Monte Carlo studies of the spontaneous rotational symmetry breaking in a matrix model with the complex action

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1. Introduction

Matrix models as a constructive definition of superstring theory

iKKT model (IIB matrix model)

$\Rightarrow$ Promising candidate for constructive definition of superstring theory.

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115.

$$S = N\left(-\frac{1}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2 + \frac{1}{2}\mathrm{tr}\,\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]\right).$$

- A_μ (10d vector) and ψ_α (10d MW spinor) $\Rightarrow$ $N \times N$ matrices .
- Evidences for spontaneous breakdown of SO(10) $\longrightarrow$ SO(4).
  J. Nishimura and F. Sugino, hep-th/0111102,
- Complex fermion determinant:
  - * Crucial for rotational symmetry breaking.
  - * Difficulty of Monte Carlo simulation.

2. Simplified IKKT matrix model

Simplified model with spontaneous rotational symmetry breakdown,
J. Nishimura, hep-th/0108070.

$$S = \underbrace{\frac{N}{2}\mathrm{tr}\,A_\mu^2}_{=S_b} \underbrace{-\bar{\psi}^f_\alpha(\Gamma_\mu)_{\alpha\beta}A_\mu\psi^f_\beta}_{=S_f}$$

- A_μ: $N \times N$ hermitian matrices ($\mu = 1, \cdots, 4$)
  $\bar{\psi}^f_\alpha, \psi^f_\alpha$: N-dim vector ($\alpha = 1, 2, f = 1, \cdots, N_f$),
  $\Rightarrow$ CPU cost $O(N^3)$ (instead of $O(N^6)$ in IKKT)
  N_f = (number of flavors).
- $SO(4)$ rotational symmetry. No supersymmetry.
- Partition function:

$$Z = \int dA e^{-S_B}(\det \mathcal{D})^{N_f} = \int dA e^{-S_0} e^{i\Gamma}, \text{ where}$$
$$\mathcal{D} = \Gamma_\mu A_\mu = (2N \times 2N \text{ matrices}),$$

Phase-quenched one: $Z_0 = \int dA e^{-S_0} = \int dA e^{-S_B} |\det \mathcal{D}|^{N_f}$.

Analytical studies of the model

Gaussian expansion analysis up to 9th order:
T. Okubo, J. Nishimura and F. Sugino, hep-th/0412194.

Observable for probing dimensionality : $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$.

λ_i $(i = 1, 2, 3, 4)$: eigenvalues of $T_{\mu\nu}$ $(\lambda_1 \geqq \lambda_2 \geqq \lambda_3 \geqq \lambda_4)$

Spontaneous breakdown of $SO(4)$ to $SO(2)$ at finite r $\left(=\frac{N_f}{N}\right)$.

3. Monte Carlo simulation

Factorization method

Numerical approach to the complex action problem.
K. N. Anagnostopoulos and J. Nishimura, hep-th/0108041 ,

Distribution function

$$\rho_i(x) \stackrel{\text{def}}{=} \langle \delta(x - \tilde{\lambda}_i)\rangle = \frac{1}{C}\rho^{(0)}_i(x) w_i(x), \text{ where}$$
$$\tilde{\lambda}_i = \lambda_i / \langle \lambda_i \rangle_0, \quad C = \langle \cos\Gamma\rangle_0,$$
$$\rho^{(0)}_i(x) = \langle \delta(x - \tilde{\lambda}_i)\rangle_0, \quad w_i(x) = \langle \cos\Gamma\rangle_{i,x},$$
$$\langle * \rangle_{i,x} = [\text{V.E.V. for the partition function } Z_{i,x}]$$
$$Z_{i,x} = \int dA e^{-S_0}\delta(x - \tilde{\lambda}_i)].$$

The position of the peak x_p for the distribution function $\rho_{i,V}(x)$:

$$0 = \frac{\partial}{\partial x}\log \rho_{i,V}(x) = f^{(0)}_i(x) - \langle \lambda_i\rangle_0 V'(\langle \lambda_i \rangle_0 x), \text{ where}$$
$$f^{(0)}_i(x) \stackrel{\text{def}}{=} \frac{\partial}{\partial x}\log \rho^{(0)}_i(x).$$

Monte Carlo evaluation of $\langle \tilde{\lambda}_i \rangle$

$w_i(x) > 0 \Rightarrow \langle \tilde{\lambda}_i \rangle$ is the minimum of $\mathcal{F}_i(x)$:

$$\mathcal{F}_i(x) = (\text{free energy density}) = -\frac{1}{N^2}\log \rho_i(x).$$

We solve $\mathcal{F}'_i(x) = 0$, namely $\frac{1}{N^2}f^{(0)}_i(x) = -\frac{d}{dx}\left\{\frac{1}{N^2}\log w_i(x)\right\}$.

