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# Emergence of an expanding (3+1)-dimensional spacetime in the type IIB matrix model

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Challenges and opportunities in Lattice QCD simulations and related fields, Feb 15th 2023, 15:10～15:40

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# 1. Introduction

Type IIB matrix model (a.k.a. IKKT model)
⇒Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115]

$$\boxed{S = \underbrace{\frac{-N}{4}\mathrm{tr}[A_\mu, A_\nu]^2}_{=S_\mathrm{b}} + \underbrace{\frac{-N}{2}\mathrm{tr}\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_\mathrm{f}}}$$

- Dimensional reduction of the D=10 super-Yang-Mills theory to 0 dimension
- $A_\mu(\mu=0,1,\ldots,9)$, $\Psi_\alpha$ ($\alpha=1,2,\ldots,16$ after Weyl projection)
  ⇒N × N Hermitian traceless matrices.
- N=2 supersymmetry ⇒ eigenvalues of $A_\mu$ are interpreted as the spacetime coordinates.

How does our (3+1)-dim spacetime emerge dynamically?

# 2. Lorentzian type IIB matrix model

## Sign problem of the type IIB matrix model

Euclidean version : Wick rotation $A_{10} = -iA_0,\ \Gamma_{10} = i\Gamma^0$
contracted by the Euclidean metric $\delta_{\mu\nu} = \mathrm{diag}(1, 1, 1, \cdots, 1)$

SSB SO(10)→SO(3)
(3dim space*time*)

$$Z = \int dA \left( \underset{\text{real}}{\boxed{e^{-S_{\mathrm{b}}}}} \boxed{\underbrace{\underset{\text{complex}}{\int d\psi e^{-S_{\mathrm{f}}}}}_{=\mathrm{Pf}\ \mathcal{M}}} \right)$$

[K.N. Anagnostopoulos, T. Azuma, Y. Ito J. Nishimura, T. Okubo and S.K. Papadoudis arXiv:2002.07410]

$\mathcal{M}$ is a $16(N^2-1) \times 16(N^2-1)$ sparse matrix

Lorentzian version : no Wick rotation
contracted by the Lorentzian metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, \cdots, 1)$

$$Z = \int dA \left( \underset{\text{complex}}{\boxed{e^{iS_{\mathrm{b}}}}} \boxed{\underbrace{\underset{\text{real}}{\int d\psi e^{iS_{\mathrm{f}}}}}_{=\mathrm{Pf}\ \mathcal{M}}} \right)$$

[J. Nishimura and A. Tsuchiya, arXiv:1904.05919]

We employ the Complex Langevin Method (CLM)

# 2. Lorentzian type IIB matrix model

Time development of the Lorentzian version:

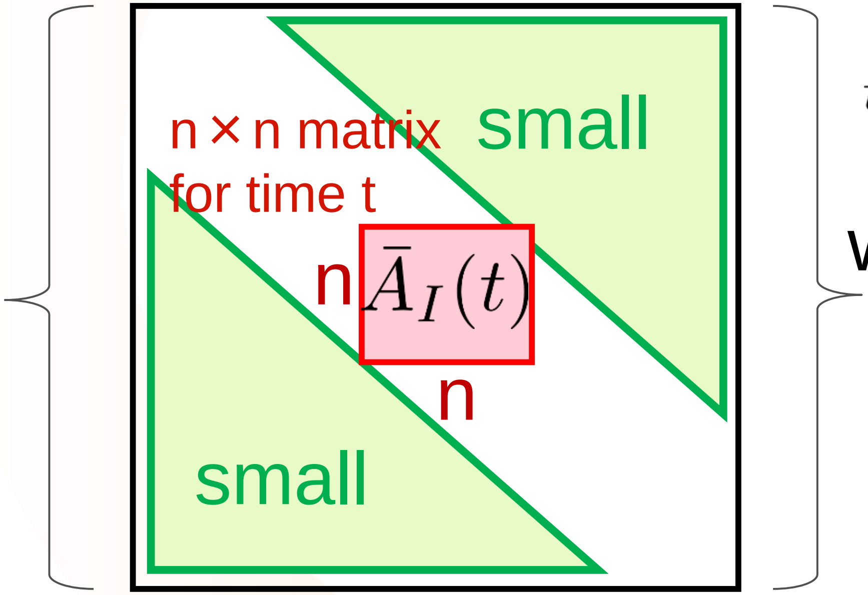
⇒gauge fixing to diagonalize $A_0$

[S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

$A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N)$, where $\alpha_1 < \alpha_2 < \cdots < \alpha_N$.

