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Monte Carlo Studies of Dynamical Compactification of Extra Dimensions in a Model of Non-perturbative String Theory

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1. Introduction

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Difficulties in putting complex partition functions on computers.

$$Z = \int dA \exp(-S_0 + i\Gamma), \quad Z_0 = \int dA e^{-S_0}$$

e.g. lattice QCD, matrix models for superstring theory

1. Sign problem:

The reweighting $\langle \mathscr{O} \rangle = \frac{\langle \mathscr{O} e^{i\Gamma} \rangle_0}{\langle e^{i\Gamma} \rangle_0}$ requires configs. $\exp[O(N^2)]$

$<*>_0$ = (V.E.V. for the phase-quenched partition function Z_0)

2. Overlap problem:

Discrepancy of important configs. between Z_0 and Z.

2. Factorization method

Method to sample important configurations for Z.

[J. Nishimura and K.N. Anagnostopoulos, hep-th/0108041
K.N. Anagnostopoulos, T.A. and J. Nishimura, arXiv:1009.4504]

We constrain the observables $\Sigma = \{\mathscr{O}_k | k = 1, 2, \cdots, n\}$
correlated with the phase Γ.

They are normalized as $\tilde{\mathscr{O}}_k = \mathscr{O}_k / \langle \mathscr{O}_k \rangle_0$

The distribution function factorizes as

$$\rho(x_1, \cdots, x_n) \stackrel{\text{def}}{=} \left\langle \prod_{k=1}^{n} \delta(x_k - \tilde{\mathscr{O}}_k) \right\rangle = \underbrace{\frac{1}{\langle e^{i\Gamma} \rangle_0}}_{=1/C} \times \underbrace{\left\langle \prod_{k=1}^{n} \delta(x_k - \tilde{\mathscr{O}}_k) \right\rangle_0}_{\stackrel{\text{def}}{=} \rho^{(0)}(x_1, \cdots, x_n)} \times \underbrace{\frac{\langle \prod_{k=1}^{n} \delta(x_k - \tilde{\mathscr{O}}_k) e^{i\Gamma} \rangle_0}{\langle \prod_{k=1}^{n} \delta(x_k - \tilde{\mathscr{O}}_k) \rangle_0}}_{\stackrel{\text{def}}{=} w(x_1, \cdots, x_n)}$$

$$w(x_1, \cdots, x_n) = \langle e^{i\Gamma} \rangle_x \left(\langle * \rangle_x = \left\{ \text{V.E.V. for } Z_x = \int dA e^{-S_0} \prod_{k=1}^{n} \delta(x_k - \tilde{\mathscr{O}}_k) \right\} \right)$$

Simulation of Z_x with a proper choice of the set Σ
⇒sample the important region for Z.

Evaluation of the observables $\langle \tilde{\mathcal{O}}_k \rangle$

Peak of the distribution function ρ at V=(system size)→∞.
= Minimum of the free energy $\mathscr{F} = -\frac{1}{N^2} \log \rho$

⇒Solve the saddle-point equation $\frac{1}{N^2} \frac{\partial}{\partial x_n} \log \rho^{(0)} = -\frac{\partial}{\partial x_n} \frac{1}{N^2} \log w$

Applicable to general systems with sign problem.

3. The model

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IKKT model (or the IIB matrix model)

⇒Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$S = \underbrace{-\frac{N}{4}\mathrm{tr}[A_\mu, A_\nu]^2}_{=S_B} + \underbrace{\frac{N}{2}\mathrm{tr}\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_F}$$

- *Euclidean* case after the Wick rotation $A_0 \to iA_{10}$, $\Gamma^0 \to -i\Gamma_{10}$.
- A_μ, Ψ_α ⇒N×N Hemitian matrices (μ=1,2,…,d=10, α,β=1,2,…,16)
- Eigenvalues of A_μ ⇒spacetime coordinate
- Dynamical emergence of the spacetime due to the Spontaneous Symmetry Breaking (SSB) of SO(10).