Both $\frac{1}{N^2}\log w_i(x)$ and $\frac{1}{N^2}f^{(0)}_i(x)$ scale at large N as

$$\frac{1}{N^2}\log w_i(x) \to \Phi_i(x), \qquad \frac{1}{N^2}f^{(0)}_i(x) \to F_i(x).$$

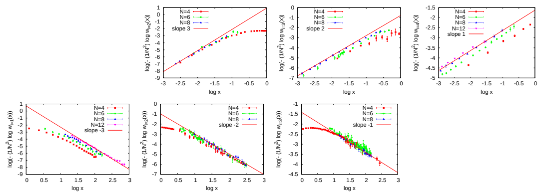
Behavior of $\Phi_i(x)$

Expected power behaviors:

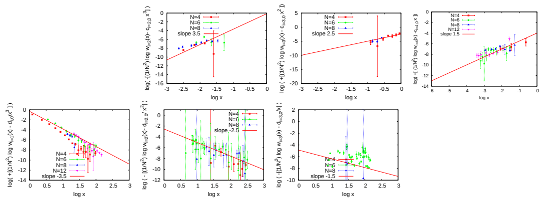
$$\Phi_i(x) \propto \begin{cases} c_{i,0}x^{5-i} + c_{i,1}x^{\frac{11}{2}-i} + \cdots & (x \ll 1, i = 2,3,4) \\ \frac{d_{i,0}}{x^{4-i}} + \frac{d_{i,1}}{x^{\frac{9}{2}-i}} + \cdots & (x \gg 1, i = 1,2,3) \end{cases}$$

Simulation for $r = 1$

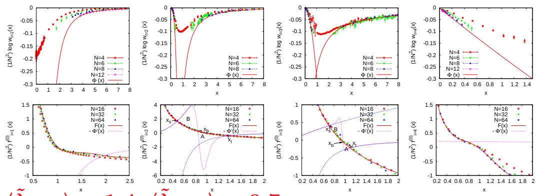
Contribution of the leading order



Contribution of the next-leading order



Evaluation of $\langle \tilde{\lambda}_i \rangle$



$\langle \tilde{\lambda}_{i=2}\rangle = 1.4$, $\langle \tilde{\lambda}_{i=3}\rangle = 0.7$

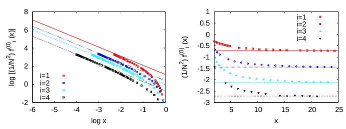
$\Rightarrow$ Rotational symmetry breaking SO(4) $\longrightarrow$ SO(2).

Result of 9th-order Gaussian expansion:

$\tilde{\lambda}_{i=1} \simeq 1.4$, $\tilde{\lambda}_{i=2} \simeq 1.4$, $\tilde{\lambda}_{i=3} \simeq 0.7$, $\tilde{\lambda}_{i=4} \simeq 0.5$.

Behavior of $\frac{1}{N^2}f^{(0)}_i(x)$

Simulation for $r = 1, N = 64$



Small x $(x \ll 1) \longrightarrow (5 - i)$ directions are shrunk.

- $i = 2, 3, 4$: $\rho^{(0)}_i(x) \simeq (\sqrt{x})^{N^2(5-i)}$
  $\Rightarrow \frac{1}{N^2}f^{(0)}_i(x) \simeq \left(\frac{5-i}{2x}\right)$
- $i = 1$: Eigenvalues of A_μ are collapsed to zero.
  $\Rightarrow$ Add the effect of fermionic determinant
  (polynomial of A_μ with degree $2N^2 r$).
  $\Rightarrow \rho^{(0)}_{i=1}(x) = (\sqrt{x})^{2N^2(1+r)} \Rightarrow \frac{1}{N^2}f^{(0)}_i(x) \simeq \left(\frac{2+r}{x}\right)$

$$\log\left(\frac{1}{N^2}f^{(0)}_i(x)\right) = \begin{cases} -\log x + \log(2+r), & i = 1, \\ -\log x + \log\left(\frac{5-i}{2}\right), & i = 2,3,4. \end{cases}$$

Large x $(x \gg 1)$: $\frac{1}{N^2}f^{(0)}_i(x) \stackrel{x\to+\infty}{\longrightarrow}$ (costant)

4. Simulation of IKKT model

6d version of IKKT model
K.N. Anagnostopoulos, T. Aoyama, T.A., M .Hanada and J. Nishimura

Fermion is not vector but adjoint $\Rightarrow$ More CPU cost $O(N^6)$

Supersymmetry $\Rightarrow$ Solution of $\mathcal{F}'_i(x) = 0$ at $x \ll 1$ and $x \gg 1$.

Asymptotic behaviors of $\frac{1}{N^2}\log w_i(x)$ and $\frac{1}{N^2}f^{(0)}_i(x)$ are important.