Gravity theory on Poisson manifold with R-flux

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References

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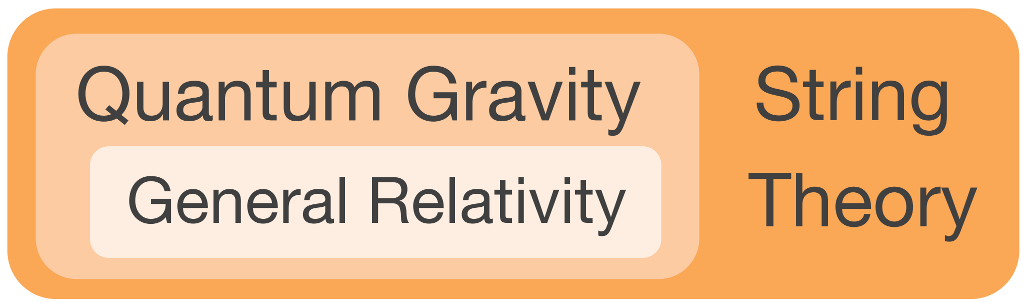
1508.05706 [hep-th] (Fortsch.Phys. 63 (2015) 683-704)

Space-time Geometry probed with strings

String theory would describe quantum gravity



Fundamental Objects	Point Particles	Strings
Space-time Geometry	Riemannian	? ? ? ? ?

Typical Example of T-duality

Compactifying on $X^9 \sim X^9 + 2\pi R$ (periodic)

Contribution to KK tower (Mass spectrum of reduced theory)

$=$ KK momenta $+$ Windings

$$P_9 = \frac{K}{R}$$

$$W_9 = \frac{1}{2\pi\alpha'} 2\pi N R = \frac{R}{\alpha'} N$$

$$R \leftrightarrow \frac{\alpha'}{R}$$

Reduced theories are physically equivalent:
T-duality [`84 Kikkawa, Yamasaki]

Why Generalized Geometry ?

T-duality implies a physical equivalence

between two different background geometries

(configurations of space-time metric and NSNS *B*-field)

suggesting the appearances of

- -Strange metric (T-folds) [`04 Hull], ...
- -Non-geometric fluxes [`05 Shelton, Taylor, Wecht], ...
- -Exotic branes [`10 de Boer, Shigemori], ...

➢ Machinery treating these as "geometry"

⇨ (Poisson) Generalized Geometry

Plan of Today's Talk

- Introduction & Motivations
- A Little Bit More on T-duality
- Generalized Geometry
  - -Definitions & Properties
  - -Generalized Riemannian Geometry
- Poisson Generalized Geometry
  - -Definitions & Properties
  - -Poisson Generalized Riemannian Geometry

A Little Bit More on T-duality: Buscher rule

T-duality: Physical equiv. between two backgrounds

$$(g, B) \sim (\tilde{g}, \tilde{B})$$

given by Buscher rule ("0": isometry) ['87 Buscher]

$$\tilde{g}_{00} = \frac{1}{g_{00}}, \quad \tilde{g}_{0i} = \frac{B_{0i}}{g_{00}}, \quad \tilde{g}_{ij} = g_{ij} - \frac{g_{0i}g_{0j} - B_{0i}B_{0j}}{g_{00}},$$

$$\tilde{B}_{0i} = \frac{g_{0i}}{g_{00}}, \qquad \tilde{B}_{ij} = B_{ij} - \frac{g_{0i}B_{0j} - g_{0j}B_{0i}}{g_{00}}.$$

Metric and B-field should be on equal footing

Generalized Geometry

['04 Hitchin, '04 Gualtieri]

Consider tangent and cotangent bundles at the same time!

- Canonical Inner Product

$$\langle X+\xi, Y+\eta \rangle = \frac{1}{2}(i_X\eta + i_Y\xi)$$

  - ➢ Invariant under an action of O(D,D)：T-duality transf.

- Operations corresponding to diffeo.＋B-field gauge transf.

$$[X+\xi, Y+\eta] = [X,Y] + \mathcal{L}_X\eta - \mathcal{L}_Y\xi - \frac{1}{2}d(i_X\eta - i_Y\xi)$$

Riemannian Geom. based on Gen. Geom.

- *O(D,D)*-invariant inner product

  ⇒ Decompose in Positive-/Negative-definite subbundles

$$C_{\pm} = \{X + (\pm g + B)(X) | X \in \Gamma(TM)\}$$

- Define a connection ∇ on positive-def. subbundle C_+ :

  Coefficients = Christoffel Symbol + *H*-flux (*B*-field's field strength)

$$\nabla_{\partial_i}(\partial_j)^+ = g^{lk}\left(2\underline{\Gamma_{kij}} + \underline{H_{kij}}\right)(\partial_l)^+$$

Gravity based on Gen. Geom.

