Master's Thesis

On Orbifolds in Closed Superstring Field Theory

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Zur Orbifolds in der geschlossenen Superstring-Feld-Theorie

Abstract

In this thesis we study the deformation of orbifold conformal field theories (CFT) in the framework of closed superstring field theory (superSFT). In particular, we consider the type II theory on $\mathbb{C}^4/\mathbb{Z}^2$ and its possible deformation of massless modes to the nearby theory on a smoothed-out manifold. This process is described by so-called ``resolving'' or ``blowing-up'' in algebraic geometry.

Description of orbifold CFT is based on twist field [1] that changes the periodic boundary condition to anti-periodic one. This allows us to explore the moduli space of marginal deformations corresponding to the twist modes. By analysing the equation of motion of closed bosonic SFT, we show that the blowing-up is obstructed at the second order.

Furtherly, we move on to superstrings. Since computation shows the second-order obstruction vanishes, it is possible to derive the blown-up metric in terms of the twisted moduli. We note that, although the initial orbifold theory posesses a $\mathcal{N} = 2$ superconformal symmetry, the blown-up mode is not necessarily Kähler, indicating a broken superconformal symmetry. This makes sense because the closed superSFT is defined on a generic $\mathcal{N} = 1$ superconformal string background.

We carry on the analysis to the third order, where we find out that due to the chiral structure of the closed superSFT, the third-order obstruction is simply reduced to zero. This is unlike the open-string case [2].

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Chapter 1

Introduction

1.1 Motivation

Theoretical physicists seek a mathematical description of the nature, in particular, a mathematical model of the fundamental forces and particles. This led to two major milestones in the last century: quantum mechanics and general relativity. In spite of their huge experimental success, it is still not known how the nature coorperates the principles of quantum mechanics into the theory of gravity.

String theory, although not originally proposed for it, is a theory of quantum gravity. It is a theoretical framework in which the point-like particles are blown-up into one-dimensional objects called strings. While string theory may or may not be the dreamt theory of everything, the understanding of string theory and its dualities has cast profound insights into quantum field theories and mathematics. To name a few, AdS/CFT correspondence, Non-abelian gauge theories, supersymmetry, modular forms, etc. A proper understanding of string theory is thus still relevant.

A string theory can be broken down into a sum of all different conformal field theories on Riemann surfaces, called the worldsheet. A sensible theory of fundamental particles should contain fermions in its spectrum. This is achieved by the NSR formalism of the superstring theory replacing the ordinary Riemann surfaces with super Riemann surfaces. There are five consistent superstring theories: type I, type IIA, type IIB, heterotic $SO(32)/\mathbb{Z}_2$ and heterotic $E_8 \times E_8$. However, all those theories require a dimensionality dramatically larger than four of our observed universe. Therefore, it's been a major task of string phenomenology to study the compactification of these extra dimentions into micro scale.

The basic example would be compactifying on a torus, however, it seems difficult to go very far beyond that. One thus needs a systematical method to build new phenomenological models from the old ones. One such way is the orbifold theory, which is a classical geometrical method of constructing new spaces that can be implemented directly into string theory. On the worldsheet it corresponds to the twisted boundary conditions. The mathematical consequence of allowing twisted strings is that string theory can somehow ``probe'' the theory where the singularities in the orbifold theoy are resolved. This raises an interesting question on the interplay between string theory and the geometry of the resolved orbifold, that is, to study the latter from a stringy point of view.

The deformation among different target spaces is realized in string theory by shifting the string vacuum. This process is most suitable to be described in the second quantized picture known as string field theory. The textbook approach to string theory is known to be perturbative. Conceptually it plays a similar role to the Feynman rules and propagators in particle physics. One may wonder, however, if there is an action principle governing the string interaction, and hence comes the string field theory. The field theory approach is especially helpful for our purpose, since different string backgrounds are understood to be the classical solutions to the string field equation of motion. That is to say, string field theory provides a unified viewpoint of all the string backgrounds that serves as a standpoint to start our analysis of the deformation.

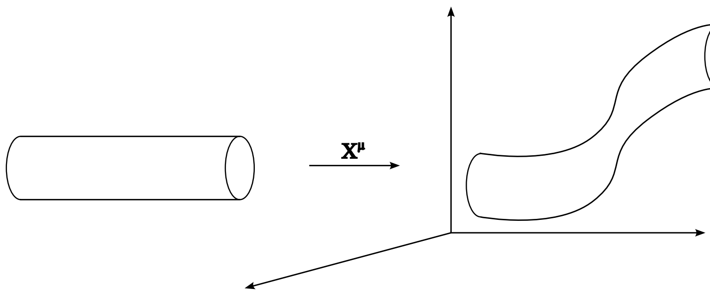

Figure 1.1: String theory studies the map from worldsheet into targe spacetime.

1.2 Content of the Thesis

The main goal of the thesis is to exploit the viewpoint stated above. More specifically, we try to apply the type II string field theory to understand the spacetime geometry deformed from an orbifold model. Before really tackling the problem, we will develop necessary machineries. The outline of the thesis is as follows:

In chapter 2 we review the worldsheet approach of string theory. We will see how path-integral of Polyakov action gives a measure on the moduli space of Riemann surfaces and amplitudes of a proper worldsheet conformal field theory. We then review common CFT techniques to treat the latter, including the CFT resulted from an orbifold construction.

In chapter 3 we review the construction of the type II closed string field theory action. The measure on the moduli space provided by string theory should be extended off-shell. Geometrically this corresponds to pulling back to an appropriate bundle over the moduli space. We explain the construction of string field vertices in such a geometric setup, yielding a geometric Batalin-Vilkovisky structure. Passing to the worldsheet CFT we obtained the algebraic vertices appearing in tha action, satisfying a L_∞ relation in the classical part. We then move on to the theory of superstrings. We see that integrating out the odd modulus results in dressing the vertices with the picture-changing operators, which is achieved in the NS-NS sector by a recursive procedure.

Chapter 4 contains all the calculations of the main problem. The problem of whether we can find an exactly marginal deformation is formulated in a form of perturbative equations. We show that, in the bosonic case, the deformation is obstructed already at the second order. However, this obstruction is resolved in superstring theory. We are thus able to compute the second-order deformation of the target spacetime geometry. In particular we compute the spacetime metric, and find out that it is not Kähler anymore. Moreover, we are capable of moving further, by showing that the third-order obstruction is still vanishing.

Chapter 2

Worldsheet Content

In this chapter we review the basics of string theory and how it is reduced to an integrated worldsheet CFT amplitude. We do it in a manner that treats bosonic strings and superstrings similarly, namely from a (super)Riemann surface point of view. As we will see, this approach is readily generalized to string field theory. The main reference we follow here is [3]. Moreover, we review the notion of an orbifold CFT introduced in [1], which is of particular interest in this thesis.

2.1 String Theory

2.1.1 Path Integral Construction of Amplitudes

The string worldsheet is a Riemann surface Σ_h of genus h with a Riemannian metric g. It is worth noting that the complex structure on such a surface is uniquely determined by the conformal structure $[g]$ induced by the metric, and vice versa. The time slice of these Riemann surface is typically depicted as one or a few loops of string, propagating through the Minkowski spacetime $M^{1,D-1}$.

We start with the well-known Polyakov action as a functional on the mapping space $\mathcal{C}^\infty(\Sigma_h, M^{1,D-1})$

$$S_P = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\sqrt{g}g^{ab}G_{\mu\nu}(X)\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}, \tag{2.1}$$

where $\sigma^a = (\tau, \sigma)$ are local coordinates on the worldsheet and $X \in \mathcal{C}^\infty(\Sigma_h, M^{1,D-1})$. The action itself is nothing more than a simplest example of σ-model. However, the metric $g_{ab}(\sigma)$ is taken to be an auxiliary field and needs to be integrated out in the path integral

$$Z_h = \int \mathrm{d}g \int \mathrm{d}X \mathrm{e}^{-S_P[g,X]}. \tag{2.2}$$

The fact that action S_P possesses a infinite-dimensional gauge symmetry gives divergence. We denote the gauge group as $\mathcal{D}iff \rtimes \mathcal{W}eyl$. The physically meaningful partition function should be modified as

$$Z_h = \int \frac{\mathrm{d}g}{\mathcal{V}ol}\int \mathrm{d}X \mathrm{e}^{-S_P[g,X]}, \tag{2.3}$$

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where $\mathcal{V}ol$ is the volumn of the gauge group.

The path integral is evaluated via the standard Faddeev-Popov procedure, and furtherly replace the Faddeev-Popov determinent with an integral over two anticommuting ghost fields, the ghost c^a field and the symmetric traceless anti-ghost b_{ab} field. The path integral can be rewritten as

$$Z_h = \int_{\mathcal{M}_h} \mathrm{d}t \frac{\det(\phi_i, \hat{\mu}_j)_{\hat{g}}}{\sqrt{\det(\phi_i, \phi_j)_{\hat{g}}}} \frac{\Omega_{ckv}[\hat{g}]^{-1}}{\sqrt{\det(\psi_i, \psi_j)_{\hat{g}}}} \int \mathrm{d}_{\hat{g}} X \mathrm{d}'_{\hat{g}} b \mathrm{d}'_{\hat{g}} c \mathrm{e}^{-S_P[\hat{g}, X] - S_{gh}[\hat{g}, b, c]}, \tag{2.4}$$

where d' denotes integration without the zero modes that give infinite result in Gaussian integral, ψ_i and ϕ_i are respectively a basis for zero-modes of the CKV operator and its adjoint, and μ_i are Beltrami differentials that correspond to traceless Teichmüller deformations. One may refer textbooks like [4]. The benefit is that the integrand depends only on a fixed reference metric we choose, so it is sufficient to study a fixed conformal field theory given by $S_P[\hat{g}, X] + S_{gh}[\hat{g}, b, c]$, the conformal background, then integrate on the moduli space.

2.1.2 Measures on the (Super)moduli

It would be beneficial to write the integral (2.4) as an integral of superfunctions. One needs to choose such a function that the Berezin integral along the odd variables gives the right measure on the moduli space. Here we merely quote the conclusion and key properties, details can be found in [3].

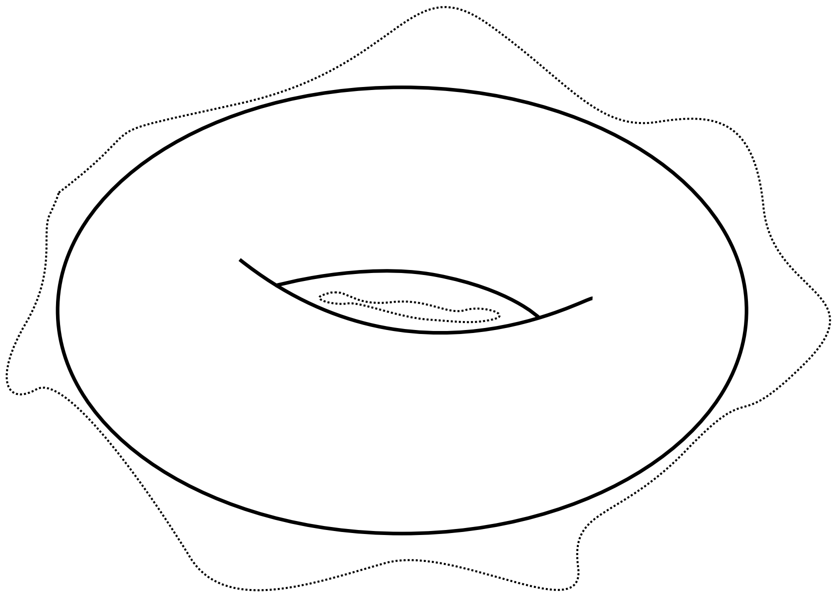

Figure 2.1: Supermanifolds can be thought of as spaces with a ``mist'' of odd coordinates

One initiates the re-formulating by defining a superfunction $F_{\mathcal{V}}(g, \delta g)$ on the superspace $(g, \delta g)$, where the vertex operators $\mathcal{V}_1, \cdots, \mathcal{V}_n$ are included in the definition. The natural requirements on the function would be Weyl invariance and diffeomorphism invariance, making it into a pullback from the n-point, genus h moduli space $\mathcal{M}_{h,n}$. One thus defines

$$\hat{S}[X,b,c,g,\delta g] = S_P + S_{gh} + \frac{1}{4\pi}\int \mathrm{d}^2\sigma\sqrt{g}\delta g_{ij}b^{ij} \tag{2.5}$$

and

$$F_{\mathcal{V}}(g,\delta g) = \int \mathrm{d}X\mathrm{d}b\mathrm{d}c\mathrm{e}^{-\hat{S}[X,b,c,g,\delta g]}\mathcal{V}_1\cdots\mathcal{V}_n. \tag{2.6}$$

The introducing of $\hat{S}$ actually gives the correct Jacobian passing from g to the modular parameters $\{m_{\dim(\mathcal{M}_{h,n})}\}$. Therefore, $F_{\mathcal{V}}(g,\delta g)$ provides a form on the moduli space $\mathcal{M}_{h,n}$ that will be denoted by $\Omega_{\mathcal{V};h,n}$.

