Non-lattice simulation for supersymmetric gauge theories in one dimension

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Takehiro Azuma,[1] Jul. 13th 2007, 16:00 ∼ 17:00

Contents

[1] This slide is used for Takehiro Azuma's presentation in the Tea-duality Seminar at TIFR. It is not the authors but *the speaker Takehiro Azuma* that is responsible for any flaw in this slide.

1 Introduction

Matrix models as a constructive definition of superstring theory

IKKT model (IIB matrix model) ⇒ Promising candidate for the constructive definition of superstring theory.

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115.

$$S = \frac{1}{g^2}\left(-\frac{1}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2 + \frac{1}{2}\mathrm{tr}\,\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right).$$

- **Dimensional reduction of $\mathcal{N} = 1$ 10d Super-Yang-Mills (SYM) theory to 0d.**
  A_μ (10d vector) and ψ (10d Majorana-Weyl spinor) are $N \times N$ matrices.
  Eigenvalues of A_μ ⇒ spacetime coordinate.
- **Matrix regularization of Green-Schwarz action of type IIB superstring theory.**
- **$\mathcal{N} = 2$ supersymmetry in 10 dimensions.**
- **Matrices describe the many-body system.**

- **No free parameters:** $A_\mu \to g^{\frac{1}{2}} A_\mu$, $\psi \to g^{\frac{3}{4}} \psi$.

- **Evidences for spontaneous breakdown of SO(10) symmetry to SO(4).**
  J. Nishimura and F. Sugino, hep-th/0111102, H. Kawai, et. al. hep-th/0204240,0211272,0602044,0603146.

- **Complex action (after integrating out fermions) :**

  * **Crucial for spontaneous breakdown of rotational symmetry:**
    J. Nishimura and G. Vernizzi, hep-th/0003223.

  * **Difficulty of Monte Carlo simulation**

Fuzzy sphere studies in large-N reduced models

- **Dynamical generation of spacetime and gauge group**

- **New regularization scheme alternative to lattice regularization**

2 Simulation of 0-dim supersymmetric matrix model

4d supersymmetric Yang-Mills-Chern-Simons (YMCS) model:

K.N. Anagnostopoulos, T. Azuma, K. Nagao and J. Nishimura, hep-th/0506062.

$$S = \underbrace{-\frac{N}{4}\mathrm{tr} \sum_{\mu,\nu=1}^{4} [A_\mu, A_\nu]^2 + \frac{2iN\alpha}{3} \sum_{i,j,k=1}^{3} \epsilon_{ijk} A_i A_j A_k}_{=S_B} + \underbrace{N\bar{\psi}_\alpha \sum_{\mu=1}^{4} (\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_F}.$$

- A_μ (ψ_α): $N \times N$ traceless hermitian (complex) matrices.

$$\Gamma_1 = \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \Gamma_2 = \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \Gamma_3 = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ \Gamma_4 = i\mathbf{1}_{2\times 2} = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}.$$

- $\alpha \neq 0 \Rightarrow$ SO(4) is broken to SO(3). Supersymmetry is softly broken.
- 3d (4d) model $\Rightarrow$ Partition function is divergent (convergent).

  P. Austing and J. Wheater, hep-th/0310170

- Classical equation of motion:

$$[A_\nu, [A_\nu, A_\mu]] + i\alpha\epsilon_{\mu\nu\rho}[A_\nu, A_\rho] = 0, \text{ where } \epsilon_{\mu\nu\rho} = 0 \text{ (if any one of } \mu, \nu, \rho \text{ is 4).}$$

Fuzzy S^2 classical solution:

$$A_\mu^{(S^2)} = \begin{cases} \oplus_{I=1}^{s} \alpha (L_\mu^{(n_I)} \otimes 1_{k_I}), & (\text{for } \mu = 1,2,3), \\ 0, & (\text{for } \mu = 4), \end{cases} \quad (\sum_{I=1}^{s} n_I k_I = N).$$

