Improved GLSM for Exotic Five-brane

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Work in progress

In collaboration with Shin SASAKI (Kitasato Univ.)

THE Technique We Proposed in 2013

10D spacetime description

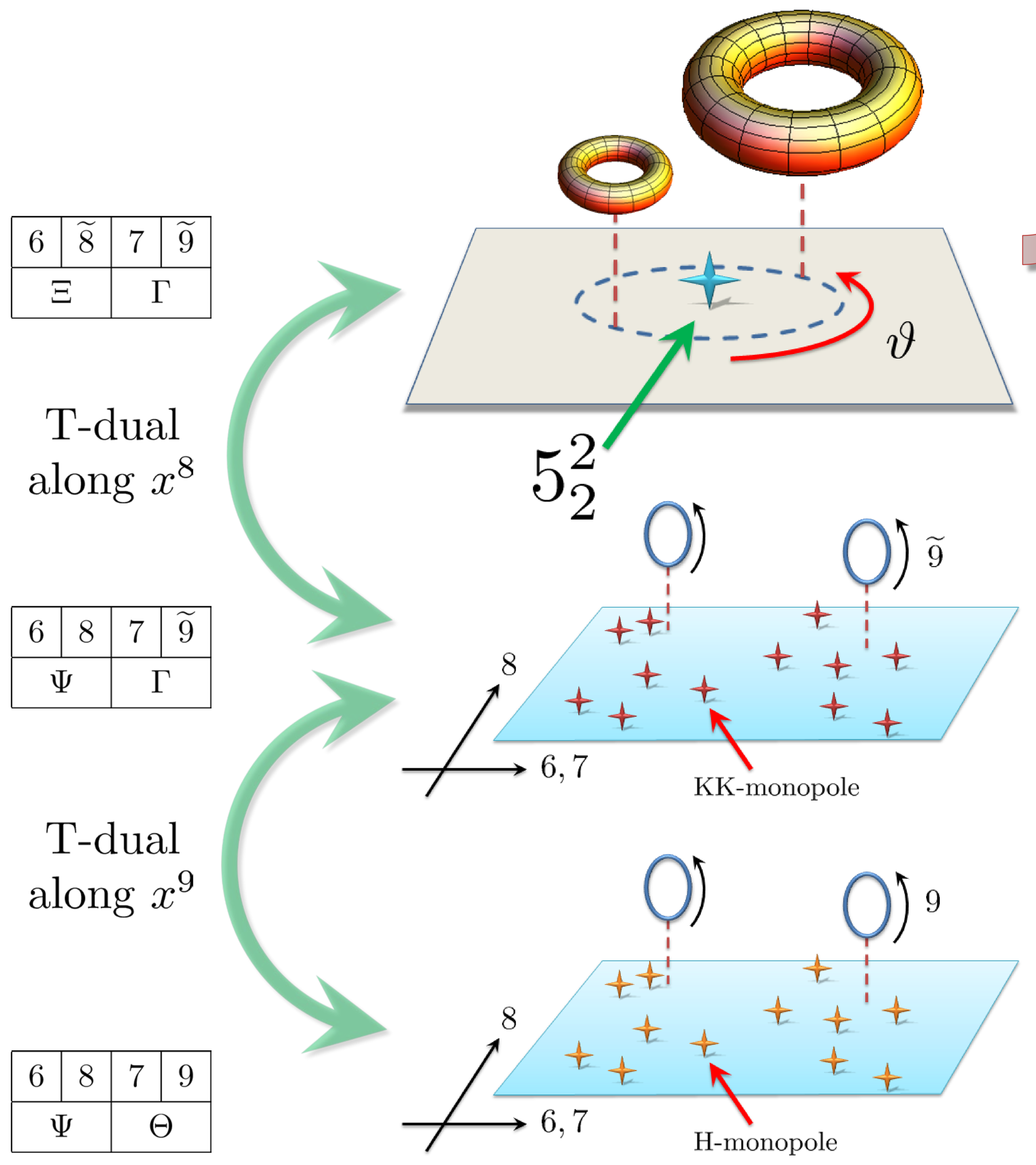
$$ds^2 = dx^2_{012345} + H\big[(d\varrho)^2 + \varrho^2 (d\vartheta)^2\big] + HK^{-1}\big[(d\widetilde{x}^8)^2 + (d\widetilde{x}^9)^2\big]$$

$$H = h + \sigma \log\left(\frac{\mu}{\varrho}\right) \qquad K = H^2 + (\sigma\vartheta)^2$$

