Proceedings of the 2nd Workshop on Phenomenology for Particle and Anti-Particle 2019 (PPAP2019)

25th-27th March 2019 , Graduate School of Science, Hiroshima University

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Abstract

This is a proceedings for PPAP 2019. Thanks to speakers and participants, the workshop was timely and lively. The topics cover the uniqueness of the standard model and beyond, flavor and modular symmetries from string compactification, D meson mixing, Dark matter and Dark energy, Lorentz invariant and CPT breaking effect, modular symmetry and flavor mixing and masses, relation between CP violation of quark and lepton, and Higgs mass formulas in the left-right model.

1 Program

2nd workshop on Phenomenology for Particle and Anti-Particle 2019 (March 25-27)

Place: Room E002 (Graduate School of Science, Hiroshima Univ.)

Schedule March 25 (Mon.)

Time	Title of Talk	Speaker
10:00-10:30	Registration	
10:30-10:35	Opening	Organizer
10:35-12:05	Uniqueness of the standard model and beyond	Chao-Qiang. Geng (National Tsing Hua Univ.)
12:05-14:00	Lunch	
14:00-15:30	Introduction to string phenomenology	Tatsuo Kobayashi (Hokkaido University)
15:30-16:00	Break	
16:00-16:30	Novel inclusive analysis for D meson mixing	Hiroyuki Umeeda (Academia Sinica)
16:30-17:00	Dark matter mass and anisotropy in directional detector	Keiko Nagao (Okayama University of Science)
17:00-17:30	Lorentz invariant CPT breaking in the Standard Model: Neutrino oscillation and baryogenesis	Kazuo Fujikawa (Interdisciplinary Theoretical and Mathematical Sciences Program, RIKEN)

Schedule March 26 (Tue.)

Time	Title of Talk	Speaker
10:30-12:00	Flavor and modular symmetries from string compactification	Tatsuo Kobayashi (Hokkaido University)
12:00-14:00	Lunch	
14:00-15:30	Dark Energy and Gravitational Waves (I)	Chao-Qiang Geng (National Tsing Hua Univ.)
15:30-16:00	Break	
16:00-16:30	super-curvature-mode dark energy model	Kazuhiro Yamamoto (Hiroshima University)
16:30-17:00	Fermion masses and baryon and lepton number violation from finite modular groups	Hikaru Uchida (Hokkaido University)
17:00-17:30	Modular symmetric flavor model for quark and lepton mixing	Kenta Takagi (Hiroshima University)

Schedule March 27 (Wed.)

Time	Title of Talk	Speaker
10:30-11:30	Dark energy and Gravitational Waves (II)	Chao-Qiang. Geng (National Tsing HuaUniv.)
11:30-12:00	Sign of CP Violating Phase in Quarks and Leptons	Shunya Takahashi (Hiroshima University)
12:00-12:30	New mixing angles and Higgs mass formulas in the left-right symmetric model	Takesi Saito (Kwansei Gakuin Univ.)

All Short Talks include discussion time (5-10 min.)

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Phenomenology for Particle-Anti-Particle 2019 Uniqueness of the Standard Model and Beyond

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Abstract

Firstly, we review the three known chiral anomalies in four dimensions and use the anomaly free conditions to study the uniqueness of quark and lepton representations and charge quantizations in the standard model. We also extend our results to the theory with an arbitrary number of color and the family problem. Secondly, we examine the neutrino mass problem. Thirdly, we explore dark matter and dark energy. Finally, we present the future prospectives.

1 Introduction

Although the standard model (SM) [1] of $SU(3)_C \times SU(2)_L \times U(1)_Y$ has been remarkably successful experimentally, there are seven theoretical puzzles, which can be listed as follows:

(1) Why are there 15 states of quarks and leptons for each family?

(2) Why are the electric charges of particles quantized?

(3) Are these quantum numbers unique?

(4) Why are there three fermion generations?

(5) How to generate the fermion masses?

(6) Is there any new physics beyond the SM?

(7) What are the real natures of Dark Matter and Dark Energy?

In this talk, I would like to study these puzzles. In particular, I will answer the first three in the viewpoint of the chiral gauge anomaly cancellations and then explore the other four.

2 Anomalies in four-dimension

It is well-known that the anomaly free conditions arising from the theoretical requirements of renormalizability and self-consistency are the most elegant tool to test the gauge theory. Three anomalies thus far have been identified for chiral gauge theories in four dimensions: (1) The triangular (perturbative) chiral gauge anomaly [2], which must be canceled to avoid the breakdown of gauge invariance and renormalizability of the theory; we call this the *triangular* anomaly. (2) The global (non-perturbative) $SU(2)$ chiral gauge anomaly [3], which must be

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absent in order to define the fermion integral in a gauge invariant way; we call this the *global* anomaly. This anomaly was first pointed out by Witten [3], and is known as the Witten $SU(2)$ anomaly. He showed in 1982 that any $SU(2)$ gauge theory with an odd number of left-handed fermion (Weyl) doublets is mathematically inconsistent. (3) The mixed (perturbative) chiral gauge-gravitational anomaly [4, 5], which must be canceled in order to ensure general covariance of the theory; we call this the *mixed* anomaly. This anomaly was first discussed by Kimura in 1969 [4] and its consequences studied by Alvarez-Gaumé and Witten [5] in 1983, who concluded that a necessary condition for consistency of the theory coupled to gravity is that the sum of the $U(1)$ charges of the left-handed fermions vanishes, i.e., $TrQ = 0$.

We now review the three chiral anomalies for the simple Lie groups.

The Triangular Anomaly: It has been shown [6] that the simple Lie groups: $SU(2)$, $SO(2k+1)(k>2)$, $SO(4k)(k\geq 2)$, $SO(4k+2)(k\geq 2)$, $Sp(2k)$, G_2, F_4, E_6, E_7, and E_8 are safe groups. The only simple groups with possible triangular anomaly are the unitary groups $SU(n)(n\geq 3)$. Therefore, if we start with the groups which do not contain $SU(n)(n\geq 3)$ group, the theory will be free of triangular anomaly.

The Global Anomaly: We classify the simple Lie groups G into the following two classes. (I) $Sp(2k)(Sp(2)\simeq SU(2))$. These groups have the property of $\mathbf{\Pi}_4(Sp(2k)) = \mathbf{Z}_2$ [3, 7], where $\mathbf{\Pi}_4$ is the fourth homotopy group and $\mathbf{Z}_2$ is the two-valued discrete group (like parity). According to Witten [3], the group $G^{(I)} = Sp(2k)$ has global anomaly if the number of fermion zero modes (for $SU(2)$ group, it is equal to the number of fermion doublets) is odd. (II) $SU(n)(n\geq 3), SO(2k+1)(k>2), SO(4k)(k\geq 2), SO(4k+2)(k\geq 2), G_2, F_4, E_6, E_7$, and E_8. These groups ($G^{(II)}$) have no global anomaly since their fourth homotopy groups are trivial [3, 7], i.e.,

$$\mathbf{\Pi}_4(G^{(II)}) = 0 \ . \tag{1}$$

However, the interesting question arises as to how one can know at the level of $G^{(II)}$ whether such a theory is global anomaly-free when $G^{(II)}$ breaks down to groups which contain $G^{(I)}$. This question has been answered in the work [8] with a sufficient condition that for any simple group G, containing $Sp(2k)$ as a subgroup, and for which $\mathbf{\Pi}_4(G) = 0$, the vanishing of the triangular perturbative anomaly for Weyl representations of G will guarantee the absence of the global non-perturbative $Sp(2k)$ anomaly.

The Mixed Anomaly: This anomaly is non-trivial only for the theory in which there is $U(1)$ symmetry with non-zero total charges [4, 5]. Obviously, all the simple Lie groups ($G^{(I),(II)}$) are safe groups. Furthermore, when these groups break down to groups which contain $U(1)$, e.g.,

$$G \to g \times \prod_i U(1)_i \ , \tag{2}$$

unlike the previous case, there is no mixed anomaly since the $U(1)$ operators are the generators of G and must be traceless.

3 Uniqueness of fermion representations and charges in the standard model

The triangular anomaly-free of the standard model was first noted [9] in 1972 for each quark-lepton family. It was clear that with only the triangular anomaly-free condition [10] one could not explain the empirically determined quark-lepton representations and their quantized hypercharges. We now study [11] the question of the uniqueness of quarks and leptons in the standard model by insisting on all three anomaly-free conditions. We begin by allowing an arbitrary number of (left-handed) Weyl representations under the group of $SU(3)\times SU(2)\times U(1)$, i.e.,

$$\begin{array}{cccl} SU(3) & \times SU(2) & \times U(1) & \\ 3 & 2 & Q_i\,, & i=1,\cdots,j \\ 3 & 1 & Q'_i\,, & i=1,\cdots,k \\ \overline{3} & 1 & \overline{Q}_i\,, & i=1,\cdots,l \\ \overline{3} & 2 & \overline{Q}'_i\,, & i=1,\cdots,m \\ 1 & 2 & q_i\,, & i=1,\cdots,n \\ 1 & 1 & \overline{q}_i\,, & i=1,\cdots,p \\ .. & .. & .. & \end{array} \tag{3}$$

where the integers j,k,l,m,n and p and the $U(1)$ charges are all arbitrary. The triangular anomaly free conditions then lead to the following equations:

$$\begin{aligned} [SU(3)]^3 \quad &: \quad \sum_{i=1}^{j} 2 + \sum_{i=1}^{k} 2 - \sum_{i=1}^{l} 1 - \sum_{i=1}^{m} 2 + \cdots = 0\,, \\ [SU(3)]^2\,U(1) \quad &: \quad 2\sum_{i=1}^{j} Q_i + \sum_{i=1}^{k} Q'_i + \sum_{i=1}^{l} \overline{Q}'_i + 2\sum_{i=1}^{m} \overline{Q}'_i + \cdots = 0\,, \\ [SU(2)]^2\,U(1) \quad &: \quad 3\sum_{i=1}^{j} Q_i + 3\sum_{i=1}^{m} \overline{Q}'_i + \sum_{i=1}^{n} q_i + \cdots = 0\,, \\ [U(1)]^3 : 3\sum_{i=1}^{j} Q_i^3 \quad &+ \quad \frac{N}{2}\sum_{i=1}^{k} Q_i'^3 + \frac{3}{2}\sum_{i=1}^{l} \overline{Q}_i^3 + 3\sum_{i=1}^{m} \overline{Q}_i'^3 + \sum_{i=1}^{n} q_i^3 + \frac{1}{2}\sum_{i=1}^{p} \overline{q}_i^3 + \cdots = 0\,. \end{aligned} \tag{4}$$

The global $SU(2)$ anomaly-free condition is

$$3\,j + 3\,m + n + \cdots = 0 \; mod \; 2\,. \tag{5}$$

Finally the mixed anomaly-free condition is

$$[U(1)] : 3\sum_{i=1}^{j} Q_i + \frac{3}{2}\sum_{i=1}^{k} Q'_i + \frac{3}{2}\sum_{i=1}^{l} \overline{Q}_i + 3\sum_{i=1}^{m} \overline{Q}'_i + \sum_{i=1}^{n} q_i + \frac{1}{2}\sum_{i=1}^{p} \overline{q}_i + \cdots = 0\,. \tag{6}$$

Here, the ellipses represent all other possible representations and charges.

The minimality condition with chiral fermions and the three anomaly-free conditions [Eqs. (5)-(7)] lead to a unique solution $j=1$, $k=0$, $l=2$, $m=0$, $n=1$, $p=1$, and two solutions of $U(1)$ charges

$$Q_1 = \frac{1}{3}\,,\; \overline{Q}_1 = -\frac{4}{3}\,,\; \overline{Q}_2 = \frac{2}{3}\,,\; \overline{q}_1 = -2q_1 = -2,, \tag{7}$$

$$Q_1 = q_1 = \overline{q}_1 = 0\,,\; \overline{Q}_1 = -\overline{Q}_2\,, \tag{8}$$

where we have chosen the normalization $q_1 = -1$ in Eq. (9). This unique solution corresponds to 15 states of quarks and leptons for each family, which answers the first puzzle stated early. The charge solutions in Eqs. (9) and (10) are the "standard model" and the so called "bizarre" ones, respectively. We note that the "inert" state (1,1,0) for the "bizarre" solution [12] is a non-chiral representation and it must be excluded.

Without considering the "bizarre" solution, all the $U(1)$ charges are uniquely determined. In this case, the resulting Weyl representations of $SU(3)$ and $SU(2)$ and their $U(1)$ charges are those in the standard model (cf. Table 1). The electric charges of quarks and leptons are given in Table 1 where the electroweak symmetry is spontaneously broken down to $U(1)_{EM}$ by the Higgs mechanism.

Table 1. The quantum numbers of quark and lepton representations under $SU(3)_C \times SU(2)_L \times U(1)_Y$ and $SU(3)_C \times U(1)_{EM}$ $(i = 1, 2, 3)$.

Particles	$SU(3)_C$	$\times SU(2)_L$	$\times U(1)_Y$	$\rightarrow$	$SU(3)_C$	$\times U(1)_{EM}$
$\begin{pmatrix} u \\ d \end{pmatrix}_L^i$	3	2	$\frac{1}{3}$		$\begin{pmatrix} 3 \\ 3 \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
$u_L^{c\,i}$	$\overline{3}$	1	$-\frac{4}{3}$		$\overline{3}$	$-\frac{2}{3}$
$d_L^{c\,i}$	$\overline{3}$	1	$\frac{2}{3}$		$\overline{3}$	$\frac{1}{3}$
$\begin{pmatrix} \nu \\ e \end{pmatrix}_L^i$	1	2	-1		$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$e_L^{c\,i}$	1	1	2		1	1

For the standard model of $SU(3)_C \times SU(2)_L \times U(1)_Y$, we thus find that the requirements of minimality and freedom from all three chiral gauge anomalies lead to a unique set of Weyl representations (and their $U(1)_Y$ charges) of the standard group that correspond to the observed quarks and leptons of one family. Furthermore, the $U(1)_Y$ charges of these quarks and leptons are quantized and correctly determined by adding the mixed anomaly-free condition and thus a long-standing puzzle of the electric charge quantization of quark and lepton can be solved within the content of the standard model. As a result, the second and third puzzles are resolved.