Partition function of the model

Introduce a complete basis for general complex $N \times N$ matrices:

$$t^a = E_{i_a j_a}, \text{ where } a = (i_a - 1)N + j_a, \ E_{i_a j_a} = \begin{cases} 1, & (i_a, j_a) \text{ component} \\ 0, & \text{otherwise} \end{cases}$$

Decomposition with respect to this basis:

$$(\psi_\alpha)_{i_a j_a} = \sum_{a=1}^{N^2} (\psi_{a,\alpha})(t^a)_{i_a j_a}, \ (\bar{\psi}_\alpha)_{i_a j_a} = \sum_{a=1}^{N^2} (\bar{\psi}_{a,\alpha})(t^a)_{i_a j_a}.$$

Tracelessness condition:

$$\underbrace{\psi_{1,\alpha}}_{(i_a,j_a)=(1,1)} + \underbrace{\psi_{N+2,\alpha}}_{(i_a,j_a)=(2,2)} + \cdots + \underbrace{\psi_{N^2,\alpha}}_{(i_a,j_a)=(N,N)} = 0, \ \psi_{1,\alpha} + \psi_{N+2,\alpha} + \cdots + \psi_{N^2,\alpha} = 0.$$

Integrate out $\psi_{N^2,\alpha}$, $\bar{\psi}_{N^2,\alpha}$:

$$\mathrm{tr}\,(N\bar{\psi}_{\alpha}(\Gamma_{\mu})_{\alpha\beta}[A_{\mu},\psi_{\beta}]) = \sum_{a=1}^{N^2}\sum_{b=1}^{N^2}\sum_{\alpha,\beta=1}^{2}\bar{\psi}_{a,\alpha}\mathcal{M}'_{a\alpha,b\beta}\psi_{b,\beta} = \sum_{a=1}^{N^2-1}\sum_{b=1}^{N^2-1}\sum_{\alpha,\beta=1}^{2}\bar{\psi}_{a,\alpha}\mathcal{M}_{a\alpha,b\beta}\psi_{b,\beta}, \text{ where}$$

$$\mathcal{M}'_{a\alpha,b\beta} = N(\Gamma_{\mu})_{\alpha\beta}\mathrm{tr}\,(t^{a}[A_{\mu},t^{b}]) = N(\delta_{i_a j_b}(A_{\mu})_{j_a i_b} - \delta_{i_b j_a}(A_{\mu})_{j_b i_a}),$$

$$\mathcal{M}_{a\alpha,b\beta} = \mathcal{M}'_{a\alpha,b\beta} - \delta_{i_a j_a}\mathcal{M}'_{N^2\alpha,b\beta} - \delta_{i_b j_b}\mathcal{M}'_{a\alpha,N^2\beta}.$$

Partition function of the model:

$$Z = \int dA e^{-S_{\text{eff}}}, \text{ where } S_{\text{eff}} = S_B - \log\det\mathcal{M}.$$

In 4d SUSY model, det $\mathcal{M}$ is real positive.

(Proof) From the structure of gamma matrices, $\sigma_2\Gamma_{\mu}\sigma_2 = (\Gamma_{\mu})^{*}$. Thus, $\sigma_2\mathcal{M}\sigma_2 = \mathcal{M}^{*}$.

$v_{a\alpha}$: eigenvector of M satisfying $\mathcal{M}v_{a\alpha} = \lambda v_{a\alpha}$.

$w_{a\alpha} = (\sigma_2)_{\alpha\beta}(v_{a\beta})^{*}$ is also an eigenvector:

$$\mathcal{M}w = \sigma_2^2\mathcal{M}\sigma_2 v^{*} = \sigma_2\mathcal{M}^{*}v^{*} = \sigma_2\lambda^{*}v^{*} = \lambda^{*}w.$$

The eigenvalue λ and λ^{*} always comes in pair. Thus, det $\mathcal{M}$ is real positive. (Q.E.D.)