Band-diagonal structure of $A_I$

$A_I =$



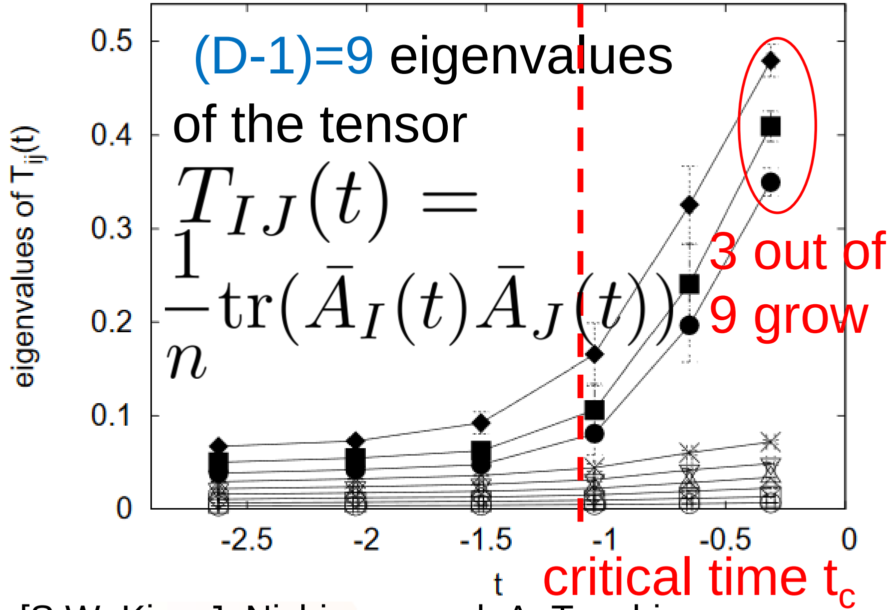
$$t_\nu = \sum_{k=1}^{\nu} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|,$$

where $$\bar{\alpha}_k = \frac{1}{n} \sum_{i=1}^{n} \alpha_{k+i}$$

# 2. Lorentzian type IIB matrix model

## Results with the approximation to avoid the sign problem

SUSY (include $S_f$) D=10, N=16, n=4, HMC (not CLM)



(D-1)=9 eigenvalues of the tensor

$$T_{IJ}(t) = \frac{1}{n}\mathrm{tr}(\bar{A}_I(t)\bar{A}_J(t))$$

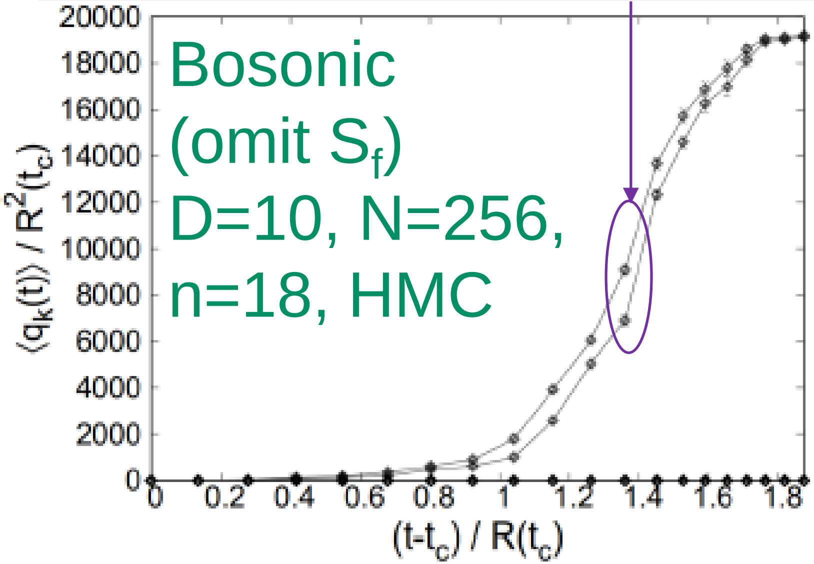
3 out of 9 grow

critical time $t_c$

[S.W. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540]

**Dynamical emergence of (3+1)-dim spacetime.**

2 of the n eigenvalues of $Q(t) = \sum_{I=1}^{9}(\bar{A}_I(t))^2$ grow.