The superstring amplitudes are more or less similarly given by

$$F_{\mathcal{V}}(\mathcal{J},\delta\mathcal{J}) = \int \mathrm{d}X\mathrm{d}b\mathrm{d}c\mathrm{d}\beta\mathrm{d}\gamma\mathrm{e}^{-\hat{S}[X,b,c,\beta,\gamma,\mathcal{J},\delta\mathcal{J}]}\mathcal{V}_1\cdots\mathcal{V}_n, \tag{2.7}$$

where

$$\hat{S} = S + \frac{1}{2\pi}\int \mathrm{d}z\mathrm{d}\bar{z}\mathrm{d}\theta\mathrm{d}\bar{\theta}(\delta\mathcal{J}B - \delta\bar{\mathcal{J}}\bar{B}) \tag{2.8}$$

with $C^z = c^z + \theta\gamma^\theta$ and $B_{z\theta} = \beta_{z\theta} + \theta b_{zz}$ being the complexified ghost superfields introduced to fix the super-diffeomorphic gauge redundancy and $\delta\mathcal{J}$ being the variation of the complex structure on the worldsheet.

2.1.3 Reducing to the Bosonic Moduli

Integrating out all the fermionic degrees of freedom in (2.7) needs a basis chosen for the odd moduli. By turning on a gravitino field $\chi_{\bar{z}}^{\theta}$ we can deform the super Riemann surface Σ by

$$\delta\mathcal{J}_{\bar{z}}^{z} = \theta\chi_{\bar{z}}^{\theta}. \tag{2.9}$$

The construction is most often considered when the gravitino modes are delta functions supported at some point $p_i \in \Sigma_{red}$, and then the integration gives a factor that can be represented by

$$\mathcal{X}(z) = \delta(\beta(z))S_{z\theta}(z), \tag{2.10}$$

the picture-changing operator.

2.2 CFT Systems

In the previous section we have identified the string amplitudes with the amplitudes of conformally invariant vertex operators on the worldsheet, provided that they have the right ghost and picture number and an integration over the moduli space. This allows us to use the standard CFT techniques in the evaluation of the string amplitudes. In this section we review some CFT systems that serve as the building blocks of string theory. All the CFT systems will be discussed in the radial quantization picture, where the radial direction represents time.

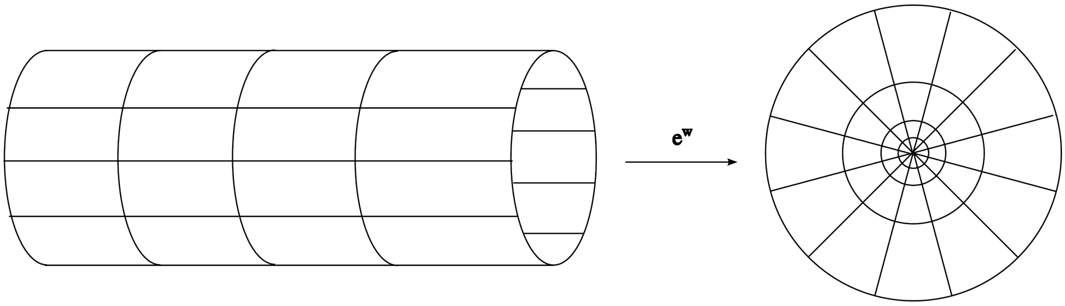

Figure 2.2: Radial quantization mapping from a cylinder worldsheet into the complex plane

2.2.1 Free Boson

Each of the D copies of free scalar fields has the action

$$S = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2x\sqrt{g}g^{\mu\nu}\partial_\mu X\partial_\nu X, \tag{2.11}$$

with the identification $X(\sigma) = X(\sigma + 2\pi)$. The cylinder can be conformally mapped into $\mathbb{C}\setminus\{0\}$ via radial quantization. In complex coordinates, the action on flat space reads

$$S = \frac{1}{2\pi\alpha'}\int \mathrm{d}z\mathrm{d}\bar{z}\,\partial_z X\partial_{\bar{z}} X. \tag{2.12}$$

The energy-momentum tensor of the theory is

$$T(z) = \frac{1}{\alpha'} : jj : (z). \tag{2.13}$$

The propagator can be easily derived from the path integral identity

$$0 = \int \mathrm{d}X \frac{\delta}{\delta X(z)}\mathrm{e}^{-S[X]X(w)} \tag{2.14}$$

as

$$\langle X(z)X(w)\rangle = -\frac{\alpha'}{2}\ln(z-w). \tag{2.15}$$

The operator product expansion can also be read off

$$X(z)X(w) = -\frac{\alpha'}{2}\ln(z-w) + O(z-w). \tag{2.16}$$

To prevent the logarithm we introduce the current

$$j(z) = i\partial X(z) = \sum_{n\in\mathbb{Z}} j_n z^{-n-1}, \tag{2.17}$$

where the modes satisfy the commutation relation $[j_n, j_m] = n\delta_{n+m}$ encoded in the OPE

$$j(z)j(w) = \frac{\alpha'}{2}\frac{1}{(z-w)^2} + O(z-2). \tag{2.18}$$

To construct the Hilbert space of the theory one hopes to identify all the primary fields. They are exactly the current $j(z)$ satisfying

$$T(z)j(w) = \frac{j(w)}{(z-w)^2} + \frac{\partial j(w)}{z-w} + O(z-w) \tag{2.19}$$

and $V_p(z) =: \mathrm{e}^{ipX} : (z)$ satisfying

$$T(z)V_p(w) = \frac{p^2}{2}\frac{V_p(w)}{(z-w)^2} + \frac{\partial V_p(w)}{z-w} + O(z-w). \tag{2.20}$$

Then one can construct the Verma module as usual, starting with a unique vacuum $|0\rangle$. The modes j_n are used to construct the Fock space:

$$j_n|0\rangle = 0, \quad n \geq 0, \tag{2.21}$$

$$\langle 0|j_n = 0, \quad n \leq 0. \tag{2.22}$$

2.2.2 Ghost Systems

A ghost system, or a first-order system describes two symmetric traceless fields $b_{\mu_1\cdots\mu_\lambda}$ and $c^{\mu_1\cdots\mu_{\lambda-1}}$ via the action

$$S = \frac{1}{4\pi}\int \mathrm{d}^2x\sqrt{g}g^{\mu\nu}b_{\mu\mu_1\cdots\mu_{\lambda-1}}\nabla_\nu c^{\mu_1\cdots\mu_{\lambda-1}}. \tag{2.23}$$

In complex coordinates the action writes

$$S = \frac{1}{2\pi}\int \mathrm{d}z\mathrm{d}\bar{z}\left(b\partial_z c + \bar{b}\partial_{\bar{z}}\bar{c}\right), \tag{2.24}$$

where the parameter λ can actually be generalized to include half-integers that should be written as spinors in real coordinates. Ghost fields carry wrong Grassmann parity against spin-statistic theorem, which we denote by ϵ. The conformal weights of the fields are

$$h(b) = 1-\lambda, \quad h(c) = \lambda. \tag{2.25}$$

The energy-momentum tensor of the theory is

$$T(z) = -\lambda : b\partial c : (z) + (1-\lambda) : \partial bc : (z). \tag{2.26}$$

Again, we can obtain the propagator and thus the OPE

$$c(z)b(w) = \frac{1}{z-w} + O(z-w), \quad b(z)c(w) = \frac{\epsilon}{z-w} + O(z-w). \tag{2.27}$$

In string theory we will encounter two types of ghost systems. The ususal bc-ghost system is parameterized by $(\epsilon, \lambda) = (1, 2)$. The $\beta\gamma$ superghost system is parameterized by $(\epsilon, \lambda) = (-1, \frac{3}{2})$.

2.2.3 Free Fermion

In the super Riemann surface formulation of superstring theory, one derives the worldsheet action introducing the odd variable θ. This results in an worldsheet fermion ψ with the action

$$S[\psi] = \frac{1}{2\pi} \int \mathrm{d}^2 z \left(\psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi} \right). \tag{2.28}$$

With this action at hand one can derive the equation of motion, which implies $\psi(z)$ and $\bar{\psi}(\bar{z})$ are holomorphic and anti-holomorphic field respectively. The OPE can also be write down straightforwardly,

$$\psi(z)\psi(w) = \frac{1}{z-w} + O(z-w). \tag{2.29}$$

The subtlety here is that for the worldsheet fermions, there is no canonical choice of the spin structure or, in terms of super Riemann surface, the superschemes that reduce to a Riemann surface are not unique. This results in different sectors for the fermions. A consistent superstring theory requires a proper summation of these sectors in the path integral, and the consequence is known as the GSO projection.

For fermions on the complex plane there are two inequivalent spin structures that are encoded in the boundary conditions

$$\psi(\mathrm{e}^{2\pi i}) = \pm \psi(z). \tag{2.30}$$

The boundary condition can be periodic or antiperiodic under a rotation around the origin, and the fermion sujecting to these conditions are called in the Neveu-Schwarz (NS) sector and the Ramond (R) sector respectively.

Solving the equation of motion with different boundary conditions actually gives different mode expansion. In the NS sector we have

$$\psi(z) = \sum_{r \in \mathbb{Z}+1/2} \psi_r z^{-r-1/2}, \tag{2.31}$$

where the modes ψ_r satisfy the anticommutation relation $\{\psi_r, \psi_s\} = \delta_{r+s}$.

The energy-momentum tensor is

$$T(z) = \frac{1}{2} : \psi \partial \psi : (z). \tag{2.32}$$

In the Ramond sector we have

$$\psi(z) = \sum_{r \in \mathbb{Z}} \psi_r z^{-r-1/2}, \tag{2.33}$$

where the modes ψ_r again satisfy the relation $\{\psi_r, \psi_s\} = \delta_{r+s}$. Note the presence of zero-mode here. The Ramond Hilbert space is built from a different (degenerate) vacuum $|S\rangle$. Some correlation functions are thus different from the NS sector, even though the fields share the same OPE, for example we have

$$\langle \psi(z)\psi(w) \rangle_S = \frac{1}{2(z-w)} \left(\sqrt{\frac{z}{w}} + \sqrt{\frac{w}{z}} \right), \tag{2.34}$$

with the subscript S indicates a different vacuum.

It is possible to connect the two vacuua $|0\rangle$ and $|S\rangle$ by defining that

$$|S\rangle = \lim_{z\to 0} S(z)|0\rangle. \tag{2.35}$$

The field $S(z)$ is called the fermionic twist field and is with conformal dimension 1/16.

We see, in this subsection, that half-integer modes come into play because of the boundary conditions determined by different spin structures, and it results in two distinct vacua that can be connected by introducting a new field corresponding the operation of ``twisting'' the boundary condition. We will show that similar things need to be done for the bosons too, but for a slightly different reason.

2.2.4 Bosonization of the Superghosts

The $\beta\gamma$-ghost system is obtained by taking $\epsilon = -1$ and $\lambda = \frac{3}{2}$. It can be bosonized into another fermionic first-order system (ξ, η) with $(\epsilon, \lambda) = (1, 1)$ and a scalar boson ϕ. which are decoupled. The bosonization is explicitly written as

$$\gamma = \eta e^{\phi}, \quad \beta = \partial\xi e^{-\phi}. \tag{2.36}$$

The energy-momentum tensor is

$$T(z) = T^{\xi\eta}(z) + T^{\phi}(z) \tag{2.37}$$

with

$$T^{\xi\eta}(z) = - : \eta\partial\xi : (z), \quad T^{\phi}(z) = -\frac{1}{2} : \partial\phi\partial\phi : (z) - \partial^2\phi(z). \tag{2.38}$$

The OPE's of the new fields are listed below

$$\xi(z)\eta(w) = \frac{1}{z-w} + O(z-w) \tag{2.39}$$

$$e^{q_1\phi(z)}e^{q_2\phi(w)} = \frac{e^{(q_1+q_2)\phi(w)}}{(z-w)^{q_1+q_2}} + O(z-w) \tag{2.40}$$

$$\partial\phi(z)\partial\phi(w) = -\frac{1}{(z-w)^2} + O(z-w). \tag{2.41}$$

Upon bosonization the PCO in (2.10) becomes

$$\mathcal{X}(z) = \{Q, \xi(z)\}, \tag{2.42}$$

and similar for the anti-holomorphic sector.

2.2.5 Spin Fields

Bosonization can also be applied to the free fermion systems. For a D-dimensional target spacetime there are D copies of the free fermion systems denoted by ψ^i. There exists a equivalence between two free fermion systems ψ^{2i-1}, ψ^{2i} and a free boson ϕ^i realized by

$$\Psi^{\pm,i} = \frac{1}{\sqrt{2}}(\psi^1(z) \pm i\psi^2(z)), \tag{2.43}$$

$$\Psi^{\pm,i} =: \mathrm{e}^{\pm i\phi^i} : . \tag{2.44}$$

As indicated by the half-integer modes, for the Ramond vacuum the aforementioned fermionic twist field (2.35) is bosonized as

$$S^i(z) =: \mathrm{e}^{\frac{1}{2}i\phi^i} : . \tag{2.45}$$

In order to get spacetime spinor from the worldsheet spinor, one has to construct representations of $SO(D)$ group (in the Euclidean case). This is done by introducing indices

$$A = (\underbrace{\pm\frac{1}{2}, \cdots, \pm\frac{1}{2}}_{D/2\,\text{times}}) \tag{2.46}$$

for the spin fields

$$S^A = \prod_{i=1}^{D/2} : \mathrm{e}^{iA_i\phi^i} : . \tag{2.47}$$

All the $2^{D/2}$ spin fields split into two irreducible chiral representation: The left handed spinor S_α with an even number of minus signs and the right handed spinor $S^{\dot\alpha}$ with an odd number of minus signs.