Our results can be extended to the theory with an arbitrary number of color, N, as shown in Table 2.

Table 2. The quantum numbers of quark and lepton representations under $SU(N)_C \times SU(2)_L \times U(1)_Y$ and $SU(N)_C \times U(1)_{EM}$ $(i = 1, 2, 3)$.

Particles	$SU(N)_C$	$\times SU(2)_L$	$\times U(1)_Y$	$\rightarrow$	$SU(N)_C$	$\times U(1)_{EM}$
$\begin{pmatrix} u \\ d \end{pmatrix}_L^i$	N	2	$\frac{1}{N}$		$\begin{pmatrix} N \\ N \end{pmatrix}$	$\begin{pmatrix} \frac{N+1}{2N} \\ -\frac{N-1}{2N} \end{pmatrix}$
$u_L^{c\,i}$	$\overline{N}$	1	$-\frac{N+1}{N}$		$\overline{N}$	$-\frac{N+1}{2N}$
$d_L^{c\,i}$	$\overline{N}$	1	$\frac{N-1}{N}$		$\overline{N}$	$\frac{N-1}{2N}$
$\begin{pmatrix} \nu \\ e \end{pmatrix}_L^i$	1	2	-1		$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$e_L^{c\,i}$	1	1	2		1	1

We see that the "bizarre" solution for the quark sector may be viewed as the standard one when $N \rightarrow \infty$. It is interesting to note that the decay rate for the anomalous process of $\pi^0 \rightarrow \gamma\gamma$, given by

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) \propto N(Q_u^2 - Q_d^2)e^2 \equiv e^2, \tag{9}$$

is independent of the color number N due to $Q_u^2 - Q_d^2 = 1/N$ from the electric charges in Table 2. In fact, one can show that the above statement is true for any decay rate of an anomalous process.

In spite of the success of the standard model, it is still a mystery why the three anomaly cancellations, especially the global and the mixed ones, should be satisfied. Naturally one hopes that new physics beyond the standard model can provide us an explanation to this question. From the above studies we see that the three anomaly-free conditions in the standard model may be automatically satisfied if it comes from a large group, especially, a grand unification group. For example, with the E_6 grand unification theory, the triangular, the global, and the mixed anomalies are trivial at the level of E_6 which guarantees their freedom at the standard group level. We thus conclude that the resolution of the question of the uniqueness of the massless fermion representations and $U(1)_Y$ charges for the standard group – when viewed from the standpoint of the perturbative triangular and mixed chiral gauge-gravitational anomalies and the absence of the non-perturbative global $SU(2)$ chiral gauge anomaly in four dimensions – argues strongly for some new physics beyond the standard model.

4 Family problem

We now discuss the family issue. It is clear that, as one can see from the above study, the imposition of all three anomaly-free conditions for the standard model does not shed any immediate light on the "generation problem". In fact, the quantum numbers in Table 1 are generation blind. Moreover, if one enlarges the standard group to include an $SU(2)$ or $SU(3)$ group, one can show that the theories are precisely the one family fermion structure of the left-right symmetric model $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$ [11] and the chiral-color model $SU(3)_{CL} \times SU(3)_{CR} \times SU(2)_L \times U(1)_Y$ [13], respectively, instead of having a family group. Clearly, some new ideas [14] are needed to constrain on the number of families which would be a key to the new physics. We now present a toy model which gives rise to three families of quarks and leptons. In the standard model, in each family there are 15 Weyl spinors. With a right handed neutrino, the number becomes 16. For three families, the total numbers are 48. One may put all these 48 Weyl spinors into a flavor box to form a large global symmetry as $U(48)$ [14] We can extend the group of $SU(N)\times SU(2)\times U(1)$ with both even and odd numbers of N to a larger group of $SU(N) \times SU(2) \times SU(2)$ in which N has to be an even number. For N=4, it is just the Pati-Salam model [15], which contains a right-handed neutrino. We remark that the representations under $SU(N) \times SU(2) \times SU(2)$ are unique unlike the case with a $U(1)$ symmetry and there is no more "bizarre" solution like the one in Eq. (9).

We now take the global flavor symmetry $U(48)$ and gauge its subgroup $SU(12) \times SU(2) \times SU(2)$ so that the fermions transform according to the representations given in Table 3 with $N = 12$. Thus, the model is a generalized Pati-Salam theory with the color being 12. The symmetry breaking chains by various suitable scalars are given as follows:

$$\begin{array}{c}
SU(12)_C \times SU(2)_L \times SU(2)_R \\
\begin{array}{ccc} 12 & 2 & 1 \\ \overline{12} & 1 & 2 \end{array} \\
\downarrow \\
SU(4)_{C3} \times SU(4)_{C2} \times SU(4)_{C1} \times SU(2)_L \times SU(2)_R \times U(1) \times U(1) \\
\downarrow \\
SU(4)_C \times SU(2)_L \times SU(2)_R \\
\downarrow \\
SU(3)_C \times SU(2)_L \times U(1)_Y \\
\underbrace{\textit{three quark and lepton families}}
\end{array}$$

Therefore, there are three generations of quarks and leptons under the standard group of $SU(3)_C \times SU(2)_L \times U(1)_Y$. However, before taking this model seriously, more works have to be done.

5 Neutrino mass generation

Neutrino oscillations observed by the solar, atmospheric, and reactor neutrino experiments [16] have revealed that neutrinos are massive but tiny and mix with each other [17]. The neutrino mixing matrix $V_{\rm PMNS}$ [18] can be parametrized as follows [17]:

$$V_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}, \quad (10)$$

where $s_{ij}(c_{ij})$=$\sin\theta_{ij}$ $(\cos\theta_{ij})$ with θ_{ij} being the mixing angles, δ is the Dirac CP violation phase, and α_{21} and α_{31} are the two Majorana CP violation phases. The recent global best-fit $(\pm 1\sigma)$ values from the neutrino oscillation data are given by [17]

$$\begin{aligned} \sin^2\theta_{12} &= 0.297^{+0.057}_{-0.047}\,,\ \sin^2\theta_{23} = 0.425^{+0.190}_{-0.044}\ \left(0.589^{+0.047}_{-0.205}\right), \\ \sin^2\theta_{13} &= 0.0215^{+0.0025}_{-0.0025}\ \left(0.0216^{+0.0026}_{-0.0026}\right),\ \ \delta/\pi = 1.38^{+0.52}_{-0.38}\ \left(1.31^{+0.57}_{-0.39}\right), \\ \Delta m^2_{21} &= \left(7.37^{+0.59}_{-0.44}\right)\times 10^{-5}\,{\rm eV}^2\,, \\ \Delta m^2_{32} &= \left(2.56^{+0.15}_{-0.11}\right)\times 10^{-3}\,{\rm eV}^2\ \left((2.54\pm 0.12)\times 10^{-3}\,{\rm eV}^2\right), \end{aligned} \quad (11)$$

with 3σ and 2σ ranges for the normal (inverted) neutrino mass hierarchy and the Dirac CP violation phase δ, respectively.

From the results in Eq. (11), it is clear that at least two neutrinos carry nonzero masses. However, the origin of these small masses is still a mystery. In the standard model, the neutrino masses have to be all zero duo to the absence of the right-handed neutrinos (ν_R) and the chiral nature of the left-handed ones (ν_L). For theories beyond the standard model, the simplest way to obtain nonzero neutrino masses is to include ν_R so that the Yukawa interaction for neutrinos exists, leading to Dirac neutrino masses after the electroweak symmetry breaking, just like the charged fermions. In order to account for the neutrino data in Eq. (11), extreme small Yukawa couplings of $O(10^{-13}-10^{-12})$ are inevitably required, which are commonly believed to be too small to be natural.

Apart from the mass generation of Dirac neutrinos with ν_R, seesaw mechanisms with type-I [19], type-II [20] and type-III [21] have been proposed to generate masses for Majorana neutrinos by realizing the Weinberg operator [22] $(\bar{L}^c_L H)(H^T L_L)$ at tree-level, where H and L_L are the doublets of Higgs and left-handed lepton fields, respectively. In these scenarios, either heavy degrees of freedom or tiny coupling constants are needed in order to conceive the small neutrino masses. On the other hand, models with the Majorana neutrino masses generated at one-loop [23], two-loop [24, 25] and higher loop [26] diagrams have also been proposed without introducing ν_R. Due to the loop suppression factors, the strong bounds on the coupling constants and heavy states are relaxed, resulting in a somewhat natural explanation for the smallness of neutrino masses. However, in most of the above radiative neutrino models, since only $SU(2)_L$ singlet scalars are introduced, new physics effects are limited in the lepton sector, whereas those involving hadrons, such as the neutrinoless double beta decay ($0\nu\beta\beta$) believed as a benchmark of the Majorana nature of neutrinos, do not show up. On the other hand, a special type of neutrino models has been proposed [25], in which a doubly charged singlet

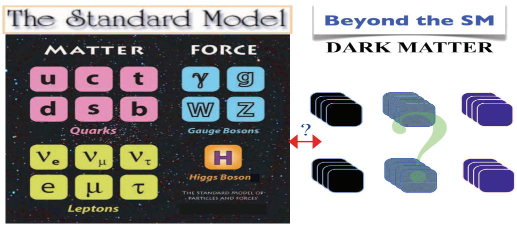

Figure 1: What is the nature of DM?

scalar Ψ : $(1, 4)$ and a triplet Δ : $(3, 2)$ under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ are introduced to yield the new Yukawa coupling $\Psi \bar{\ell}^c_R \ell_R$ with the right-handed charged lepton ℓ_R as well as the effective gauge coupling $\Psi^{\pm\pm} W^{\mp} W^{\mp}$ due to the mixing between $\Psi^{\pm\pm}$ and $\Delta^{\pm\pm}$, leading to the neutrino masses through two-loop diagrams [25]. It is interesting to note that $\Psi^{\pm\pm} W^{\mp} W^{\mp}$ can also be induced from non-renomalizable high-order operators [27]. One of the most interesting features of these models is that $0\nu\beta\beta$ is dominated by the short-range contribution at tree level due to the effective coupling of $\Psi^{\pm\pm} W^{\mp} W^{\mp}$, unlike other radiative Majorana neutrino mass models in which $0\nu\beta\beta$ is suppressed as it arises from the traditional long-range one proportional to neutrino masses from loops.

6 Dark matter and dark energy

Dark Matter (DM) and Dark Energy (DE) consist of about 27% and 68% of the energy densities of the Universe, respectively. Although they are the main constituents (95%) of the Universe, they are dark as we cannot see them directly though the normal ways based on electric and magnetics interactions. Around 1930s, Zwicky used the radial velocity dispersion in the Coma cluster to conclude that the mass of the luminous mass is only about 10% of the gravitation mass needed to have the stable cluster. This kind of the missing gravitation mass is now called DM. In these almost 90 years, there has been an overwhelming evidence [28] for the existence of DM in our universe through gravitational interactions in Galactic, Galaxy cluster and Cosmological scales, respectively. In particular, the experimental result on DM is given by

$$\Omega_{DM} h^2 \quad = \quad 0.1196 \pm 0.0031 \,, \tag{12}$$

which is clearly a measurement with great precision. Despite the success, the real nature of DM is still a mystery. It is known that DM cannot be the particle in the SM, which has to be: (a) Massive; (b) Non baryonic; (c) No (electric or color) charge; and (d) Stable with $\tau_{DM} > 10^{26} s$ while $\tau_{universe} \sim 10^{17} s$. Nowadays, from the particle physics point of view, the most popular DM candidates are Weakly Interacting Massive Particles (WIMPs), Axions and Sterile Neutrinos. In the SM, there are three generations of quarks and leptons along with the Higgs boson as well as strong and electroweak interactions. What is about DM? Is it a single particle or a bunch or a few classes of ones like those in the SM (see Fig. 1)?

How to detect DM? It is clear that it is hopeless if it only involves the gravitation interaction. Beyond it, there exist four types of searches: direct, indirect and collider detections and astrophysical probes. Since DM is distributed in our galaxy and the universe, it is widely believed that its annihilations and decays would give rise to the visible signals in terms of light stable particles, such as positrons/electrons, (anti)protons, photons and neutrinos, which can be observed in the sky and around the Earth. As a result, such an indirect search for DM [29] is regarded as one of the most promising ways to detect DM. On the other hand, One promising way to search for WIMPs is direct DM detections [30], which try to measure the nuclear recoil energies in detectors deposited by WIMP collisions. So far, there is still no indication in either indirect or direct DM detections.

According to the recent cosmological observations, our universe is undergoing a late-time accelerating expansion phase, which can be realized by introducing a time independent vacuum energy, called DE, built in the ΛCDM model [31]. Although this standard model of cosmology fits well with the observational data, it fails to solve the cosmological constant problem, related to the hierarchy [32] and coincidence [33] ones. These problems have motivated people to explore new theories beyond ΛCDM, such as those with the dynamical DE [34]. A typical model of such theories is to modify the standard general relativity (GR) by promoting the Ricci scalar of R in the Einstein-Hilbert action to an arbitrary function, i.e., $f(R)$ [35]. Besides describing gravity as the behavior of curvature, in general the torsion effect should be considered in gravity theory. There are several torsion gravities, such as teleparallel gravity [36] and $f(T)$ [37]. The teleparallel gravity is the gravity, which is constructed by the curvature-free Weitzenböck connection and forms an alternative and equivalent gravity of GR. Due to equivalent property, one can extend the the non-minimally coupled idea from GR to teleparallel gravity, called teleparallel dark energy [38]. Even though GR and teleparallel gravity follow the same Einstein equation, teleparallel DE has been proved to be different from the scalar-tensor theory at the background, and linear scalar perturbation level [39].

