Monte Carlo studies of the six-dimensional IKKT model

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1. Introduction

Matrix models as a constructive definition of superstring theory

IKKT model (IIB matrix model)
$\Rightarrow$ Promising candidate for constructive definition of superstring theory.
N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115.

- Evidences for spontaneous breakdown of SO(10) $\longrightarrow$ SO(4).
  J. Nishimura and F. Sugino, hep-th/0111102,
- Complex fermion determinant:
  - * Crucial for rotational symmetry breaking.
    J. Nishimura and G. Vernizzi, hep-th/0003223.
  - * Difficulty of Monte Carlo simulation.

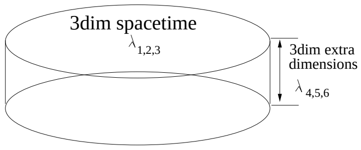
2. 6d IKKT matrix model

Toy model for studying rotational symmetry breaking.

$$S = \underbrace{-\frac{N}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2}_{=S_B} + \underbrace{\frac{N}{2}\mathrm{tr}\,\bar{\psi}_\alpha (\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_F}.$$

- A_μ (6d vector) and ψ (6d Weyl spinor) are $N \times N$ matrices .
- SO(6) rotational symmetry and SU(N) gauge symmetry.
- Presence of $\mathcal{N}=2$ supersymmetry.
- $Z = \int dA e^{-S_B}(\det \mathcal{M}) = \int dA e^{-S_0} e^{i\Gamma}$. CPU cost is $O(N^6)$.
  4d $\longrightarrow$ $\det \mathcal{M}$ is real positive
  6d and 10d $\longrightarrow$ $\det \mathcal{M}$ is complex.
  Complex phase is important in SO(6) breakdown.
- Previous works on this model:
  - * Simulation of phase-quenched 6d and 10d IKKT model
    $\Rightarrow$ no symmetry breakdown of SO(6) (and SO(10)).
    J. Ambjorn et. al., hep-th/0005147
  - * Gaussian Expansion Method
    $\Rightarrow$ symmetry breakdown of SO(6) to SO(3).
    T. Aoyama, J. Nishimura and T. Okubo

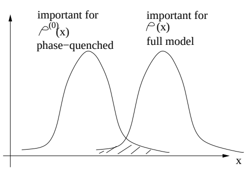
    Observable for probing dimensionality : $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$.
    λ_n $(n = 1, \cdots, 6)$: eigenvalues of $T_{\mu\nu}$ $(\lambda_1 \geqq \lambda_2 \geqq \cdots \geqq \lambda_6)$
    At large N, $\langle \lambda_{1,2,3} \rangle \gg \langle \lambda_{4,5,6} \rangle$
    $\langle \lambda_4 \rangle = 0.18 \longrightarrow$ finite extent of extra dimension.



3. Monte Carlo simulation

Factorization method

Numerical approach to the complex action problem.
K. N. Anagnostopoulos and J. Nishimura, hep-th/0108041 ,



Distribution function

$$\rho_n(x) \stackrel{\rm def}{=} \langle \delta(x - \tilde{\lambda}_n) \rangle = \frac{1}{C}\rho_n^{(0)}(x) w_n(x), \text{ where}$$
$$\tilde{\lambda}_n = \lambda_n / \langle \lambda_n \rangle_0, \quad C = \langle \cos \Gamma \rangle_0,$$
$$\rho_n^{(0)}(x) = \langle \delta(x - \tilde{\lambda}_n) \rangle_0.$$
$$\langle * \rangle_0 = (\text{ V.E.V. for the phase-quenched model } Z_0 = \int dA e^{-S_0}).$$

no symmetry breakdown of SO(6) for $Z_0 \longrightarrow \langle \lambda_1 \rangle_0 = \cdots = \langle \lambda_6 \rangle_0$.
$\tilde{\lambda}_n = \lambda_n / \langle \lambda_n \rangle_0 \neq 1 \longrightarrow$ SO(6)'s breakdown.

$$w_n(x) = \langle \cos \Gamma \rangle_{n,x},$$
$$\langle * \rangle_{n,x} = (\text{V.E.V. for the partition function } Z_{n,x})$$
$$Z_{n,x} = \int dA e^{-S_0}\delta(x - \tilde{\lambda}_n).$$

Resolution of the overlap problem:
The system is forced to visit the configurations where $\rho_n(x)$ is important.