To be precise the bosonization procedure needs to be modified according to [5], in order to enforce the right anticommutation relations between different fermions.

The ϕ^i system is a free CFT of bosons, its correlation function obeys

$$\begin{aligned} &\left\langle \mathrm{e}^{ip_1\phi^{i_1}}(z_1) \cdots \mathrm{e}^{ip_n\phi^{i_n}}(z_n) \right\rangle \\ &= \mathrm{e}^{\sum_{i<j} p_i p_j G(z_i,z_j)} \delta\left(\sum_{i=1}^n p_i\right) \\ &= \prod_{i<j} (z_i - z_j)^{p_i p_j} \delta\left(\sum_{i=1}^n p_i\right), \end{aligned} \tag{2.48}$$

where $G(z_i, z_j)$ is just the Green's function

$$G(z_i, z_j) = -\left\langle \phi(z_i)\phi(z_j) \right\rangle = \ln(z_i - z_j). \tag{2.49}$$

Therefore, spin fields also simplify some calculation of the correlation functions, it will be useful to know

$$\left\langle S^{\dot\alpha}(z_1)\psi_\mu(z_2)S^{\dot\beta}(z_3) \right\rangle = -\frac{(\bar\sigma_\mu)^{\dot\alpha}_{\dot\beta}}{\sqrt{2}} \frac{1}{z_{12}^{1/2} z_{23}^{1/2}} \tag{2.50}$$

and

$$\left\langle S^{\dot{\alpha}}(z_1)\psi_{\mu\nu}(z_2)S^{\dot{\beta}}(z_3)\right\rangle = -\frac{1}{2}(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}\dot{\beta}}\frac{z_{13}^{1/2}}{z_{12}z_{23}}, \tag{2.51}$$

with

$$\psi_{\mu\nu} =: \psi_\mu\psi_\nu : . \tag{2.52}$$

2.3 Orbifold Theory

Orbifold CFT's are introduced in [1] for model-building in string theory. In fact, other than string theory, it is a general procedure to obtain new CFT from an old one.

An orbifold CFT is constructed as follow. Consider an automorphism σ of the original CFT $\mathcal{H}$ such that $\sigma^n = \mathrm{id}_{\mathcal{H}}$. We have the twisted sector generated by the fractional power field

$$\psi(z) = \sum_{r\in\frac{\mathbb{Z}}{n}} \psi_r z^r. \tag{2.53}$$

This is nothing more than the notion of a σ-twisted representation of the orinal CFT operator algebra [6]. For a finite group, rather than a single generator, this notion is easily generalized by assigning a twisted sector for each conjugacy class.

2.3.1 CFT on $\mathbb{Z}_2$ Orbifolds

The simplest example for the orbifold construction, which also has a clear geometric picture, is the CFT with a $\mathbb{Z}_2 = \{1, \sigma\}$-orbifold. One can simply view it as the theory of a string propagating in the quotient spacetime $M/\sim$ identifying $X \sim -X$. In this case there is only one twisted sector, $\mathcal{H}'$, and it is generated by the half-integer modes j_n with $n \in \mathbb{Z} + \frac{1}{2}$, satisfying the commutation relations $[j_n, j_m] = n\delta_{n+m}$.

Again, different sectors correspond to different vacua. For the twisted vacuum $|\sigma\rangle$. one can define, using the state-field correspondence, the bosonic twist field $\sigma(z)$ of conformal dimension $\frac{1}{16}$ such that

$$|\sigma\rangle = \lim_{z\to 0}\sigma(z)\,|0\rangle\,. \tag{2.54}$$

The correlation functions in the twisted sector should be evaluated in the twisted vacuum $|\sigma\rangle$. For example, we have the propagators

$$\langle X(z)X(w)\rangle_\sigma = -\ln\left(\frac{1-\sqrt{\frac{z}{w}}}{1+\sqrt{\frac{z}{w}}}\right) \tag{2.55}$$

$$\langle j(z)j(w)\rangle_\sigma = -\frac{1}{2(z-w)^2}\left(\sqrt{\frac{z}{w}}+\sqrt{\frac{z}{w}}\right). \tag{2.56}$$

The OPE of the twist field contians a branch cut. For example, we write

$$j(z)\sigma(w) = \frac{s(w)}{(z-w)^{\frac{1}{2}}} + O(z-w) \tag{2.57}$$

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$$j(z)s(w) = \frac{1}{2}\frac{\sigma(w)}{(z-w)^{\frac{3}{2}}} + \frac{2\partial\sigma(w)}{(z-w)^{\frac{1}{2}}} + O(z-w) \tag{2.58}$$

$$\tilde{\sigma}(z)\sigma(w) = \frac{1}{(z-w)^{\frac{1}{8}}} + O(z-w), \tag{2.59}$$

where $s(z)$ is the excited twist field (see [1] for details) and $\tilde{\sigma}(z)$ is the conjugate field defined by BPZ inner product (not to be confused with operator in the anti-holomorphic sector).

2.4 Deformations of the Background

Conformal field theories are not isolated. In a given CFT, operators are classified as relevant, irrelevant and marginal according to their conformal dimensions. Among those the marginal operators of conformal dimension one are related to the possible deformations of a conformal field theory. Such a deformation generated by $V_i(z)$ is of the form

$$\delta S \propto \int \mathrm{d}^2 z \sum_i g_i V_i(z), \tag{2.60}$$

where g_i's are constants corresponding to coordinates in the moduli space of CFT's (rigorously speaking they're the coordinates in the tangent space). We can easily seen that the operator V_i should be of conformal dimension 1. Those operators are called the marginal operators. However, the marginality of $V(z)$ can only preserve the classical conformal symmetry of the action. At the quantum level, one needs a refined notion of exactly marginal deformation, meaning that the deformations generated by them are tangent vectors on the moduli space of conformal field theories, which is also called the conformal manifold in the literature. Checking exact marginality ususally breaks into an order-by-order procedure to see if the deformation preserves the conformal dimension of itself. One should compute the exact beta-function to see if it vanishes. Intuitively, we need to require the vanishing of integrals of the three-point functions

$$\left\langle V_i(z)V_i(w)\int \mathrm{d}^2 z' V_i(z')\right\rangle = 0 \tag{2.61}$$

to guarantee the two-point function remains unperturbed. If all those perturbation, including higher order terms, vanish, then the perturbation is exactly marginal. In general, it is difficult to verify by examination of $(n+2)$-point functions that an operator remains marginal to all orders.

Studying the conformal manifold of two-dimensional conformal field theory is of particular interest as it describes the landscape of string theory vacua. In such cases, these conformal manifold is endowed with a spacetime interpretation of the conformal field theory when we view it as a classical solution of string theory [7]. Various works has been done in this direction. However, it is difficult in general without an extended symmetry. For example, the $\mathcal{N}=2$ superconformal field theories are fairly understood. One can identify all the exactly marginal fields, and, even better, find out that the conformal manifold have locally the structure of the product of the complexified Kähler moduli and the complex moduli.

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The case with $\mathcal{N} = (1,1)$ supersymmetry is, however, elusive. The main purpose of this thesis is to explore the exactly maringal deformation of $\mathcal{N} = (1,1)$ theory around the orbifold point. The string theory picture again plays an important role here, since it is known that the string field theory provide a somehow unified framework to treat different backgrounds as the classical solution to the string field equation of motion. This would be the central topic of the rest of the thesis. Therefore, it is worthwhile to spend the next chapter in introducing the formulation of closed string field theory.

Chapter 3

Closed Superstring Field Theory

The goal of this chapter is to present the geometric construction of the Type II superstring field theory, as outlined by [8], to initiate our calculation in the next chapter. We explain the geometric setup in the bosonic case first, where the mathematical structures such as the BV structure and strong homotopy Lie algebras (L_∞ algebras) shows up (details can be found in [9] and references therein). Going to the super case requires detailed study of super Riemann surfaces and their sewing property. Finally, we explain the explicit construction of the classical string field action in the NS-NS sector given by [10], which is needed for our calculations in the next chapter.

3.1 String Field Theory

String field theory (SFT) is originally proposed as a logical completion to the perturbative formulation of worldsheet string theory. One imetates the procedure of second quantization to obtain an interacting theory of free propagating strings. Like the case of quantum field theory, the summation over topologically different worldsheet is supposed to be derived as Feynman rules from the string field theory action.

However, the fact that there is in general no natrual choice of the field variables leads to a complecated non-polynomial action and various different construction, related by field redefinition and/or gauge fixing. It thus remains elusive that how the string field theory plays the fundamental role like QFT. Nevertheless, recent research has shown that SFT is indispensable in the non-perturbative region such as the infrared behaviour of string theory and D-brane backgrounds.

String field theory is assumed to be background independent (and proven so in, e.g., [11]), in a sense that the landscape of different string backgrounds are organized in the form of classical solutions to the string field theory equation of motion. We shall exploit this point in Type II string field theory.

3.2 Geometric BV Structure

The construction of string field theory requires full-fledged Batalin-Vilkovisky formalism, though many intuitions can be gained from the second quantization from point particle

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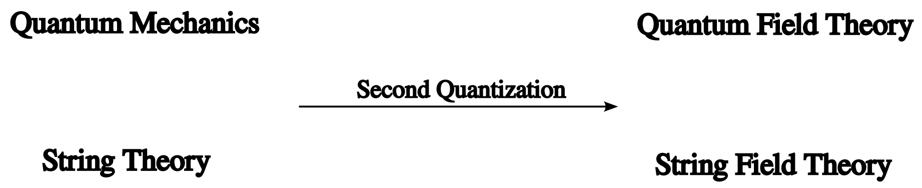

Figure 3.1: String field theory plays the role of quantum field theory in the second quantized picture

to ordinary quantum field theory. Such a generalization requires off-shell amplitudes in string theory. Vertex operators $\mathcal{V}$ in the on-shell amplitudes (2.6) are subject to certain conditions. Primarily, they are taken to be BRST closed

$$Q\mathcal{V}_i = 0, \tag{3.1}$$

which is analogous to the (linearized) equation of motion in QFT, for it reduces to the Klein-Gordon equation for point particles [9].

Generalizing to an arbitrary operator in the background CFT faces an immediate difficulty that the conformal invariance is lost because we lose control of the conformal weights of the operator products. One compensates it by introducing local coordicate curves near the punctures

$$f_i : D \to \Sigma_{h,n} \tag{3.2}$$

such that $f_i(0) = z_i$. The off-shell amplitude thus reads

$$\int_{m_{h,n}} \mathrm{d}(g|\delta g) F_{\mathcal{V}}(g, \delta g) = \int_{m_{h,n}} \mathrm{d}(g|\delta g) \int \mathrm{d}X \mathrm{d}b \mathrm{d}c \mathrm{e}^{-\hat{S}[X,b,c,g,\delta g]} \left(f_1 \circ \mathcal{V}_1\right) \cdots \left(f_n \circ \mathcal{V}_n\right). \tag{3.3}$$

All the data for defining the off-shell amplitudes can be collected into a bundle strucutre $\hat{\mathcal{P}}_{h,n} \to m_{h,n}$ with each fiber the local curves around the punctures. A choice of the global section $\mathcal{A}_{h,n}$ of $\hat{\mathcal{P}}_{h,n}$ fixes the off-shell amplitudes. However, the existence of global sections is obstructed in general, even for $\hat{\mathcal{P}}_{0,4}$. The established way to deal with this problem is to restrict the class of states to those which have no sense of ``direction'' on the unit disk. That is, we will only attempt to define off-shell amplitudes for vertex operators satisfying

$$f \circ \mathcal{V}(0) = \mathcal{V}(0) \tag{3.4}$$

with $f(z) = \mathrm{e}^{i\theta} z$ a rotation around the origin. The local coordinate systems up to rotation are called coordinate curves for that they can be uniquely characterized by an circle embedded around the punctured point $S^1 \to \Sigma : z \mapsto f(z), |z| = 1$. We denote the set of all the choices of such coordinate curves by the bundle $\mathcal{P}_{h,n}$.