Result of Gaussian Expansion Method (GEM)

Order parameter of the SO(10) rotational symmetry breaking

$$\lambda_n (\lambda_1 \geq \cdots \geq \lambda_{10}) : \text{eigenvalues of } T_{\mu\nu} = \frac{1}{N} \mathrm{tr}(A_\mu A_\nu)$$

$$\langle \lambda_1 \rangle = \cdots = \langle \lambda_d \rangle (= R^2) \gg \langle \lambda_{d+1} \rangle = \cdots = \langle \lambda_{10} \rangle (= r^2)$$

Extended d-dim. and shrunken (10-d) dim. at N→∞

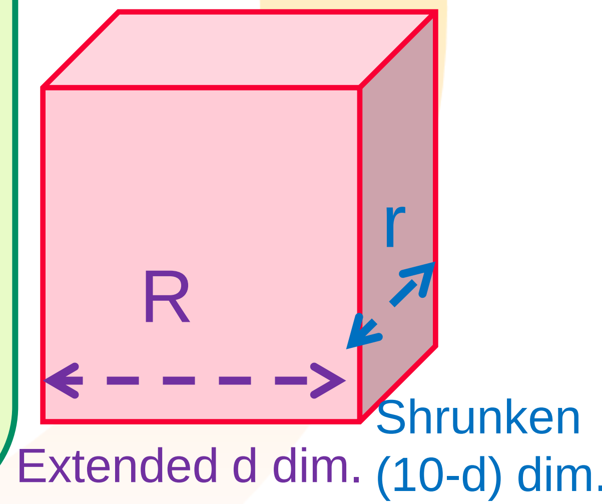
SSB SO(10)→SO(d)

Main Results of GEM

[J. Nishimura, T. Okubo and F. Sugino, arXiv:1108.1293]

- Universal compactification scale $r^2 \cong 0.15$ for SO(d) ansatz (d=2,3,…7).
- Constant volume property except d=2 $V=R^d \times r^{10-d}=l^{10}$, $l^2 \cong 0.38$
- SSB SO(10)→SO(3).

10 dim. volume $V=R^d \times r^{10-d}$



Mechanism of SSB in Euclidean case

Partition function of the model:

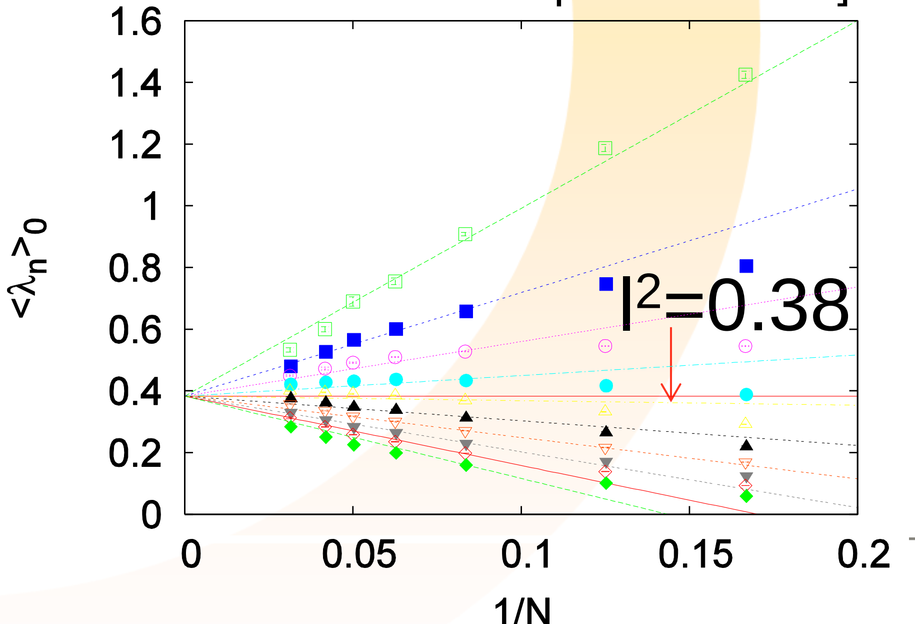
$$Z = \int dA e^{-S_B} \underbrace{\left(\int d\psi e^{-S_F} \right)}_{= \mathrm{Pf}\mathcal{M} = |\mathrm{Pf}\mathcal{M}| e^{i\Gamma}} = \int dA \underbrace{e^{-S_0}}_{= e^{-S_B} |\mathrm{Pf}\mathcal{M}|} e^{i\Gamma}$$