Bosonic (omit $S_f$) D=10, N=256, n=18, HMC

[T. Aoki, M. Hirasawa, Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1904.05914]

$$\bar{A}_I(t) \propto \sigma_I \oplus 0_{n-2} \quad (I=1,2,3)$$

**Pauli-matrix space structure**

# 2. Lorentzian type IIB matrix model

## Equivalence between the Lorentzian and Euclidean version

[Y. Asano, private communication]

Contour deformation $A_0 = e^{-3i\pi u/8}\tilde{A}_0,\ \ A_I = e^{i\pi u/8}\tilde{A}_I$

$$S_{\rm b} \to \tilde{S}_{\rm b} = \boxed{e^{i\pi u/2}} \left\{ \frac{-N}{4}\mathrm{tr}[\tilde{A}_I, \tilde{A}_J]^2 + \frac{N}{2}\boxed{e^{-i\pi u}}\mathrm{tr}[\tilde{A}_0, \tilde{A}_I]^2 \right\}$$

w.r.t. worldsheet

w.r.t. target space

u=0: Lorentzian, u=1: Euclidean

$$e^{iS_{\rm b}(A)} = e^{-S(\tilde{A})}, \quad S(\tilde{A}) = -\frac{N}{4}\boxed{e^{-i\pi(1-u)/2}}\mathrm{tr}[\tilde{A}_I, \tilde{A}_J]^2 - \frac{N}{2}\boxed{e^{i\pi(1-u)/2}}\mathrm{tr}[\tilde{A}_0, \tilde{A}_I]^2$$

real part is positive (0<u≦1).

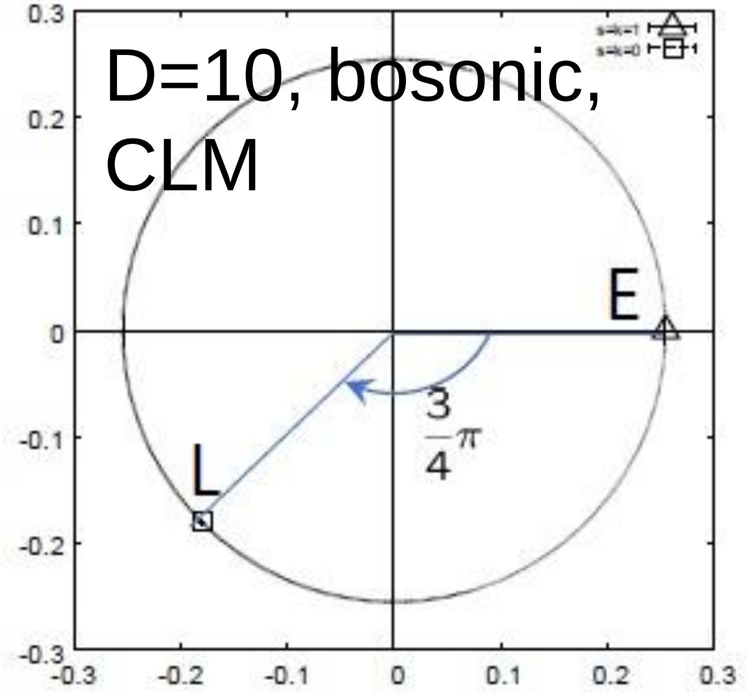
Cauchy's theorem:

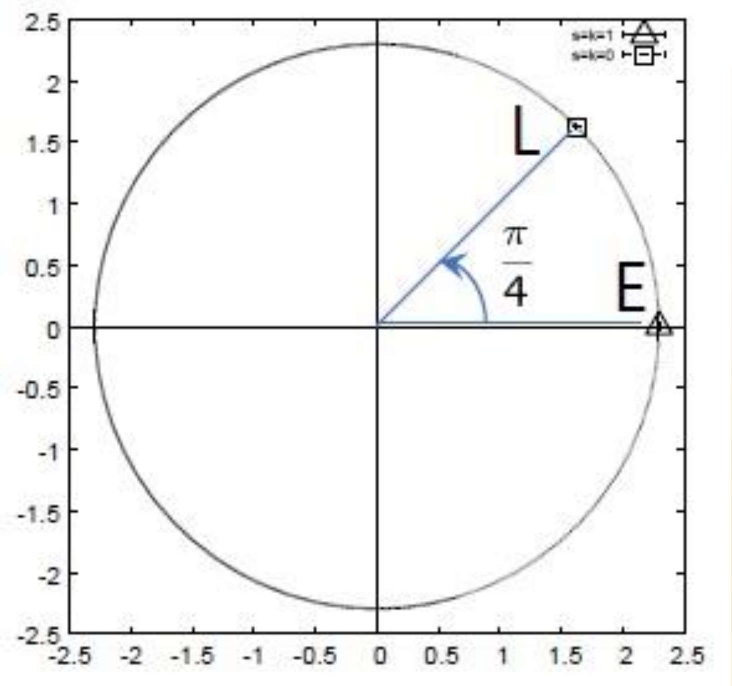
$\langle \mathcal{O}(e^{-3i\pi u/8}\tilde{A}_0, e^{i\pi u/8}\tilde{A}_I)\rangle_u$ is independent of u.