New punctured Riemann surfaces with coordinate curves can be generated from old ones. This is done by the sewing operation. Given two punctures p_1 and p_2 with local coordinates z_1 and z_2, one can glue these two by identifying via the map

$$I(z_1) = -\frac{1}{z_1} = z_2. \tag{3.5}$$

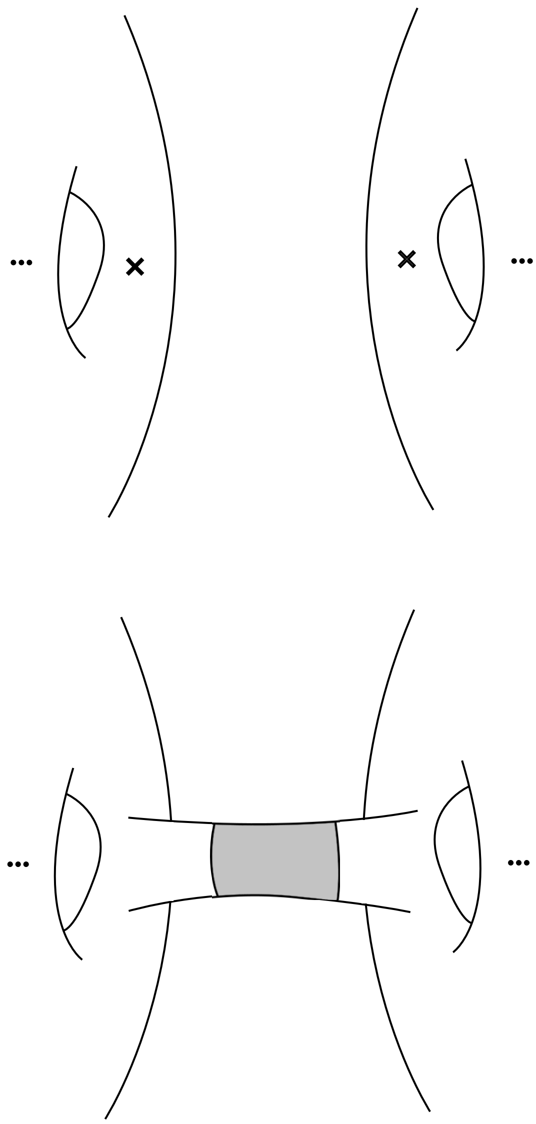

Figure 3.2: The identification creates a ``tube'' connecting two punctures

The sewing operation provides two maps on the bundle $\mathcal{P}_{h,n}$, by gluing two pieces of different Riemann surfaces

$$\Phi : \mathcal{P}_{h_1,n_1+1} \times \mathcal{P}_{h_2,n_2+1} \to \mathcal{P}_{h_1+h_2,n_1+n_2}, \tag{3.6}$$

and by gluing two punctures on the same Riemann surface

$$\zeta : \mathcal{P}_{h,n} \to \mathcal{P}_{h+1,n-2}. \tag{3.7}$$

In order to extract the mathematical structures related to the moduli space, one must introduce the geometric chains $C^*(\mathcal{P}_{h,n})$ on $\mathcal{P}_{h,n}$. The definition of this chain complex, however, requires mathematical technical that we would not like to involve, so we refer [12] for interested readers. Intuitively speaking, we can think of the generators $a_{h,n}$ as some $\dim(a_{h,n})$-dimensional space mapping into $\mathcal{P}_{h,n}$, with the grading

$$\deg(a_{h,n}) = \dim(\mathcal{M}_{h,n}) - \dim(a_{h,n}). \tag{3.8}$$

Therefore, one can choose some zero-degree generators $\nu_{h,n} \in C^0(\mathcal{P}_{h,n})$ and think of them as some spaces with the same dimension as the moduli space $\mathcal{M}_{h,n}$ mapping into the bundle $\mathcal{P}_{h,n}$, that is, some local sections characterizing continuous choices of coordinate curves along the moduli. They will be called the geometric vertices in the following context.

The $\nu_{h,n}$'s contain all the geometric data to construct a string field theory. But we must consider some compatibility conditions among them. The operations (3.6) and (3.7) induce operations on the (co)chain complex, that, by abuse of notations, we still denote as Φ and ζ,

$$\Phi : C^{k_1}(\mathcal{P}_{h_1,n_1+1}) \times C^{k_2}(\mathcal{P}_{h_2,n_2+1}) \to C^{k_1+k_2+1}(\mathcal{P}_{h_1+h_2,n_1+n_2}), \tag{3.9}$$

$$\zeta : C^k(\mathcal{P}_{h,n}) \to C^{k+1}(\mathcal{P}_{h+1,n-2}). \tag{3.10}$$

From now on we restrict to the subchains $C^*_{\text{inv}}(\mathcal{P}_{h,n})$ that are invariant under the permutation of punctures, as required by the indistinguishability of particles. Together with the boundary operator ∂ of the chain complex, we have three operations ∂, $\{-,-\}$ and Δ. The latter two are induced by Φ and ζ,

$$\begin{aligned} \{-,-\} : C^{k_1}_{\text{inv}}(\mathcal{P}_{h_1,n_1+1}) \times C^{k_2}_{\text{inv}}(\mathcal{P}_{h_2,n_2+1}) &\to C^{k_1+k_2+1}_{\text{inv}}(\mathcal{P}_{h_1+h_2,n_1+n_2}) \\ (b_{h_1,n_1+1}, b_{h_2,n_2+1}) &\mapsto \sum_{\sigma \in \text{sh}(n_1,n_2)} \sigma \cdot \Phi(b_{h_1,n_1+1}, b_{h_2,n_2+1}), \end{aligned} \tag{3.11}$$

where $\text{sh}(n_1, n_2)$ denotes the shuffles, that is, the subset of the n_1+n_2-permutation group under the constraint that $\sigma_1 < \cdots < \sigma_{n_1}$ and $\sigma_{n_1+1} < \cdots < \sigma_{n_1+n_2}$.

$$\begin{aligned} \Delta : C^k_{\text{inv}}(\mathcal{P}_{h,n}) &\to C^{k+1}_{\text{inv}}(\mathcal{P}_{h+1,n-2}) \\ b_{h,n} &\mapsto \zeta(b_{h+1,n-2}). \end{aligned} \tag{3.12}$$

Together with ∂, Δ, and $\{-,-\}$, $C^*_{\text{inv}}(\mathcal{P}_{h,n})$ yields a structure of BV algebra:

$$\begin{aligned} \partial^2 &= 0 \\ \Delta^2 &= 0 \\ \partial\Delta + \Delta\partial &= 0 \\ \partial\{-,-\} &= \{\partial,-\} - \{-,\partial\} \\ \Delta\{-,-\} &= \{\Delta,-\} - \{-,\Delta\} \\ \{a,b\} &= (-1)^{(\deg(a)+1)(\deg(b)+1)}\{b,a\} \\ (-1)^{(\deg(a)+1)(c+1)}\{\{a,b\},c\} + \text{cycl.} &= 0. \end{aligned} \tag{3.13}$$

Constructing a string field theory action is then an decomposition of the moduli space $\mathcal{M}_{h,n}$ into pieces corresponding different Feynman diagrams. It can be achieved by sewing geometric vertices $\nu_{h,n}$ in a way that reproduces all the entire off-shell amplitudes. For instance, one can sew two cubic vertices $\nu_{0,3}$ in all possible, nonequivalent ways gives the boundary of a region the moduli space $\mathcal{M}_{0,4}$. In order to cover the whole moduli space one needs to add an elementary four-vertex $\nu_{0,4}$. This can be cast into the form of a geometric BV equation

$$\partial\nu_{0,4} + \frac{1}{2}\left\{\nu_{0,3}, \nu_{0,3}\right\} = 0. \tag{3.14}$$

In general, the moduli space $\mathcal{M}_{h,n}$ can be covered by the elementary vertex $\nu_{h,n}$, sewing two different vertices by $\{-,-\}$ and self-sewing one vertex by Δ. More thorough consideration of the geometric vertices shows that

$$\partial\nu_{h,n} + \frac{1}{2}\sum_{\substack{n_1\leq n_2;\, h_1\leq h_2\\ n_1+n_2=n;\, h_1+h_2=h}}\left\{\nu_{h_1,n_1+1}, \nu_{h_2,n_2+1}\right\} + \Delta\nu_{h-1,n+2} = 0. \tag{3.15}$$

The geometric vertices are data that are independent of the background. One introduces the background dependence via the worldsheet CFT. More specifically, via the off-shell amplitude (3.3). The algebraic vertices corresponding to a given background are now defined by integrating the geometric vertices $\nu_{h.n}$ over the appropriate differential forms:

$$f_{h,n} \equiv \int_{\nu_{h,n}} \pi^*\Omega_{h,n}, \tag{3.16}$$

where $\Omega_{h,n} \in \Omega^*(\mathcal{M}_{h,n}, \mathrm{Hom}_{\mathrm{inv}}(\mathcal{H}_{SFT}^{\otimes n}, \mathbb{C}))$ is obtained from the form

$$\Omega_{h,n} : (\mathcal{V}_1, \cdots, \mathcal{V}_n) \mapsto \Omega_{\mathcal{V};h,n}. \tag{3.17}$$

The corresponding BV master action is written as

$$S[\Psi] = \sum_{h=0}^{\infty} \hbar^h f_h(\Psi). \tag{3.18}$$

where f_h represents the genus h interaction and has the structure of a sum of n point vertices,

$$f_h(\Psi) = \sum_{n=1}^{\infty} \frac{1}{(n+1)!} f_{(h,n)}(\underbrace{\Psi, \cdots, \Psi}_{n\,\text{times}}), \tag{3.19}$$

and

The consequence of passing from $\hat{\mathcal{P}}_{h,n}$ to $\mathcal{P}$ is to further restrict the space of string fields into a subspace satisfying

$$(b_0 - \bar{b}_0)\Psi = 0. \tag{3.20}$$

Note that Q is a valid operator on the resticted state space since $\left\{Q, b_0 - \bar{b}_0\right\} = L_0 - \bar{L}_0$, and $\left(L_0 - \bar{L}_0\right)\Psi = 0$ by the level-matching condition.

Simliar to the BV structure introduced in (3.11) and (3.12), we define a BV structure on $\mathrm{Hom}_{\mathrm{inv}}(\mathcal{H}_{SFT}^{\otimes n}, \mathbb{C})$, which is induced by the odd symplectic structure. Again, we have the algebraic BV master equation

$$\partial f_{h,n} + \frac{1}{2} \sum_{\substack{n_1 \le n_2;\, h_1 \le h_2 \\ n_1+n_2=n;\, h_1+h_2=h}} \left\{ f_{h_1,n_1+1}, f_{h_2,n_2+1} \right\} + \Delta f_{h-1,n+2} = 0. \tag{3.21}$$

Constructing explicitly these vertices is highly non-trivial even in the bosonic case (and more so for closed strings). For closed string field theory, a general construction procedure can be given by the minimal area metric [13], but writing down the explicit forms are still a highly tedious task.

3.3 L_∞ Algebras

Now we can summerize what we have obtained for the bosonic closed string field theory. From the background CFT we get a Hilbert space $\mathscr{H}$. The space of string fields is a refinement on $\mathscr{H}$. A state $\Psi \in \mathscr{H}$ is said to be a string field if it satisfies:

1. $\mathrm{gh}(\Psi) = 2$,
2. $(b_0 - \bar{b}_0)\Psi = 0$,
3. $(L_0 - \bar{L}_0)\Psi = 0$.

To get meaningful amplitudes one has to keep track of the total grading.

The consequence of (3.15) is the quantum BV master equation (3.21) on the algebraic vertices, or the string products. It is sometimes also written in a form that fits into the general BV quantization framework

$$\frac{1}{2} \{S[\Psi], S[\Psi]\} + \hbar \Delta S[\Psi] = 0. \tag{3.22}$$

proposed as a consistency requirement of the action.

From a physical point of view, introducing the symplectic form, namely a graded antisymmetric, non-degenerate bilinear form defined via the BPZ inner product

$$\omega(\Psi_1, \Psi_2) = (-1)^{\deg(\Psi_1)} \langle \Psi_1, c_0^- \Psi_2 \rangle , \tag{3.23}$$

helps us to write down the free field action in terms of the propagator Q.

Considering only the genus zero contribution to the string interaction, that is, ignoring all higher-genus effects, we obtain the classical master equation

$$\{S[\Psi], S[\Psi]\} = 0. \tag{3.24}$$

We can define, since ω is non-degenerate

$$\omega(-, L_n) \equiv f_{n+1,0}, \tag{3.25}$$

which is compatible with the identification $L_1 = Q$. The classical action is then of the form

$$S = \frac{1}{2}\omega(\Psi, Q\Psi) + \sum_{n=1}^{\infty} \frac{1}{(n+2)!}\omega(\Psi, L_{n+1}(\Psi, \cdots, \Psi)). \tag{3.26}$$

It appears to be quite astonishing that the classical master equation ranslates exactly into the L_∞ relations

$$\sum_{p=1}^{n}[L_p, L_{n-p+1}] = 0. \tag{3.27}$$

We will show that, however, in the appendix A that it is because BV structures and homotopy algebras are unified under the theory of operads.

The construction of all closed bosonic products $L_n^{(0,0)}$ is another story which can be organized under a minimal area metric (see [13]).

3.4 Closed Superstring Field Theory

Closed superstring field theory takes the Hilbert space of the closed superstring as it space of field configuration. We'll focus on Type II theory in the RNS formalism here. The procedure is very similar to the bosonic case, except for two points: The moduli space is replaced by the super moduli of super Riemann surfaces, and one must distinguish between punctures in Neveu-Schwarz and Ramond sectors. The geometric setup is the same in spirit, except for that now we have to types of punctures, NS and R. And we still have the result that the vertices of the classical BV master equation of closed superstring field theory satisfy the axioms of a L_∞-algebra.

Now let's take a look at the L_∞ algebra structures in the classical closed superstring field theory action in details. Once more, we begin by the space of states $\mathcal{H}$ in superstring perturbation theory. Now the space $\mathcal{H}$ is bi-graded by ghost and picture number, and the BRST charge Q together with ghost number grading yields a cochain complex, whose first cohomology has been as with the physical states.