The Pfaffian PfM is complex in the Euclidean case
⇒Complex phase Γ is crucial for the SSB of SO(10).
[J. Nishimura and G. Vernizzi hep-th/0003223]

No SSB with the phase-quenched partition function.

$$Z_0 = \int dA e^{-S_0}$$

$\langle * \rangle_0$=V.E.V. for Z_0



4. Results of the Monte Carlo simulation

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It turns out sufficient to constrain only one eigenvalue λ_{d+1}

$\Sigma = \{\lambda_{d+1} \text{ only}\}$ Corresponds to the SO(d) vacuum

$$\langle\lambda_1\rangle = \cdots = \langle\lambda_d\rangle (= R^2) \gg \langle\lambda_{d+1}\rangle = \cdots = \langle\lambda_{10}\rangle (= r^2)$$

$\tilde{\lambda}_n \stackrel{\text{def}}{=} \lambda_n / \langle\lambda_n\rangle_0$ corresponds to $(r/l)^2$[$\simeq$0.15/0.38=0.40 (GEM)]

Solve the saddle-point equation for n=d+1.

$$\frac{1}{N^2} f_n^{(0)}(x) = -\frac{d}{dx}\frac{1}{N^2}\log w_n(x) \quad \text{where}$$

$$f_n^{(0)}(x) \stackrel{\text{def}}{=} \frac{d}{dx}\log\langle\delta(x-\tilde{\lambda}_n)\rangle_0, \quad w_n(x) \stackrel{\text{def}}{=} \langle e^{i\Gamma}\rangle_{n,x} = \langle\cos\Gamma\rangle_{n,x}$$

$$\langle * \rangle_{n,x} = \left\{ \text{V.E.V. for } Z_{n,x} = \int dA e^{-S_0}\delta(x-\tilde{\lambda}_n) \right\}$$

The solution $\bar{x}_n$ corresponds to $\bar{x}_n = \langle\tilde{\lambda}_{d+1}\rangle_{\mathrm{SO(d)}}$
in the SO(d) vacuum.

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The phase $w_n(x)$ scales at large N as

$$\Phi_n(x) = \lim_{N\to\infty} \frac{1}{N^2} \log w_n(x) \simeq -a_n x^{10-(n-1)} - b_n \ \ (x<1)$$

Around x≅1: $f_n^{(0)}(x)/N$ scales at large N:

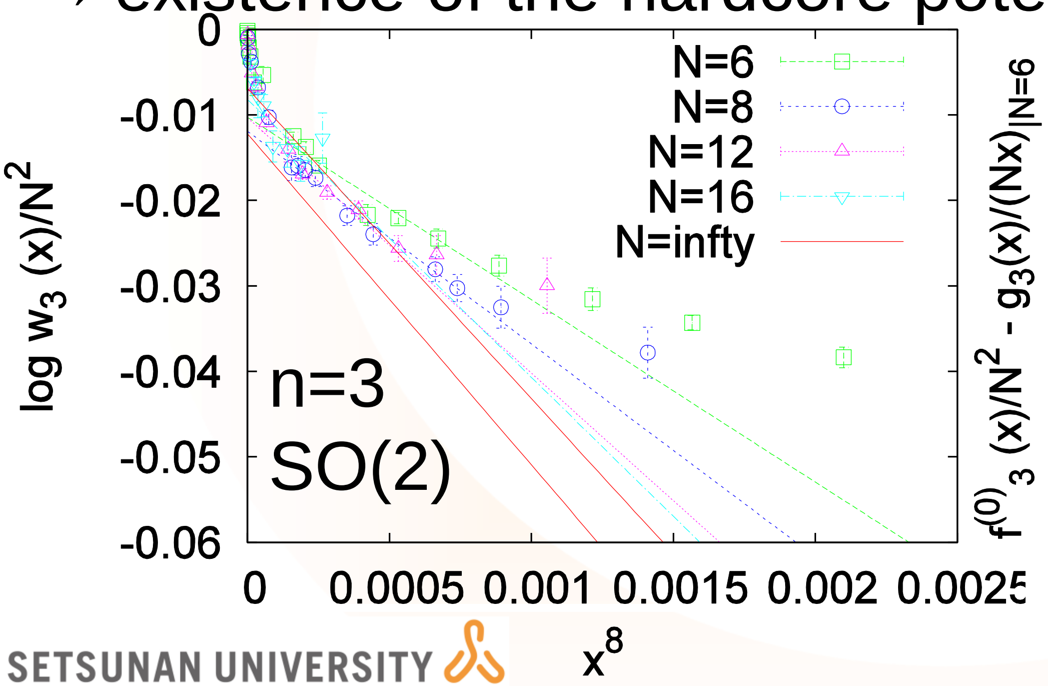
$$\frac{x}{N} f_n^{(0)}(x) \simeq g_n(x) = c_{1,n}(x-1) + c_{2,n}(x-1)^2$$