$$B_{89} = -(\sigma\vartheta)K^{-1} \qquad e^{2\Phi} = HK^{-1}$$

2D N=(4,4) Gauge Theory

$$\begin{aligned}\mathscr{L}_{\rm GLSM} \ =\ & \sum_{a=1}^{k} \int d^4\theta \, \frac{1}{e_a^2}\Big(-\overline{\Sigma}_a \Sigma_a + \overline{\Phi}_a \Phi_a \Big) + \sum_{a=1}^{k} \int d^4\theta \Big\{ \overline{Q}_a e^{2V_a} Q_a + \overline{\widetilde{Q}}_a e^{-2V_a} \widetilde{Q}_a \Big\} \\ & + \int d^4\theta \Big\{ -\frac{g^2}{2}\Big(\Xi + \overline{\Xi} - \sqrt{2}\sum_{a=1}^{k}(C_a + \overline{C}_a)\Big)^2 \Big\} + \int d^4\theta \, \frac{g^2}{2}\Big(\Gamma + \overline{\Gamma} + 2\sum_{a=1}^{k} V_a \Big)^2 \\ & - \sqrt{2}\sum_{a=1}^{k} \int d^4\theta \, (\Psi - \overline{\Psi})(C_a - \overline{C}_a) \\ & + \sqrt{2}\sum_{a=1}^{k} \Big\{ \int d^2\theta \Big(\widetilde{Q}_a \Phi_a Q_a + s_a \Phi_a \Big) + \int d^2\widetilde{\theta} \, (t_a \Sigma_a) + (\text{h.c.}) \Big\} - \sqrt{2}\,\varepsilon^{mn} \sum_{a=1}^{k} \partial_m (\theta^9 A_{n,a})\end{aligned}$$



T-duality	
(free) string	sign-flip (parity) in right-mover momentum ↔ winding
spacetime	Buscher rule $(G_{mn}, B_{mn}) \to (G'_{mn}, B'_{mn})$
SUSY sigma model	Roček-Verlinde formula chiral ↔ twisted chiral

5^2_2-brane is from H-monopoles (smeared NS5-branes) or from KK-monopoles under string T-duality (see the left pictures). This never appears in gravitational theories *without B-field*. This object is exotic because :

- logarithmic harmonic function (co-dim. 2)
  → no well-defined asymptotic behaviors
- non-trivial monodromy charges
  → globally non-geometric structure
- etc...

5^2_2-brane has been investigated in supergravity from the "conventional spacetime viewpoint". Now we are ready to analyze it from the **string worldsheet** viewpoint, because the sigma model for NS5-branes/KK-monopoles is well established. String worldsheet theory will tell us much richer property of the 5^2_2-brane, because the theory naturally contains B-field on the target space. Here, we report that we found the worldsheet model ***as a 2D N=(4,4) SUSY gauge theory (GLSM)*** for the 5^2_2-brane (see the above), though it **seemed hard to construct** the worldsheet model caused by the above exotic features themselves.

Two Techniques

1: F-terms → D-terms chiral superfield → general superfield

$$\boxed{\Phi_a = \overline{D}_+ \overline{D}_- C_a}$$

$$\begin{aligned}\mathscr{L}_{\Psi} \ =\ & \int d^4\theta \, \frac{1}{g^2}\overline{\Psi}\Psi + \sqrt{2}\sum_{a=1}^{k}\Big\{ \int d^2\theta \, (-\Psi\Phi_a) + (\text{h.c.}) \Big\} \\ =\ & \int d^4\theta \Big\{ \frac{c}{g^2}(\Psi + \overline{\Psi})^2 - \sqrt{2}(\Psi + \overline{\Psi}) \sum_{a=1}^{k} (C_a + \overline{C}_a) + \frac{2c-1}{2g^2}(\Psi - \overline{\Psi})^2 - \sqrt{2}(\Psi - \overline{\Psi}) \sum_{a=1}^{k} (C_a - \overline{C}_a) \Big\}\end{aligned}$$

↓

$$\begin{aligned}\mathscr{L}_{RSX\Xi} \ \equiv\ & \int d^4\theta \Big\{ \frac{c}{g^2} R^2 - \sqrt{2} R \sum_{a=1}^{k}(C_a + \overline{C}_a) + R(\Xi_1 + \overline{\Xi}_1) + R(X + \overline{X}) \Big\} \\ & + \int d^4\theta \Big\{ \frac{2c-1}{2g^2}(iS)^2 - \sqrt{2}(iS) \sum_{a=1}^{k}(C_a - \overline{C}_a) + iS(\Xi_2 - \overline{\Xi}_2) + iS(X - \overline{X}) \Big\}\end{aligned}$$

integrate out Ξ_1, Ξ_2 and X → GLSM for KK-monopoles

integrate out R and Ξ_2 → new GLSM (see the right above)