7 Future prospectives

To give the future prospectives, we have to understand the history of particle physics. We may divide Modern Particle Physics into the following 7 Periods with each period being 15 years.
(1) Period before 1945: *Pre-Modern Particle Physics Period*;
(2) Startup Period (1945–1960): *Early contributions to the basic concepts of modern particle physics*;
(3) Heroic Period (1960–1975): *Formulation of the standard model of strong and electroweak interactions*;
(4) Period of Consolidation and Speculation (1975–1990): *Precision tests of the standard model and theories beyond the standard model*;
(5) Frustration and Waiting Period (1990–2005): *No evidence for new physics in experimental searches, but three dark clouds: (i) cosmic microwave fluctuations (2006 NP), (ii) dark energy (2011 NP) and (iii) neutrino oscillations (2015 NP) showing up*;
(6) Preparation Period (2005–2020): *The current period with the discovery of the Higgs particle as well as many proposed experiments*; and
(7) Super-Heroic Period (2020–2035): *The upgrade LHC searches for something unexpected along with the future 100 TeV great collider as well as gravitational wave detections, such as LISA.*

In this talk, we have discussed the most important seven questions in particle physics, in which some of them are still unsolved. In the Super-Heroic Period between 2020–2035, these unsolved questions would be answered, while many Nobel Prize works would be produced just like those during the Heroic Period (1960–1975). In sum, the party of particle physics is clearly not over yet. In fact, its *Super Party* is coming.

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Phenomenology for Particle-Anti-Particle 2019
Flavor and modular symmetries from string compactification

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Abstract

We review recent studies on the modular symmetry. It is shown that the modular symmetry transforms zero-modes each other in certain compactification. That is a sort of flavor symmetries. The modular forms for finite sub-groups of the modular symmetries are also constructed. A new type of flavor models, which use these finite sub-groups of the modular symmetry, is also outlined.

1 Introduction

Superstring theory is a promising candidate for the unified theory of our Nature, that is, all of interactions including gravity and matter such as quarks and leptons as well as the Higgs field. Superstring thoery could realize the standard model of particle physics, and solve several mysteries of particle physics and cosmology. Superstring theory predicts six-dimensional compact space in addition to our four-dimensional space-time. It is expected the compactification scale to be much higher than the weak scale. Thus, we have a huge energy scale gap between the compactification scale and the weak scale. Symmetry would be a good tool to make a bridge between high energy physics and low energy physics. Then, it is important to study mysteries by both top-down and bottom-up approaches complementarily.

One of important mysteries in particle physics is the origin of the flavor structure. Why there are three generations of quarks and leptons ? Why are quark and lepton masses so hierarchical ? What determines mixing angles and CP phases in the quark and lepton sectors ? It is still a challenging issue to understand the origin of the flavor structure.

Non-Abelian discrete flavor symmetries are an interesting approach to derive the flavor structure. Indeed, many studies have been done by use of various non-Abelian discrete symmetries such as S_N, A_N, $\Delta(3N^2)$, $\Delta(6N^2)$, etc. [1, 2, 3, 4]. A compact space has a geometrical symmetry. Thus, such geometrical symmetries could be origins of non-Abelian discrete flavor symmetries. In addition to such geometrical symmetries, stringy coupling selection rules can lead to certain discrete flavor symmetries [5, 6]. (See also Refs. [7, 8, 9].)

The torus compactification as well as the orbifold compactification has the so-called modular symmetry. The modular symmetry includes S_3, A_4, S_4 as finite sub-groups [10], while these symmetries have been used as flavor symmetries in many flavor models. Thus, it would be interesting to study the modular symmetry and its finite sub-groups by both top-down and bottom-up approaches complementarily in order to solve the flavor mystery.

In this talk, we review on recent studies on the modular symmetry. We study modular transformation properties of zero-modes in the torus compactification with magnetic fluxes. We also construct modular forms of weight 2 for finite sub-groups of the modular symmetry. Moreover, we outline a new type of flavor model building using the finite sub-groups of the modular symmetry and modular forms.

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2 Magnetic flux compactification

Here, we give a brief review on the torus compactification with magnetic fluxes [11]. The higher dimensional super Yang-Mills theory appears as effective field theory of D-brane models. For simplicity, let us consider the six-dimensional $U(1)$ theory on two-dimensional torus T^2. We use the complex coordinate $z = x^1 + \tau x^2$ on T^2, where τ is the complex modulus parameter, and x^1 x^2 are real coordinates. The metric on T^2 is given by

$$g_{\alpha\beta} = \begin{pmatrix} g_{zz} & g_{z\bar{z}} \\ g_{\bar{z}z} & g_{\bar{z}\bar{z}} \end{pmatrix} = (2\pi R)^2 \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}. \tag{1}$$

We identify $z \sim z+1$ and $z \sim z+\tau$ on T^2.

We introduce the magnetic flux,

$$F = i\frac{\pi M}{\mathrm{Im}\tau} dz \wedge d\bar{z}, \tag{2}$$

which corresponds to the vector potential,

$$A(z) = \frac{\pi M}{\mathrm{Im}\tau}\mathrm{Im}(\bar{z}dz), \tag{3}$$

where M must be integer.

Now, let us consider the spinor with $U(1)$ $q = 1$. The spinor on T^2 has two componets, $\psi_\pm$. Its zero-mode equation with the above gauge background

$$i \not{D}\psi_\pm = 0, \tag{4}$$

has chiral solutions for either ψ_+ or ψ_- depending on the sign of M. When M is positive, ψ_- has no solution, but ψ_+ has M degenerate zero-mode solutions, whose profiles are written by

$$\psi^{j,M}(z) = \mathcal{N} e^{i\pi M z \frac{\mathrm{Im}z}{\mathrm{Im}\tau}} \cdot \vartheta \begin{bmatrix} \frac{j}{M} \\ 0 \end{bmatrix} (Mz, M\tau), \tag{5}$$

with $j = 0, 1, \cdots, (M-1)$, where ϑ denotes the Jacobi theta function,

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) = \sum_{l \in \mathbf{Z}} e^{\pi i (a+l)^2 \tau} e^{2\pi i (a+l)(\nu+b)}. \tag{6}$$

Here, $\mathcal{N}$ denotes the normalization factor given by

$$\mathcal{N} = \left(\frac{2\mathrm{Im}\tau M}{\mathcal{A}^2} \right)^{1/4}, \tag{7}$$

with $\mathcal{A} = 4\pi^2 R^2 \mathrm{Im}\tau$. This degenerate number would correspond to the family number. When M is negative, ψ_- have $|M|$ degenerate zero-mode solutions, but ψ_+ has no zero-mode solution. Thus, we can realize a four-dimensional chiral theory. In what follows, we restrict ourselves to models with $M > 0$.

Similarly, we can study the T^2/Z_2 orbifold compactification with magnetic fluxes, where the number of zero-modes is different from one on T^2 [12]. One can also introduce discrete Wilson lines [13]. Staring with higher dimensional super $U(N)$ Yang-Mills theory, we compactify the extra dimensiona to orbifolds like T^2/Z_2, T^4/Z_2, T^6/Z_2, or $T^6/(Z_2 \times Z_2)$. Then, we can break the $U(N)$ group to smaller groups such as $SU(3) \times SU(2) \times U(1)$ and $SU(4) \times SU(2) \times SU(2)$ by magnetic fluxes and the Z_2 orbifold projection. As results, we can construct three-generation models, where the generation number is realized as the above degeneracy by magnetic fluxes with and without the Z_2 orbifold projection. Hence, we can construct realistic models. (See e.g. Refs. [14, 15, 16].)

3 Modular symmetry

Here, we study the modular transformation of zero-mode wavefunctions [11, 17, 18, 19]. Recently, its anomalies were also studied [20]. Following [17], we restrict ourselves to even magnetic fluxes M.

3.1 Modular symmetry of zero-modes

The torus T^2 is the division of two-dimensional real flat space by a lattice Λ, which is spanned by two lattice vectors, α_1 and α_2. The same lattice Λ is spanned by changing the lattice basis vectors, i.e. $SL(2,\mathbb{Z})$. Such transformation is represented by the modular transformation of the modulus parameter,

$$\tau \longrightarrow \frac{a\tau+b}{c\tau+d}. \tag{8}$$

This group includes two important generators, S and T,

$$S : \tau \longrightarrow -\frac{1}{\tau}, \tag{9}$$

$$T : \tau \longrightarrow \tau + 1. \tag{10}$$

Under S, the zero-mode wavefunctions $\psi^{j,M}(z)$ on T^2 with the magnetic flux M transform as

$$\psi^{j,M} \to \frac{1}{\sqrt{M}} \sum_k e^{2\pi i jk/M} \psi^{k,M}. \tag{11}$$

On the other hand, the zero-mode wavefunctions $\psi^{j,M}(z)$ transform as

$$\psi^{j,M} \to e^{\pi i j^2/M} \psi^{j,M}, \tag{12}$$

under T. Note that the modular symmetry transforms zero-modes each other, and that is the flavor symmetry in a sense.

Generically, the T-transformation satisfies [17]

$$T^{2M} = \mathbb{I}_{M\times M}, \tag{13}$$

on the zero-modes, $\psi^{j,M}$, where $\mathbb{I}_{M\times M}$ denotes the $(M\times M)$ unit matrix. Furthermore, in Ref. [17] it is shown that

$$(ST)^3 = e^{\pi i/4}\mathbb{I}_{M\times M}, \tag{14}$$

on the zero-modes, $\psi^{j,M}$. Hence, on $\psi^{j,M}$, T and $(ST)^3$ are represented by diagonal matrices, and they are Z_{2M} and Z_8 symmetries, respectively.

For example, we study the model with $M=2$. The S-transformation acts on the zero-modes,

$$\begin{pmatrix} \psi^{0,2} \\ \psi^{1,2} \end{pmatrix} \longrightarrow S \begin{pmatrix} \psi^{0,2} \\ \psi^{1,2} \end{pmatrix}, \qquad S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \tag{15}$$

and the T-transformation acts

$$\begin{pmatrix} \psi^{0,2} \\ \psi^{1,2} \end{pmatrix} \longrightarrow T \begin{pmatrix} \psi^{0,2} \\ \psi^{1,2} \end{pmatrix}, \qquad T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}. \tag{16}$$

They satisfy the following algebraic relations,

$$S^2 = \mathbb{I}, \qquad T^4 = \mathbb{I}, \qquad (ST)^3 = e^{\pi i/4}\mathbb{I}. \tag{17}$$

Their closed algebra corresponds to $(Z_8 \times Z_4) \rtimes S_3$ [18].

In Ref. [6], it was shown that the model with $M = 2$ has the D_4 flavor symmetry, and two zero modes, $\psi^{0,2}(z)$ and $\psi^{1,2}(z)$, correspond to the D_4 doublet. Indeed, the above modular group includes

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = T^2, \qquad C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = ST^2S. \tag{18}$$

The closed algebra including Z and C correspond to D_4. However, the difference between D_4 and the modular symmetry is that while Yukawa couplings as well as higher order couplings are invariant under D_4, those couplings transform non-trivially under the modular symmetry. See for details of its implication Ref. [18].

Similarly, we can study the modular transformation of zero-modes for larger M. For example, the modular group on zero-modes for $M = 4$ is $(Z_8 \times Z_8) \rtimes A_4$ [18].

3.2 Modular form

As mentioned in the previous section, Yukawa couplings transform non-trivially under the modular transformation. The holomorphic part of Yukawa coupling is written by

$$X^{i,M}(\tau) = \vartheta \begin{bmatrix} \frac{i}{M} \\ 0 \end{bmatrix} (0, M\tau)\,. \tag{19}$$

They transform

$$X^{j,M} \to \sqrt{\frac{-i\tau}{M}} \sum_k e^{2\pi i jk/M} X^{k,M}, \tag{20}$$

under S, and

$$X^{j,M} \to e^{\pi i j^2/M} X^{j,M}, \tag{21}$$

under T.

Using the above properties, we can construct the modular forms by products of $X^{i,M}$ [19]. For example, the following products,

$$Z_1 = (X^{0,2})^4 + (X^{1,2})^4, \qquad Z_2 = 2\sqrt{3}(X^{0,2})^2(X^{1,2})^2, \tag{22}$$

correspond to the S_3 doublet modular form of weight 2. Indeed, on $(Z_1, Z_2)^T$, S and T are represented by

$$\rho(S) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{23}$$

They satisfy $\rho(S^2) = \mathbb{I}_{2\times 2}$, $\rho(T^2) = \mathbb{I}_{2\times 2}$, and $\rho((ST)^3) = \mathbb{I}_{2\times 2}$. That is nothing but S_3. Note that the modular forms of weight 2 are fundamental. By their products, the modular forms of larger weights can be constructed.

Similarly, the following products,

$$Z_3 = (X^{0,2})^4 - (X^{1,2})^4, \qquad Z_4 = 2\sqrt{2}(X^{0,2})^3 X^{1,2}, \qquad Z_5 = 2\sqrt{2}X^{0,2}(X^{1,2})^3, \tag{24}$$

correspond to the S_4 triplet modular forms of weight 2. On $(Z_3, Z_4, Z_5)^T$, S and T are represented by

$$\rho(S) = \frac{1}{2}\begin{pmatrix} 0 & -\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & -1 & 1 \\ -\sqrt{2} & 1 & -1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -i \end{pmatrix}. \tag{25}$$

They satisfy $\rho(S^2) = \mathbb{I}_{3\times3}$, $\rho(T^4) = \mathbb{I}_{3\times3}$, and $\rho((ST)^3) = \mathbb{I}_{3\times3}$. That is isomorphic to S_4. Thus, (Z_3, Z_4, Z_5) corresponds to the S_4 triplet.