# 2. Lorentzian type IIB matrix model

$$\left\langle \frac{1}{N}\mathrm{tr}(A_0^{(L)})^2 \right\rangle = e^{-3i\pi/4} \left\langle \frac{1}{N}\mathrm{tr}(A_0^{(E)})^2 \right\rangle$$

$$\left\langle \frac{1}{N}\mathrm{tr}(A_I^{(L)})^2 \right\rangle = e^{i\pi/4} \left\langle \frac{1}{N}\mathrm{tr}(A_I^{(E)})^2 \right\rangle$$


D=10, bosonic, CLM



Equivalence of the Euclidean (u=1) and Lorentzian (u→+0) model.

⇒The spacetime is Euclidean.

[K. Hatakeyama, et. al. , arXiv:2112.15368]

# 2. Lorentzian type IIB matrix model

## Adding the Lorentzian mass term

$$\boxed{Z = \int dA d\psi e^{i(S+S_\gamma)}, \; S_\gamma = \frac{-N\gamma}{2}\mathrm{tr}(A^\mu A_\mu) = \frac{N\gamma}{2}\{\mathrm{tr}(A_0)^2 - \mathrm{tr}(A_I)^2\}}$$

$$e^{iS_\gamma(A)} = e^{-S_\gamma(\tilde{A})}, \quad S_\gamma(\tilde{A}) = \frac{N\gamma}{2}\{\boxed{e^{-i\pi(2+3u)/4}}\mathrm{tr}(\tilde{A}_0)^2 + \boxed{e^{i\pi(2+u)/4}}\mathrm{tr}(\tilde{A}_I)^2\}$$

real part is negative (0<u≦1).

At γ>0, we cannot define the model by contour deformation

⇒Equivalence to the Euclidean model (u=1) is violated.

We consider the limit N→∞ ⇒γ→0.

# 3. Complex Langevin Method

## Complex Langevin Method (CLM)

⇒Promising method to solve complex-action systems.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

- Introducing $\boxed{\tau_a}$

⇒time order $\alpha_1<\alpha_2<\ldots<\alpha_N$ for complexified $\alpha_i$ (k=2,・・・,N)

$$\boxed{\alpha_1 = 0,\ \alpha_k = \sum_{i=1}^{k-1} e^{\tau_i}}$$

$$\boxed{A_0 = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N)}$$

- Mass term to avoid the near-zero modes of the Dirac operator: $\mathrm{Pf}\ \mathcal{M}_{m_\mathrm{f}} = \int d\psi e^{iS_{m_\mathrm{f}}},$

$$S_{m_\mathrm{f}} = \frac{-N}{2}\mathrm{tr}\{\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta] + \boxed{m_\mathrm{f}\bar{\psi}_\alpha(\Gamma^7\Gamma^{8\dagger}\Gamma^9)_{\alpha\beta}\psi_\beta\}}$$

**Effective action** $Z = \int dA e^{-S_\mathrm{eff}},$

$$S_\mathrm{eff} = -i(S_\mathrm{b} + S_\gamma) - \log \mathrm{Pf}\ \mathcal{M}_{m_\mathrm{f}} \boxed{- \log \prod_{1\le a<b\le N} (\alpha_a - \alpha_b)^2} \boxed{- \sum_{a=1}^{N-1} \tau_a}$$

# 3. Complex Langevin Method

$$\frac{d(A_I)_{ab}(\sigma)}{d\underset{\text{fictitious Langevin time}}{\boxed{\sigma}}} = -\overset{\text{drift term}}{\boxed{\frac{dS_{\text{eff}}}{d(A_I)_{ba}}}} + \overset{\text{Hermitian-matrix white noise}}{\boxed{\eta_{I,ab}(\sigma)}}, \quad \frac{d\tau_a(\sigma)}{d\underset{\text{fictitious Langevin time}}{\boxed{\sigma}}} = -\overset{\text{drift term}}{\boxed{\frac{dS_{\text{eff}}}{d\tau_a}}} + \overset{\text{real-number white noise}}{\boxed{\eta_a(\sigma)}}$$

- $A_I$ : Hermitian→general complex traceless matrices.
- $\tau_a$ : Real number → complex number.