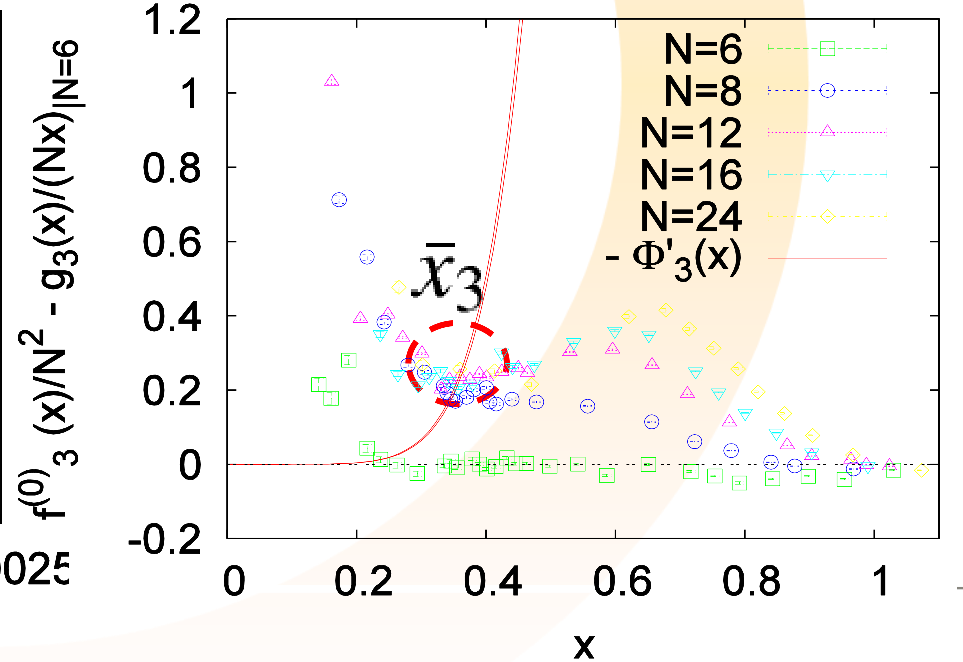
Around x<0.4: $f_n^{(0)}(x)/N^2$ scales at large N