- Curvature tensor:

$$R(X,Y)u := (\nabla_X \nabla_Y - \nabla_Y \nabla_X - \nabla_{[X,Y]_C})u$$

- Ricci scalar:

$$\underline{\mathcal{R}} - \frac{1}{4}\underline{H^{ijk}H_{ijk}}$$

- Einstein-Hilbert-like action：

$$S = \int d^D x \sqrt{g}\left(\mathcal{R} - \frac{1}{4}H^2\right)$$

  - This is the same as NSNS-sector of SUGRA

G.G. would be a good tool to "geometrize" string theory

Variant of Generalized Geometry

Slightly modifying structures of GG would be interesting：

- ❖ Vector + 1-form ⇨ unchanged
- ❖ Inner product (T-dual) ⇨ unchanged
- ❖ diffeo.＋B-field gauge trnsf. ⇨ changed!

$$[X+\xi, Y+\eta] = [X,Y] + \mathcal{L}_X\eta - \mathcal{L}_Y\xi - \frac{1}{2}d(i_X\eta - i_Y\xi)$$

⇨ Then how do we change it ???

Hint: A chain of T-duality

String action $\mathcal{L} \sim g(\dot{X}^2 + X'^2) + B\dot{X}X'$

⇨ conjugate momenta : $\Pi_i \sim g_{ij}\dot{X}^j + B_{ij}X'^j$

$[p_i, p_j] \sim H_{ijk}w^k$

momentum winding

T-duality : $p_i \leftrightarrow w^i = X'^i$

T

$[p_i, w^k] \sim F^k_{ij} w^j$

T

$[p_i, w^k] \sim Q_i^{kj} p_j$

T

$[w^i, w^j] \sim R^{ijk} p_k$

Fluxes associated with non-geom. background

Fluxes in Generalized Geometry

$$H_{ijk} \longleftrightarrow F_{ij}^{k} \longleftrightarrow Q_{k}^{ij} \longleftrightarrow R^{ijk}$$

$[\partial_i, \partial_j]$ $\sim H_{ijk}dx^k$

$[\partial_i, dx^k]$ $\sim F_{ij}^{k}dx^j$

Understood in terms of GG

$[\partial_i, dx^j]$ $\sim Q_{ik}^{j}\partial_k$??

$[dx^i, dx^j]$ $\sim R^{ijk}\partial_k$??

MISSING in GG!!

$[X, Y] \neq 0 \quad [\xi, \eta] = 0$

$[\xi, \eta] \neq 0 \quad [X, Y] = 0$

⇨ Variant of GG interchanging roles of Vector and 1-form

Poisson Geometry

Poisson bi-vector $\theta = \frac{1}{2}\theta^{ij}\partial_i \wedge \partial_j$

-Poisson bracket $\{f,g\} = \theta^{ij}\partial_i f \partial_j g$

-Jacobi id. $\{\{f,g\},h\} + \{\{g,h\},f\} + \{\{h,f\},g\} = 0$

$\Leftrightarrow$ $[\theta,\theta]_S = 0$ Poisson cond. $\theta^{l[i}\partial_l\theta^{jk]} = 0$

Schouten bracket: *e.g.* $[X \wedge Y, Z]_S = [X,Z] \wedge Y - [Y,Z] \wedge X$

Extension of Lie br. to multi-vector

Lie algebra on 1-forms

Lie bracket: $[\xi,\eta]_\theta = \mathcal{L}_{\theta(\xi)}\eta - \mathcal{L}_{\theta(\eta)}\xi + d(\theta(\eta,\xi))$

Cartan Algebra on Poly-Vector Fields

"Interior product" $\bar{\imath}_\xi$ $\qquad \bar{\imath}_\xi(X\wedge Y) = (i_X\xi)Y - (i_Y\xi)X$

"Exterior derivative" $d_\theta = [\theta, \cdot]_S$

-Nilpotent $d_\theta^2 = 0 \Leftarrow [\theta,\theta]_S = 0$

"Lie derivative"

$$\overline{\mathcal{L}}_\zeta f := \bar{\imath}_\zeta d_\theta f$$

$$\overline{\mathcal{L}}_\zeta \xi := [\zeta, \xi]_\theta \qquad \left([\xi,\eta]_\theta = \mathcal{L}_{\theta(\xi)}\eta - i_{\theta(\eta)}d\xi\right)$$

$$\overline{\mathcal{L}}_\zeta X := (d_\theta \bar{\imath}_\zeta + \bar{\imath}_\zeta d_\theta)X$$

"Cartan algebra": enables diff. calculus induced by 1-form

$$\{\bar{\imath}_\xi, \bar{\imath}_\eta\} = 0, \quad \{d_\theta, \bar{\imath}_\xi\} = \overline{\mathcal{L}}_\xi, \quad [\overline{\mathcal{L}}_\xi, \bar{\imath}_\eta] = \bar{\imath}_{[\xi,\eta]_\theta},$$
$$[\overline{\mathcal{L}}_\xi, \overline{\mathcal{L}}_\eta] = \overline{\mathcal{L}}_{[\xi,\eta]_\theta}, \quad [d_\theta, \overline{\mathcal{L}}_\xi] = 0.$$