Algorithm of Hybrid Monte Carlo (HMC) simulation

Hybrid Monte Carlo simulation ⇒ standard technique to incorporate fermions.

P_μ : **(auxiliary bosonic hermitian matrix → conjugate momentum of** A_μ**)**

$$S_{\rm HMC}[P,A] = \frac{1}{2}{\rm tr}\,(P_\mu^2) + S_{\rm eff}[A].$$

1. **Update** $P_\mu(\tau=0)$ **with a Gaussian random number.**

   Inherit $A_\mu(\tau=0)$ **from the previous sweep.**

   τ: **fictitious time of the classical system** $(0 \leq \tau \leq T)$.

2. **Solve the Hamiltonian equation of motion. CPU power for** $\mathcal{M}^{-1} \Rightarrow \mathcal{O}(N^6)$.

$$\frac{d(A_\mu)_{ij}}{d\tau} = \frac{\partial S_{\rm HMC}}{\partial (P_\mu)_{ij}} = (P_\mu)_{ji},$$
$$\frac{d(P_\mu)_{ij}}{d\tau} = -\frac{\partial S_{\rm HMC}}{\partial (A_\mu)_{ij}} = N(-[A_\nu,[A_\mu,A_\nu]] + 2i\alpha\epsilon_{\mu\nu\rho}A_\nu A_\rho)_{ji} - {\rm Tr}(\mathcal{M}^{-1}\frac{d\mathcal{M}}{d(A_\mu)_{ij}}), \text{ where}$$
$$\frac{d\mathcal{M}}{d(A_\mu)_{ij}} = \delta_{i_a j_b}\delta_{i j_a}\delta_{j i_b} - \delta_{i_b j_a}\delta_{i j_b}\delta_{j i_a}.$$

3. $[P_\mu^{\rm (old)}, A_\mu^{\rm (old)}] = [P_\mu(\tau=0), A_\mu(\tau=0)]$,

   $$[P_\mu^{\rm (new)}, A_\mu^{\rm (new)}] = [P_\mu(\tau=T), \underbrace{A_\mu(\tau=T) - \frac{1}{N}({\rm tr}\,A_\mu(\tau=T))1_{N\times N}}_{\text{tracelessness condition}}].$$

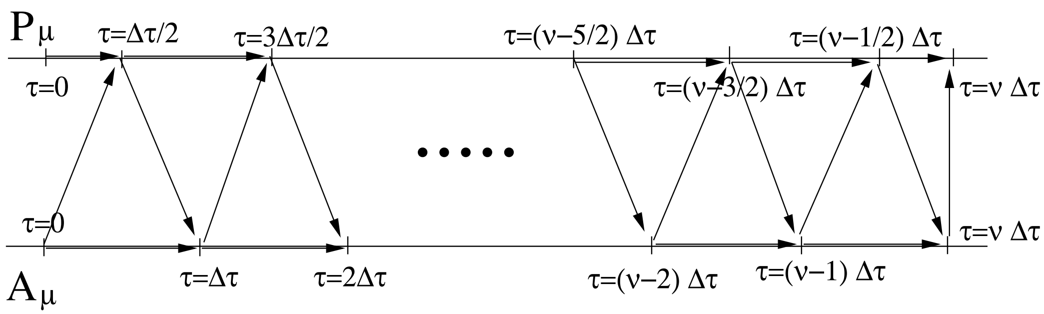
Metropolis accept/reject procedure:

Accept the new configuration with the probability $\max(1, e^{-\Delta S_{\text{HMC}}})$,

$$\Delta S_{\text{HMC}} = S_{\text{HMC}}[P_\mu^{(\text{new})}, A_\mu^{(\text{new})}] - S_{\text{HMC}}[P_\mu^{(\text{old})}, A_\mu^{(\text{old})}].$$

Leap frog discretization:

Discretized Hamiltonian equation of motion. ($\Delta\tau$: step size, $T = \nu\Delta\tau$).



$$
\begin{aligned}
(P_\mu^{(1/2)})_{ij} &= (P_\mu^{(0)})_{ij} - \frac{\Delta\tau}{2} \frac{dS_{\text{HMC}}}{d(A_\mu)_{ij}}(A_\mu^{(0)}) , \\
(A_\mu^{(1)})_{ij} &= (A_\mu^{(0)})_{ij} + \Delta\tau \, (P_\mu^{(1/2)})_{ji} , \\
(P_\mu^{(n+1/2)})_{ij} &= (P_\mu^{(n-1/2)})_{ij} - \Delta\tau \frac{dS_{\text{HMC}}}{d(A_\mu)_{ij}}(A_\mu^{(n)}) , \\
(A_\mu^{(n+1)})_{ij} &= (A_\mu^{(n)})_{ij} + \Delta\tau \, (P_\mu^{(n+1/2)})_{ji} , \\
(P_\mu^{(\nu)})_{ij} &= (P_\mu^{(\nu-1/2)})_{ij} - \frac{\Delta\tau}{2} \frac{dS_{\text{HMC}}}{d(A_\mu)_{ij}}(A_\mu^{(\nu)}) ,
\end{aligned}
$$

Result of numerical studies

Fuzzy-sphere initial condition of the Monte Carlo simulation:

$$A_\mu^{(0)} = \alpha L_\mu^{(N)}.$$

The observables to study ($\tilde{\alpha} = \alpha\sqrt{N}$): (one-loop at large N)

$$\frac{1}{N^2}\langle S\rangle \simeq -\frac{\tilde{\alpha}^4}{24} \underbrace{-\frac{1}{2}}_{\text{one-loop}},$$

