Smart and Human

常翔学園

摂南大学

Studies of the space-time emerging from the matrix model for superstrings (Project ID: hp200106)

Takehiro Azuma (Setsunan Univ.)

The 8th Project Report Meeting of the HPCI System

Oct 29th 2021, 16:15〜18:20

with Konstantinos N. Anagnostopoulos (NTUA), Kohta Hatakeyama (KEK),
Mitsuaki Hirasawa (KEK→INFN), Yuta Ito (Tokuyama College),
Jun Nishimura (KEK, SOKENDAI), Stratos Kovalkov Papadoudis (NTUA)
and Asato Tsuchiya (Shizuoka Univ.)

1. Introduction

SETSUDAI

Type IIB matrix model (a.k.a. IKKT model)

⇒Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$S = \underbrace{\frac{-1}{4g^2}\mathrm{tr}[A_\mu, A_\nu]^2}_{=S_\mathrm{b}} + \underbrace{\frac{-1}{2g^2}\mathrm{tr}\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_\mathrm{f}}$$

- Dimensional reduction of the D=10 super-Yang-Mills theory to 0 dimension
- A_μ, Ψ_α ⇒N×N Hermitian traceless matrices.
- N=2 supersymmetry ⇒ eigenvalues of A_μ are interpreted as the spacetime coordinate.

How does our (3+1)-dim spacetime emerge dynamically?

2. Lorentzian type IIB matrix model

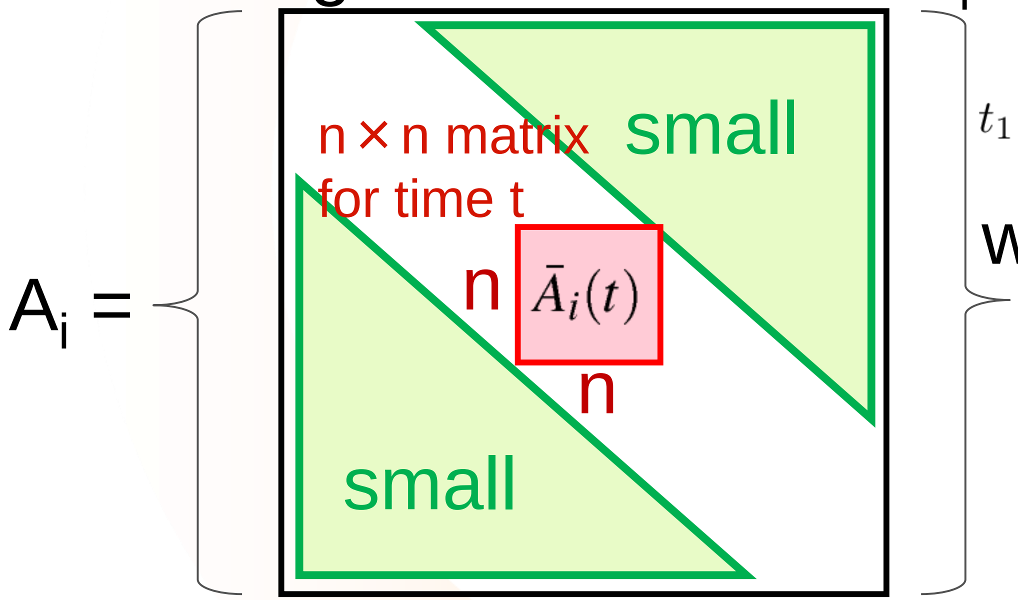
Lorentzian version [S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

⇒contracted by the Lorentzian metric $\eta = \mathrm{diag}(-1, 1, 1, \cdots, 1)$

Time development: gauge fixing to diagonalize A_0

$$A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N), \text{ where } \alpha_1 < \alpha_2 < \cdots < \alpha_N.$$

Band-diagonal structure of A_i



$$t_1 = 0, \quad t_\nu = \sum_{k=1}^{\nu-1} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$$

where $\bar{\alpha}_k = \frac{1}{n} \sum_{i=1}^{n} \alpha_{i+k}$

2. Lorentzian type IIB matrix model

SETSUDAI

Sign problem of the Lorentzian version

$$Z = \int dA \left(e^{iS_b} \underbrace{\int d\psi e^{iS_f}}_{=\mathrm{Pf}\mathcal{M}} \right)$$

complex real

[J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

- We employ the Complex Langevin Method (CLM)
- We introduce parameters of Wick rotation:

[J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

- multiply overall $e^{is\pi/2}$ (w.r.t. world sheet)
- $A_0 \to A_0 e^{-ik\pi/2}$ (w.r.t. target space)

2. Lorentzian type IIB matrix model

- multiply overall $e^{is\pi/2}$ (w.r.t. world sheet)
- $A_0 \to A_0 e^{-ik\pi/2}$ (w.r.t. target space)

Original Lorentzian

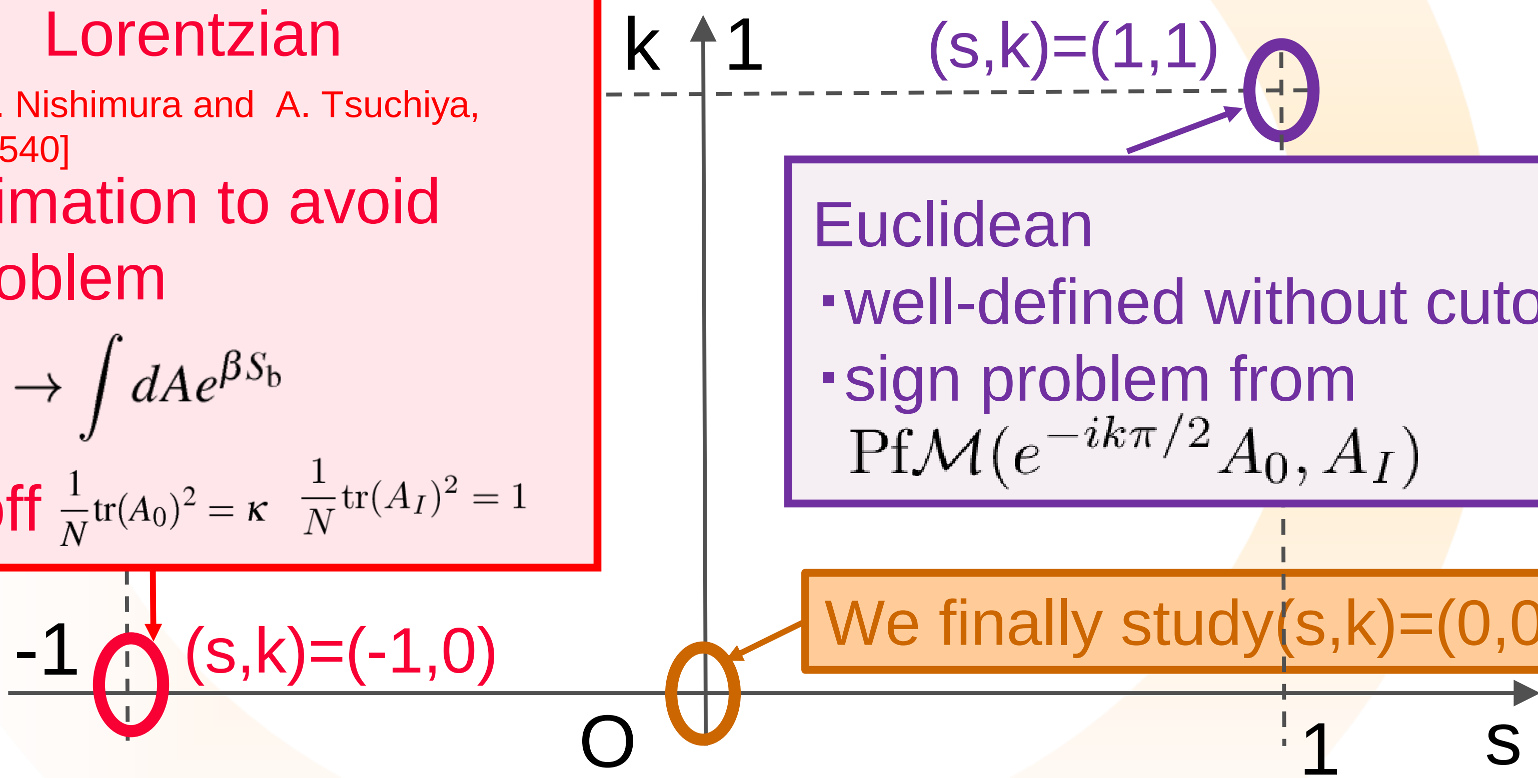
[S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