In Ref. [19], the modular forms of weight 2 corresponding to $\Delta(96)$ and $\Delta(384)$ triplets were also explicitly constructed. $\Delta(96)$ and $\Delta(384)$ are finite sub-groups of the modular symmetry.

4 New flavor models

The modular symmetry on τ satisfy the following algebraic relations,

$$S^2 = (ST)^3 = 1. \tag{26}$$

On top of that, when we impose T^N, such subgroups are isomorphic to S_3, A_4, S_4, A_5 for $N = 2, 3, 4, 5$, respectively [10]. As shown in the previous section, zero-modes transform each other under the modular symmetry, that is, the flavor symmetry. Yukawa couplings as well as other couplings are functions of the modulus and transform non-trivially under the modular symmetry. Note that even couplings of non-perturbatively induced terms also depend on the modulus and transform non-trivially under the modular symmetry. (See e.g. Ref. [21].)

Inspired by these aspects, a new type of flavor models were proposed in Ref. [22]. In this new approach, the three families of leptons are assigned to non-trivial representations of the A_4 flavor symmetry as the conventional flavor symmetry model. However, the A_4 symmetry is assumed to be the finite sub-group of the modular symmetry. The Yukawa couplings and right-handed Majorana masses as well as higher order couplings are required to be modular forms. The lepton and Higgs fields also have proper modular weights such that the Lagrangian is invariant under the A_4 modular symmetry. Then, realization of lepton masses and mixing angles was studied. Such a new approach to flavor models were extended by use of several groups such as S_3, A_4, S_4, A_5 [23, 24, 25, 26, 27, 28, 29, 30, 31]. In these studies, experimental values of quark and lepton masses and mixing angles as well as the CP phase were reproduced. See for explicit models and realization of quark and lepton masses and mixing angles as well as the CP phases [32, 33].

It is remarkable that when one fixes the vacuum expectation value of the modulus τ, the flavor symmetry is broken. In the conventional flavor symmetry models, scalar fields, which are the so-called flavon fields, are introduced and their vacuum expectation values break flavor symmetries. Thus, one can construct flavor symmetry models without flavon fields in the new type of approaches.

5 Conclusion

We have reviewed on recent studies on the modular symmetry. In the compactified theory, the modular symmetry transforms zero-modes each other. That is, the modular symmetry is a sort of flavor symmetries and zero-modes are their non-trivial representations. Furthermore, Yukawa coulings as well as higher order couplings transform non-trivially under the modular symmetry. That is different from the conventional flavor symmetry models.

Inspired by these aspects, the new type of flavor models was proposed and extensively studied. We can realize experimental values of quark and lepton masses and mixing angles as well as the CP phase. Hence, this approach is quite interesting. In this approach, one can construct flavor symmetry models without flavon fields.

We have shown zero-mode modular transformation in the simple torus compactification with magnetic fluxes. They represent not simple finite groups such as S_3, A_4, S_4, A_5, but larger groups including them. However, the simple groups such as S_3, A_4, S_4, A_5 have been used to realize quark and lepton masses and mixing angles. There is still a gap between the compactified theory and the bottom-up approach of flavor models. It is quite important to study more about the modular symmetry from both sides, the top-down and bottom-up approaches. In the string theoretical side, it is important to study geometrical symmetries like the modular symmetry for other compactifications and which compactifications can lead to simple finite groups such as S_3, A_4, S_4, A_5.

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Novel inclusive analysis for D meson mixing

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Abstract

For $D^0 - \bar{D}^0$ mixing parameters, the prediction of the standard model (SM) based on the operator product expansion (OPE) gives an order of magnitude smaller than experimental data. In this study, we consider methodology in which the dispersion relation providing strong constraints on the mixing parameters is used. By introducing a fictitious D meson that is sufficiently heavy, one ensures that the OPE is reliable. The mixing parameters associated with the low mass scale are predicted as a solution of the constraining equation.

1 Introduction

Theoretically, charm physics is a challenging field due to its unique mass scale; it is too heavy to apply the chiral perturbation theory and possibly too light to rely on the heavy quark expansion. Furthermore, perturbative QCD prediction cannot be applied since the coupling, α_s, is relatively large at the charm mass scale.

In the SM, it is known that the amplitude of $D^0 - \bar{D}^0$ mixing is suppressed due to the Glashow-Iliopoulos-Maiani (GIM) mechanism [1]. The only surviving contributions are from SU(3) breaking that is characterized by the mass difference between strange and down quarks and from CKM-suppressed diagrams with bottom quarks in the loop.

There exist two methods to calculate the mixing parameters: the exclusive analysis and the inclusive one. The former utilizes the hadronic level calculation, which relies on the data of D meson decays, while the latter is carried out on the basis of the quark-gluon picture. In this sense, the inclusive method offers a purely theoretical methodology. In the exclusive way, the topological approach [2] and the factorization-assisted topological approach [3] took into account two-body decays, reproducing half of the y value measured in experiments. These results indicate that the other multibody channels may need to be fully considered to describe the observables quantitatively.

In the inclusive analysis, however, the situation is more sutle: the quark level analysis is implemented in a languege of the OPE, relying on the quark-hadron duality. How large the duality violation effect is in the D meson system is a non-trivial issue, while the OPE method results in a successful descpription on the $B^0_{d,s} - \bar{B}^0_{d,s}$ mixing. The prediction of the inclusive analysis, including next-to-leading order QCD correction, exhibits $x \sim y \simeq 6 \times 10^{-7}$ [4], much smaller than the experimental values, $x = (0.46^{+0.14}_{-0.15})\%$ and $y = (0.62 \pm 0.08)\%$ [5] with no CP violation.

In this context, the D meson mixing based on the inclusive analysis requires a further investigation for clarifying the applicability of the OPE. Specifically, novel features in this work are:

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- We adopt the OPE only if quark is heavy enough, in which case the expansion in terms of Λ/m_h with Λ being a scale of QCD and m_h being a heavy quark mass is reliable.
- Diagrams with bottom quark in the loop are taken into account since they give considerable contribution when a charm quark gets heavy.
- The dispersion relation defined over a wide range of mass scale is considered, providing constrains whose form is given by an integral equation.
- Since the methodology in this work relies only on the dispersion relation, the prediction is independent of the choice of duality violation models.

2 Method

We introduce a fictitious D^0 ($\bar{D^0}$) meson, a pseudoscalar that consists of $h\bar{u}$ ($\bar{h}u$) with h being an up-type quark. Although the quantum numbers of h are totally identical to those of a charm quark, their masses are not necessarily equal to one another. Likewise, the fictitious D meson has a property similar to that of a physical D meson. When the mass of the fictitious D meson is fixed to be m_D, it should coincide a physical D meson that is observed in experiments. In this work, we restrict ourselves to the leading order in power of $1/m_c$ as analyzing the mixing parameters, at which masses of h and the fictitious D meson are degenerate as demonstrated in the heavy quark effective theory.

In the CP conserving limit, mass/width difference for the fictitious D meson system is characterized by,

$$\Delta M(s) = 2M_{12}, \quad \Delta\Gamma(s) = 2\Gamma_{12}, \tag{1}$$

where $M_{12}(s)$ ($\Gamma_{12}(s)$) represents the dispersive (absorptive) part of the $\bar{D^0} \to D^0$ transition amplitude with s being a mass squared of the fictitious charm quark. The specific experssions for M_{12} and Γ_{12} for generic mass of h are obtainable from Ref. [6], in which the box-diagrams were calculated. Instead of Eq. (1), one can introduce the dimensionless variables,

$$x(s) = \Delta M(s)/\Gamma, \quad y = \Delta\Gamma(s)/(2\Gamma). \tag{2}$$

The total width for a D meson, Γ, is experimentally fixed to $(1.6050 \pm 0.0059) \times 10^{-12}$GeV [7].

By exploiting properties of the $\bar{D^0} \to D^0$ amplitude, $A(s+i\epsilon) = A^*(s-i\epsilon)$ along the branch cut and $A(s) = 0$ for large $|s|$, one can derive the dispersion relation,

$$x(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{R} \frac{y(s')}{s-s'}ds, \tag{3}$$

where r.h.s. is defined via the principle value integral. The conventional dispersion relation corresponds to the case with $R \to \infty$ in Eq. (3). One can confirm that r.h.s of Eq. (3) is stable under the variation of R if R is sufficiently large.

Equation (3) can be rewritten by the following form,

$$\int_{4m_\pi^2}^{\Lambda^2} \frac{y(s')}{s-s'}ds' = \omega(s), \tag{4}$$

$$\omega(s) = \pi x(s) - \int_{\Lambda^2}^{R} \frac{y(s')}{s-s'}ds'. \tag{5}$$

Here, Λ^2 is a scale low enough to ensure the validity of the OPE. In Eq. (4), we split the integral region $[4m_\pi^2,\ R]$ into $[4m_\pi^2,\ \Lambda^2]$ and $[\Lambda^2,\ R]$. Given that s is defined in a range $[\Lambda^2,\ (\Lambda')^2]$, where Λ' is a scale satisfying $\Lambda < \Lambda' \leq \sqrt{R}$, it is evident that $\omega(s)$ contains x and y evaluated only in the high mass region so that they are calculable by means of the OPE. On the other hand, l.h.s. of Eq. (4) contains y only in the low mass region, which is treated as an unknown. By solving Eq. (4), one can extract s dependence of $y(s)$ so that the observable, $y(m_c^2)$, can be predicted. After implementing this procedure, one can also know about $x(m_c^2)$ through Eq. (3) by fixing $s = m_c^2$ in both sides.

One can implement the change of variables to gain dimensionless variables, $t^{(\prime)} = s^{(\prime)}/\Lambda^2$. By defining $\tilde{x}(t) = x(s)$ and $\tilde{y}(t') = y(s')$, one can rewrite the dispersion relation in Eq. (4) as,

$$\int_0^1 \frac{\tilde{y}(t')}{t-t'}dt' = \tilde{\omega}(t), \tag{6}$$

$$\tilde{\omega}(t) = \pi\tilde{x}(t) - \int_1^{R/\Lambda^2} \frac{\tilde{y}(t')}{t-t'}dt'. \tag{7}$$

On l.h.s. of Eq. (6), we took the limit where $4m_\pi^2/\Lambda^2 \to 0$. We parametrize the solution of the integral relation (6) as a rational function,

$$\tilde{y}(t) = N(d,m)\frac{t[b_1 + b_2(t-m) + b_3(t-m)^2]}{[(t-m)^2 + d^2]^n}. \tag{8}$$

Hereafter, we restrict ourselves on $n = 2$, the simplest case. In the above equation, t is multiplied as an overall factor in order to satisfy the boundary condtion $\tilde{y}(0) = 0$. Also, $N(d,m)$ is a factor defined so as to normalize the first term to unity, *i.e.*,

$$N(d,m) = \left[\int_0^1 \frac{t\mathrm{d}t}{[(t-m)^2 + d^2]^2}\right]^{-1}. \tag{9}$$

The normalizations for the second (third) term is taken into account by a relative size between b_1 and b_2 (b_3). Moreover, if $b_2 \neq 0$ and/or $b_3 \neq 0$, the distribution can exhibit a shape with multiple peaks. By performing integration, one can find that l.h.s. of Eq. (6) is given by,

$$\int_0^1 \frac{\tilde{y}(t')}{t-t'}dt' = b_1 f_1(t,d,m) + b_2 f_2(t,d,m) + b_3 f_3(t,d,m), \tag{10}$$

$$f_i(t,d,m) = N(d,m)\int_0^1 \frac{t'(t'-m)^{i-1}}{(t-t')[(t'-m)^2 + d^2]^2}dt'. \quad (i = 1,2,3) \tag{11}$$

The unknow constants, (b_1, b_2, b_3, d, m), are determined so as to reproduce the input of the OPE, which is valid for the high mass region. After implementing this fitting, s dependence of y is fixed by Eq. (8), enabling us to identify the shape of the width difference.

To summarize, we proposed a novel method for calculating the D meson mixing by introducing a fictitious D meson that provides the constraining equation for the mixing paramers. The width difference as a solution, exhibihiting resonance(s) for the low mass region, is parametrized in such a way that $\tilde{y}(0) = 0$ is ensured. A dedicated analysis based on this method will be presented elsewhere.

Acknowledgement

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Dark Matter Mass and Anisotropy in Directional Detector

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Abstract

Velocity distribution of dark matter is supposed to be isotropic Maxwell-Boltzmann distribution in most cases, however, other distribution models including anisotropic one are suggested by simulations. Directional direct detection of dark matter is expected to be a hopeful way to discriminate isotropic distribution from anisotropic one. We investigate the possibility to obtain the anisotropy and WIMP mass by Monte-Carlo simulation supposing the directional detector.

1 Introduction

Cosmological and astrophysical observations have showed that dark matter consists 27% of the energy density of the Universe, which corresponds to about 5 times as much as baryonic matter. Weakly interacting massive particles (WIMPs) are a hopeful candidate of dark matter. In spite of many projects to hunt dark matter such as direct, indirect detections and collider search, we still know very little about the dark matter. Velocity distribution is one of essential quantity of dark matter. Especially, it can affect derivation of constraints for dark matter mass and cross section in the direct detection. Direct detection which has directional sensitivity is hopeful experiment to reach the local velocity distribution of WIMPs [2]. Especially, it is suitable to investigate anisotropic components of the velocity distribution suggested by N-body simulation [3]. In the reference, the velocity distribution associated with tangential velocity v_ϕ in the Galactic rest frame is indicated as

$$f(v_\phi) = \frac{1-r}{N(v_{0,\mathrm{iso.}})} \exp\left[-v_\phi^2/v_{0,\mathrm{iso.}}^2\right] + \frac{r}{N(v_{0,\mathrm{ani.}})} \exp\left[-(v_\phi - \mu)^2/v_{0,\mathrm{ani.}}^2\right], \tag{1}$$

where $N(v_{0,\mathrm{iso.}})$ and $N(v_{0,\mathrm{ani.}})$ are normalization factors, $r = 0.25$ is anisotropic parameter, $v_{0,\mathrm{iso.}} = 250$ km/s, $v_{0,\mathrm{ani.}} = 120$ km/s and $\mu = 150$ km/s. Note that the velocity distribution is isotropic if $r = 0.00$. We investigate the possibility to discriminate anisotropic velocity distribution Eq. (1) from isotropic distribution by the directional direct detections including a solid type detector.