The space of superstring fields is a refinement on $\mathcal{H}$. A state $\Psi \in \mathcal{H}$ is said to be a string field if it satisfies:

1. $\mathrm{gh}(\Psi) = 2, \quad \mathrm{pic}(\Psi) = (-1, -1)$,
2. $(b_0 - \bar{b}_0)\Psi = 0$,
3. $(L_0 - \bar{L}_0)\Psi = 0$.

The first condition is usually called the ``natural'' ghost and picture number to compute generic amplitudes in literature, for example see [14]. To get meaningful amplitudes one has to keep track of the total grading, since the symplectic form ω is nonvanishing only on states whose ghost number adds up to five and whose picture number ads up to $(-2, -2)$. The string field action is defined by an L_∞ algebra consisting of a series of multilinear products of ghost degree 1

$$L_n^{(n-1,n-1)} : \mathcal{H}^{\wedge n} \to \mathcal{H}, \tag{3.28}$$

where the 1-product is just Q. The superscript indicates that the picture number of the n-product is $(n-1, n-1)$ to ensure the output has the natrual picture $(-1, -1)$ as a string field. The full classical action thus reads

$$S = \frac{1}{2}\omega(\Psi, Q\Psi) + \sum_{n=1}^{\infty}\frac{1}{(n+2)!}\omega(\Psi, L_{n+1}^{(n,n)}(\Psi, \cdots, \Psi)). \tag{3.29}$$

The problem of constructing superstring products is then how to place picture changing operators on the bosonic string products.

After construction in the supermoduli one would naturally come to the question that whether it is possible to split the integration over the even and odd moduli, integrate out the latter, and reduce the integral to the one on the bosonic moduli. Unfortunately, this is shown not to be true in [15]. One can not find a global holomorphic projection from the supermoduli to the bosonic one. We've seen that the picture-changing operators must be inserted as a result of integration over the odd moduli. The problem is closely related to the so-called spurious singularity of those PCO's, where it is not possible to find a global way to insert the PCO's for all the moduli space. There is a workaround, though, by the method of vertical integration [16], whose idea is to assgin the PCO's locally, with discontinuity and carefully consider the ``vertical'' sections connecting them. Therefore, one can still construct the theory by properly dressing the bosonic products with PCO's locally. This will be the topic of the next section.

3.5 Assigning PCO's

A construction of Type II superstring field theory in the NS-NS sector [10] is summarized briefly here. As explained before, the string field Ψ has ghost number 2 and picture number -1, and satisfies $(b_0-\bar{b}_0)\Psi=(L_0-\bar{L}_0)\Psi=0$. An on-shell state in Siegel gauge $b_0\Psi=\bar{b}_0\Psi=0$ takes the form

$$\Psi\sim c\bar{c}\mathrm{e}^{-\phi}\mathrm{e}^{-\bar{\phi}}V(0,0), \tag{3.30}$$

where V is a worldsheet matter primary field with conformal weight $(\frac{1}{2},\frac{1}{2})$. The amplitudes (2.7) is nonvanishing only if the states are of ghost numbers adding up to 5 and picture numbers adding up to -2.

The picture changing operator has a zero mode

$$\mathcal{X}=\oint_{|z|=1}\frac{\mathrm{d}z}{2\pi i}\frac{1}{z}\mathcal{X}(z), \tag{3.31}$$

which is to be used in the string field theory vertices. Unlike the open strings, the b constraint prohibits a more general charge $\xi=\oint_{|z|=1}\frac{\mathrm{d}z}{2\pi i}\frac{1}{z}\xi(z)$ to construct $\mathcal{X}=\{Q,\xi\}$.

We start by correctly assigning picture changing operators in the holomorphic sector. By collecting superstring products into a generating function

$$L^{[0]}(t)=\sum_{n=0}^{\infty}t^nL_{n+1}^{(n,0)}, \tag{3.32}$$

where the $(n+1)$-st superstring product is the n-th coefficient of the formal variable t. The upper index on the generating function is to indicate the picture dificit relative to what is needed in the superstring amplitudes. The products should be independent of the ξ zero-mode,

$$\left[\eta,L^{[0]}(t)\right]=0. \tag{3.33}$$

In our notation the L_∞ relations can be written as

$$\left[L^{[0]}(t), L^{[0]}(t)\right] = 0. \tag{3.34}$$

Expanding it in powers of t gives the L_∞ relations. Such a series of products can be realized by a differential equation postulate on the generating function

$$\frac{\partial}{\partial t} L^{[0]}(t) = \left[L^{[0]}(t), \lambda^{[0]}(t)\right], \tag{3.35}$$

where

$$\lambda^{[0]}(t) = \sum_{n=0}^{\infty} t^n \lambda_{n+2}^{(n+1,0)}, \tag{3.36}$$

is a generating function for deficit-free auxiliary gauge products.

An non-trivial condition is proposed on the products by requiring are in the small Hilbert space. Taking η of the differential equation (3.35) we see

$$\left[L^{[0]}(t), L^{[1]}(t)\right] = 0, \tag{3.37}$$

where

$$L^{[1]}(t) = \left[\eta, \lambda^{[0]}(t)\right] = \sum_{n=0}^{\infty} t^n L_{n+2}^{(n,0)} \tag{3.38}$$

is the single deficit generating function. It can be solved by introducing new auxiliary products with proper deficit, that is,

$$\frac{\partial}{\partial t} L^{[1]}(t) = \left[L^{[0]}(t), \lambda^{[1]}(t)\right] + \left[L^{[1]}(t), \lambda^{[0]}(t)\right], \tag{3.39}$$

where

$$\eta^{[1]}(t) = \sum_{n=0}^{\infty} t^n \eta_{n+3}^{(n+1,0)}. \tag{3.40}$$

This iterative procedure on picture deficit can be carried out as follows. Taking η of each differential equation implies a constraint on the generating function for products with higher deficits, which can be solved by yet another differential equation. Introducing another formal variable s for the recursion on deficit, the procedure can be compactly expressed by the generating functions

$$L(s,t) = \sum_{m,n=0}^{\infty} s^m t^n L_{m+n+1}^{(n)}, \tag{3.41}$$

$$\lambda(s,t) = \sum_{m,n=0}^{\infty} s^m t^n \lambda_{m+n+2}^{(n+1)}. \tag{3.42}$$

Note that at $t = 0$, $L(s,t)$ reduces to a generating function of the bosonic string products, and at $s = 0$ it reduces to a generating function for the desired superstring products.

The recursion is dictated by the differential equations

$$\frac{\partial}{\partial t} L(s,t) = \left[L(s,t), \lambda(s,t)\right], \tag{3.43}$$

$$\frac{\partial}{\partial s}L(s,t) = [\eta, \lambda(s,t)]\,, \tag{3.44}$$

whose solutions automatically determines a set of superstring products which live in the small Hilbert space and satisfy the L_∞ relations.

Expanding (3.43) is s, t and comparing the coefficients give

$$L^{(n+1,0)}_{m+n+2} = \frac{1}{n+1}\sum_{k=0}^{n}\sum_{l=0}^{m}\left[L^{(k,0)}_{k+l+1}, \lambda^{(n-k+1,0)}_{m+n-k-l+2}\right], \tag{3.45}$$

which determines the explicit form of the product $L^{(n+1,0)}_{m+n+2}$ provided that we know all the gauge products $\lambda^{(k,0)}_l$. The gauge products can be read off from (3.44) by expanding,

$$\left[\eta, \lambda^{(n+1,0)}_{m+n+2}\right] = (m+1)L^{(n,0)}_{m+n+2}. \tag{3.46}$$

This equation will determine $\lambda^{(n+1,0)}_{m+n+2}$ in terms of $L^{(m,0)}_{m+n+2}$ and the solution to it is not unique. One naturally suggested solution similar to the resolving procedure in [10] would be

$$L^{(m+1,0)}_{m+n+2} = \frac{1}{m+1}\sum_{k=0}^{m}\sum_{l=0}^{n}\left[L^{(k,0)}_{k+l+1}, \lambda^{(m-k+1,0)}_{m+n-k-l+2}\right]. \tag{3.47}$$

The construction is recursive and provides a specific way to assgin correct holomorphic picture number to the superstring products. Similar needs to be done to the anti-holomorphic sector. In conclusion, we now have a way to construction the $(n+1)$-st closed superstring field theory product by climbing a ladder of products and auxiliary products

$$L^{(0,0)}_{n+1}, \lambda^{(1,0)}_{n+1}, L^{(1,0)}_{n+1}, \cdots, L^{(n,0)}_{n+1}, \lambda^{(n,1)}_{n+1}, L^{(n,1)}_{n+1}, \cdots, L^{(n,n)}_{n+1}, \tag{3.48}$$

where the iteration is given as

$$L^{(m+1,0)}_{m+n+2} = \frac{1}{m+1}\sum_{k=0}^{m}\sum_{l=0}^{n}\left[L^{(k,0)}_{k+l+1}, \lambda^{(m-k+1,0)}_{m+n-k-l+2}\right], \tag{3.49}$$

$$\lambda^{(m+1,0)}_{m+n+2} = \frac{n+1}{m+n+3}\left(\xi L^{(m,0)}_{m+n+2} - L^{(m,0)}_{m+n+2}(\xi \wedge \mathbb{I}_{m+n+1})\right), \tag{3.50}$$

$$L^{(m+n+1,m+1)}_{m+n+2} = \frac{1}{m+1}\sum_{k=0}^{m}\sum_{l=0}^{n}\left[L^{(k+l,k)}_{k+l+1}, \lambda^{(m+n-k-l+1,m-k+1)}_{m+n-k-l+2}\right], \tag{3.51}$$

$$\lambda^{(m+n+2,m+1)}_{m+n+2} = \frac{n+1}{m+n+3}\left(\bar{\xi} L^{(m+n+1,m)}_{m+n+2} - L^{(m+n+1,m)}_{m+n+2}(\bar{\xi} \wedge \mathbb{I}_{m+n+1})\right). \tag{3.52}$$

For our purpose it is enough to know these relations up to 3-products, which are listed below

$$L^{(2,2)}_3 = \frac{1}{2}\left[Q, \lambda^{(2,2)}_3\right] + \frac{1}{2}\left[L^{(1,1)}_2, \lambda^{(1,1)}_2\right], \tag{3.53}$$

$$\lambda^{(2,2)}_3 = \frac{1}{4}\left(\bar{\xi} L^{(2,1)}_3 - L^{(2,1)}_3(\bar{\xi} \wedge \mathbb{I}_2)\right), \tag{3.54}$$

$$\left[Q, L^{(1,1)}_2\right] = 0, \tag{3.55}$$

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$$\left[Q, L_3^{(2,2)}\right] + \frac{1}{2}\left[L_2^{(1,1)}, L_2^{(1,1)}\right] = 0. \tag{3.56}$$

In this chapter, we described the construction of closed string field theory, other realizations of string field theory can be described similarly, for example, the open-closed theory [17]. The most successful realization of string field theory is definitely Witten's open string field theory [18]. It is special in the sense that besides the kinetic term it involves only a cubic vertex – the star product. Witten constructed this theory in a completely different manner than described above, by seeking for a Chern-Simons like action which possesses decent gauge symmetries. Anyhow, it has been realized later on that Witten's cubic string field theory arises indeed form the geometrical approach with appropriate minimal area metrics [19].

We conclude this chapter with the fact that the string field theory action is not limited to the NS-NS sector which we discussed here. The definition of the products should be extended to include the Ramond sectors, but it is beyond the scope of this thesis. For a complete theory of closed superstrings see, for example, [20].

Chapter 4

Deformation via SFT Action

In this chapter we study the possibility of a marginal deformation in the orbifold theory to be exactly marginal corresponding to the blow-up of the orbifold singularity. The framework of (bosonic or supersymmetric) SFT is needed and exploited in this problem. We study the marginality up to the third order, and derive the perturbated metric at the second order.

4.1 Perturbative Approach

In this section we formulate the problem of exact marginality corresponding to twist fields in the framework of string field theory. Suppose we have identified the form of marginal operators in both bosonic and supersymmetric cases that we denote as $\Psi^{(1)}$ here. The equation of motion of closed string field theory can be simply derive from the action as

$$Q\Psi + L_2(\Psi, \Psi) + L_3(\Psi, \Psi, \Psi) + \cdots = 0. \tag{4.1}$$

Note that the on-shell condition $Q\Psi^{(1)} = 0$ can be understood as the linearization of the full equation of motion. The question of exact marginality is then translated into whether this solution can be lifted to a full solution, written as a perturbative series $\Psi = \lambda\Psi^{(1)} + \lambda^2\Psi^{(2)} + \lambda^3\Psi^{(3)} + \cdots$.