The drift term involves $\frac{d}{d(A_I)_{ba}}\{-\log \text{Pf}\, \mathcal{M}_{m_f}\} = -\frac{1}{2}\text{Tr}\left(\frac{d\mathcal{M}_{m_f}}{d(A_I)_{ba}}\mathcal{M}_{m_f}^{-1}\right)$

- $\mathcal{M}_{m_f}$'s near-zero modes⇒singular drift problem.
- We use conjugate gradient (CG) method and noisy estimator.

Large-scale numerical simulation using supercomputers.

# 3. Complex Langevin Method

The condition to justify the CLM: [K. Nagata, J. Nishimura and S. Shimasaki, arXiv:1606.07627]

The probability distribution of the drift norms falls exponentially or faster.

$$u_A = \sqrt{\frac{1}{9N^3}\sum_{I=1}^{9}\sum_{a,b=1}^{N}\left|\frac{dS_{\text{eff}}}{d(A_I)_{ba}}\right|^2},\quad u_\alpha = \sqrt{\frac{1}{N}\sum_{a=1}^{N-1}\left|\frac{dS_{\text{eff}}}{d\tau_a}\right|^2}$$

Look at the drift terms⇒Get the drift of the CLM.

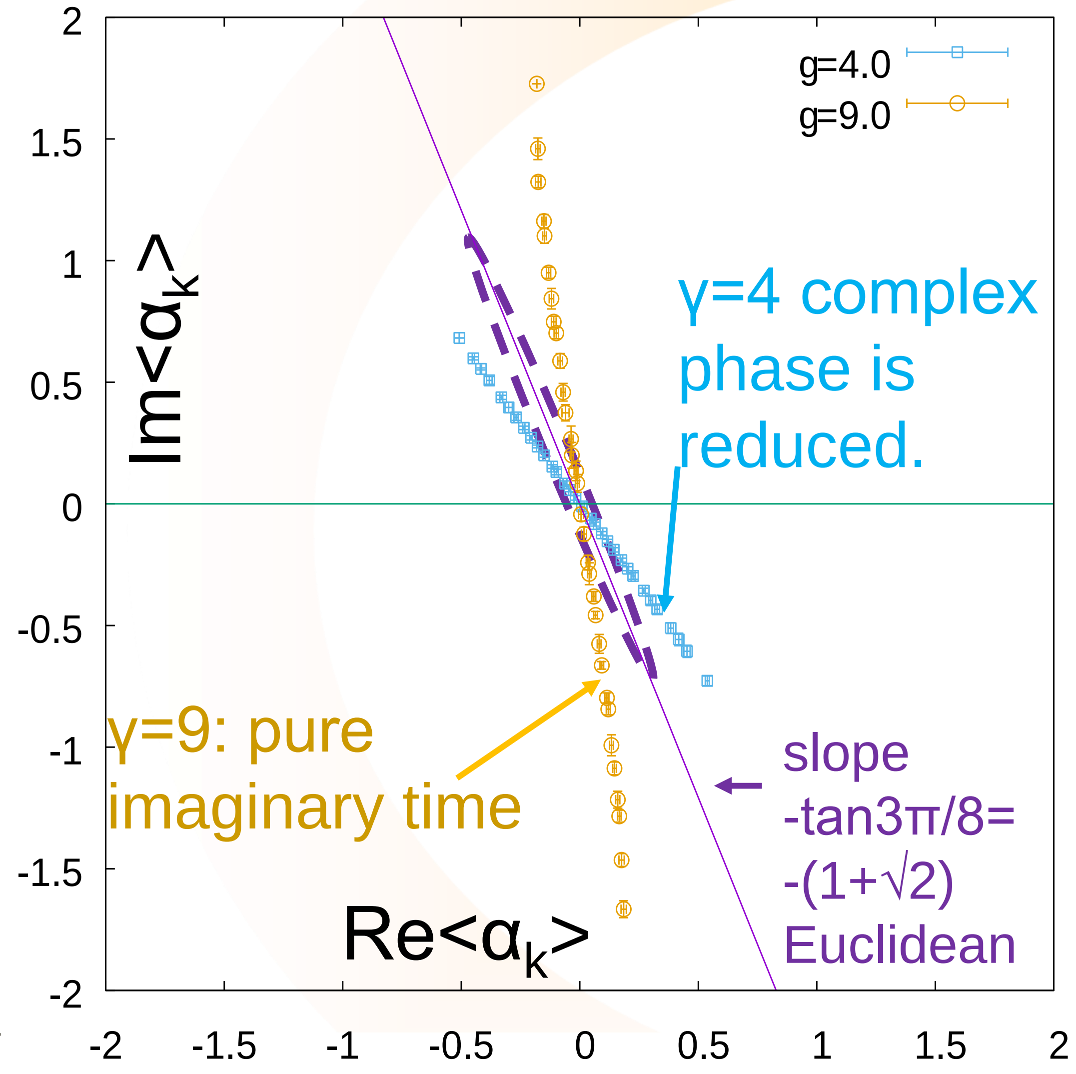
Dynamical stabilization: [F. Attanasio and B. Jäger arXiv:1808.04400]

After each Langevin step, $A_I \to \frac{A_I + \eta A_I^\dagger}{1+\eta}$ Here, η=0.01.

