};$$

$$(n+3)+(n+5)+5+(n+9)+(n+13)-1 = 4n+34$$

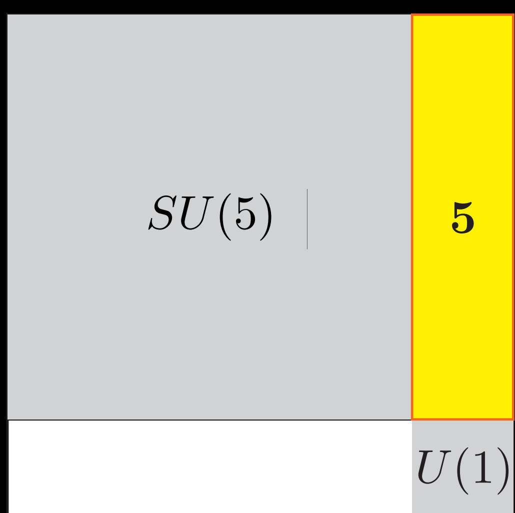
$$(n+2)\,(\mathbf{5}\oplus\mathbf{5}\oplus\mathbf{10}\oplus\mathbf{1})\oplus(n+12)\mathbf{5} = (n+2)\mathbf{10}\oplus(3n+16)\mathbf{5}\oplus(n+2)\mathbf{1},$$

⇨ in all $(n+2)\mathbf{10},\quad (3n+16)\mathbf{5},\quad (5n+36)\mathbf{1}.$ Anomaly free!

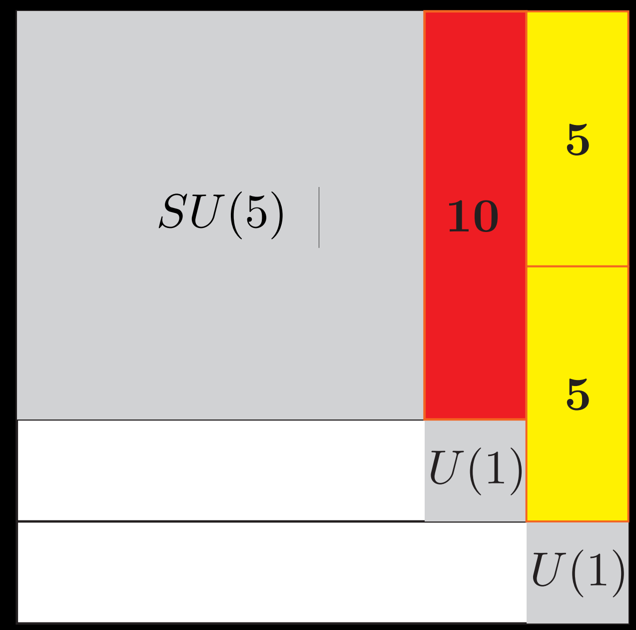
Where does the extra matter come from?