- approximation to avoid sign problem
  $$\int dA e^{iS_b} \to \int dA e^{\beta S_b}$$
- IR cutoff $\frac{1}{N}\mathrm{tr}(A_0)^2 = \kappa \quad \frac{1}{N}\mathrm{tr}(A_I)^2 = 1$

Euclidean

- well-defined without cutoff
- sign problem from $\mathrm{Pf}\mathcal{M}(e^{-ik\pi/2}A_0, A_I)$

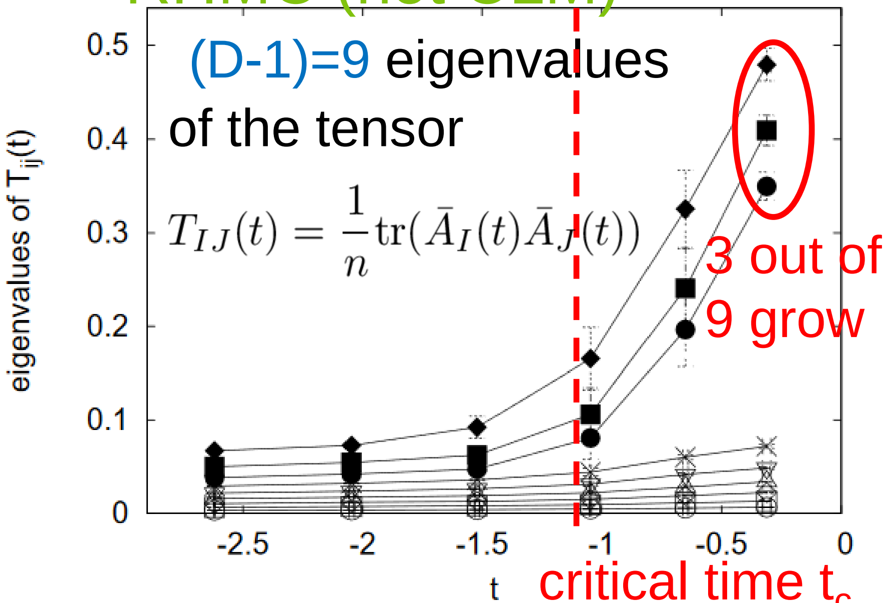
We finally study (s,k)=(0,0)



2. Lorentzian type IIB matrix model

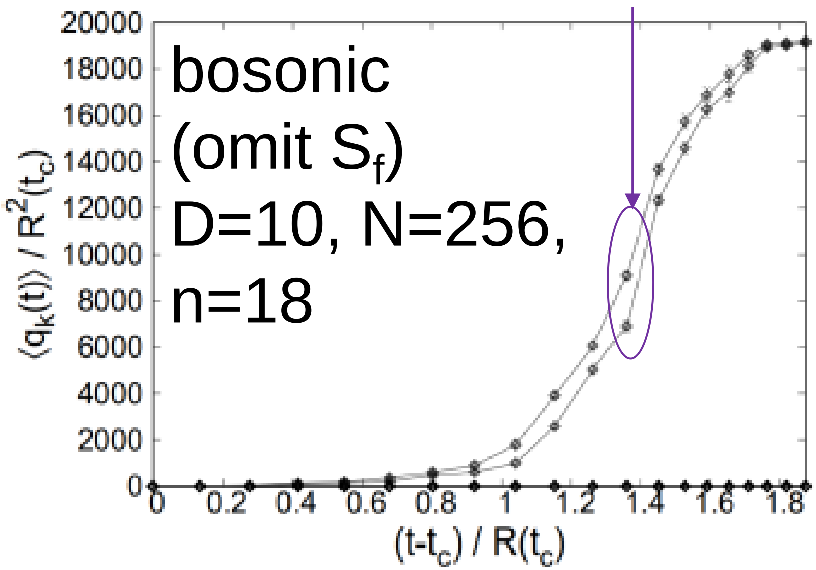
Results at (s,k)=(-1,0), with the approximation to avoid the sign problem:

D=10, N=16, n=4,
RHMC (not CLM)



[S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

2 of the n eigenvalues of $Q(t) = \sum_{I=1}^{D-1} (\bar{A}_I(t))^2$ grow.