2: shift symmetry vs dual coordinate analysis

The term $\int d^2\theta \, (\Psi - \overline{\Psi})(C_a - \overline{C}_a)$ looks pathological because this breaks the shift symmetry, i.e., the isometry on the geometry. BUT, this term plays an **essential** role! *If absent, ...*

- *IR theory is reduced to a chiral model: conflict w/ N=(4,4) SUSY,*
- *Target space metric is single-valued: trivial monodromy,*
- *Target space B-field does not appear: conflict w/ Buscher rule.*

This term yields the T-dual (non-geometric) coordinate, which is inevitable to derive the exotic brane geometry!

Worldsheet Model in IR

IR limit of the gauge theory is the NLSM on the 5^2_2-brane with B-field. The procedure is parallel to the one in the case of KK-monopoles :

1. find a SUSY vacuum $\mathscr{L}^{\rm pot}_{\rm GLSM} = 0$
2. solve the constraints on charged fields in $(Q_a, \widetilde{Q}_a)$
3. take IR limit $e_a \to \infty$ and integrate out the gauge fields
4. integrate out the T-dual Coordinate Fields

$\int d^2\theta \, (\Psi - \overline{\Psi})(C_a - \overline{C}_a)$ plays a crucial role in process 4.

We successfully produced the exotic five-brane metric with B-field !

(see **the left above "10D spacetime description"**)

But, very complicated...

- ✓ We have to prepare many auxiliary superfields in the first order Lagrangian such as R, iS, X, Ξ_1, Ξ_2 , so noisy!!
- ✓ Integration rule is so complicated.
- ✓ It is hard to applied this method to other nongeometric systems.

We must improve it!

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THE Technique We Show in 2015

Duality transformation without isometry
by virtue of **reducible** superfields
(reviewed by M.Grisaru, M.Massar, A.Sevrin, J.Troost in hep-th9801080)

$$\begin{aligned}\mathscr{L} &= K(A,\overline{A},L,\overline{L}) - AL - \overline{A}\,\overline{L} \\ &= K(A,\overline{A},L,\overline{L}) - \tfrac{1}{2}(A+\overline{A})(L+\overline{L}) - \tfrac{1}{2}(A-\overline{A})(L-\overline{L})\end{aligned}$$

chiral to complex linear

$$\frac{1}{g^2}\Psi \sim \overline{L} + 2\sqrt{2}\overline{C} \qquad 0 = \overline{D}_+\overline{D}_- L$$

twisted chiral to complex twisted linear

$$\frac{1}{g^2}\Theta \sim -\overline{\widetilde{L}} - 2V \qquad 0 = \overline{D}_+ D_- \widetilde{L}$$

$$\begin{aligned}\mathscr{L}_{\Psi\Theta} &= \frac{1}{g^2}\int \mathrm{d}^4\theta \Big\{ + |\Psi|^2 - |\Theta|^2 \Big\} \\ &\quad - \Big\{ \sqrt{2}\int \mathrm{d}^2\theta\, \Psi\Phi + (\mathrm{h.c.}) \Big\} - \Big\{ \sqrt{2}\int \mathrm{d}^2\widetilde{\theta}\, \Theta\Sigma + (\mathrm{h.c.}) \Big\} \\ &\sim g^2 \int \mathrm{d}^4\theta \Big\{ - \big|L + 2\sqrt{2}C\big|^2 + \big|\widetilde{L} + 2V\big|^2 \Big\}\end{aligned}$$

It is quite simple and easy to formulate N=(2,2) systems.
Of course, we can apply this method to systems with isometry.

The complex (twisted) linear superfields are reducible, i.e., they can be described as a sum of irreducible superfields such as chiral and twisted chiral superfields.

$$\begin{cases} L = X + Y + \overline{Z} & \text{complex linear} \\ \widetilde{L} = X + Y + \overline{W} & \text{complex twisted linear} \end{cases}$$

where $\begin{cases} X, W : & \text{chiral} \\ Y, Z : & \text{twisted chiral} \end{cases}$

This method will admit further T-duality transformations, and give us new descriptions of globally nongeometric objects (maybe).

NEXT Quests

- ✓ Analyze worldsheet instantons via gauge theory instantons (see our work arXiv:1305.4439)
- ✓ Explore quantum moduli space as in N=(4,4) GLSM
- ✓ Relation to DFT and β-supergravity (see works by Hull and Zwiebach, Andriot, and many guys)
- ✓ S-duality
- ✓ Etc., etc…