(η=0: do nothing, η=1:Hermitize completely)

# 4. Result

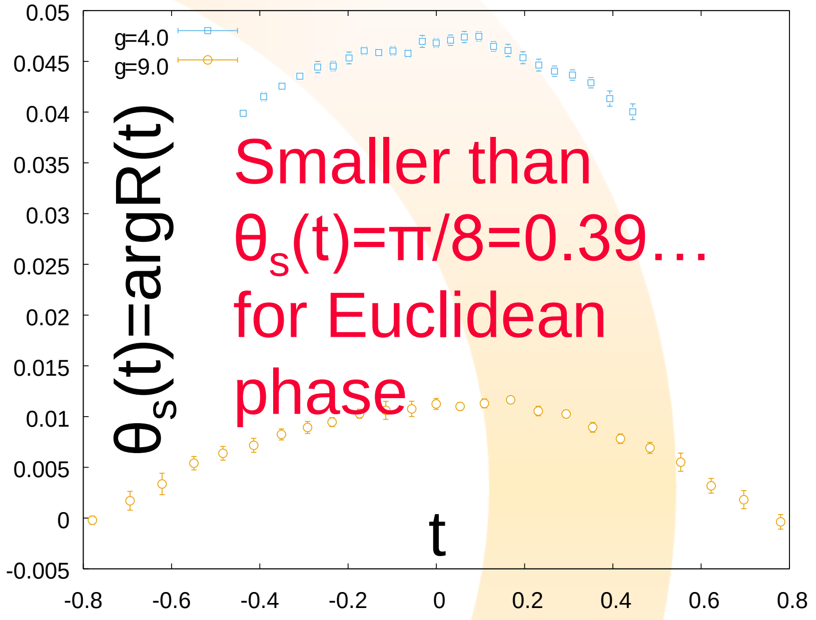
## SUSY (include $S_f$), N=32, n=8, u=0, $m_f$=3.5 (preliminary)



γ=4 complex phase is reduced.

γ=9: pure imaginary time

slope -tan3π/8= -(1+√2) Euclidean



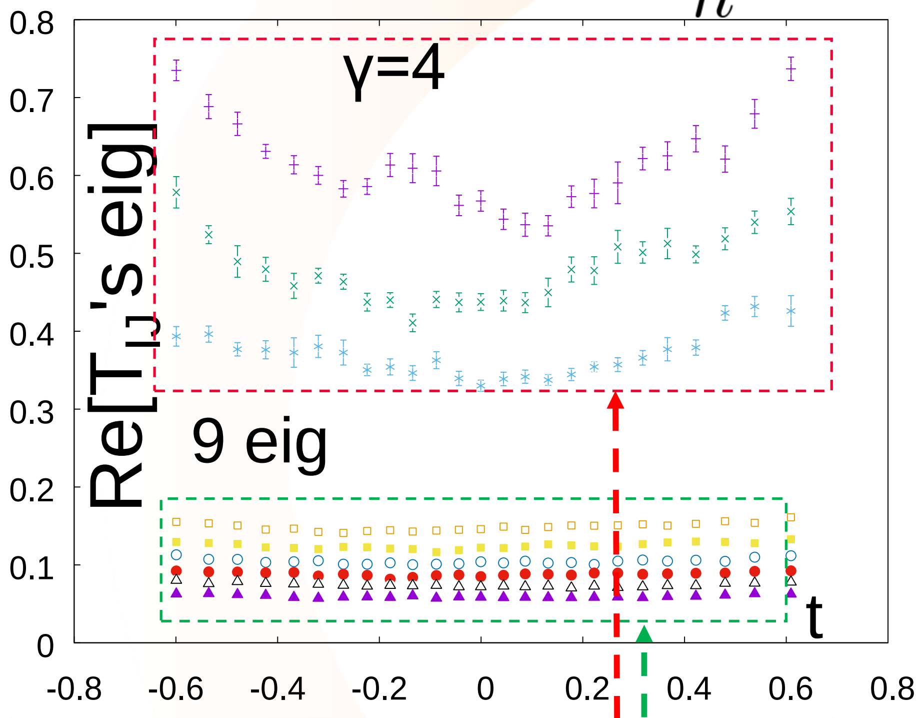
Smaller than $\theta_s(t)=\pi/8=0.39\ldots$ for Euclidean phase