2 Detection in Directional Detector

In Figure 1, WIMP - nucleon scattering in the directional detector is shown. The z-axis is taken as the direction of WIMP wind towards the Solar system. Supposing the velocity distribution of corresponding anisotropy r, a WIMP is generated following a probability of the distribution and the recoil energy and the scattering angle are obtained in the Monte-Carlo simulation. Most of the directional direct detector are gas detector in which fluorine (F) is adopted as a target atom. In the solid directional detector, there are several target atoms including silver

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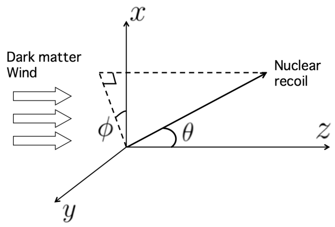

Figure 1: WIMP-nucleon scattering in the Galactic rest frame.

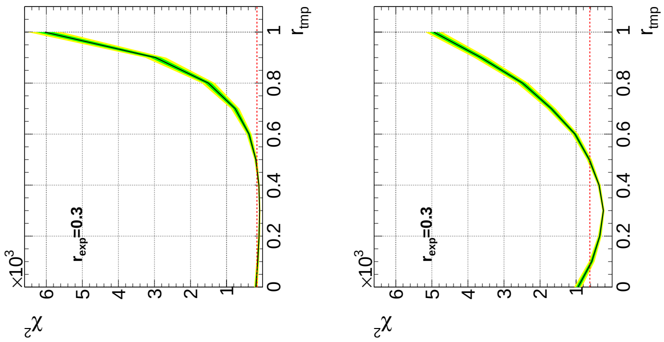

Figure 2: (Left) The chi-squared test of the energy-angular distributions for target F. Red dotted lines show 90% C.L. The event number of pseudo-experiment is set as 6×10^3, and the energy threshold of the detector is 20 KeV. (Right) The chi-squared test for target Ag case. The event number of pseudo-experiment is set as 6×10^4, and the energy threshold of the detector is 50 KeV.

(Ag), bromine and carbon. Among them we suppose F and Ag as target atoms in the numerical simulation.

In order to distinguish the energy-angular distribution obtained by supposing anisotropic velocity distribution from that by supposing anisotropic one, two kinds of data is generated by the Monte-Carlo simulation.

3 Numerical Result

3.1 Case 1 : Supposing the WIMP mass is known.

In the case that the WIMP mass is obtained by other experiment, the energy threshold can be optimized to distinguish the anisotropy of the velocity distribution. In Figure 2, the chi-squared test for target F and Ag are shown. If the anisotropic case is realized, the required event number is 6×10^3 (target F) and 6×10^4 (target Ag) in order to reject the isotropic distribution by 90 % confidence level (C.L.).

3.2 Case 2 : Constraining both WIMP mass and velocity distribution.

Even if information of WIMP mass is not provided, a constraint for both WIMP mass and anisotropy can be obtained. In Figure 3 and 4, the probability distribution for target F and target Ag obtained by likelihood method are shown, respectively. With only directional data or only the recoil energy data, indication for the parameters is unreliable compared to the case that both data are used in the analysis.

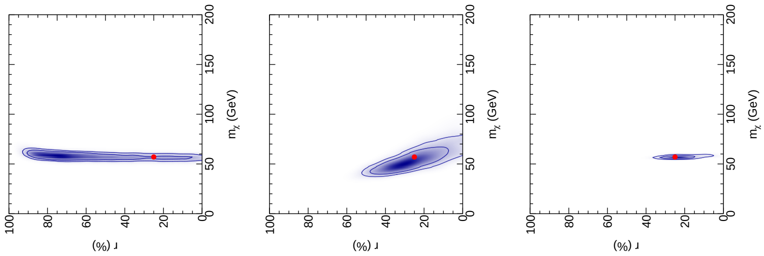

Figure 3: The 2D posterior probability distributions in the WIMP mass and anisotropy space for target F. Left: Only data of recoil energy E_R is used. Center: Only data of scattering angle $\cos\theta$ is used. Right: Both recoil energy and scattering angle are used.

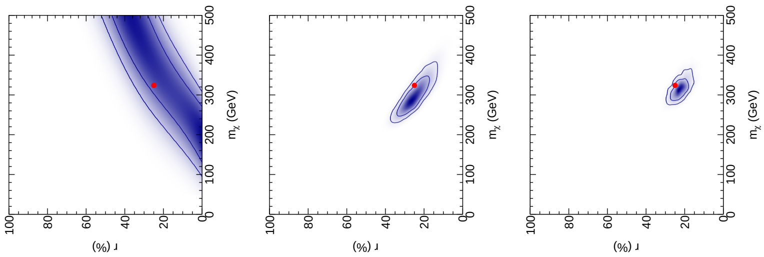

Figure 4: Legend is same as Figure 3 but for the target Ag.

4 Summary

Discrimination of the anisotropy of the WIMP velocity distribution in the directional detector is investigated. Once the WIMP mass is obtained, supposing 30% of anisotropy component of the velocity distribution, the required event numbers for discriminate the anisotropy are O(1000) for light target and O(10000) for heavy one, respectively. Even without the WIMP mass information, the WIMP mass and anisotropy can be obtained with less uncertainty by using both the recoil energy and the scattering angle data, than in the analysis by using only one of the data.

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Lorentz invariant CPT breaking in the Standard Model; neutrino oscillation and baryogenesis

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Abstract

The CPT symmetry is the fundamental symmetry in Lorentz invariant local field theory. It is nevertheless interesting to entertain the idea of the possible breaking of CPT symmetry and discuss its implications. We explain the general idea of the possible Lorentz invariant CPT breaking, since it is often erroneously stated in the literature that CPT breaking implies the Lorentz symmetry breaking.

1 Introduction

The CPT symmetry is valid for any Lorentz invariant local theory with normal spin-statistics and hermitian Lagrangian. The CPT theorem was introduced in the following articles:
W. Pauli, *Niels Bohr and the Development of Physics*, W. Pauli (ed.), Pergamon Press, New York, 1955.
G. Lüders, *On the Equivalence of Invariance under Time-Reversal and under Particle-Antiparticle Conjugation for Relativistic Field Theories*, Det. Kong. Danske Videnskabernes Selskab, Mat.-fys. Medd. **28** (5) (1954).

The title of this second paper clearly shows what the CPT theorem means, namely, T invarince is equivalent to C invariance if parity (P) is preserved since P was believed to be preserved at that time.

One may seek the possible CPT violation at the Planck scale, for example, due to the following specific properties of the Lagrangian;
1. Non-local Lagrangian,
2. Lorentz non-invariant Lagrangian.
The conventional argument, which is attributed to O.W. Greenberg, Phys. Rev. Lett. **89**, 231602 (2002), *erroneously* asserts that the CPT violation inevitably implies the Lorentz symmetry violation. An explicit *counter example* to the above claim, using a non-local Lorentz invariant Lagrangian which breaks CPT symmetry, was given by M. Chaichian, K. Fujikawa and A. Tureanu, Phys. Lett. B**712** (2012) 115.

2 Non-local Lorentz invariant theory with CPT violation

The essence of the idea of the CPT breaking in non-local Lorentz invariant theory is explained by the modified Dirac equation in momentum space

$$[\gamma^\mu p_\mu - m - \Delta m(\theta(p_0) - \theta(-p_0))\theta(p_\mu^2)]\psi(p) = 0. \quad (1)$$

The symmetry of positive and negative energy eigenvalues is lifted by $m \pm \Delta m$ for a small Δm and thus breaking CPT symmetry. The mass degeneracy of the particle and antiparticle is thus lifted in a Lorentz invariant manner since the combinations

$$\theta(\pm p_0)\theta(p_\mu^2) \quad (2)$$

with step functions (signature of the time-like vector) are manifestly Lorentz invariant.

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3 Lorentz invariant Lagrangian with particle-antiparticle mass splitting

We start with the hermitian Lorentz invariant combination in a space-time description

$$\int d^4xd^4y[\theta(x^0-y^0)-\theta(y^0-x^0)] \\ \times\delta((x-y)^2-l^2)[i\mu\bar{\psi}(x)\psi(y)]. \tag{3}$$

Transformation properties of the operator part are given using *spin-statistics theorem*,

$$\begin{aligned} &\mathrm{C}: \ i\mu\bar{\psi}(x)\psi(y)\rightarrow i\mu\bar{\psi}(y)\psi(x), \\ &\mathrm{P}: \ i\mu\bar{\psi}(x^0,\vec{x})\psi(y^0,\vec{y})\rightarrow i\mu\bar{\psi}(x^0,-\vec{x})\psi(y^0,-\vec{y}), \\ &\mathrm{T}: \ i\mu\bar{\psi}(x^0,\vec{x})\psi(y^0,\vec{y})\rightarrow -i\mu\bar{\psi}(-x^0,\vec{x})\psi(-y^0,\vec{y}), \end{aligned} \tag{4}$$

and thus the overall transformation property of (3) is C$=-1$, P$=1$, T$=1$. Namely, all the discrete symmetries are broken C=CP=CPT$=-1$.

It is then interesting to examine a new modified Dirac action

$$\begin{aligned} S &= \int d^4x\{\bar{\psi}(x)i\gamma^\mu\partial_\mu\psi(x)-m\bar{\psi}(x)\psi(x) \\ &\quad -\int d^4y[\theta(x^0-y^0)-\theta(y^0-x^0)]\delta((x-y)^2-l^2)[i\mu\bar{\psi}(x)\psi(y)]\}, \end{aligned} \tag{5}$$

which is Lorentz invariant and hermitian.

The Dirac equation is then replaced by

$$\begin{aligned} &i\gamma^\mu\partial_\mu\psi(x)=m\psi(x) \\ &+i\mu\int d^4y[\theta(x^0-y^0)-\theta(y^0-x^0)]\delta((x-y)^2-l^2)\psi(y). \end{aligned} \tag{6}$$

By inserting an ansatz for the possible solution

$$\psi(x)=e^{-ipx}U(p), \tag{7}$$

we have

$$\begin{aligned} \not{p}U(p) &= mU(p) \\ &+ i\mu\int d^4y[\theta(x^0-y^0)-\theta(y^0-x^0)] \\ &\times \delta((x-y)^2-l^2)e^{-ip(y-x)}U(p) \\ &= mU(p)+i\mu[f_+(p)-f_-(p)]U(p), \end{aligned} \tag{8}$$

where

$$f_\pm(p)=\int d^4z_1e^{\pm ipz_1}\theta(z_1^0)\delta((z_1)^2-l^2), \tag{9}$$

is the Lorentz invariant form factor.

For the space-like p, we have $f_+(p)=f_-(p)$ and no mass splitting, *which implies no tachyon.*

For time-like p, we go to the frame where $\vec{p}=0$ and the eigenvalue equation becomes

$$p_0 = \gamma_0\{m+i\mu[f_+(p_0)-f_-(p_0)]\}, \tag{10}$$

namely,

$$p_0 \;=\; \gamma_0 \left[m - 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}} \right]. \tag{11}$$

This eigenvalue equation under $p_0 \to -p_0$ becomes:

$$-p_0 \;=\; \gamma_0 \left[m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}} \right]. \tag{12}$$

By sandwiching this equation by γ_5, we have

$$p_0 \;=\; \gamma_0 \left[m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}} \right], \tag{13}$$

which is not identical to the original equation. In other words, if p_0 is the solution of the original equation, $-p_0$ cannot be the solution of the original equation for $\mu \neq 0$. The last term in the Lagrangian with C=CP=CPT=-1 splits the particle and antiparticle masses.

As a crude estimate of the mass splitting, one may assume $\mu \ll m$ and solve these equations iteratively. If the particle mass is chosen at

$$p_0 \simeq m - 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[m\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}}, \tag{14}$$

then the antiparticle mass is estimated at

$$p_0 \simeq m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[m\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}}. \tag{15}$$

Thus CPT breaking term with μ splitts the particle and antiparticle masses.

4 Neutrino oscillation and baryogenesis

One can incorporate the above CPT breaking scheme in the neutrino mass sector of a minimal extension of the Standard Model by maintaining
a)C, CP and CPT breaking
b)Lorentz invariance
c)SU(2)xU(1) gauge invariance
d)Non-locality within a distance scale of the Planck length.

Neutrino oscilllation is very sensitive to a small mass and mass splitting, but it is shown that the neutrino-antineutrino mass splitting generated by the above mechanism is consistent with experiments. It is also shown that the *induced electron-positron* mass splitting, for example, is negligibly small. The possible implication of CPT breaking on the *thermal equilibrium baryogenesis* has been briefly mentioned.

The details of the above analyses are found in [1, 2].

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Supercurvature-mode dark energy model

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Abstract

We investigate large-scale inhomogeneity of the supercurvature-mode dark energy mode proposed by Aoki, Iso, Lee, Sekino, Yeh in Ref. [2]. In this scenario, the dark energy is explained by a scalar ϕ field coupled to another scalar field ψ which gives rise to an open inflation scenario induced by a bubble nucleation. During the bubble nucleation, supercurvature modes of the scalar field ϕ can be produced, and may remain until present epoch without decaying; thus they can play a role of the dark energy, if the mass of the scalar field is sufficiently light in the present universe. The supercurvature modes fluctuate at a very large spatial scale, much longer than the Hubble length in the present universe. Thus they create large-scale inhomogeneities of the dark energy, and generate large-scale anisotropies in the cosmic microwave background fluctuations.