[T. Aoki, M. Hirasawa, Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1904.05914]

Dynamical emergence of (3+1)-dim spacetime.

$$\bar{A}_I(t) \propto \sigma_I \oplus 0_{n-2} \quad (I=1,2,3)$$

Pauli-matrix space structure
⇒hollow 3-dim sphere

3. Complex Langevin Method

Complex Langevin Method (CLM)

⇒Promising method to solve complex-action systems.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

$$\frac{d(A_i)_{ab}(t_l)}{dt_l} = -\frac{dS_{\text{eff}}}{d(A_i)_{ba}} + \eta_{i,ab}(t_l) \qquad \frac{d\tau_a(t_l)}{dt_l} = -\frac{dS_{\text{eff}}}{d\tau_a} + \eta_a(t_l)$$

drift term; Hermitian-matrix white noise; fictitious Langevin time (for t_l)

drift term; real-number white noise; fictitious Langevin time (for t_l)

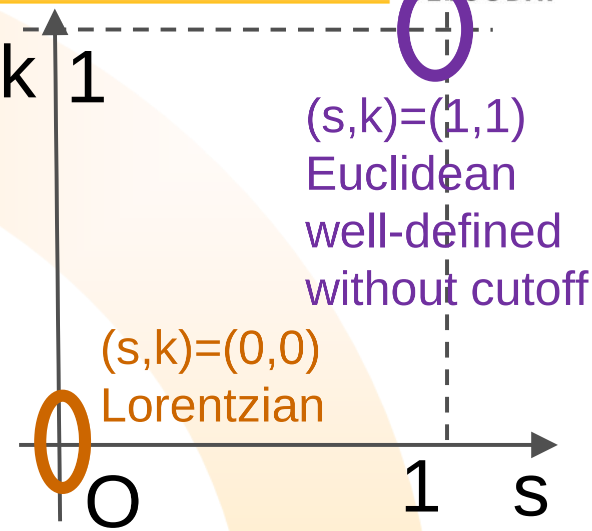
- A_i : Hermitian→general complex traceless matrices.
- τ_a : Real number → complex number.

Introducing time order $\alpha_1 < \alpha_2 < \ldots < \alpha_N$ for complexified α_i

$$\alpha_1 = 0, \ \alpha_k = \sum_{i=1}^{k-1} e^{\tau_i} \ (k = 2, 3, \cdots, N) \qquad t_1 = 0, \ \ t_\nu = \sum_{k=1}^{\nu-1} |\bar{\alpha}_{k+1} - \bar{\alpha}_k| \qquad \bar{\alpha}_k = \frac{1}{n}\sum_{i=1}^{n} \alpha_{i+k}$$

4. Result

Bosonic action after the Wick rotation



$A_0 \to A_0 e^{-ik\pi/2}$ multiply overall $e^{is\pi/2}$

$\Downarrow$

$$Z = \int dA e^{-\tilde{S}_{\mathrm{b}(s,k)}(A_0, A_I)}$$, where

$$\tilde{S}_{\mathrm{b}(s,k)}(A_0, A_I) = N\mathrm{tr}\left(-\frac{1}{2}e^{i(1+s-2k)\pi/2}\sum_{I=1}^{D-1}[A_0, A_I]^2 - \frac{1}{4}e^{i(s-1)\pi/2}\sum_{I,J=1}^{D-1}[A_I, A_J]^2\right)$$