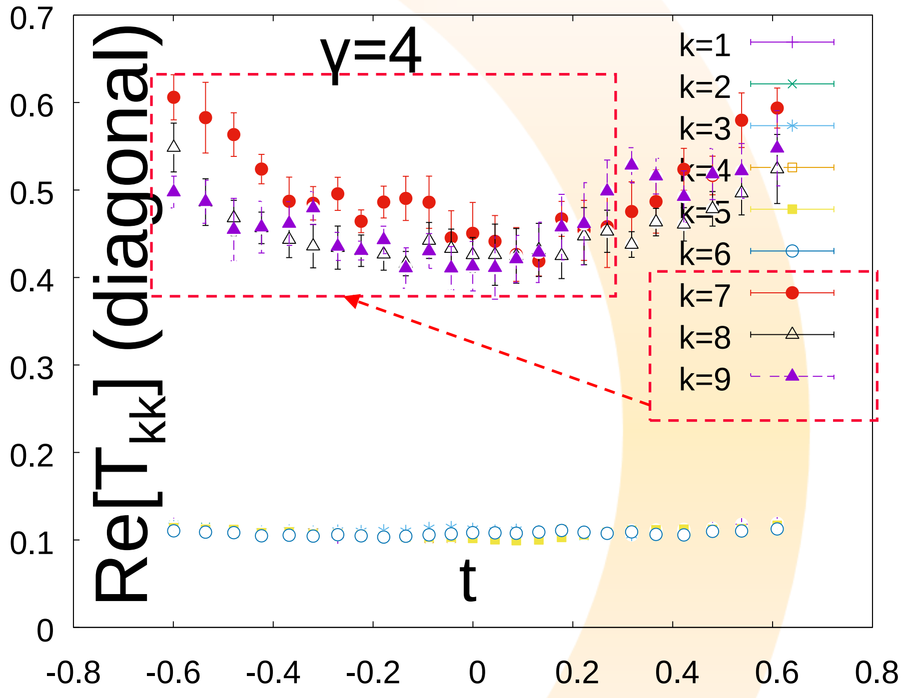
$$R^2(t) = \left\langle \frac{1}{n} \sum_{I=1}^{9} \mathrm{tr}\left(\bar{A}_I(t)\right)^2 \right\rangle$$

# 4. Result

SUSY (include $S_f$), N=32, n=8, u=0, $m_f$=3.5 (preliminary)

9 × 9 tensor $T_{IJ} = \frac{1}{n}\mathrm{tr}(\bar{A}_I(t)\bar{A}_J(t))$





expanding 3dim space(similarly for γ=9)

shrunken 6dim space

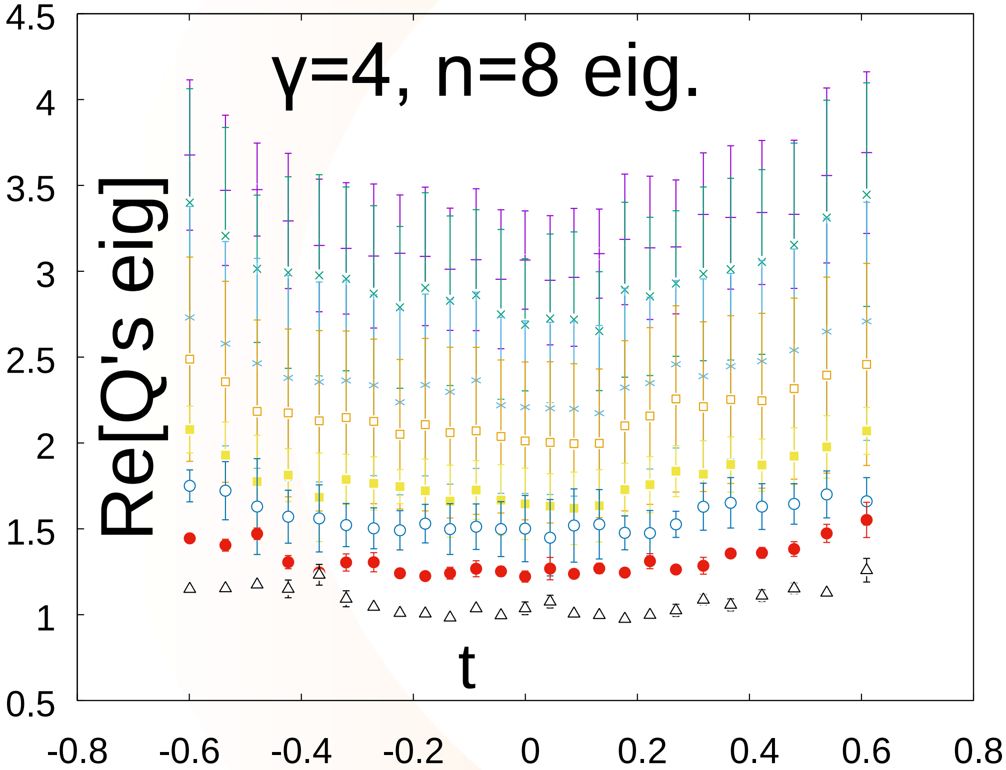
effect of $m_f\bar{\psi}_\alpha(\Gamma^7\Gamma^{8\dagger}\Gamma^9)_{\alpha\beta}\psi_\beta$