1 Introduction

Our Universe is in an accelerating expansion phase, which is one of the biggest mystery of modern cosmology. A cosmological constant, which explains the accelerating expansion, is one of the important factors of the standard model of cosmology. However, there have been proposed many models of dark energy, which are dynamical models to explain the cosmic accelerated expansions. Quintessence is well-known as a dark energy model. Isotropy and homogeneity of the background universe are the fundamental assumption of the standard cosmological model. We investigate a observational effect in a model of dark energy with spatially varying properties, which we call supercurvature-mode dark energy model [2], which may violate the isotropy and homogeneity of the background Universe on superhorizon scales.

Following the supercuvature-mode dark energy model, we consider a scalar field ϕ [2], which is coupled to another field ψ that derives a false vacuum decay and an open inflation [5]. Open inflation scenario is induced by a bubble nucleation, i.e., a tunneling process from a false vacuum in de Sitter space, then a slow roll inflation follows inside the bubble. The tunneling process of the bubble nucleation is described by the Coleman de Luccia bounce solution, assuming the O(4) symmetry in the Euclidean metric. The analytic continuation of the bounce solution to Lorentzian region describes the solution of an expanding bubble. Due to the O(4) symmetry of the bounce solution, an open universe is realized inside the bubble [6].

We consider the quantum field theory of a massive scalar field ϕ on the background of an expanding bubble. It has been shown that a discrete mode appears besides the continuous modes for the expression of the quantum field in an open universe when the mass of ϕ is smaller than the Hubble parameter [4]. The discrete mode is characterized by a negative eignevalue $p_*^2(<0)$, which we write $p_* = i(1-\epsilon)$ with $\epsilon = \mathcal{O}(1)(m_A/H_A)^2$, where m_A and H_A are the mass

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of ϕ and the Hubble parameter of the false vacuum. Here we assume $\epsilon \ll 1$. As this discrete mode has the length scale much longer than the spatial curvature scale, then the discrete mode is called supercurvature mode.

The evolution of the supercurvature mode is different from that of the continuous modes. The continuous modes decay after inflation, but the supercurvature mode is almost constant as long as ϵ is small, $\epsilon \ll 1$. Then the supercurvature mode freezes. The amplitude of the quantum fluctuation of the supercurvature mode is determined as $\langle \phi^2 \rangle \simeq H_A^2/m_A^2$. If the mass of the field is sufficiently light in the present universe, $m_0 \sim H_0$, of the same order of the present Hubble parameter, the potential energy density of the supercurvature mode can explain the dark energy, although we need a further assumption for the parameters $M_{pl}/H_A \sim H_A/m_A$.

This dark energy model is interesting as an inhomogeneous dark energy model, and we investigate the spatially varying property. The energy density of the dark energy is approximately expressed by $\rho_{DE} = \frac{1}{2}m_0^2\phi^2$, then the correlation function of the dark energy density contrast $\delta_{DE}(x) = (\rho_{DE}(x) - \langle \rho_{DE} \rangle)/\langle \rho_{DE} \rangle$ is estimated

$$\langle \delta(\eta, \vec{x})\delta(\eta, \vec{y}) \rangle = \frac{2\langle \phi(\eta, \vec{x})\phi(\eta, \vec{y})\rangle^2}{\langle \phi^2(\eta, \vec{x})\rangle^2} = 2\left(\frac{\sinh(1-\epsilon)R}{(1-\epsilon)\sinh R}\right)^2, \tag{1}$$

with $R = \sqrt{-K}|\vec{x}-\vec{y}|$ where K is the spatial curvature and we used $\langle \phi(\eta, \vec{x})\phi(\eta', \vec{y})\rangle \propto \frac{\sinh(1-\epsilon)R}{(1-\epsilon)\sinh R}$ for the supercurvature mode. The result means that the dark energy density contrast has the spatial inhomogeneity of $\mathcal{O}(1)$ on the (comoving) scale of the supercurvature $1/\epsilon\sqrt{-K}$, which is much longer than the curvature scale $1/\sqrt{-K}$ when $\epsilon \ll 1$. We note that the curvature scale must be longer than the present Hubble scale $1/\sqrt{-K} \gg 1/H_0$.

Because the supercurvature scale is much longer than the present horizon scale, then the dark energy density is almost constant in our observable universe, but observational signature of the inhomogeneity of dark energy may appear. We investigated the observational effect on the cosmic microwave background (CMB) anisotropy, imprinted through the integrated Sachs-Wolfe effect,

$$\frac{\Delta T}{T}(\vec{\gamma}) = \int_{\eta_*}^{\eta_0} d\eta \left(\frac{\partial \Psi(\eta, \chi, \vec{\gamma})}{\partial \eta} - \frac{\partial \Phi(\eta, \chi, \vec{\gamma})}{\partial \eta}\right)_{\chi=\eta_0-\eta}, \tag{2}$$

where Ψ and Φ are the gravitational potential and the curvature potential in the Newtonian gauge on an open universe background.

The evolution of Ψ and Φ from the supercurvature mode is obtained by solving the perturbed equations of the Einstein equation and the matter's fluid equations, under the condition that the supercurvature mode is much longer than the curvature and the horizon scale,

$$\Phi(\eta, \chi, \vec{\gamma}) = -\Psi(\eta, \chi, \vec{\gamma}) \simeq \frac{1}{F(\eta)}\int_0^\eta d\eta_1 \frac{4\pi G F(\eta_1)}{B(\eta_1)} m_0^2 \left(\phi(\eta_1, \chi, \vec{\gamma})^2 - \phi(\eta_1, 0)^2\right), \tag{3}$$

where we defined $F(a) = a^{5/2}/\sqrt{\Omega_m + (1-\Omega_m)a^3}$ and $B(a) = 6H_0 a^{-2}\sqrt{\Omega_m/a^3 + 1 - \Omega_m}$ (See Ref.[1]). The two point function of the CMB temperature anisotropy due to the integrated Sachs-Wolfe effect is computed using the two point function of the field, which is written in the form of the multipole expansion

$$\left\langle \frac{\Delta T}{T}(\vec{\gamma})\frac{\Delta T}{T}(\vec{\gamma}') \right\rangle = \frac{1}{4\pi}\sum_\ell (2\ell+1)C_\ell P_\ell(\cos\theta), \tag{4}$$

with $\cos\theta = \vec{\gamma}\cdot\vec{\gamma}'$. The multipole coefficients are expressed in the form $C_\ell = \alpha_\ell S_\ell^2$, where we defined $S_\ell = \int_0^1 da \left(\sqrt{-K}(\eta_0 - \eta(a))\right)^\ell \frac{\partial}{\partial a}\left(\frac{1}{F(a)}\int_0^a da' \frac{8\pi G\rho_{\rm DE}(a')F(a')}{3a'H^2(a')}\right)$, and α_ℓ is $\alpha_1 = 32\pi/9$ and

$\alpha_2 = 32\pi/75$, for $\ell = 1, 2$. Numerical computation gives $C_1 = 0.14 \times \epsilon\Omega_K$, $C_2 = 0.01 \times \epsilon\Omega_K^2$, and $C_\ell \sim \mathcal{O}(\epsilon\Omega_K^\ell)$. Then, we find that the dipole and the quadrupole are the components relevant to observational signature for $\epsilon \ll 1$ and $\Omega_K \ll 1$.

Comparison with observations puts a constraint on the model parameters. Adopting the observed values of the dipole and the quadrupole component of the CMB temperature anisotropies (e.g., [7]), $C_1^{\rm obs} \simeq 6.3 \times 10^{-6}$ and $C_2^{\rm obs} \lesssim 1.0 \times 10^{-10}$, we obtain these constraints on the model parameters $\epsilon\Omega_K \lesssim 4.9 \times 10^{-5}$ and $\epsilon\Omega_K \lesssim 1.0 \times 10^{-8}$. Recent observation by Planck satellite put upper bound on the spatial curvature parameter $\Omega_K \lesssim 10^{-2} \sim 10^{-3}$. When $\Omega_k = 10^{-3}$, we have $\epsilon < 10^{-2}$, but if Ω_k is smaller than 10^{-3}, the constraint on ϵ becomes weaker.

2 Conclusions

We investigated the spatially varying property of the supercurvature mode dark energy model, induced in an open inflation scenario. This property is a notable feature of this scenario, where quantum fluctuations of a scalar field are responsible for the dark energy. We calculate imprints of the scenario on the CMB anisotropies through the integrated Sachs-Wolfe effect, and give observational constraints on the curvature parameter Ω_K and on an additional parameter ϵ describing some properties of the false vacuum. We need further investigations to check the predictions of the model. Possible deviation from a cosmological constant model may appear in the equation of state parameter when the kinetic term of the energy density and pressure is not negligible (See Ref.[3]), which we assumed to be small here. Also we need check constraints from the tensor perturbations and the large scale structure.

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Fermion masses and baryon and lepton number violation from finite modular groups

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Abstract

We study a flavor model that the quark sector has the S_3 modular symmetry, while the lepton sector has the A_4 modular symmetry. Our model reproduces the masses and mixing angles of both quarks and leptons. We also study baryon and lepton number violation, in particular, a proton decay. This talk is based on [1].

Many flavor models with non-Abelian discrete flavor symmetries [2–4] succeeded in reproducing realistic mixing angles of quarks and leptons. In these models, the flavor symmetries are broken by the extra gauge singlet scalars which are called flavons. Instead, the flavons produce many unknown parameters. We try to reproduce realistic mixing angles of both quarks and leptons without flavons.

Superstring theory requires ten-dimensional space-time, though we observe four-dimensional space-time. The additional six-dimensional space must be compactified. The three generations of quarks and leptons can appear from certain compact spaces, for example, the compact space in which suitable magnetic fluxes are inserted. As a six-dimensional compact space, in particular, we assume the six-dimensional torus T^6 which is decomposed of a product of three two-dimensional tori T^2, i.e. $T^6 = T_1^2 \times T_2^2 \times T_3^2$. It is because that each T_i^2 has modular symmetry which is a geometrical symmetry and the modular symmetry contains certain non-Abelian discrete symmetries as finite subgroups [5].

First, I review the modular symmetry of T^2 and the modular finite subgroups. T^2 can be constructed as division of the complex plane C by a tow-dimensional lattice Λ, i.e. $T^2 = C/\Lambda$. The lattice is spanned by two-lattice vectors, $\alpha_1 = 2\pi R$ and $\alpha_2 = 2\pi R\tau$, where R is a real radius parameter which decides the torus size and τ is a complex modulus parameter which decides the torus shape. There is some ambiguity in choice of the basis vectors. The same lattice can be spanned by the following basis vectors,

$$\begin{pmatrix} \alpha_2' \\ \alpha_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix}, \tag{1}$$

where a, b, c, d are integer with satisfying $ad - bc = 1$. That is the $SL(2, Z)$ transformation. Under the above transformation, the modulus parameter $\tau = \alpha_2/\alpha_1$ transforms as

$$\tau \to \tau' = \frac{a\tau + b}{c\tau + d}. \tag{2}$$

This transformation is called modular transformation. The holomorphic function of modulus parameter τ, which transforms as

$$f(\tau) \to (c\tau + d)^k f(\tau) \tag{3}$$

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under the modular transformation Eq. (2), is called the modular form of modular weight k. The modular transformation Eq. (2) is generated by S and T transformations,

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \in SL(2,Z) \quad \longrightarrow \quad S : \tau \to -\frac{1}{\tau} \,, \tag{4}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in SL(2,Z) \quad \longrightarrow \quad T : \tau \to \tau + 1 \,. \tag{5}$$

They satisfy $S^2 = \mathbf{1}$, $(ST)^3 = \mathbf{1}$. In addition, if we impose $T^2 = \mathbf{1}$ or $T^3 = \mathbf{1}$, these algebraic relations are the same as S_3 or A_4 group, respectively [5]. Therefore, in that case, the modular form is transformed non-trivially under S_3 or A_4 modular transformation [6]. (See for the S_3 modular forms Ref. [7].) We apply these results for the flavor symmetries of matters.

We use $T^6 = T_1^2 \times T_2^2 \times T_3^2$ as a six-dimensional compact space in the superstring theory, then assume that three generations of quarks appear on T_1^2 which has S_3 modular symmetry while three generations of leptons appear on T_2^2 which has A_4 modular symmetry. Note that the reason of assuming the different modular flavor symmetries for quarks and leptons is that the mixing angles of quarks and leptons are different, which means ones of quarks are small but ones of leptons are large. Table 1 and Table 2 show the assignments for quarks and leptons, respectively. In the quark sector, we assign S_3 doublets with modular weight -2 for the first and second families of both left-handed and right-handed quarks, and assign S_3 non-trivial singlet with modular weight 0 for the third family of both left-handed and right-handed quarks. On the other hand, in the lepton sector, we assign A_4 triplets with modular weight -1 for the three left-handed leptons and the right-handed neutrinos, and assign the different A_4 singlets with modular weight -1 for the three right-handed charged leptons, respectively. As for the Higgs sector, we assign S_3 and A_4 singlet with modular weight 0. In addition, Yukawa couplings are obtained by integrating the six-dimensional part of the ten-dimensional space-time in the superstring theory. Then, the couplings depend on the geometry of the compact space, that is size and shape. Here, we assume that the couplings of quarks and leptons are written by modular forms and transformed non-trivially under S_3 and A_4 modular transformations, respectively. That is different from the models with flavons.

	(Q_1,Q_2)	Q_3	(q_1,q_2)	q_3	H_u	H_d
$SU(2)$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$
S_3	$\mathbf{2}$	$\mathbf{1'}$	$\mathbf{2}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{1}$
$-k_I$	-2	0	-2	0	0	0

Table 1: The assignments of S_3 representations and modular weights $-k_I$ to the MSSM fields.

	L	$e,\ \mu,\ \tau$	ν	H_u	H_d
$SU(2)$	2	1	1	2	2
A_4	3	$1,\ 1'',\ 1'$	3	1	1
$-k_I$	-1	-1	-1	0	0

Table 2: The assignments of A_4 representations and modular weights $-k_I$ to the MSSM fields.