This gives a non-homogeneous linear equation at each order of λ. In particular, we have

$$Q\Psi^{(2)} + L_2(\Psi^{(1)}, \Psi^{(1)}) = 0, \tag{4.2}$$

and

$$Q\Psi^{(3)} + L_2(\Psi^{(2)}, \Psi^{(1)}) + L_3(\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}) = 0, \tag{4.3}$$

which will be of primary interest and serve as a starting point of our perturbative analysis. At any order, to solve the equation we need to check if any obstruction appear, namely if the BRST charge Q is invertible on the linear subspace where the non-homogeneous part lies. For example, at the second order, the solution

$$\Psi^{(2)} = Q^{-1}L_2(\Psi^{(1)}, \Psi^{(1)}) \tag{4.4}$$

is well-defined only if $\mathcal{O}_2 = P_0[L_2(\Psi^{(1)}, \Psi^{(1)})] = 0$, where P_0 is the projector on the space on which Q is not invertible. Whenever the obstruction is non-vanishing, the lifting procedure is blocked.

4.2 Second-order Deformation

In this section we explicitly evaluate the obstruction appears in the second order equation, in both bosonic and supersymmetric SFT. However, as we'll see immediately, lifting to the full exactly marginal operator requires us to consider the theory of superstrings.

4.2.1 Bosonic String

The massless field in the twist sector is $\mathbb{V} = \prod_{k=10}^{25} |0\rangle \equiv \Delta |0\rangle$. Together with its right-moving partner $\bar{\mathbb{V}}$ it describe a tangent vector in the moduli space of the target spacetime at the orbifold point.Considering the ghost contribution, the marginal operator reads $\Psi^{(1)} = c\mathbb{V}\bar{c}\bar{\mathbb{V}}$.

The second order obstruction $\mathcal{O}_2$ is expressed by the projection

$$P_0 L_2(\Psi^{(1)}, \Psi^{(1)}) = \lim_{z\to 0} P_0[c\mathbb{V}(z)c\mathbb{V}(-z)] \otimes \lim_{\bar{z}\to 0} P_0[\bar{c}\bar{\mathbb{V}}(\bar{z})\bar{c}\bar{\mathbb{V}}(-\bar{z})]. \tag{4.5}$$

We now show the holomorphic part is non-zero by noting that the OPE of two ghost fields selects out in the OPE of matter field the term to z^{-1}. The OPE is given by

$$\bar{\Delta}(z)\Delta(w) \sim \frac{1}{(z-w)^2} - \frac{: e^{i\sqrt{2}X} : - : e^{-i\sqrt{2}X} :}{z-w} + \cdots, \tag{4.6}$$

and contributes to the projection

$$\lim_{z\to 0} P_0[c\mathbb{V}(z)c\mathbb{V}(-z)] = c\partial c(: e^{i\sqrt{2}X} : - : e^{-i\sqrt{2}X} :)(0) \tag{4.7}$$

as shown in [21]. The anti-holomorphic part of the projection is similar, and in conclusion we will have a non-vanishing obstruction in the bosonic case.

The presence of this obstruction means that it is impossible to find a bosonic string background near the orbifold theory, and a geometric picture of blown up sigma model is absent. As a consequence, we are led to the superstring field theory. This is necessary anyhow, since the bosonic string field theory is not considered a consistent theory in many senses, both for the presence of tachyon that indicates a wrong choice of vacuum and for being unable to describe fermions. In the next subsection we will see that this obstruction $\mathcal{O}_2$ is not there anymore in the superstring theory, and it will allow us to calculation the graviton contribution of the perturbative solution, and thus compute the metric of the blown up model.

4.2.2 Superstring

The existence of obstruction leads us to consider the Type II superstring field theory. The method of constructing such a theory was reviewed in the last chapter. There are various approaches to a consistent SFT action. We will use the one that was proposed in [22], which we've introduced in 3.5. In the NS-NS sector, the twisted deformation moduli is given by the spin fields as

$$w_\alpha S^\alpha \Delta |0\rangle_{NS} \otimes \bar{w}_\beta \bar{S}^\beta \bar{\Delta} |\bar{0}\rangle_{NS}. \tag{4.8}$$

In order to test exact marginality, we define the massless field in the $(-1,-1)$ picture $\Psi^{(1)} = c\bar{c}\mathbb{V}_{\frac{1}{2}}\bar{\mathbb{V}}_{\frac{1}{2}}e^{-\phi}e^{-\bar{\phi}}$, where $\mathbb{V}_{\frac{1}{2}} = w_\alpha S^\alpha \Delta$ and $\bar{\mathbb{V}}_{\frac{1}{2}} = \bar{w}_\beta \bar{S}^\beta \bar{\Delta}$.

Taking into account the picture-changing operator arising from integrating over odd moduli, the two-product in superstring field theory is defined by (see [22])

$$\begin{aligned} &L_2^{(1,1)}(\Psi^{(1)}, \Psi^{(1)}) \\ &= \frac{1}{9}\mathcal{X}\left(\bar{\mathcal{X}}L_2^{(0,0)}(\Psi^{(1)}, \Psi^{(1)}) + L_2^{(0,0)}(\bar{\mathcal{X}}\Psi^{(1)}, \Psi^{(1)}) + L_2^{(0,0)}(\Psi^{(1)}, \bar{\mathcal{X}}\Psi^{(1)})\right) \\ &+ \frac{1}{9}\left(\bar{\mathcal{X}}L_2^{(0,0)}(\mathcal{X}\Psi^{(1)}, \Psi^{(1)}) + L_2^{(0,0)}(\bar{\mathcal{X}}\mathcal{X}\Psi^{(1)}, \Psi^{(1)}) + L_2^{(0,0)}(\mathcal{X}\Psi^{(1)}, \bar{\mathcal{X}}\Psi^{(1)})\right) \\ &+ \frac{1}{9}\left(\bar{\mathcal{X}}L_2^{(0,0)}(\Psi^{(1)}, \mathcal{X}\Psi^{(1)}) + L_2^{(0,0)}(\bar{\mathcal{X}}\Psi^{(1)}, \mathcal{X}\Psi^{(1)}) + L_2^{(0,0)}(\Psi^{(1)}, \mathcal{X}\bar{\mathcal{X}}\Psi^{(1)})\right), \end{aligned} \tag{4.9}$$

where the bare product or the ``buttom-of-the-ladder'' product $L_n^{(0,0)}$ is just the bosonic product L_n by construction. This serves as a starting point of the recursive construction of all superstring product decribed in 3.5. Making use of the factorization property (4.5) we can factorize each term into left- and right-movers

$$\begin{aligned} P_0 L_2^{(1,1)}(\Psi^{(1)}, \Psi^{(1)}) = \lim_{z\to 0} P_0 \big(&\mathcal{X}\cdot c\mathbb{V}e^{-\phi}(z)c\mathbb{V}e^{-\phi}(-z) \\ &+ \mathcal{X}c\mathbb{V}e^{-\phi}(z)c\mathbb{V}e^{-\phi}(-z) \\ &+ c\mathbb{V}e^{-\phi}(z)\mathcal{X}c\mathbb{V}e^{-\phi}(-z)\big) \\ \otimes \lim_{\bar{z}\to 0} P_0 \big(&\bar{\mathcal{X}}\cdot \bar{c}\bar{\mathbb{V}}e^{-\bar{\phi}}(\bar{z})\bar{c}\bar{\mathbb{V}}e^{-\bar{\phi}}(-\bar{z}) \\ &+ \bar{\mathcal{X}}\bar{c}\bar{\mathbb{V}}e^{-\bar{\phi}}(\bar{z})\bar{c}\bar{\mathbb{V}}e^{-\bar{\phi}}(-\bar{z}) \\ &+ \bar{c}\bar{\mathbb{V}}e^{-\bar{\phi}}(\bar{z})\bar{\mathcal{X}}\bar{c}\bar{\mathbb{V}}e^{-\bar{\phi}}(-\bar{z})\big), \end{aligned} \tag{4.10}$$

and deal with them separately.

The holomorphic part vanishes by the following argument, which is essentially the same as for the open string in [22]. By noting that the picture-changed marginal operator takes the form

$$\mathcal{X}c\mathbb{V}_{\frac{1}{2}}e^{-\phi} = -c\mathbb{V}_1 + \frac{1}{4}\gamma\mathbb{V}_{\frac{1}{2}}, \tag{4.11}$$

where $\mathbb{V}_1$ is a Grassmann-even superconformal primary of weight one defined by

$$T_F(z)\mathbb{V}_{\frac{1}{2}}(0) = \frac{\mathbb{V}_1(0)}{z} + O(1), \tag{4.12}$$

we compute the last two terms in the holomorphic part as

$$\begin{aligned} &\lim_{z\to 0} P_0 \left(\mathcal{X}c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(z)c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(-z) + c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(z)\mathcal{X}c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(-z)\right) \\ &= \lim_{z\to 0} P_0 \left((-c\mathbb{V}_1 + \frac{1}{4}\gamma\mathbb{V}_{\frac{1}{2}})(z)c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(-z) + c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(z)(-c\mathbb{V}_1 + \frac{1}{4}\gamma\mathbb{V}_{\frac{1}{2}})(-z)\right). \end{aligned} \tag{4.13}$$

Since the OPE of $\mathbb{V}_{\frac{1}{2}}(z)\mathbb{V}_{\frac{1}{2}}(-z)$ contains a single pole while that of $\mathbb{V}_{\frac{1}{2}}(z)\mathbb{V}_1(-z)$ doesn't, we conclude that

$$\begin{aligned}&\lim_{z\to 0} P_0\left(\mathcal{X}c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(z)c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(-z)+c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(z)\mathcal{X}c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(-z)\right)\\&=\lim_{z\to 0} P_0\left(\frac{1}{4}\gamma\mathbb{V}_{\frac{1}{2}}(z)c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(-z)+c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(z)\frac{1}{4}\gamma\mathbb{V}_{\frac{1}{2}}(-z)\right).\end{aligned}\tag{4.14}$$

Observe that the two terms have the same operator products in both the matter and ghost sector, but commuting c with $\mathbb{V}_{\frac{1}{2}}$ results in an extra minus sign. Therefore the two terms cancel out and give a vanishing result.

The remaining term can be evaluated as follows. First notice that $\mathcal{X}$ and P_0 commute

$$\lim_{z\to 0} P_0\left(\mathcal{X}\cdot c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(z)c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(-z)\right)=\mathcal{X}\lim_{z\to 0} P_0\left(c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(z)c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(-z)\right).\tag{4.15}$$

The un-picture-changed part is obtained via the OPE

$$\lim_{z\to 0} P_0\left(c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(z)c\mathbb{V}_{\frac{1}{2}}e^{-\phi}(-z)\right)=P_0[\partial(c\partial ce^{-2\phi}\mathbb{V}_0)+c\partial c\mathbb{V}_1e^{-2\phi}].\tag{4.16}$$

It's not hard to see that the first term $\partial(c\partial ce^{-2\phi}\mathbb{V}_0)=Q(\partial ce^{-2\phi}\mathbb{V}_0)$ is Q-exact and thus projected out, leaving

$$P_0L_2^{(0,0)}(\Psi^{(1)},\Psi^{(1)})=c\partial c\mathbb{V}_1e^{-2\phi}\otimes\bar{c}\bar{\partial}\bar{c}\bar{\mathbb{V}}_1e^{-2\bar{\phi}}.\tag{4.17}$$

We proceed using the identity $\mathcal{X}=\{Q,\xi\}$ in the large Hilbert space. The BRST charge is given by $Q=Q_0+Q_1+Q_2$, where

$$Q_0=\oint\frac{\mathrm{d}z}{2\pi i}(cT+c\partial cb)(z),\tag{4.18}$$

$$Q_1=-\oint\frac{\mathrm{d}z}{2\pi i}\gamma T_F(z),\tag{4.19}$$

$$Q_2=-\frac{1}{4}\oint\frac{\mathrm{d}z}{2\pi i}b\gamma^2(z).\tag{4.20}$$

Since $QP_0=0$, we have $\mathcal{X}P_0L_2^{(0,0)}(\Psi^{(1)},\Psi^{(1)})=Q\xi P_0L_2^{(0,0)}$. And then

$$P_0L_2^{(1,1)}(\Psi^{(1)},\Psi^{(1)})=(Q_0+Q_1+Q_2)\left(\xi c\partial c\mathbb{V}_1e^{-2\phi}\otimes\bar{\xi}\bar{c}\bar{\partial}\bar{c}\bar{\mathbb{V}}_1e^{-2\bar{\phi}}\right),\tag{4.21}$$

where the only non-trivial term is the second one. It is proportional to the double pole of the OPE explicitly evaluated below

$$T_F(z)\mathbb{V}_1(0)\sim\frac{1}{z}(\partial X^\mu\psi^\nu-\partial X^\nu\psi^\mu)+O(1),\tag{4.22}$$

where T_F is the supercurrent defined by

$$T_F(z)=\frac{i}{\sqrt{2\alpha'}}\psi_\mu\partial X^\mu.\tag{4.23}$$

The double pole is absent in the OPE, therefore, we establish that

$$P_0L_2^{(1,1)}(\Psi^{(1)},\Psi^{(1)})=0.\tag{4.24}$$

4.3 Metric

We've seen that in a superstring setup, the perturbative solution

$$\Psi^{(2)} = Q^{-1} L_2^{(1,1)}(\Psi^{(1)}, \Psi^{(1)}) \tag{4.25}$$

is well-defined. Using this we can compute the perturbated metric by evaluating the solution against the vertex operator for the metric

$$G^{(2)}_{I\bar{J}} = \left\langle V^{(-1,-1)}_{G_{I\bar{J}}}, Q^{-1} L_2^{(1,1)}(\Psi^{(1)}, \Psi^{(1)}) \right\rangle , \tag{4.26}$$

where the vertex operator $V^{(-1,-1)}_{G_{I\bar{J}}}$ is taken to be in the $(-1,-1)$ picture. However, the effect of assgined picture-changing operator in $L_2^{(1,1)}$ can be moved around, resulting in a $(0,0)$ picture number for the metric operator. Consequently we have

$$G^{(2)}_{I\bar{J}} = \left\langle \left(f_\infty \circ V^{(-1)}\bar{V}^{(-1)}\right)(\infty) \left(f_1 \circ V^{(0,0)}_{G_{I\bar{J}}}\right)(0;-k) \left(f_0 \circ V^{(-1)}\bar{V}^{(-1)}\right)(0) \right\rangle , \tag{4.27}$$

where the operator in natural picture number

$$V^{(-1,-1)}_{G_{I\bar{J}}} = \frac{1}{4} c\bar{c}\psi_I \bar{\psi}_{\bar{J}} \mathrm{e}^{-\phi}\mathrm{e}^{-\bar{\phi}}\mathrm{e}^{-ik\cdot X}\mathrm{e}^{-ik\cdot\bar{X}} \tag{4.28}$$

needs to be assigned the picture-changing operator $\mathcal{X}\bar{\mathcal{X}}$ and turns into

$$V^{(0,0)}_{G_{I\bar{J}}} = \frac{1}{4} c\bar{c} \left(i\partial X_I - \frac{\alpha'}{2}(k\cdot\psi)\psi_I \right) \left(i\bar{\partial} X_{\bar{J}} - \frac{\alpha'}{2}(k\cdot\bar{\psi})\bar{\psi}_{\bar{J}} \right) \mathrm{e}^{-ik\cdot X}\mathrm{e}^{-ik\cdot\bar{X}} . \tag{4.29}$$