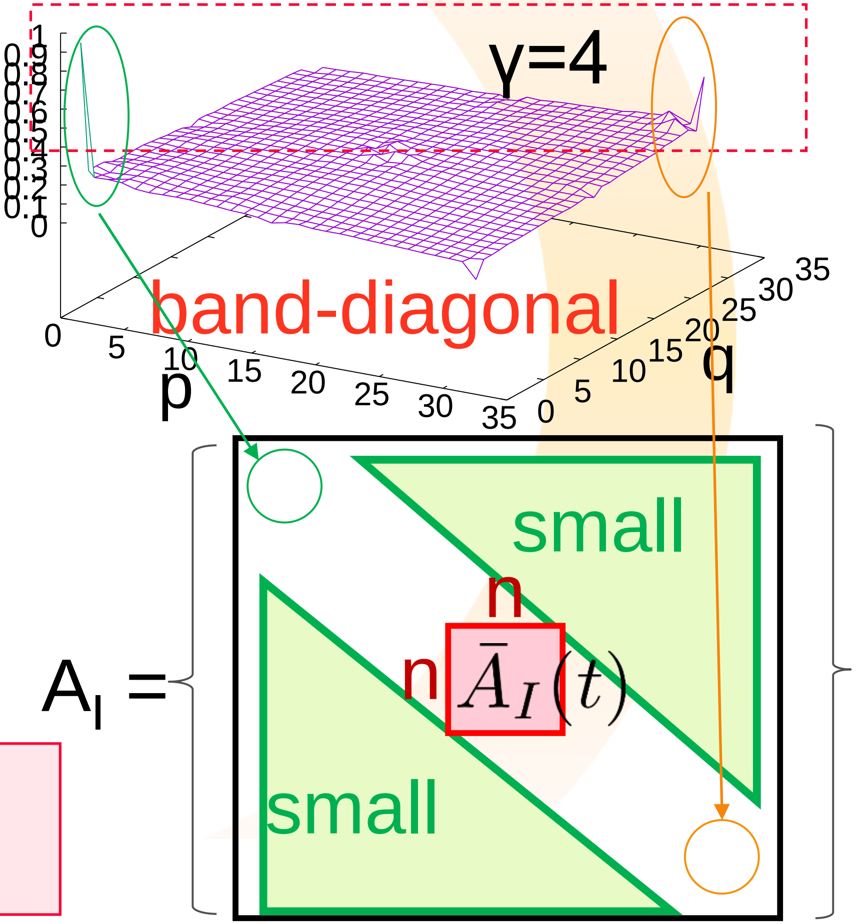
# 4. Result

SUSY (include $S_f$), N=32, n=8, u=0, $m_f$=3.5 (preliminary)

n × n matrix $Q(t)=\sum_{I=1}^{9}(\bar{A}_I(t))^2$

$$A_{pq}=\sum_{I=1}^{9}|(A_I)_{pq}|^2$$





departure from Paulian structure (similarly for γ=9)

# 5. Conclusion

Complex Langevin Method (CLM) for the type IIB matrix model.

Equivalence of the Euclidean and Lorentzian model without the Lorentzian mass term.

Introduce a Lorentzian mass term
⇒emergence of 3dim space.

At larger N, and smaller γ and $m_f$:

Can we observe the transition from Euclidean to Lorentzian geometry?

Large-scale numerical simulation at larger N is important.