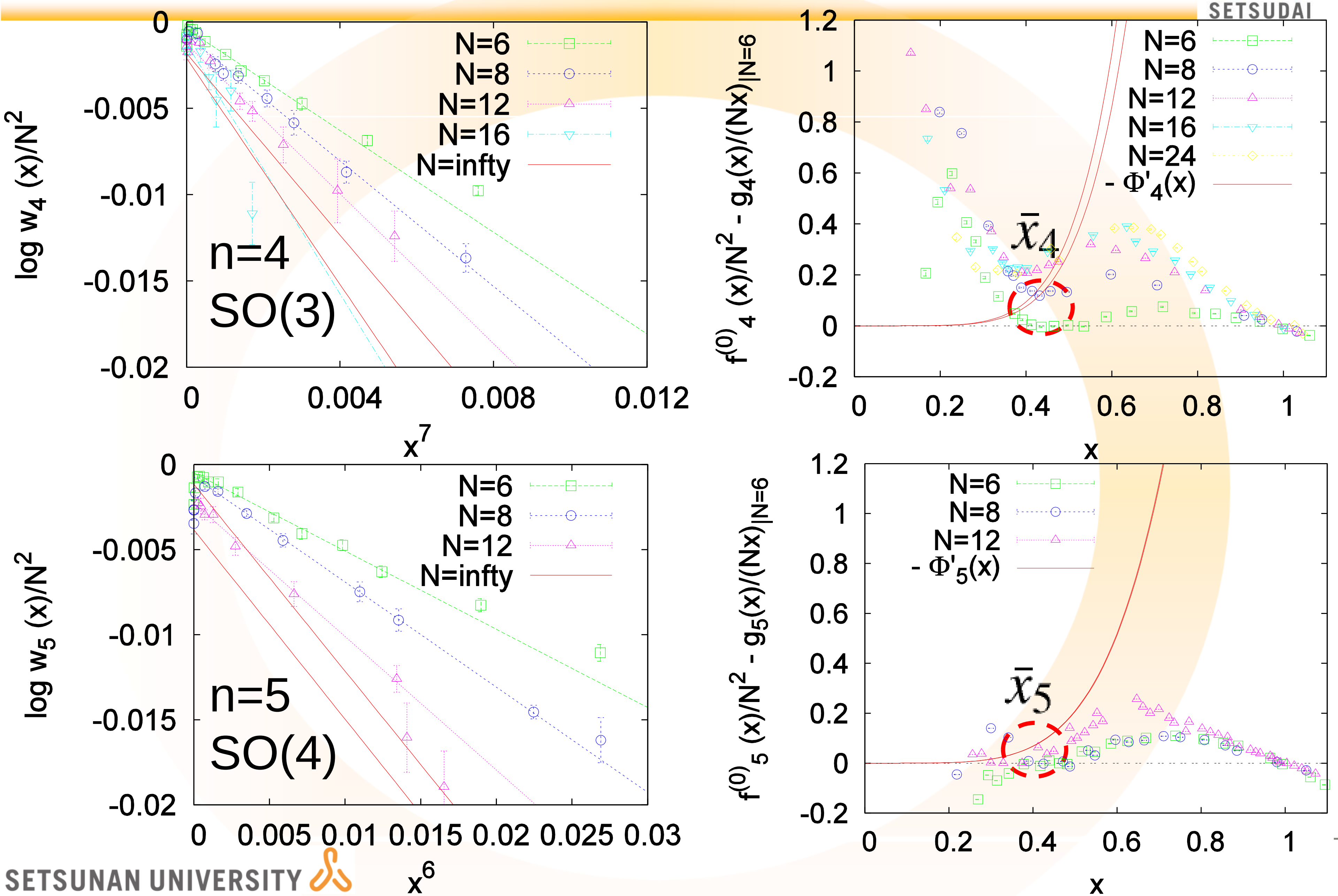
→ existence of the hardcore potential.

GEM suggests

$$\bar{x}_n = \langle \tilde{\lambda}_{d+1} \rangle_{\mathrm{SO(d)}} \simeq 0.40$$