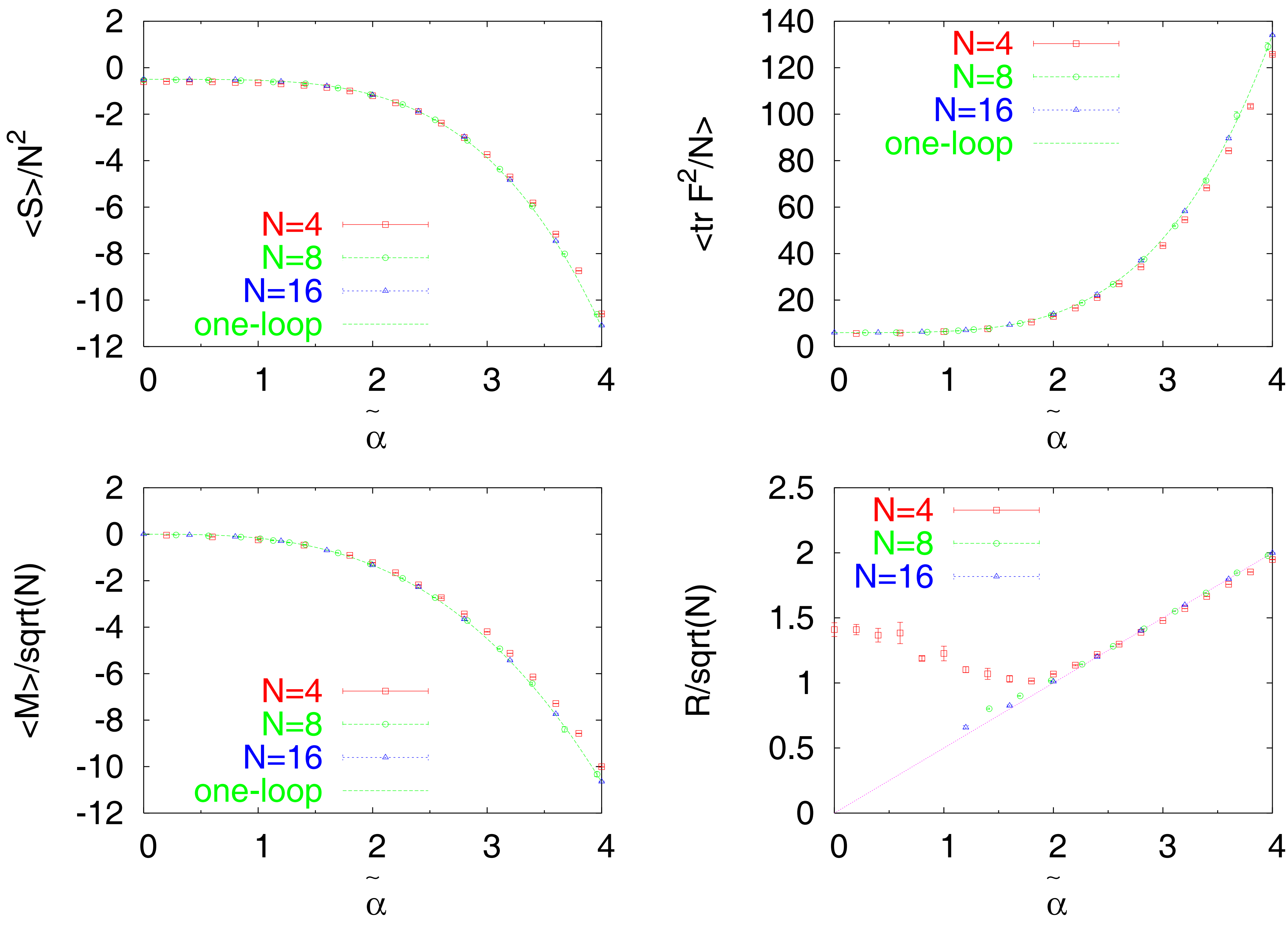
$$\langle \frac{1}{N}\text{tr}\,(F_{\mu\nu}^2)\rangle \simeq \langle \frac{1}{N}\text{tr}\sum_{\mu,\nu=1}^{4}(i[A_\mu, A_\nu])^2\rangle = \frac{\tilde{\alpha}^4}{2} \underbrace{+6}_{\text{one-loop}},$$

$$\frac{1}{\sqrt{N}}\langle M\rangle \simeq \frac{1}{\sqrt{N}}\langle \frac{2i}{3N}\sum_{i,j,k=1}^{3}\epsilon_{ijk}\text{tr}\,A_iA_jA_k\rangle = -\frac{\tilde{\alpha}^3}{6} \underbrace{+0}_{\text{one-loop}},$$

$$R = \langle\sqrt{\frac{1}{N}\sum_{\mu=1}^{4}\text{tr}\,A_\mu^2}\rangle.$$

Perturbatively, $\frac{1}{N}\langle\frac{1}{N}\text{tr}\,A_\mu^2\rangle \simeq \frac{\tilde{\alpha}^2}{4} \underbrace{+0}_{\text{one-loop}}$ is finite.

But, nonperturbatively, this is infinite!


<S>/N2
N=4
N=8
N=16
one-loop
<tr F2/N>
N=4
N=8
N=16
one-loop
<M>/sqrt(N)
N=4
N=8
N=16
one-loop
R/sqrt(N)
N=4
N=8
N=16
α̃
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Transition point $\alpha_{\rm tr}$:

$$\alpha_{\rm tr} \simeq \begin{cases} 1.1 \quad (N=4), \\ 0.5 \quad (N=8), \\ 0.3 \quad (N=16). \end{cases} \Rightarrow \alpha_{\rm tr} \simeq O(\frac{1}{N}).$$

Behavior of the eigenvalue distribution function $f(x)$ of the Casimir operator

$$Q = A_1^2 + A_2^2 + A_3^2 + A_4^2, \quad \langle \frac{1}{N} {\rm tr} Q \rangle \simeq \frac{15}{4}\alpha^2 + \frac{35}{48\alpha^2} \text{ (for finite } N=4).$$

- $\alpha < \alpha_{\rm tr}$

  Spikes in the history of $\frac{1}{N}{\rm tr}\, A_\mu^2$.