In these conditions, we are able to reproduce observed mass hierarchies and mixing angles of both quarks and leptons without flavons [1,8,9]. Note that the lepton sector has been already studied in [8] and also the quark sector was studied in [1]. The details are discussed in [9].

Let us consider mixing between quarks and leptons. In particular, it could cause the proton decay that quarks change into leptons. It occurs in the following two cases. The first case is

- the terms violating both baryon and lepton numbers: $QQQL$, $uude$,

contribute a proton decay. The second case is both

- the term violating baryon number: udd , $QQQH_d$,

- the term violating lepton number including quarks: LQd , $QueH_d$, $d^\dagger ue$, $L^\dagger Qu$,

contribute the proton decay. The above terms are ones allowed in the minimal supersymmetric standard model without R-parity under mass dimension five. In our model, the terms including odd numbers of leptons are forbidden since the weights of the modular forms as well as quarks are even. Therefore, the proton decay cannot occur at least at the tree level. Note that the terms allowed in our model and violating R-parity under mass dimension five are just two terms which violate only baryon number. However, the modular symmetry can be anomalous. Then non-perturbative effects can break the modular symmetry. Thus, the study on anomalies are important[2].

To summarize, we have studied a flavor model that the quark sector has the S_3 modular symmetry, while the lepton sector has the A_4 modular symmetry. The masses and mixing angles of both quarks and leptons are reproduced. In addition, the proton decay is forbidden at the perturbative level. However, non-perturbative effects can break such a situation.

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[2] Recently, the modular symmetry anomalies were studied from the viewpoint of extra dimensional field theory and superstring theory [10].

A modular symmetric model for the flavor mixing

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Abstract

We study a flavor model that has S_3 and A_4 modular symmetries for the quark mixing and lepton mixing respectively. The quark sector of our model is consistent with experimental data of quark masses, mixing angles and the CP violating phase. The lepton sector is also consistent with the latest neutrino oscillation experiments. We also study baryon and lepton number violations in our flavor model.

1 Introduction

- There are many interests in discrete flavor symmetries [1, 2] promoted by early works of the quark masses and mixing angles [3, 4] in touch with the latest experiments of the neutrino oscillation.
- The modular group can have S_3, A_4, S_4 and A_5 as its subgroups Γ_N [5]. An attractive ansatz was proposed by taking $\Gamma_3 \simeq A_4$ in Ref. [6] where the Yukawa couplings and both the left-handed leptons and right-handed neutrinos are described as A_4 triplets.
- We make an alternative ansatz. We assume two different flavor symmetries: the S_3 and A_4 modular symmetries for the quark and lepton sectors respectively. They originate from the modular groups defined in two different two-dimensional compact spaces. We use the same lepton sector as given in Ref. [7].

2 Model

We construct a modular symmetric flavor model without introducing any gauge singlet scalar such as the flavon. The modular invariant mass terms of the leptons are given as the following superpotentials:

$$W_e = \alpha e H_d (L Y^{A_4})_{\mathbf{1}} + \beta \mu H_d (L Y^{A_4})_{\mathbf{1'}} + \gamma \tau H_d (L Y^{A_4})_{\mathbf{1"}} \ , \tag{1}$$

$$W_D = g(\nu H_u L Y^{A_4})_{\mathbf{1}} \ , \tag{2}$$

$$W_N = \Lambda(\nu\nu Y^{A_4})_{\mathbf{1}} \ , \tag{3}$$

where sums of the modular weights vanish in each term. The parameters α, β, γ, g, and Λ are constant coefficients. The functions $Y^{A_4}(\tau) = (Y_1^{A_4}(\tau), Y_2^{A_4}(\tau), Y_3^{A_4}(\tau))^{\mathrm{T}}$ is an A_4 triplet modular form of weight 2 defined as

$$
\begin{aligned}
Y_1^{A_4}(\tau) &= \frac{i}{2\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)}+\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)}+\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}-\frac{27\eta'(3\tau)}{\eta(3\tau)}\right), \\
Y_2^{A_4}(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)}+\omega^2\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)}+\omega\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right), \\
Y_3^{A_4}(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)}+\omega\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)}+\omega^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right),
\end{aligned} \tag{4}
$$

where $\omega = \mathrm{e}^{2\pi i/3}$ and $\eta(\tau)$ is the Dedekind's eta function of the modulus τ. The superpotential of Eq.(1) leads to the following charged leptons mass matrix:

$$
M_E = v_d\,\mathrm{diag}[\alpha,\beta,\gamma]\begin{pmatrix} Y_1 & Y_3 & Y_2 \\ Y_2 & Y_1 & Y_3 \\ Y_3 & Y_2 & Y_1 \end{pmatrix}_{RL}, \tag{5}
$$

where $v_d = \langle H_d\rangle$ and we omit the superscript A_4 of $Y_i^{A_4}$ hereafter. The superpotentials of Eqs.(2) and (3) gives the Dirac and Majorana neutrino mass matrices as:

$$
M_D = v_u\begin{pmatrix} 2g_1Y_1 & (-g_1+g_2)Y_3 & (-g_1-g_2)Y_2 \\ (-g_1-g_2)Y_3 & 2g_1Y_2 & (-g_1+g_2)Y_1 \\ (-g_1+g_2)Y_2 & (-g_1-g_2)Y_1 & 2g_1Y_3 \end{pmatrix}_{RL}, \quad M_N = \Lambda\begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix}_{RR}. \tag{6}
$$

where $v_u = \langle H_u\rangle$. The effective neutrino mass matrix is obtained through the type I seesaw as $M_\nu = -M_D^{\mathrm{T}} M_N^{-1} M_D$. The charge assignment of the fields is summarized in Tables 1 and 2.

We can also construct the Dirac mass matrix of quarks in accordance with the charge assignment of Table 2 as follows:

$$
M_{u,d} = \begin{pmatrix} c^{u,d}+c'^{u,d}(Y_1^{S_3}(\tau')^2-Y_2^{S_3}(\tau')^2) & 2c'^{u,d}Y_1^{S_3}(\tau')Y_2^{S_3}(\tau') & c_{13}^{u,d}Y_2^{S_3}(\tau') \\ 2c'^{u,d}Y_1^{S_3}(\tau')Y_2^{S_3}(\tau') & c^{u,d}-c'^{u,d}(Y_1^{S_3}(\tau')^2-Y_2^{S_3}(\tau')^2) & -c_{13}^{u,d}Y_1^{S_3}(\tau') \\ c_{31}^{u,d}Y_2^{S_3}(\tau') & -c_{31}^{u,d}Y_1^{S_3}(\tau') & c_{33}^{u,d} \end{pmatrix}, \tag{7}
$$

where $Y_1^{S_3}(\tau')$ and $Y_2^{S_3}(\tau')$ are S_3 modular forms of weight 2 defined as

$$
\begin{aligned}
Y_1^{S_3}(\tau) &= \frac{i}{4\pi}\left(\frac{\eta'(\tau/2)}{\eta(\tau/2)}+\frac{\eta'((\tau+1)/2)}{\eta((\tau+1)/2)}-\frac{8\eta'(2\tau)}{\eta(2\tau)}\right), \\
Y_2^{S_3}(\tau) &= \frac{\sqrt{3}i}{4\pi}\left(\frac{\eta'(\tau/2)}{\eta(\tau/2)}-\frac{\eta'((\tau+1)/2)}{\eta((\tau+1)/2)}\right).
\end{aligned} \tag{8}
$$

	L	$e,\ \mu,\ \tau$	ν	H_u	H_d
$SU(2)$	2	1	1	2	2
A_4	3	$1,\ 1'',\ 1'$	3	1	1
$-k_I$	-1	-1	-1	0	0

Table 1: Charge and modular weight in the lepton sector.

	(Q_1, Q_2)	Q_3	(q_1, q_2)	q_3
$SU(2)$	2	2	1	1
S_3	2	$1'$	2	$1'$
$-k_I$	-2	0	-2	0

Table 2: Charge and modular weight in the quark sector.

3 Result and summary

We show our numerical results only for the quark sector because the results of the lepton sector are same as our previous work in ref. [7]. For the quark sector, we show our prediction in Figs.1 and 2. In Fig.1, we plot the predicted correlation of θ_{23} and θ_{13}, where $\theta_{23} > 2°$ is not allowed. In Fig.2, we plot the predicted δ_{CP} versus θ_{13}, where $\delta_{CP} > 80°$ is excluded. The red lines in these figures correspond to the bounds with 3σ range at the GUT scale [8,9].

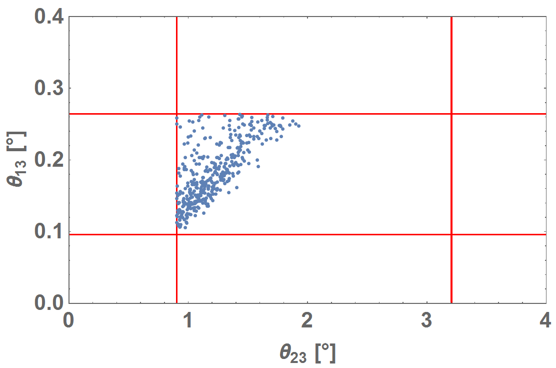

Figure 1: The prediction of θ_{13} and θ_{23}.

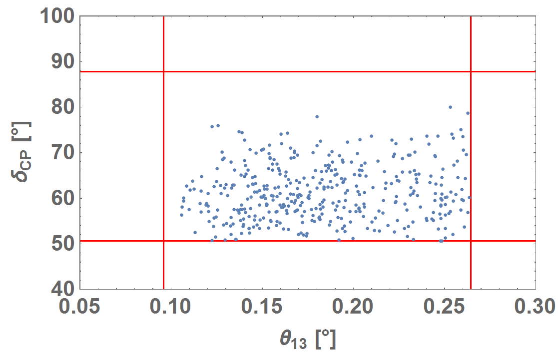

Figure 2: The prediction of δ_{CP} and θ_{13}.

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Full references and discussions are contained in [arXiv:1182.11072].

Sign of CP Violating Phase in Quarks and Leptons

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Abstract

We discuss relations between the CP violating phase of the quarks and leptons. In order to clarify the relations, we investigate CP violating observables in the standpoint of "Occam's Razor" approach. The CP violating observables of leptons are connected to that of quarks through Pati-Salam and SU(5) models. We find that the successful leptonic Jarlskog invariant are obtained in the case of the non-diagonal neutrino mass matrix.

1 Texture of down-type quark mass matrix

The flavor structure of quarks and leptons are still unknown although the standard model is well established. In order to clarify the flavor structure of quarks, the framework of "Occam's Razor" approach was considered [1]. In the "Occam's Razor" approach, the model contains minimum number of free parameters to reproduce the observed six quark masses, three CKM mixing angles and one Dirac CP phase. We embed three real parameters into the up-type quark mass matrix. The remaining six real parameters and one phase are included in the down-type quark mass matrix. For instance we consider the following quark mass matrices:

$$M_U = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}_{LR}, \qquad M_D^{(1)} = \begin{pmatrix} 0 & a_D & 0 \\ a'_D & b_D e^{-i\phi} & c_D \\ 0 & c'_D & d_D \end{pmatrix}_{LR}, \tag{1}$$

where $a_D, a'_D, b_D, c_D, c'_D$ and d_D are real parameters and ϕ is a CP violating phase. We determine the values of these model parameters by observed values of the quark masses and CKM parameters at GUT scale [2]. For $M_D^{(1)}$, we obtain central values of the model parameters as,

$$\begin{aligned} &a_D = 5.3\times 10^{-3}, \qquad a'_D = 3.6\times 10^{-3}, \qquad b_D = 36\times 10^{-3}, \\ &c_D = 32\times 10^{-3}, \qquad c'_D = 0.75, \qquad d_D = 0.93, \qquad \phi = 42^\circ, \end{aligned} \tag{2}$$

in GeV unit except ϕ. In our numerical discussions, we use these values as a benchmark.

2 CP violation in lepton sector

We discuss the flavor structure of the lepton sector especially for the CP violation in the two GUT models, Pati–Salam model and SU(5) model. The charged lepton mass matrix is related to the down-type quark mass matrix with Clebsch–Gordan (CG) coefficients. The CG coefficients

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are necessary to reproduce relevant mass ratios of the down-type quarks and charged leptons. The possible CG coefficients are given by renormalizable or non-renormalizable operators [3]:

$$\text{Pati–Salam} \rightarrow \ \text{dim.4} : (1,-3)\ , \quad \text{dim.5} : (1,-3,9)\ , \quad \text{dim.6} : (0,\frac{3}{4},1,2,-3)\ , \tag{3}$$

$$\mathrm{SU}(5) \rightarrow \ \text{dim.4} : (1,-3)\ , \quad \text{dim.5} : (-\frac{1}{2},1,\pm\frac{3}{2},-3,\frac{9}{2},6,9,-18)\ . \tag{4}$$

Next we discuss the charged lepton mass matrix $M_E^{(1)}$ corresponding to the down-type mass matrix $M_D^{(1)}$. In the Pati–Salam and SU(5) model, it is given as,

$$\text{Pati–Salam} \rightarrow M_E^{(1)} = \begin{pmatrix} 0 & a_E & 0 \\ a'_E & b_E\, e^{-i\phi} & c_E \\ 0 & c'_E & d_E \end{pmatrix}_{LR} \ , \quad \mathrm{SU}(5) \rightarrow M_E^{(1)} = \begin{pmatrix} 0 & a'_E & 0 \\ a_E & b_E\, e^{-i\phi} & c'_E \\ 0 & c_E & d_E \end{pmatrix}_{LR} \ . \tag{5}$$

We assume that one single operator dominates each matrix element. Then the elements of the charged lepton mass matrix are given in terms of the down-type quark mass matrix elements and CG coefficients as,

$$a_E = C_a a_D, \quad a'_E = C_{a'} a'_D, \quad b_E = C_b b_D, \quad c_E = C_c c_D, \quad c'_E = C_{c'} c'_D, \quad d_E = C_d d_D, \tag{6}$$

where $C_a, C_{a'}, C_b, C_c, C_{c'}$ and C_d are possible CG coefficients in Eq.(3) for the Pati–Salam model or in Eq.(4) for the SU(5) model. The phase ϕ is common between the down-type quark mass matrix and the charged lepton mass matrix. Thus the CP violating observable of leptons is correlated with that of quarks.