This satisfies

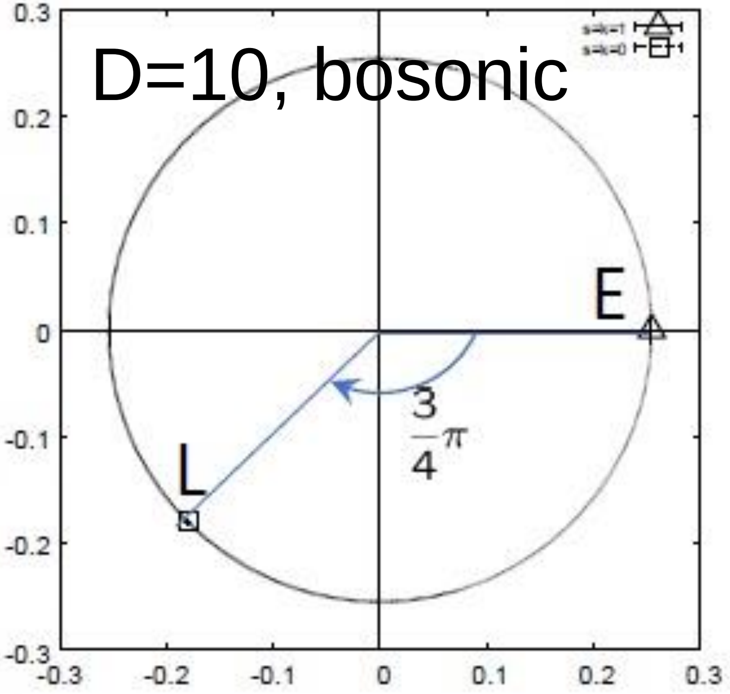
Lorentzian ... Euclidean

$$\tilde{S}_{\mathrm{b}(0,0)}(\underbrace{A_0^{(E)}e^{-3i\pi/8}}_{=A_0^{(L)}}, \underbrace{A_I^{(E)}e^{i\pi/8}}_{=A_I^{(L)}}) = \tilde{S}_{\mathrm{b}(1,1)}(A_0^{(E)}, A_I^{(E)})$$

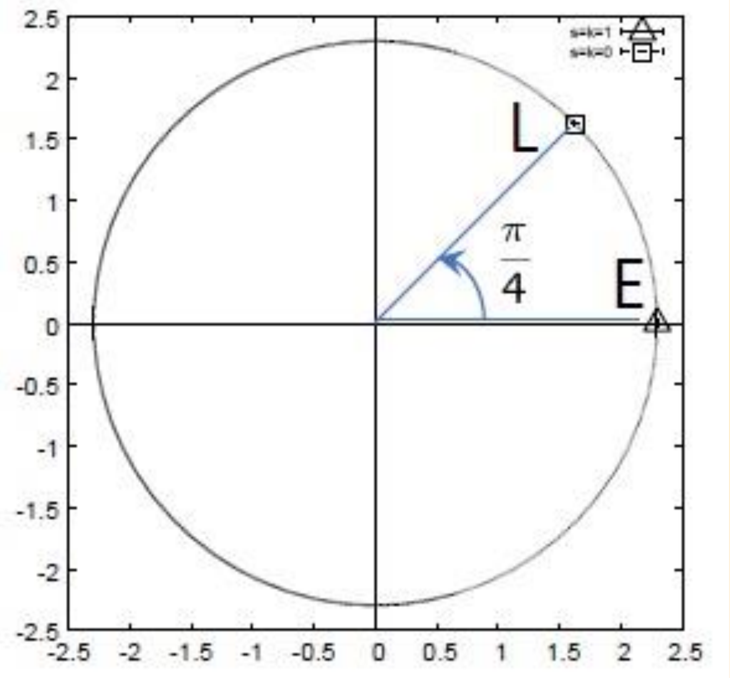
4. Result

Equivalence of the Euclidean and Lorentzian model without a cutoff

$$\left\langle \frac{1}{N}\mathrm{tr}(A_0^{(L)})^2 \right\rangle = e^{-3i\pi/4} \left\langle \frac{1}{N}\mathrm{tr}(A_0^{(E)})^2 \right\rangle$$



$$\left\langle \frac{1}{N}\mathrm{tr}(A_I^{(L)})^2 \right\rangle = e^{i\pi/4} \left\langle \frac{1}{N}\mathrm{tr}(A_I^{(E)})^2 \right\rangle$$



Neither the time nor the space is real in Lorentzian.

⇒The emergent spacetime is interpreted as Euclidean.