  Power-law tail behavior $f(x) \simeq x^{-2}$.

  W. Krauth and M. Staudacher, Phys. Lett. B 453, 253 (1999), [hep-th/9902113].

- $\alpha > \alpha_{\rm tr}$

  No spikes in the history of $\frac{1}{N}{\rm tr}\, A_\mu^2$.

  Peaks of $f(x)$ around the classical radius $Q = \frac{N^2-1}{4}\alpha^2$.

  Absence of the power-law behavior.

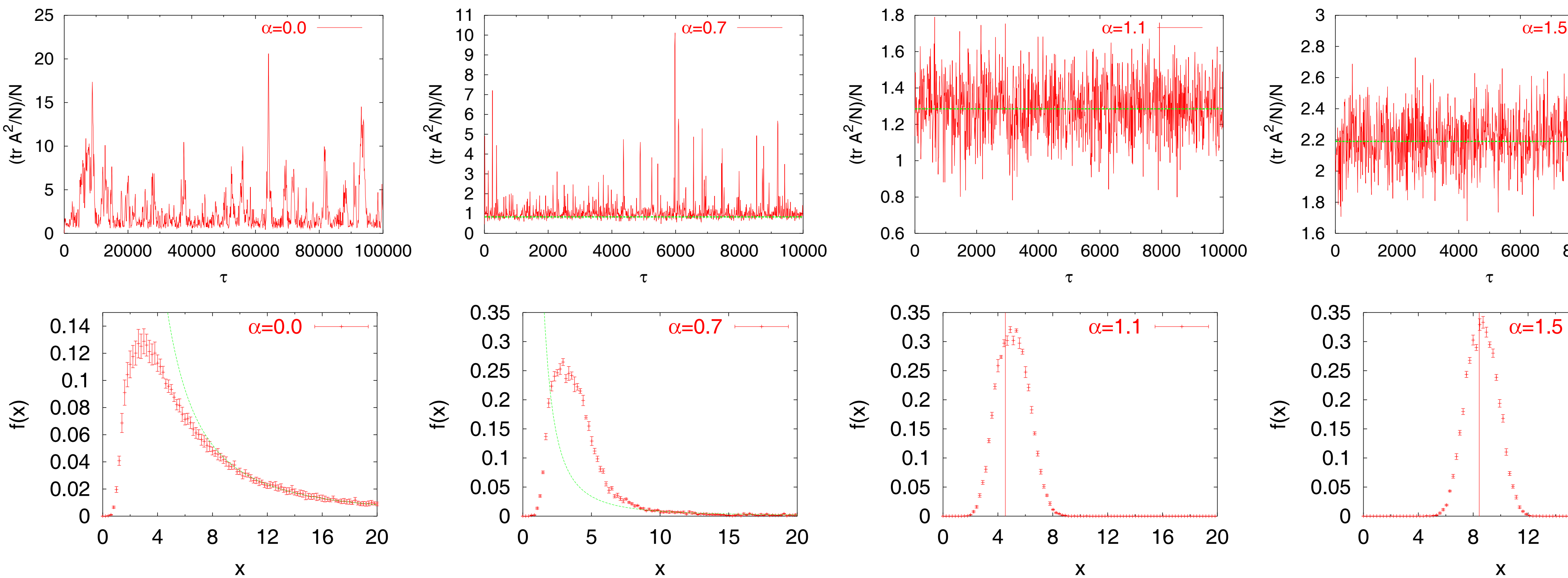
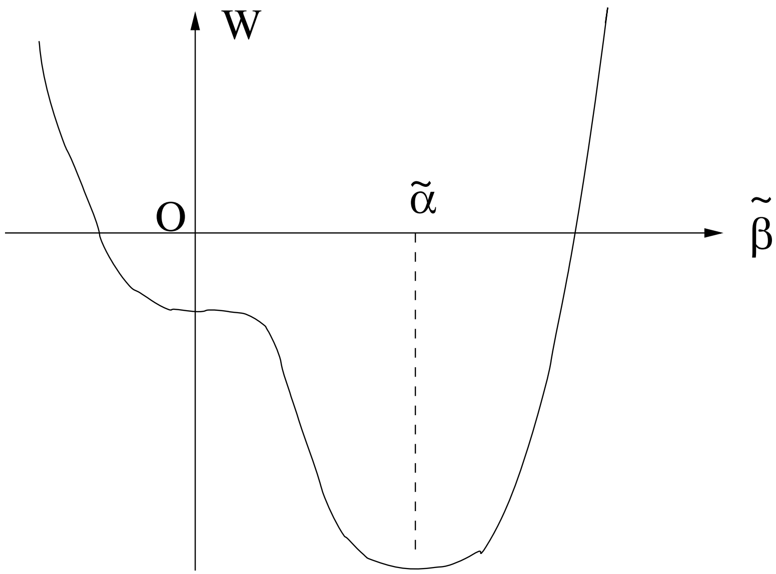