h_{n+2} locus, SO(10)singularity

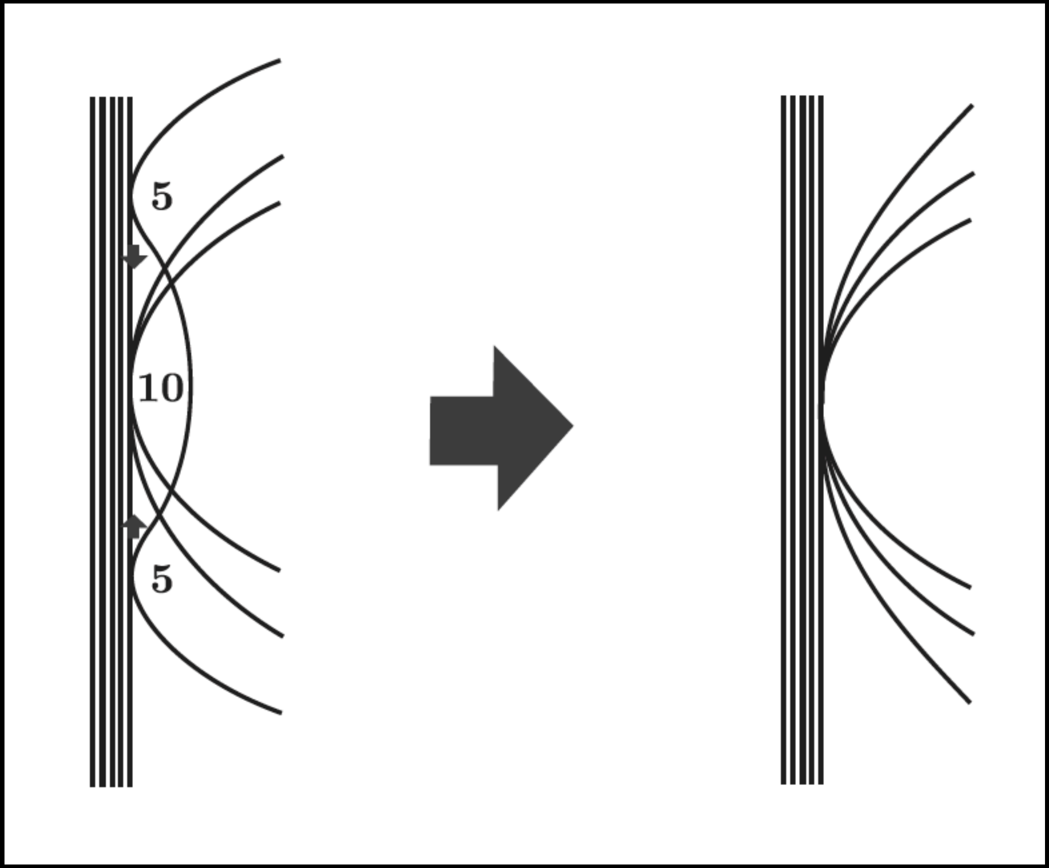


q_{n+6} locus, SU(6)singularity



h_{n+2},q_{n+6} common locus,
SO(12)singularity

Pairwise degeneration



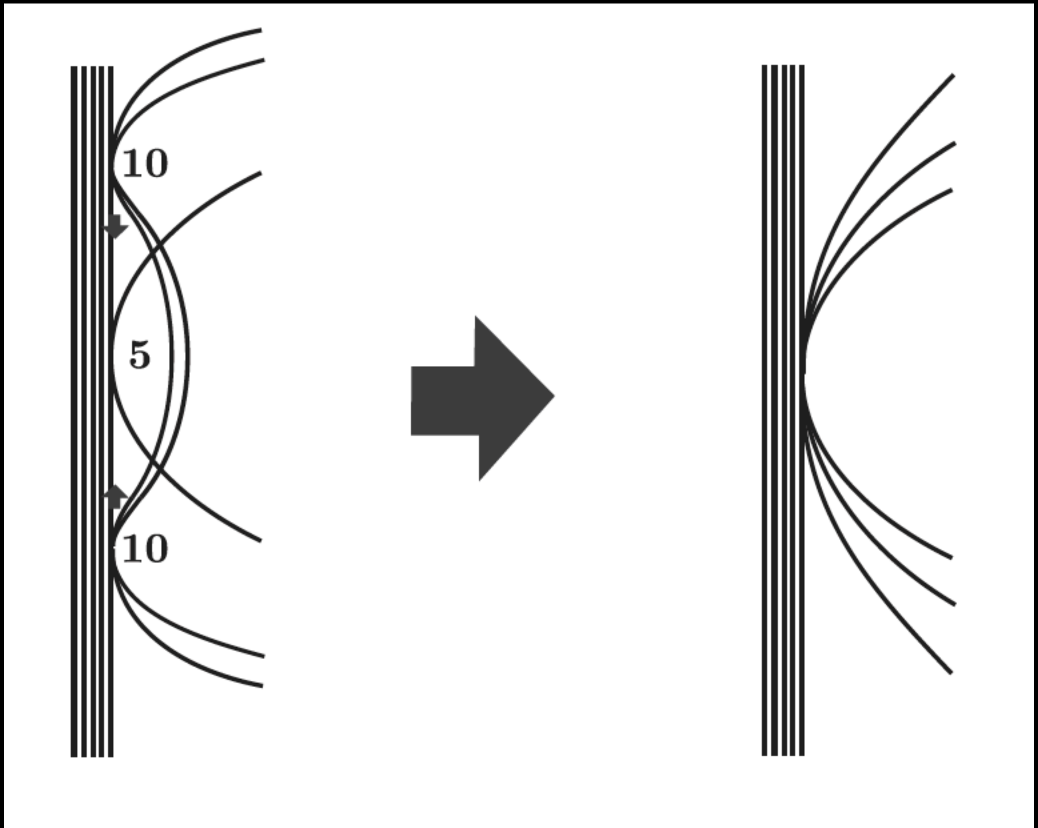
SU(5)→SO(12)


A
B
C

Enhancement to other singularities

- SU(5)→ D6 (SO(12)) is a special case because loci of 5 always pairwise coalesce with a locus of 10
- SU(5) → E6 : there are two cases
  - A Single 10 and a single 5 join ⇒ Does not form E6/(SU(5)xU(1)^2)
  - Two 10's and a single 5 join ⇒ E6/(SU(5)xU(1)^2) is realized

Pairwise degeneration(E_6)



SU(5)→E_6

Enhancement to other singularities

- SU(5)→ D6 (SO(12)) is a special case because loci of 5 always pairwise coalesce with a locus of 10
- SU(5) → E6 : there are two cases
  - A Single 10 and a single 5 join ⇒ Does not form E6/(SU(5)xU(1)^2)
  - Two 10's and a single 5 join ⇒ E6/(SU(5)xU(1)^2) is realized
- SU(5) → E7 : E7/(SU(5)xU(1)^3) is realized when and only when three 10's and four 5's coalesce
- SU(5) → E8 : E8/(SU(5)xU(1)^4) is realized when and only when five 10's and ten 5's coalesce

Such points indeed exist in the moduli space

3. CONCLUSIONS AND DISCUSSION

Conclusions

- We have proved, by an anomaly analysis, that Kugo-Yanagida-type Kahler coset spaces are indeed realized as matter spactra of localized hypermultiplets near multiple singularities in 6D F-theory compactified on a CY3 over Fn
- A multiple enhancement H→G does not always imply localized matter $G/(H\times U(1)^r)$ but only at some special points in the moduli space where enough number of matter curves simultaneously intersect

Discussion

- To generalize it to 4D F-theory we need to introduce G-fluxes
  SM,Tani in progress
- To consider the multiple singularity enhancement in F-theory has at least three virtues:

1. In general, a special point in the moduli space can be an end point of whatever flow in the moduli space after the supersymmetry is broken and potentials are generated
2. The multiple singularity may occur, in principle, in any elliptic Calabi-Yau manifold. Since the structure is universal, it may offer a potential ubiquitous mechanism for generating three generations of flavors in the framework of F-theory
3. The homogeneous Kahler structure of the spectrum of the multiple singularity is naturally endowed with conserved U(1)charges