Note that in the following calculation we may drop the terms containing the boson current since they give contributions that are diffeomoephic to zero. So there is only the contributions due to the non-vanishing momentum k^μ. We note here that the action of the maps f_{z_0} reduces for primary operator $O(z)$ with conformal dimension h to

$$f_{z_0} \circ O(z) = f'_{z_0}(z)^h O(f_{z_0}(z)). \tag{4.30}$$

For Witten's cubic action we use three maps f_0, f_1 and f_∞, however the only non-trivial action is f_1 on the off-shell operator for the graviton. Therefore the correlation function reduces to

$$G^{(2)}_{I\bar{J}}(k) = |f'_1(0)|^{\alpha' k^2} \left\langle V^{(-1)}\bar{V}^{(-1)}(\infty) V^{(0,0)}_{G_{I\bar{J}}}(1;-k) V^{(-1)}\bar{V}^{(-1)}(0) \right\rangle . \tag{4.31}$$

In the holomorphic sector, it can be split into four sub-amplitudes with generic vertex operator insertion, which belong to different CFT's and are independent from each other:

$$\begin{aligned} &w_\alpha w_\beta k^J \left\langle c(z_1)c(z_2)c(z_3)\right\rangle \left\langle \mathrm{e}^{-\phi(z_1)}\mathrm{e}^{-\phi(z_3)}\right\rangle \\ &\qquad \left\langle \Delta(z_1)\mathrm{e}^{-ik\cdot X}(z_2)\Delta(z_3)\right\rangle \left\langle S^{\dot{\alpha}}\psi_K\psi_I(z_2)S^{\dot{\beta}}(z_3)\right\rangle . \end{aligned} \tag{4.32}$$

All the correlation functions are well know and thus we can calculate

$$G^{(2)}_{I\bar{J}}(k) = \frac{|f'_1(0)|^{\alpha' k^2}}{16} \left(\frac{z_{13}}{z_{12}z_{23}} \cdot \frac{\bar{z}_{13}}{\bar{z}_{12}\bar{z}_{23}} \right)^{\alpha' k^2/2} w_\alpha (\sigma_{AI})^{\alpha\beta} w_\beta \bar{w}_\gamma (\sigma_{BJ})^{\gamma\delta} \bar{w}_\delta \mathrm{e}^{-ik\cdot x_0 - ik\cdot\bar{x}_0}, \tag{4.33}$$

where $x_0, \bar{x}_0$ are the zero-modes of $X(z), \bar{X}(\bar{z})$ corresponding to the position of the orbifold singularity and can be taken to zero.

After setting $z_1 \to \infty, z_2 = 1$ and $z_3 = 0$ we end up with

$$G^{(2)}_{I\bar{J}} = \frac{|f_1'(0)|^{\alpha' k^2}}{16} k^A k^B w_\alpha (\sigma_{AI})^{\alpha\beta} w_\beta \bar{w}_\gamma (\sigma_{BJ})^{\gamma\delta} \bar{w}_\delta. \tag{4.34}$$

By construction of Witten's cubic vertex, the map f_1 is given by the composition $f_1(z) = G(g_1(z))$ with

$$g_1(z) = \left(\frac{i-z}{i+z}\right)^{2/3} \tag{4.35}$$

and

$$G(z) = \frac{(1-\mathrm{e}^{2\pi i/3})(z-\mathrm{e}^{-2\pi i/3})}{(1-\mathrm{e}^{-2\pi i/3})(z-\mathrm{e}^{2\pi i/3})}, \tag{4.36}$$

so its derivative at the origin is

$$f_1'(0) = \frac{4}{3\sqrt{3}}. \tag{4.37}$$

Now let's take a closer look at the metric (4.34) we obtained.

We would like to comment that even though we start with the flat metric on $\mathbb{C}^2/\mathbb{Z}_2$, which is Kähler, the full metric solution to the blowing-up equation is not necessarily Kähler already at the second order. This point can be seen by taking $w_\alpha = \bar{w}_\alpha = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, then we have

$$G^{(2)}_{I\bar{J}} \sim \begin{pmatrix} 2ik_{\bar{1}}k_{\bar{1}} & 2k_{\bar{1}}k_1 & -2ik_{\bar{1}}k_{\bar{0}} & -2k_{\bar{1}}k_0 \\ 2k_{\bar{1}}k_1 & -2ik_1k_1 & -2k_{\bar{0}}k_1 & 2ik_0k_1 \\ -2ik_{\bar{1}}k_{\bar{0}} & -2k_{\bar{0}}k_1 & 2ik_{\bar{0}}k_{\bar{0}} & 2k_{\bar{0}}k_0 \\ -2k_{\bar{1}}k_0 & 2ik_0k_1 & 2k_{\bar{0}}k_0 & -2ik_0k_0. \end{pmatrix} \tag{4.38}$$

Consider a diffeomorphism

$$\kappa_0 = i\frac{k_{\bar{1}}k_{\bar{1}}}{k_0} + \frac{1}{2}k_0, \quad \kappa_{\bar{0}} = -i\frac{k_1k_1}{k_{\bar{0}}} + \frac{1}{2}k_{\bar{0}}, \tag{4.39}$$

$$\kappa_1 = i\frac{k_{\bar{0}}k_{\bar{0}}}{k_1} + \frac{1}{2}k_1, \quad \kappa_{\bar{1}} = -i\frac{k_0k_0}{k_{\bar{1}}} + \frac{1}{2}k_{\bar{1}} \tag{4.40}$$

under which the Kähler form becomes $\omega = iG_{I\bar{J}}\mathrm{d}x^I \wedge \mathrm{d}x^{\bar{J}}$. We then have

$$\mathrm{d}\omega = i\left(2(k_{\bar{I}}k_I)k_1 - i\frac{k_{\bar{1}}k_{\bar{1}}k_1k_{\bar{0}}}{k_0} + i\frac{k_{\bar{0}}k_{\bar{0}}k_{\bar{0}}k_0}{k_1}\right)\mathrm{d}x^1 \wedge \mathrm{d}x^0 \wedge \mathrm{d}x^{\bar{0}} + \cdots, \tag{4.41}$$

which is non-vanishing away from the origin. This indicates a broken of the $\mathcal{N} = 2$ spacetime supersymmetry in the shifted vacuum.

4.4 Third-order Deformation

At the third order in the blow-up parameter of the orbifold, the string field equation of motion is given by

$$Q\Psi^{(3)} - L_2^{(1,1)}(\Psi^{(2)}, \Psi^{(1)}) + L_3^{(2,2)}(\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}) = 0. \tag{4.42}$$

We thus require the projection

$$\mathcal{O}_3 = P_0 \left[L_2^{(1,1)}(Q^{-1}L_2^{(1,1)}(\Psi^{(1)}, \Psi^{(1)}), \Psi^{(1)}) - L_3^{(2,2)}(\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}) \right], \tag{4.43}$$

namely, the third order obstruction to vanish. In the same spirit of [23] we can reduce this expression to a four-point function in an SCFT which can be evaluated using standard methods, except for the projector P_0. Effectively this amount to the reverse of the construction of string field theory from amplitudes. The fact that closed string theory contains two holomorphic parts greatly simplifies our evaluation.

To see this, consider the inner product against an generic on-shell state e

$$\mathcal{O}_3^e = \left\langle e, L_2^{(1,1)} Q^{-1} L_2^{(1,1)} - L_3^{(2,2)} \right\rangle (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}). \tag{4.44}$$

It can be easily seen that the vanishing of $\mathcal{O}_3 = \sum \mathcal{O}_3^e$, where the summation is taken over a basis of $\mathrm{im}(P_0)$, is equivalent to the vanishing of component $\mathcal{O}_3^e$. To continue we use the construction $L_2^{(1,1)} = [Q, \lambda_2^{(1,1)}]$ to write

$$\begin{aligned} \mathcal{O}_3^e = \Big\langle e, \bar{\xi} \Big(& \frac{1}{2}[Q, \lambda_2^{(1,1)}] Q^{-1} L_2^{(1,1)} \\ & + \frac{1}{2} L_2^{(1,1)} Q^{-1} [Q, \lambda_2^{(1,1)}] - L_3^{(2,2)} \Big) \Big\rangle_L (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}) \end{aligned} \tag{4.45}$$

Note that we've entered the large Hilbert space in the anti-holomorphic sector by nature of the construction we use. We now use the BPZ inner product with an extra insertion of $\bar{\xi}$ to provide the extra zero-mode $\bar{\xi}_0$, i.e., $\langle \cdot, \cdot \rangle = \langle \cdot, \bar{\xi} \cdot \rangle_L$. In the following whenever the BRST charge Q hits the ghost $\bar{\xi}$ we use

$$\bar{\mathcal{X}} = \{Q, \bar{\xi}\} \tag{4.46}$$

to get a picture-changing operator, and whenever it hits the propagator Q^{-1} we use the contracting homotopy relation

$$1 - P_0 = QQ^{-1} + Q^{-1}Q \tag{4.47}$$

until it finally hits an on-shell state and vanish. After this process we are left with

$$\begin{aligned} \mathcal{O}_3^e = \Big\langle e, & \frac{1}{2}\bar{\mathcal{X}}\bar{\xi} L_2^{(1,0)} Q^{-1} L_2^{(1,1)} - \frac{1}{2}\bar{\mathcal{X}}\bar{\xi} L_2^{(1,1)} Q^{-1} L_2^{(1,0)} \\ & - \frac{1}{2}\bar{\xi} L_2^{(1,1)} P_0 \lambda_2^{(1,1)} - \frac{1}{2}\bar{\xi}\lambda_2^{(1,1)} P_0 L_2^{(1,1)} \\ & + \frac{1}{2}\bar{\xi} L_2^{(1,1)} \lambda_2^{(1,1)} + \frac{1}{2}\bar{\xi}\lambda_2^{(1,1)} L_2^{(1,1)} - \bar{\xi} L_3^{(2,2)} \Big\rangle_L (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}). \end{aligned} \tag{4.48}$$