Poisson Generalized Geometry 1408.2649

- Vector field and 1-form (same as GG)

- O(D,D)-invariant inner product (same as GG)

Poisson structure

- Operations different from GG： Vector field ⟺ 1-form

$$[X+\xi, Y+\eta] = [\xi,\eta]_\theta + \overline{\mathcal{L}}_\xi Y - \overline{\mathcal{L}}_\eta X - \frac{1}{2} d_\theta(\bar{\imath}_\xi Y - \bar{\imath}_\eta X)$$

cf. $$[X+\xi, Y+\eta]_C = [X,Y] + \mathcal{L}_X\eta - \mathcal{L}_Y\xi - \frac{1}{2} d(i_X\eta - i_Y\xi)$$

Physical intuition of New Operation

- Operation in GG :

$$[X+\xi, Y+\eta]_C = [X,Y] + \mathcal{L}_X\eta - \mathcal{L}_Y\xi - \frac{1}{2}d(i_X\eta - i_Y\xi)$$

$$\Pi_i = p_i + B_{ij}w^j$$

T-dual (*cf.* Marc's talk)

$$\widetilde{\Pi}^i = w^i + \beta^{ij}p_j$$

Poisson structure θ

- Operation in PGG :

$$[X+\xi, Y+\eta] = [\xi,\eta]_\theta + \mathcal{L}_\xi Y - \mathcal{L}_\eta X - \frac{1}{2}d_\theta(i_\xi Y - i_\eta X)$$

Riemann Geom. based on PGG 1508.05706

- *O(D,D)*-inv inner product

  ⇒ Decompose in Positive-/Negative-definite subbundles

$$C_{\pm} = \{\xi + (\pm G + \beta)(\xi) | \xi \in \Gamma(T^* M)\}$$

- Define a connection $\overline{\nabla}$ on positive-def. subbundle C_+ :

  Coefficients = Contravariant Levi-Civita + *R*-flux

$$\overline{\nabla}_{dx^i}(dx^j)^+ = \big(2\overline{\Gamma}^{ij}_k + G_{lk} R^{kij}\big)(dx^l)^+$$

$$R = d_\theta \beta = [\theta, \beta]_S$$

$$= (\theta^{l[i}\partial_l \beta^{jk]} + \beta^{l[i}\partial_l \theta^{jk]})\partial_i \wedge \partial_j \wedge \partial_k$$

Contravariant Levi-Civita conn. [`00 Fernandes]...

$$\begin{cases} \bar{\Gamma}^{\{ij\}}_k = \frac{1}{2}[\theta^{mn}(\partial_m G^{ji}) - \theta^{mi}(\partial_m G^{jn}) \\ \qquad - \theta^{mj}(\partial_m G^{in}) - G^{jl}(\partial_l \theta^{in}) - G^{il}(\partial_l \theta^{jn})]G_{nk} \\ \bar{\Gamma}^{[ij]}_k = \frac{1}{2}(\partial_k \theta^{ij}) \end{cases}$$

Two characteristic tensors G^{ij} & θ^{ij}

$$\Rightarrow \begin{cases} \overline{\nabla}_{dx^k} G^{ij} = 0 & \text{: Compatible with metric} \\ \overline{\nabla}_{dx^i} \theta^{jk} + (\text{ cyclic }) = 0 & \text{: Respecting Poisson strc.} \end{cases}$$

In particular, θ^{ij} is covariantly constant $\nabla_i \theta^{jk} = 0$ ⇨ LC conn.

Extension of the Levi-Civita respecting Poisson structure !

Gravity based on PGG

- Curvature tensor：

$$\overline{R}(\xi,\eta)u := (\overline{\nabla}_\xi \overline{\nabla}_\eta - \overline{\nabla}_\eta \overline{\nabla}_\xi - \overline{\nabla}_{[\xi,\eta]})u$$

- Ricci scalar:

$$\overline{\mathcal{R}} - \frac{1}{4} R^{ijk} R_{ijk}$$

R-flux

Curvature defined by Contra. LC

- Einstein-Hilbert-like action：

$$S = \int d^D x \sqrt{G} \left(\overline{\mathcal{R}} - \frac{1}{4} R^2 \right)$$

Summary

We gave

- a new geometric framework based on Poisson structure

  -T-dual counterpart of Generalized Geometry

- a well-defined formulation of *R*-flux

  -defined as a field strength of local bi-vector gauge potentials

- an Riemann geometry compatible with Poisson structure

Future directions

We established Riemann Geometry compatible with Poisson

- Well describable Non-geometric background ?
- Extension to quasi-Poisson, Nambu-Poisson structures
- Poisson is semi-classical limit of Non-commutativity
  $\Rightarrow$ "Riemann Geometry" on Non-commutative space
- etc...........