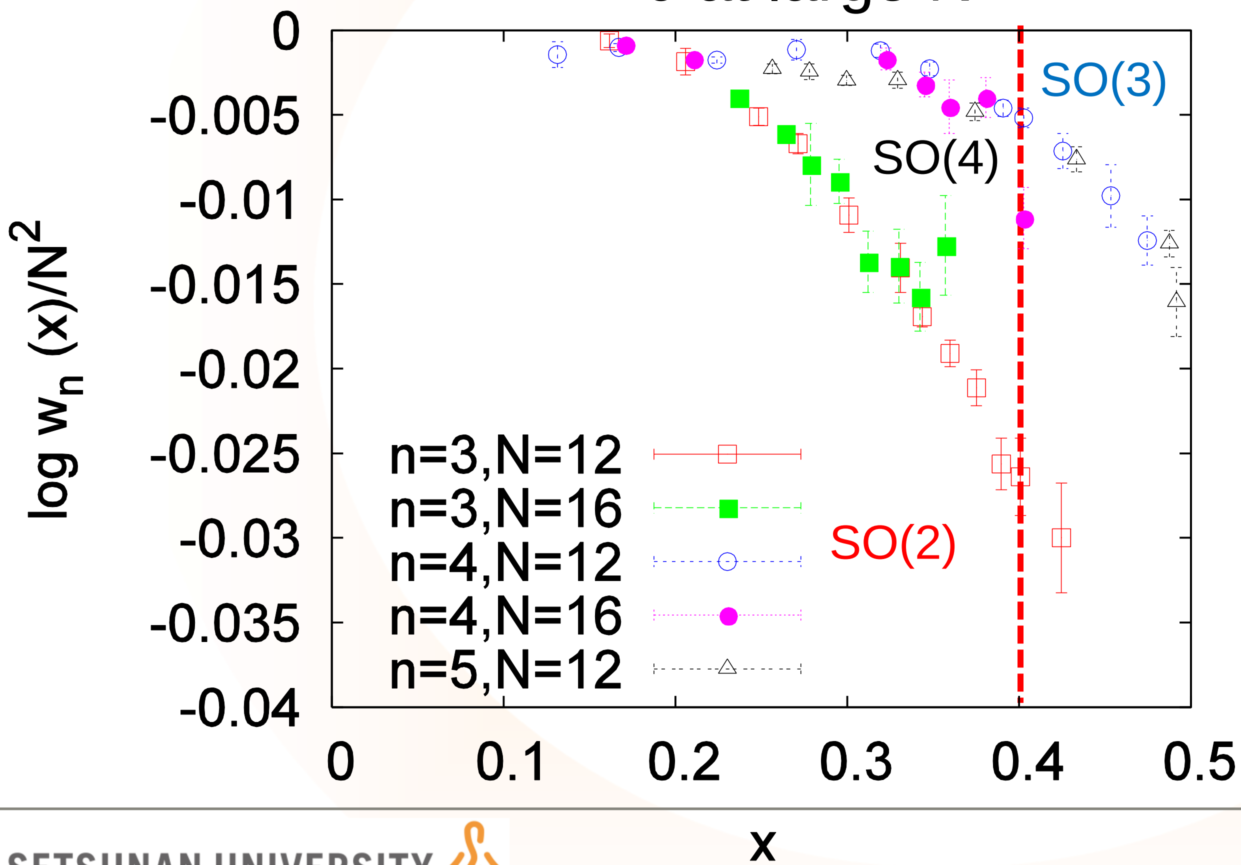
n=4
SO(3)
log w_4 (x)/N^2
x^7
N=6
N=8
N=12
N=16
N=infty
f^(0)_4 (x)/N^2 - g_4(x)/(Nx)|N=6
x
N=24
- Φ'_4(x)
x̄_4
n=5
SO(4)
log w_5 (x)/N^2
x^6
f^(0)_5 (x)/N^2 - g_5(x)/(Nx)|N=6
- Φ'_5(x)
x̄_5

Comparison of the free energy

Free energy for the SO(d) vacuum:

$$\mathscr{F}_{\mathrm{SO}(d)} = \int_{\bar{x}_n}^{1} \frac{1}{N^2} f_n^{(0)}(x)dx - \frac{1}{N^2}\log w_n(\bar{x}_n), \text{ where } n = d+1$$

→0 at large N



The SO(2) vacuum is disfavored.

$$\mathscr{F}_{\mathrm{SO}(3,4)} \ll \mathscr{F}_{\mathrm{SO}(2)}$$

5. Summary

We have studied the dynamical compactification of the spacetime in the Euclidean IKKT model.

Monte Carlo simulation via factorization method
⇒We have obtained the results consistent with GEM:

- Universal compactification scale for SO(2,3,4) vacuum.
- SO(2) vacuum is disfavored.