4. Result

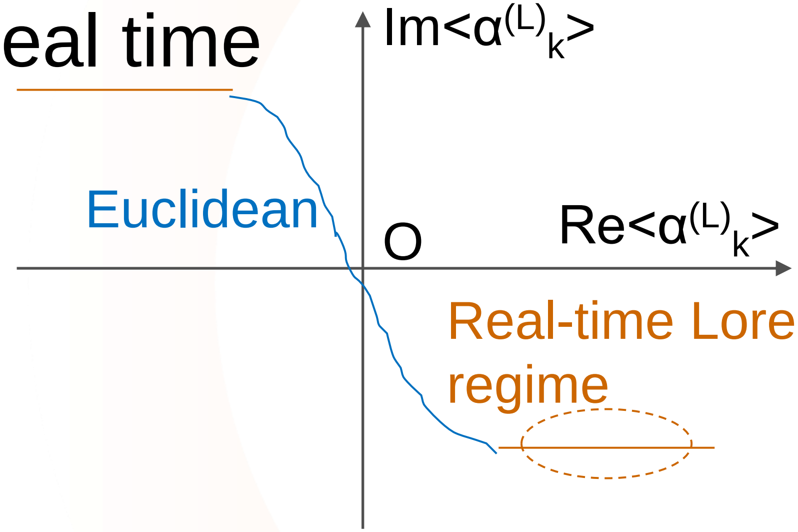
How do we realize the real spacetime?

Introduce a cutoff $\delta(\alpha_N^{(L)} - \sqrt{\kappa}), \quad \sqrt{\kappa} \in \mathbb{C}$

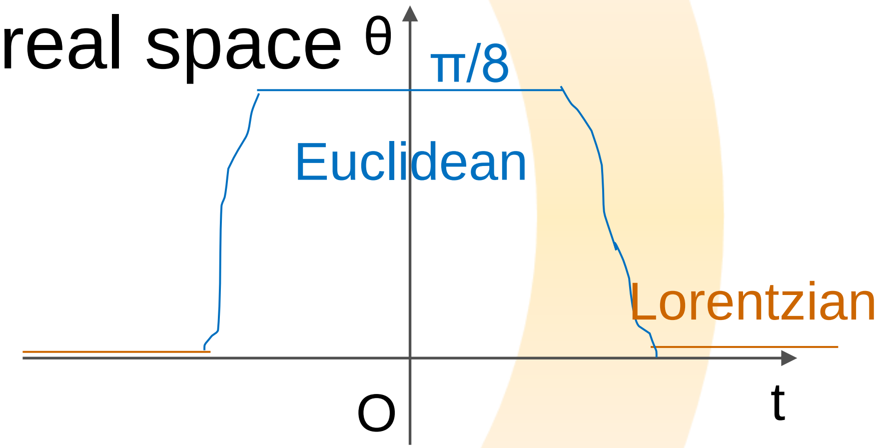
Our expectation:

$$R^2(t) = \left\langle \frac{1}{n} \mathrm{tr} \sum_{I=1}^{D-1} (\bar{A}_I^{(L)}(t))^2 \right\rangle = e^{2i\theta} |R^2(t)|$$