Finally, the Jarlskog invariant for leptons J_{CP}^l can be computed from [4–7]

$$\mathrm{Tr}([H_\nu, H_E]^3) = -6iJ_{CP}^l\ \Delta_\nu\ \Delta_e\ , \tag{7}$$

where $H_X = M_X M_X^\dagger$ with $X = \nu, E$. The matrix M_ν denotes neutrino mass matrix and

$$\Delta_\nu \equiv (m_1^2 - m_3^2)(m_1^2 - m_2^2)(m_2^2 - m_3^2)\ , \quad \Delta_e \equiv (m_e^2 - m_\tau^2)(m_e^2 - m_\mu^2)(m_\mu^2 - m_\tau^2)\ , \tag{8}$$

where m_1, m_2 and m_3 denote the neutrino masses. The present best fit value of J_{CP}^l in the global analysis [8] is $J_{CP}^l \simeq -2 \times 10^{-2}$ at EW scale.

We calculate J_{CP}^l as well as the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing angles and Dirac CP phase δ_{CP}^l in two cases for the neutrino mass matrix: (1) The neutrino mass matrix is diagonal. (2) The neutrino mass matrix is non-diagonal which leads to tri-bimaximal mixing [9,10]. In the case (2), no additional CP violating phases are introduced apart from the Majorana phases. The numerical results of the PMNS mixing angles, the Jarlskog invariant J_{CP}^l and Dirac CP phase δ_{CP}^l are given in Tables.1 and 2. We also show the CG coefficients used in our numerical analysis.

For the case (1) where the neutrino mass matrix is diagonal, the wrong prediction of the PMNS mixing angles leads to the failure of the prediction for the magnitude of J_{CP}^l. As shown in the Table.2 of [11], the relative sign among the Jarlskog invariant for quarks and leptons do not depends on the texture of the down-type quark mass matrix in the case of Pati–Salam model. On the other hand, the Jarlskog invariant of leptons vanishes for several textures in the case of SU(5) model.

For the case (2) where the neutrino mass matrix is non-diagonal, the magnitude of the leptonic Jarlskog invariant is consistent with the experimental expected value in order-of-magnitude estimate. As shown in the Table.3 of [11], the J_{CP}^l and J_{CP}^q for all textures have same sign as far as the sign of CG coefficients are positive in the case of Pati–Salam model. Therefore one negative CG coefficient -3 should be taken at least.

	J_{CP}^l	$\sin^2\theta_{12}^l$	$\sin^2\theta_{23}^l$	$\sin\theta_{13}^l$	δ_{CP}^l	$(C_a, C_{a'}, C_b, C_c, C_{c'}, C_d)$
Pati–Salam	-8.1×10^{-5}	0.021	0.012	0.015	$-20.4°$	$(2, 1, -3, -3, 1, 1)$
SU(5)	-2.5×10^{-5}	0.0022	0.39	0.038	$-1.7°$	$(1, \frac{3}{2}, \frac{9}{2}, 6, 1, 1)$

Table 1: Predicted values in the case of diagonal neutrino mass matrix. We also show the CG coefficients used in the numerical computations.

	J_{CP}^l	$\sin^2\theta_{12}^l$	$\sin^2\theta_{23}^l$	$\sin\theta_{13}^l$	δ_{CP}^l	$(C_a, C_{a'}, C_b, C_c, C_{c'}, C_d)$
Pati–Salam	-0.76×10^{-2}	0.38	0.47	0.06	$-30°$	$(2, 1, -3, \frac{3}{4}, 1, 1)$
SU(5)	-1.13×10^{-2}	0.28	0.85	0.153	$-113°$	$(1, \frac{9}{2}, \pm\frac{9}{2}, \frac{9}{2}, -\frac{3}{2}, -\frac{1}{2})$

Table 2: Predicted values in the case of non-diagonal neutrino mass matrix which leads to tri-bimaximal mixing. We also show the CG coefficients used in the numerical computations.

3 Conclusion

We investigate the correlations between CP violating observables of the quarks and leptons in the Pati–Salam and SU(5) models with the standpoint of “Occam’s Razor” approach. In the case where the neutrino mass matrix is diagonal, the wrong prediction of the PMNS mixing angles leads to the failure of the prediction for the magnitude of J_{CP}^l. For the case where the neutrino mass matrix leads to tri-bimaximal mixing, the magnitude of the leptonic Jarlskog invariant is consistent with the experimental expected value in order-of-magnitude estimate.

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New mixing angle and Higgs mass formulas in the left-right symmetric model

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In the left-right symmetric model neutral gauge fields are characterized by three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ between three gauge fields $B_\mu, W^3_{L\mu}, W^3_{R\mu}$, which produce mass eigenstates A_μ, Z_μ, Z'_μ. The mass matrix can be diagonal if $\tan\theta_{23} = -s_{12}s_{13}/c_{12} + O(\delta)$, where $s_{12} = \sin\theta_{12}$ etc. and δ is an infinitesimally small parameter associated with the spontaneously broken left-right gauge symmetry. By neglecting the δ term this is equivalent to $c'^2 + s'^2 = 1$,with $c' = c_{12}/c_{23} = \cos\theta'$ and $s' = s_{12}c_{13} = sin\theta'$. We find a new mixing angle θ', which corresponds to the Weinberg angle θ_W in the $SU(2)_L \times U(1)_Y$ gauge model, and any mixing angle is expressed in terms of θ' and $\varepsilon = g_L/g_R$ a ratio of left-right gauge coupling constants. Our general observation is that the light gauge bosons W and Z are described by θ' only, whereas heavy gauge bosons W' and Z' are described by two parameters θ' and ε. Finally we discuss the Higgs mass formulas in this model.

PACS numbers:

I. A NEW MIXING ANGLE

Let us first summarize the LRSM proposed by Mohapatra-Senjanovic[1, 2], which is invariant under the gauge group $G = SU(2)_L \times SU(2)_\times U(1)_{(B-L)}$. We introduce three kinds of Higgs fields with representations as

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \quad \tilde{\phi} = \begin{pmatrix} \bar{\phi}_2^0 & -\phi_2^+ \\ -\phi_1^- & \bar{\phi}_1^0 \end{pmatrix}, \tag{1.1}$$
$$\Delta_L = \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix}, \quad \Delta_R = \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix}.$$

The Lagrangian is given by $L = L_F + L_Y + L_B$

$$\begin{aligned} L_F &= i\bar{\psi}_L^j \gamma^\mu \Big(\partial_\mu - i\frac{1}{2}g_1 Y_F B_\mu - ig_L W_\mu^L\Big)\psi_L^j \\ &+ i\bar{\psi}_R^j \gamma^\mu \Big(\partial_\mu - i\frac{1}{2}g_1 Y_F B_\mu - ig_R W_\mu^R\Big)\psi_R^j, \quad W_\mu^{L,R} \equiv \frac{1}{2}\tau_\alpha W^\alpha_{\mu,L,R}, \end{aligned} \tag{1.2}$$

$$\begin{aligned} L_Y &= -\bar{\psi}_L^i \Big(f_{ij}\phi + \tilde{f}_{ij}\tilde{\phi}\Big)\psi_R^j + \text{H. c.} \\ &- i\psi_L^{Ti} C h_{ij}^L \tau_2 \Delta_L \psi_L^j + \text{H. c.} \\ &- i\psi_R^{Ti} C h_{ij}^R \tau_2 \Delta_R \psi_R^j + \text{H. c.}, \end{aligned} \tag{1.3}$$

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$$
\begin{aligned}
L_B = {} & tr|D_\mu \Delta_L|^2 + tr|D_\mu \Delta_R|^2 + tr|D_\mu \phi|^2 \\
& + \text{Yang-Mills terms of } B_\mu, W_\mu^L, W_\mu^R \\
& - V(\text{Higgs potential of } \phi, \Delta_L, \Delta_R) \ ,
\end{aligned} \tag{1.4}
$$

The spontaneously broken gauge symmetry of G is realized by non-zero vacuum expectation values of Higgs fields

$$
\langle \ \phi \ \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix} \ , \quad \langle \ \Delta_{L,R} \ \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ V_{L,R} & 0 \end{pmatrix} \ , \tag{1.5}
$$

with assumptions $|V_L| << |\kappa_{1,2}| << |V_R|$. Let us introduce new variables (A_μ, Z_μ, Z'_μ), which are mass eigenstates of three gauge fields $(B_\mu, W^3_{L\mu}, W^3_{R\mu})$

$$
\begin{pmatrix} B \\ W_L^3 \\ W_R^3 \end{pmatrix} = T \begin{pmatrix} A \\ Z \\ Z' \end{pmatrix} \ , \tag{1.6}
$$

where T is a 3×3 unitary matrix with three mixing angles

$$
T = \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{23} - c_{12}s_{23}s_{13} & s_{12}s_{23} - c_{12}c_{23}s_{13} \\ s_{12}c_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} \\ s_{13} & s_{23}c_{13} & c_{23}c_{13} \end{pmatrix} \ , \tag{1.7}
$$

where $c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$, In order that A should be the electric field, we have constraints to the electric charge e_0

$$
\begin{aligned}
g_1 T_{11} &= g_1 c_{12} c_{13} = e_0 \ , \\
g_L T_{21} &= g_L s_{12} c_{13} = e_0 \ , \\
g_R T_{31} &= g_R s_{13} = e_0 \ ,
\end{aligned} \tag{1.8}
$$

Gauge boson mass matrices are given by the trace parts in (1.4). We find the condition that the mass matrix of neutral gauge bosons be diagonal is $\tan\theta_{23} = -s_{12}s_{13}/c_{12} + O(\delta)$, $\delta = (\kappa_1^2 + \kappa_2^2)/V_R^2$. By neglecting the δ term this is equivalent to

$$
c'^2 + s'^2 = 1 \ , \text{with } c' = \frac{c_{12}}{c_{23}} = \cos\theta' \ , \quad s' = s_{12}c_{13} = \sin\theta' \ . \tag{1.9}
$$

where by θ' we define a new mixing angle.

II. CONCLUDING REMARKS

We see that the light gauge bosons W and Z are described by the angle θ' only as below

$$
\begin{aligned}
& M_W = \frac{37.3}{s'} \text{ GeV} \ , \\
& \frac{M_Z}{M_W} = \frac{1}{c'} \ , \\
& g_{\nu Z} = -\frac{e_0}{2c's'} \ , \\
& g(pZ)_L = \frac{e_0(c'^2 - s'^2)}{2s'c'} \ , \quad g(pZ)_R = -\frac{e_0 s'}{c'} \ ,
\end{aligned} \tag{2.1}
$$

together with

$$
\begin{aligned}
g_L &= \frac{e_0}{s_{12}c_{13}} = \frac{e_0}{s'} = g = \frac{e_0}{\sin\theta_W} = \frac{e_0}{0.48} = 0.63 \ , \\
g' &\equiv g_1 c_{13} c_{23} = \frac{e_0}{c_{12}} c_{23} = \frac{e_0}{c'} = 0.34 \ ,
\end{aligned}
\tag{2.2}
$$

where θ', g and g' are regarded as the Weinberg angle θ_W and gauge coupling constants in the $SU(2)_L \times U(1)_Y$ gauge theory. As for the heavy gauge bosons W' and Z', they are described by two parameters θ' and $\varepsilon = g_L/g_R$.

Any mixing angles are expressed in terms of θ' and ϵ

$$
\begin{aligned}
\theta' &= \theta_W \ , \\
\theta_{12} &= \sin^{-1}\Big(\frac{\sin\theta_W}{\sqrt{1-\varepsilon^2\sin^2\theta_W}}\Big) \ , \\
\theta_{13} &= \sin^{-1}\big(\varepsilon\sin\theta_W\big) \ , \\
\theta_{23} &= \cos^{-1}\sqrt{\frac{1-\varepsilon^2\tan^2\theta_W}{1-\varepsilon^2\sin^2\theta_W}}
\end{aligned}
\tag{2.3}
$$

and

$$
\frac{M_{W'}^2}{M_{Z'}^2} = \frac{1}{2}\big(1-\varepsilon^2\tan^2\theta'\big) \ . \tag{2.4}
$$

Note that we have 4 constraints (1.8) and (1.9) among 6 parameters g_1, g_L, g_R, θ_{12}, θ_{13}, θ_{23}, hence leaving two independent quantities θ' and ε.

Finally we have Higgs mass formulas

$$
\begin{aligned}
m_\phi &= 2b\kappa^2 = (125GeV) \ , \quad m_L = \sqrt{\rho'-\rho}V_R \ , \\
m_R &= 2\rho V_R \ , \quad \frac{m_L}{m_R} = \sqrt{\frac{\rho'-\rho}{2\rho}} \ ,
\end{aligned}
\tag{2.5}
$$

from the Mohapatra-Senjanovic potential

$$
\begin{aligned}
V_0 = &-\mu^2(V_L^2+V_R^2) + \frac{\rho}{4}(V_L^4+V_R^4) + \frac{\rho'}{2}V_L^2V_R^2 \\
&+ \frac{\alpha}{2}(V_L^4+V_R^4)\kappa^2 + \beta V_L V_R\kappa^2 + f(\kappa) \ ,
\end{aligned}
\tag{2.6}
$$

where $V_L V_R = \beta\kappa^2/(\rho-\rho')$ with $V_L << V_R$, and $f(\kappa) = -ak^2 + b\kappa^4/4$.

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