Figure 1: The history of $\frac{1}{N}\mathrm{tr}\, Q$ and the eigenvalue distribution $f(x)$ for $N = 4$.

Argument from one-loop effective action

One-loop perturbation around the fuzzy sphere solution

$$A_\mu = \begin{cases} \beta L_\mu^{(N)}, & (\text{for } \mu = 1,2,3), \\ 0, & (\text{for } \mu = 4), \end{cases}$$



One-loop effective action at large N:

$$W = N^2 \left(\frac{\tilde{\beta}^4}{8} - \frac{\tilde{\alpha}\tilde{\beta}^3}{6} \right) - N^2 \log N.$$

Minimum at $\tilde{\beta} = \tilde{\alpha}$.

The fuzzy sphere is always stable for fixed $\tilde{\alpha}(> 0)$ at large N.

3 Simulation of 1-dim supersymmetric matrix model

M. Hanada, J. Nishimura and S. Takeuchi, arXiv:0706.1647.

Difficulties in lattice simulation of supersymmetric gauge theories.

- **SUSY algebra contains continuous transformations: ⇒ break SUSY in lattice.**
- **Majorana nature of fermions ⇒ fermionic terms are difficult to formulate.**

Non-lattice formulation via Fourier transformation ⇒ avoids these difficulties.

SUSY recovers faster than continuum limit is achieved.

Supersymmetric anharmonic oscillator

Simple non-gauge supersymmetric theory.

$$S = \int_0^\beta dt \left(\frac{1}{2}(\partial_t \phi)^2 + \frac{1}{2} h'(\phi)^2 + \bar{\psi}(\partial_t + h''(\phi))\psi \right).$$

- $\phi(t) \Rightarrow$ real scalar field, $\psi(t) \Rightarrow$ one-component Dirac field.
- $\beta = 1/T \Rightarrow$ inverse temperature.

  Periodic boundary condition: $\phi(t+\beta) = \phi(t)$, $\psi(t+\beta) = \psi(t)$.
- Supersymmetry transformation

$$\delta^{(1)}\phi = \bar{\epsilon}\psi, \ \ \delta^{(1)}\bar{\psi} = -\bar{\epsilon}((\partial_t\phi) + h'(\phi)), \ \ \delta^{(1)}\psi = 0,$$
$$\delta^{(2)}\phi = \bar{\psi}\epsilon, \ \ \delta^{(2)}\bar{\psi} = 0, \qquad \delta^{(2)}\psi = ((\partial_t\phi) - h'(\phi))\epsilon.$$

- $h(\phi)$ can be any arbitrary function. Here, we take $h(\phi) = \frac{m}{2}\phi^2 + \frac{g}{4}\phi^4$.