real time



real space



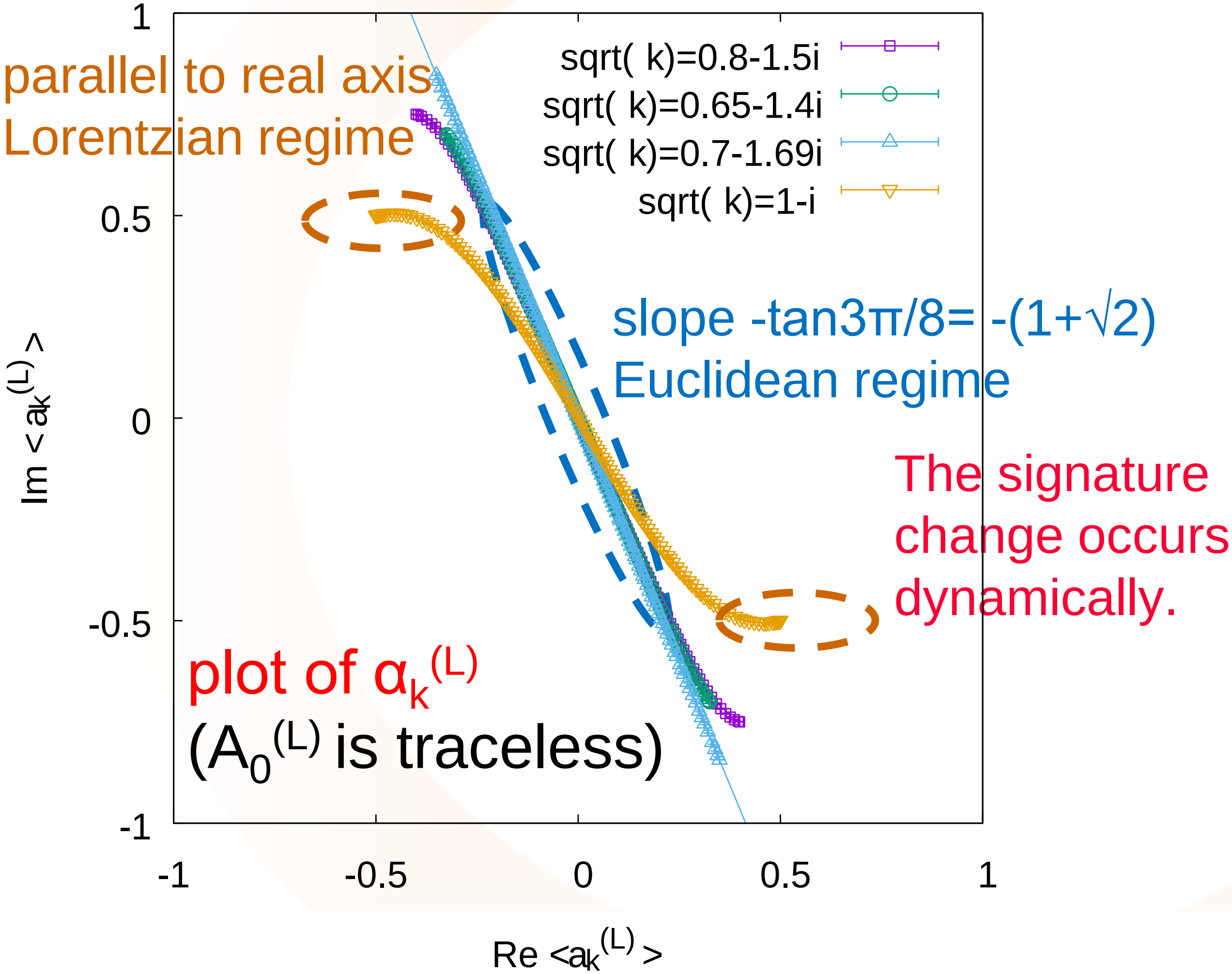
θ=argR(t)= { π/8 : Euclidean regime; 0 : Lorentzian regime

4. Result

SETSUDAI

With the cutoff $\delta(\alpha_N^{(L)} - \sqrt{\kappa}), \quad \sqrt{\kappa} \in \mathbb{C}$ (real time?)

Numerical result for D=10, N=128, bosonic (preliminary)



4. Result

SETSUDAI

With the cutoff $\delta(\alpha_N^{(L)} - \sqrt{\kappa}), \quad \sqrt{\kappa} \in \mathbb{C}$ (real space?)

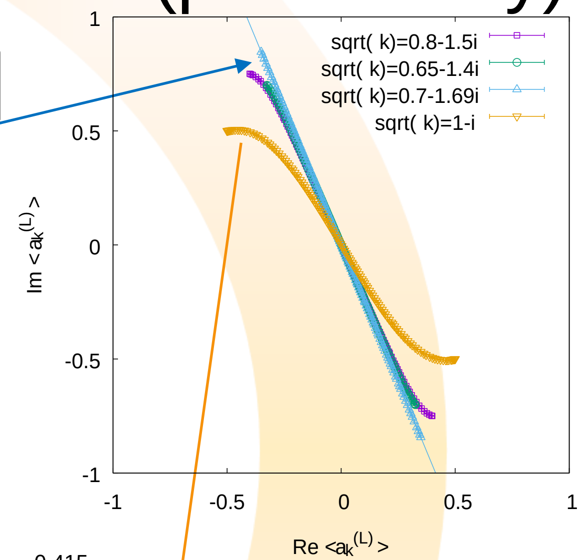
Numerical result for D=10, N=128, bosonic (preliminary)

