Destabilization of two fuzzy spheres at a distance

Takehiro Azuma (High Energy Accelerator Research Organization)
e-mail:azumat@post.kek.jp

Superstring theory is now regarded as the most powerful candidate for the quantum theory of gravity. In the late 1990's, several large-N reduced models have been proposed as the constructive definition (nonperturbative formulation) of superstring theory. Especially, the IIB matrix model[1] is regarded as one of the most promising candidates. Superstring theory requires the spacetime to be ten-dimensional due to a theoretical consistency, and it is an important question how superstring theory dynamically generates the four-dimensional spacetime and the gauge group of the Standard Model.

To this end, fuzzy spheres have been extensively studied in the context of large-N reduced models, as a simplified prototype of the spacetime. We have first studied the following simplest toy model

$$S = N\mathrm{tr}\left(-\frac{1}{4}\sum_{\mu,\nu=1}^{3}[A_\mu, A_\nu]^2 + \frac{2i\alpha}{3}\sum_{\mu,\nu,\rho=1}^{3}\epsilon_{\mu\nu\rho}A_\mu A_\nu A_\rho \right),$$

which is defined in the three-dimensional Euclidean spacetime and has the fuzzy S^2 classical solution. Here, we have focused on the dynamics of the two-fuzzy-sphere system at a distance which consists of two irreducible blocks $X_\mu^{(1)}$ and $X_\mu^{(2)}$ of size n_1 and n_2 ($n_1 + n_2 = N$), respectively:

$$X_\mu = \begin{pmatrix} X_\mu^{(1)} & \\ & X_\mu^{(2)} \end{pmatrix},$$

$$[X_\mu^{(1)}, X_\nu^{(1)}] = i\alpha\epsilon_{\mu\nu\lambda}(X_\lambda^{(1)} - R_\lambda^{(1)}), \quad [X_\mu^{(2)}, X_\nu^{(2)}] = i\alpha\epsilon_{\mu\nu\lambda}(X_\lambda^{(2)} - R_\lambda^{(2)}).$$

This represents the two fuzzy spheres with centers respectively at $r_\mu^{(1)} = \frac{1}{n_1}\mathrm{tr}\,(X_\mu^{(1)}) = \frac{1}{n_1}\mathrm{tr}\,(R_\mu^{(1)})$ and $r_\mu^{(2)} = \frac{1}{n_2}\mathrm{tr}\,(X_\mu^{(2)}) = \frac{1}{n_2}\mathrm{tr}\,(R_\mu^{(2)})$. This system has been once studied in ref. [2], and in this talk[3] we have extended this previous analysis. We have found that the two-fuzzy-sphere system retains metastability if one sphere is well inside the other or when they are well separated, while the intersecting fuzzy spheres are unstable.

We have also extended our analysis to the supersymmetric three-dimensional model and a d-dimensional model.

This talk is based on the collaboration with Subrata Bal and Jun Nishimura.

References

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[2] S. Bal and H. Takata, *"Interaction between two fuzzy spheres,"* Int. J. Mod. Phys. A **17**, 2445 (2002) [hep-th/0108002].

[3] T. Azuma, S. Bal and J. Nishimura, *"Destabilization of two fuzzy spheres at a distance"*, in preparation.