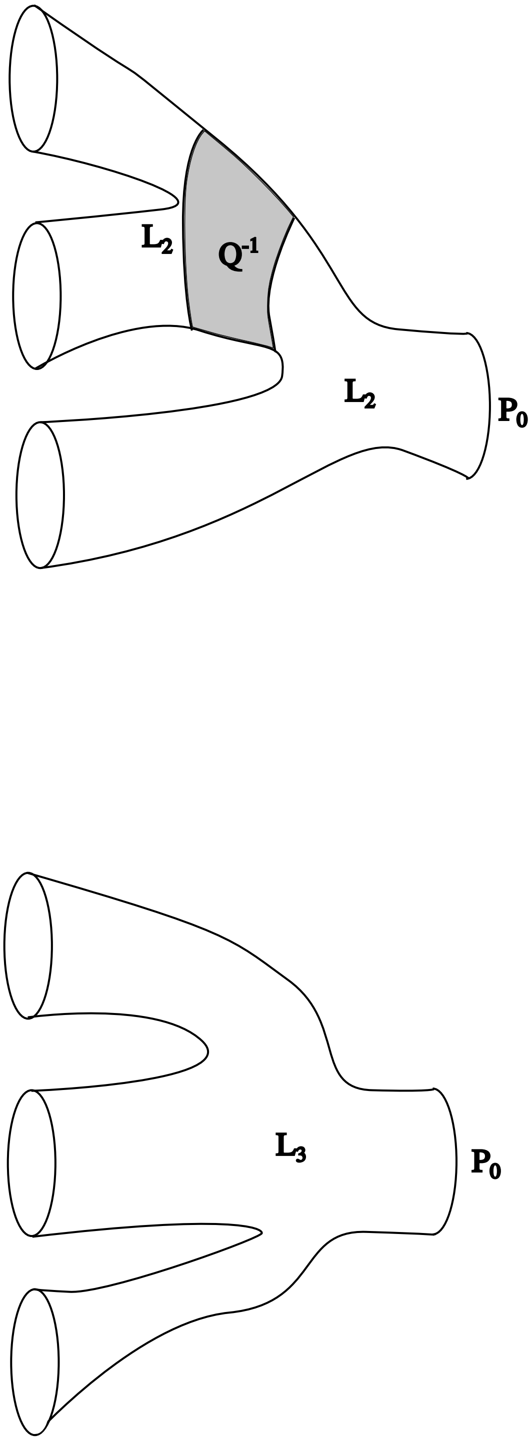

Figure 4.1: The structure of the third-order obstruction

The propagator-terms in the first line of (4.48) can be furtherly disentangled by the same procedure,

$$\begin{aligned}&\left\langle e,\frac{1}{2}\bar{\mathcal{X}}\bar{\xi}L_2^{(1,0)}Q^{-1}L_2^{(1,1)}-\frac{1}{2}\bar{\mathcal{X}}\bar{\xi}L_2^{(1,1)}Q^{-1}L_2^{(1,0)}\right\rangle_L\\&=\left\langle e,-\frac{1}{2}\bar{\mathcal{X}}^2L_2^{(1,0)}Q^{-1}\lambda_2^{(1,1)}-\frac{1}{2}\bar{\mathcal{X}}^2\lambda_2^{(1,1)}Q^{-1}L_2^{(1,0)}\right.\\&\qquad-\frac{1}{2}\bar{\mathcal{X}}\bar{\xi}L_2^{(1,0)}P_0\lambda_2^{(1,1)}-\frac{1}{2}\bar{\mathcal{X}}\bar{\xi}\lambda_2^{(1,1)}P_0L_2^{(1,0)}\\&\qquad\left.+\frac{1}{2}\bar{\mathcal{X}}\bar{\xi}L_2^{(1,0)}\lambda_2^{(1,1)}+\frac{1}{2}\bar{\mathcal{X}}\bar{\xi}\lambda_2^{(1,1)}L_2^{(1,0)}\right\rangle_L(\Psi^{(1)},\Psi^{(1)},\Psi^{(1)}).\end{aligned}\tag{4.49}$$

Regarding the projector terms in the second line of (4.48) and (4.49), note that

$$P_0L_2^{(1,1)}(\Psi^{(1)},\Psi^{(1)})=0,\tag{4.50}$$

$$P_0L_2^{(1,0)}(\Psi^{(1)},\Psi^{(1)})=0,\tag{4.51}$$

and

$$P_0\lambda_2^{(1,1)}(\Psi^{(1)},\Psi^{(1)})=\frac{1}{3}P_0\{\bar{\xi},L_2^{(1,0)}\}(\Psi^{(1)},\Psi^{(1)})=0\tag{4.52}$$

as we have shown in the previous section. We are left only with the $\bar{\mathcal{X}}^2$-terms

$$\left\langle e,-\frac{1}{2}\bar{\mathcal{X}}^2L_2^{(1,0)}Q^{-1}\lambda_2^{(1,1)}-\frac{1}{2}\bar{\mathcal{X}}^2\lambda_2^{(1,1)}Q^{-1}L_2^{(1,0)}\right\rangle_L\tag{4.53}$$

and

$$\begin{aligned}&\left\langle e,\frac{1}{2}\bar{\xi}L_2^{(1,1)}\lambda_2^{(1,1)}+\frac{1}{2}\bar{\xi}\lambda_2^{(1,1)}L_2^{(1,1)}\right.\\&\qquad\left.+\frac{1}{2}\bar{\mathcal{X}}\bar{\xi}L_2^{(1,0)}\lambda_2^{(1,1)}+\frac{1}{2}\bar{\mathcal{X}}\bar{\xi}\lambda_2^{(1,1)}L_2^{(1,0)}-\bar{\xi}L_3^{(2,2)}\right\rangle_L.\end{aligned}\tag{4.54}$$

We have in (4.54)

$$\left\langle e,\frac{1}{2}[L_2^{(1,1)},\lambda_2^{(1,1)}]-L_3^{(2,2)}+\frac{1}{2}\bar{\mathcal{X}}[L_2^{(1,0)},\lambda_2^{(1,1)}]\right\rangle(\Psi^{(1)},\Psi^{(1)},\Psi^{(1)}),\tag{4.55}$$

where we can make use of the relation

$$L_3^{(2,2)}=\frac{1}{2}([Q,\lambda_3^{(2,2)}]+[L_2^{(1,1)},\lambda_2^{(1,1)}])\tag{4.56}$$

and

$$L_3^{(2,1)}=[Q,\lambda_3^{(2,1)}]+[L_2^{(1,0)},\lambda_2^{(1,1)}].\tag{4.57}$$

In the large Hilbert state the zero-mode $\bar{\xi}_0$ is present, and thus it will be useful to note that

$$\langle e,b_n\rangle=(-1)^{\deg(b_n)}\left\langle e,b_n(\mathbb{I}^{\otimes k}\otimes\xi\otimes\mathbb{I}^{n-k})\right\rangle_L,\tag{4.58}$$

allowing us to move the ξ ghost freely. We get the remaining term in (4.54)

$$\begin{aligned}\left\langle e, \frac{1}{2}\bar{\mathcal{X}} L_3^{(2,1)}\right\rangle (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}) &= \left\langle e, \frac{1}{2}\bar{\mathcal{X}}\bar{\xi} L_3^{(2,1)}\right\rangle_L (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}) \\ &= \left\langle e, \frac{1}{2}\bar{\mathcal{X}}\lambda_3^{(2,2)}\right\rangle_L (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}).\end{aligned} \tag{4.59}$$

However, the picture-changing operator only gives Q-exact terms in large Hilbert space, namely

$$\begin{aligned}&\left\langle e, \frac{1}{2}\bar{\mathcal{X}}\lambda_3^{(2,2)}\right\rangle_L \\ &\qquad = \left\langle e, \frac{1}{2} Q\bar{\xi}\lambda_3^{(2,2)}\right\rangle_L + \left\langle e, \frac{1}{2}\bar{\xi} Q\lambda_3^{(2,2)}\right\rangle_L \\ &\qquad = \left\langle Qe, \frac{1}{2}\bar{\xi}\lambda_3^{(2,2)}\right\rangle_L + \left\langle Qe, \frac{1}{2}\lambda_3^{(2,2)}\right\rangle \\ &\qquad = 0.\end{aligned} \tag{4.60}$$

The remaining $\bar{\mathcal{X}}^2$-terms is what initially gives rise to a constraint on the open moduli (see [24] for a more general discussion). In the following we show that it is absent again by the doubling of the holomorphic sectors in closed string theory,

$$\begin{aligned}&\left\langle e, \frac{1}{2}\bar{\mathcal{X}}^2 L_2^{(1,0)} Q^{-1}\lambda_2^{(1,1)} + \frac{1}{2}\bar{\mathcal{X}}^2\lambda_2^{(1,1)} Q^{-1} L_2^{(1,0)}\right\rangle_L (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}) \\ &= \left\langle e, \bar{\mathcal{X}}^2\bar{\xi} L_2^{(1,0)} Q^{-1} L_2^{(1,0)}\right\rangle_L (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}),\end{aligned} \tag{4.61}$$

where we've moved the ghost between the inputs. We apply the strategy once more to commute Q with Q^{-1} using the contracting homotopy relation

$$\begin{aligned}&\left\langle \frac{1}{2}\bar{\mathcal{X}}^2 L_2^{(1,0)} Q^{-1} L_2^{(1,0)}\right\rangle (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}) \\ &= \left\langle \frac{1}{2}\bar{\xi}\bar{\mathcal{X}} L_2^{(1,0)} Q Q^{-1} L_2^{(1,0)}\right\rangle (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}) \\ &= \left\langle \frac{1}{2}\bar{\xi}\bar{\mathcal{X}} L_2^{(1,0)} (1 - P_0) L_2^{(1,0)}\right\rangle (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}).\end{aligned} \tag{4.62}$$

The second term vanishes by (4.51), and we have only $\left\langle \frac{1}{2}\bar{\xi}\bar{\mathcal{X}} L_2^{(1,0)} L_2^{(1,0)}\right\rangle (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)})$ left. Moving the ξ ghost in the holomorphic sector we get

$$\left\langle \frac{1}{2}\bar{\xi}\bar{\mathcal{X}} L_2^{(1,0)} L_2^{(1,0)}\right\rangle (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}) = \left\langle \frac{1}{2}\mathcal{X}^2\bar{\mathcal{X}}^2 L_2^{(0,0)} L_2^{(0,0)}\right\rangle (\Psi^{(1)}, \Psi^{(1)}, \Psi^{(1)}), \tag{4.63}$$

where the composition of the bosonic product $L_2^{(0,0)} \circ L_2^{(0,0)}$ appears. In contrast to Witten's cubic open string field theory [18], the closed string product is not associative. As a matter of fact, the L_∞ relations give

$$\frac{1}{2}[L_2^{(0,0)}, L_2^{(0,0)}] = [Q, L_3^{(0,0)}], \tag{4.64}$$

but obviously the Q-exact term vanishes when inserted into the on-shell amplitude. Consequently, we conclude that the third-order obstruction component $\mathcal{O}_3^e$ vanishes on an arbitrary state e in the image of P_0, and thus the whole obstruction $\mathcal{O}_3$ vanishes. The result is valid without assuming any other constraint analogous to the generalized ADHM constraint in [24] on the moduli. This means that any one choice of moduli corresponds to a deformation tangent at the orbifold point, at least so to the third-order correction.

The vanishing of $\mathcal{O}_3$ means that the perturbative solution to the string field theory equation of motion is well-defined up to the third order. Furthermore, it is in principle possible to compute all contributions to the metric at order $O(\lambda^3)$. The explicit calculation would be, however, rather cumbersome and not discussed here.

Chapter 5

Conclusion

In this thesis we explore exactly marginal deformations near an orbifold conformal field theory. The analysis is carried out in the string field theory framework. The vertex operator corresponding to the twist field is considered, both in the bosonic and the supersymmetric case, to generate a deformation. We are able to show that, although the bosonic deforming is blocked, it is possible to generate an exactly marginal deformation up to the third order. We also compute the metric profile generated by the deformation at the second order. The metric is not Kähler and hence indicates a breaking of $\mathcal{N} = 2$ supersymmetry of the original $\mathbb{Z}_2$ orbifold model.

Appendix A

Operadic Structures in Homotopy and BV Algebras

Definition 1. *An operad is a collection of objects of a symmetric monoidal category* $\mathcal{O}(n), n \leq 1$, *with:*
The action of the symmetric group S_n *on* $\mathcal{O}(n)$.
The composition law:

$$\gamma : \mathcal{O}(k) \times \mathcal{O}(n_1) \times \cdots \times \mathcal{O}(n_k) \to \mathcal{O}(n_1 + \cdots + n_k),$$
$$(f; f_1, \cdots, f_k) \mapsto \gamma(f; f_1, \cdots, f_k) \equiv f(f_1, \cdots, f_k).$$

The unit $e \in \mathcal{O}(1)$ *satisfying* $e(f) = f$ *and* $f(e, \cdots, e) = f$ *for any* $f \in \mathcal{O}(n)$.
The composition is equivariant with respect to the symmetric group actions:

$$(\sigma; \sigma_1, \cdots, \sigma_k) f(f_1, \cdots, f_k) = f(\sigma_{\sigma(1)} f_{\sigma(1)}, \cdots, \sigma_{\sigma(k)} f_{\sigma(k)}).$$

The composition is associative, i.e., $\gamma \circ (\mathrm{id} \times \gamma^k) = \gamma \circ (\gamma \times \mathrm{id})$.

Theorem 1. *The correspondence*

$$\begin{aligned} C^*(\mathcal{M}_{n+1}) &\to \mathrm{Hom}((H_{\mathrm{rel}})^{\otimes n}, H_{\mathrm{rel}}) \\ C &\mapsto \int_C w_{n+1} \end{aligned} \tag{A.1}$$

defines the structure of an algebra over the operad $C^*(\mathcal{M}_{n+1})$ *on the space* H_{rel}.

Theorem 2. $H_p(F_p, F_{p-1})$*is naturally isomorphic to the* n*-th component* $\mathcal{S}_n$ *of the homotopy Lie operad. Therefore the correspondence*

$$\begin{aligned} H_p(F_p, F_{p-1}) &\to \mathrm{Hom}((H_{\mathrm{rel}})^{\otimes n}, H_{\mathrm{rel}}) \\ Z &\mapsto \int_Z w_{n+1} \end{aligned} \tag{A.2}$$

defines the structure of an algebra over the homotopy Lie operad $\mathcal{S}(n)$ *on the relative state space* H_{rel}.

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Soryushiron Kenkyu

Declaration

I hereby declare that this thesis is my own work, and that I have not used any sources and aids other than those stated in the thesis.

Location, date

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