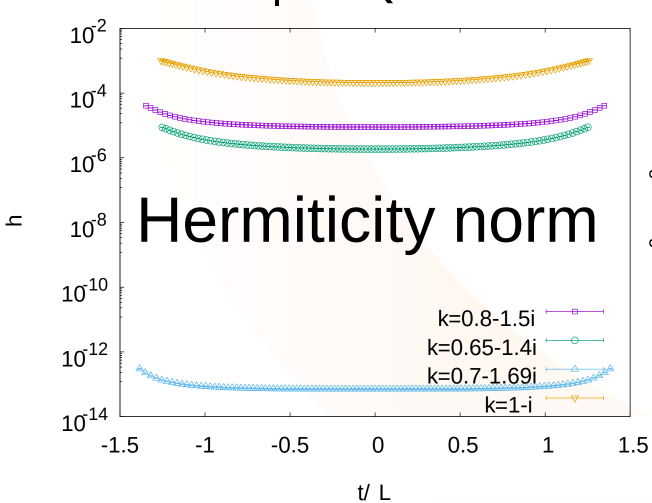
$$R^2(t) = \left\langle \frac{1}{n}\mathrm{tr} \sum_{I=1}^{D-1} (\bar{A}_I^{(L)}(t))^2 \right\rangle = e^{2i\theta}|R^2(t)| \quad \Lambda = |R(0)|$$

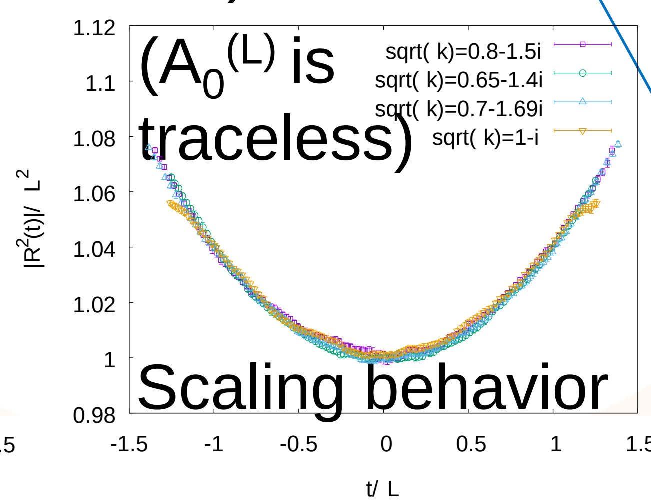
θ=argR(t)= { π/8 : Euclidean regime; 0 : Lorentzian regime }

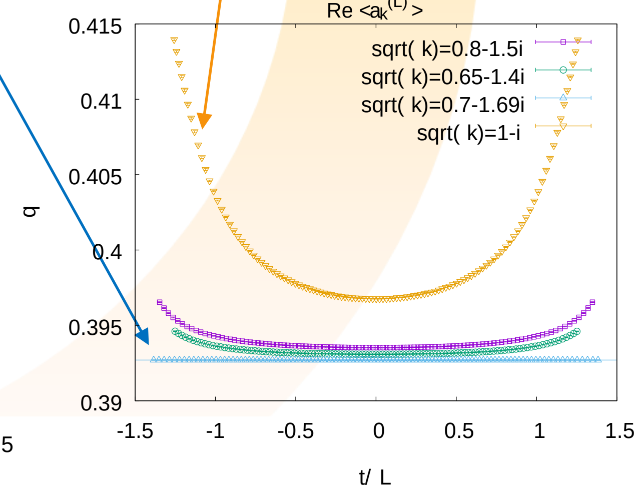
Hermiticity norm $h = \frac{1}{2}\left(1 - \frac{|\mathrm{tr}(A_I^{(L)})^2|}{\mathrm{tr}((A_I^{(L)})^\dagger A_I^{(L)})}\right)$

h=0 if $A_I^{(L)}$=(Hermitian matrix) × $e^{i\varphi}$.









5. Conclusion

SETSUDAI

Complex Langevin Method (CLM) for the type IIB matrix model.

Equivalence of the Euclidean and Lorentzian model without a cutoff.
⇒The emergent spacetime is interpreted as Euclidean.

Introduce a cutoff $\delta(\alpha_N^{(L)} - \sqrt{\kappa}), \quad \sqrt{\kappa} \in \mathbb{C}$
⇒・Scaling behavior of $|R^2(t)|$

Future directions:
Can we extend the Lorentzian-time regime and realize θ closer to 0?
⇒impact of SUSY, larger-N simulations