The position of the peak x_p for the distribution function $\rho_{n,V}(x)$:

$$0 = \frac{\partial}{\partial x}\log \rho_{n,V}(x) = f_n^{(0)}(x) - \langle \lambda_n \rangle_0 V'(\langle \lambda_n \rangle_0 x), \text{ where}$$
$$f_n^{(0)}(x) \stackrel{\rm def}{=} \frac{\partial}{\partial x}\log \rho_n^{(0)}(x).$$

Monte Carlo evaluation of $\langle \tilde{\lambda}_n \rangle$

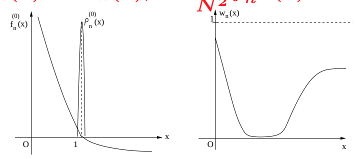
$w_n(x) > 0 \Rightarrow \langle \tilde{\lambda}_n \rangle$ is the minimum of $\mathcal{F}_n(x)$:

$$\mathcal{F}_n(x) = (\text{free energy density}) = -\frac{1}{N^2}\log \rho_n(x).$$

We solve $\mathcal{F}'_n(x) = 0$, namely $\frac{1}{N^2}f_n^{(0)}(x) = -\frac{d}{dx}\left\{\frac{1}{N^2}\log w_n(x)\right\}$.

Both $\frac{1}{N^2}\log w_n(x)$ and $\frac{1}{N^2}f_n^{(0)}(x)$ scale at large N as

$$\frac{1}{N^2}\log w_n(x) \longrightarrow \Phi_n(x), \qquad \frac{1}{N^2}f_n^{(0)}(x) \longrightarrow F_n(x).$$



Behavior of $\frac{1}{N^2}\log w_n(x)$

Expected power behaviors:

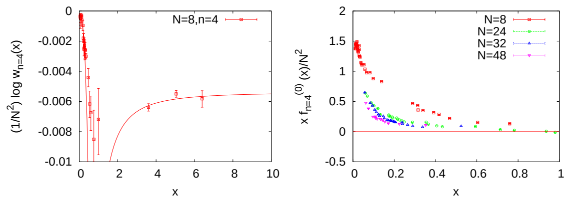
$$\frac{1}{N^2}\log w_n(x) \propto \begin{cases} c_{n,0}x^{7-n} + \cdots & (x \ll 1, n = 2, 3, \cdots, 6) \\ \text{const.} & (x \gg 1, n = 1, 2, \cdots, 5) \end{cases}$$

Behavior of $\frac{1}{N^2}f_n^{(0)}(x)$

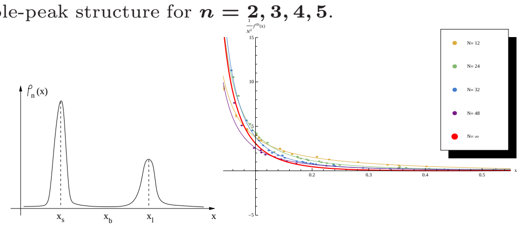
Small $x \Rightarrow (7-n)$ directions are shrunk.

- $n = 2, \cdots, 6$: $\rho_n^{(0)}(x) \simeq (\sqrt{x})^{N^2(7-n)}$
- $n = 1$: Eigenvalues of A_μ are collapsed to zero.
  $\Rightarrow$ Add the effect of fermionic determinant (polynomial of A_μ with degree $4N^2$). $\Rightarrow \rho_{i=1}^{(0)}(x) \simeq (\sqrt{x})^{(6+4)N^2}$

$$\frac{1}{N^2}f_n^{(0)}(x) = \begin{cases} \frac{7-n+4\delta_{n,1}}{2x} & (x \ll 1) \\ 0 & (x \gg 1) \end{cases}$$



Double-peak structure for $n = 2, 3, 4, 5$.



Symmetry breakdown of SO(6) to SO(3) in Gaussian Expansion Method.
x_s for $n = 4 \Rightarrow$ extent of extra dimension.

$\frac{1}{N^2}f_n^{(0)}(x)$ scales at small x in 6d full IKKT model.
(in contrast to $\frac{1}{N}f_n^{(0)}(x)$'s scaling in 6d one-loop model).
$\Rightarrow$ hard-core potential at large N (x_s is finite).
Consistent with Gaussian Expansion Method.

4. Conclusion

Monte Carlo simulation of 6d IKKT model.
Can we understand the emergence of spacetime?

- Extent of the extra dimension $\longrightarrow$ finite.
  $\Rightarrow$ Consistent with Gaussian Expansion Method.

Future works:

- Analysis of large-x regime.
- Comparison of the free energy.