Simulate the Fourier mode!

$$\phi(t) = \sum_{n=-\Lambda}^{\Lambda} \phi_n e^{i\omega n t}, \quad \psi(t) = \sum_{n=-\Lambda}^{\Lambda} \psi_n e^{i\omega n t}, \quad \bar{\psi}(t) = \sum_{n=-\Lambda}^{\Lambda} \bar{\psi}_n e^{i\omega n t}, \text{ where}$$

$$\omega = \frac{2\pi}{\beta}, \quad \Lambda = (\text{UV cutoff}).$$

Only the zero modes survive in the action $S = S_B + S_F$, where

$$S_B = \beta \left(\sum_{n=-\Lambda}^{\Lambda} \frac{1}{2}[(n\omega)^2 + m^2]\phi_n \phi_{-n} + mg(\phi^4)_0 + \frac{g^2}{2}(\phi^6)_0 \right),$$

$$S_F = \sum_{n,k=-\Lambda}^{\Lambda} \bar{\psi}_n \mathcal{M}_{nk} \psi_k, \text{ where } \mathcal{M}_{nk} = (2\Lambda \times 2\Lambda \text{ matrix}) = \beta[(in\omega + m)\delta_{nk} + 3g(\phi^2)_l \delta_{n}$$

$$(f^{(1)} f^{(2)} \cdots f^{(p)})_n = \sum_{k_1+k_2+\cdots+k_p=n} f^{(1)}_{k_1} f^{(2)}_{k_2} \cdots f^{(p)}_{k_p}.$$

Effective action for the bosons : $S_{\text{eff}} = S_B - \log\det \mathcal{M}.$

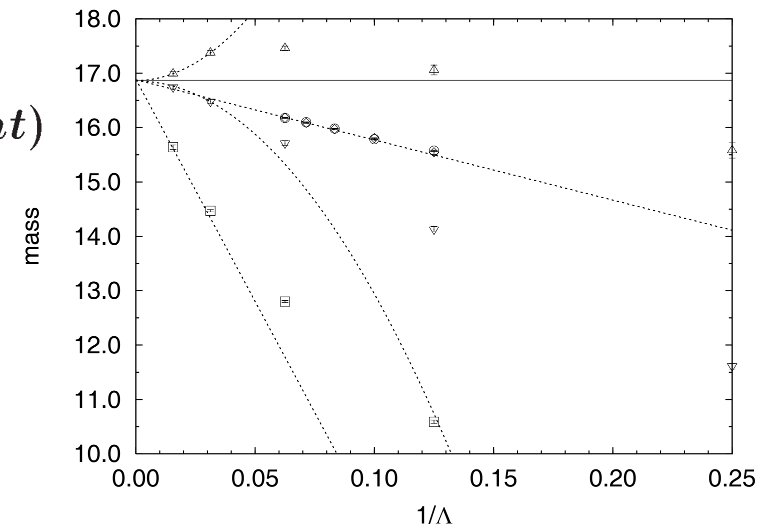
Hybrid Monte Carlo (HMC) simulation (Π_n = (conjugate momentum of ϕ_n)):

$$S_{\text{HMC}} = S_{\text{eff}} + \sum_{n=-\Lambda}^{\Lambda} \frac{1}{2}\Pi_n \Pi_{-n}.$$

CPU power is $O(\Lambda^3)$.

Extract mass from the correlators:

$$G_B(t) = \langle\phi(0)\phi(t)\rangle = b_0 + \sum_{n=1}^{\Lambda} b_n \cos(\omega n t)$$
$$\overset{\text{small}t}{\simeq} \exp(-m_{\text{bos}}t),$$
$$G_F^{(\text{sym})}(t) = \frac{1}{2}(\langle\psi(0)\bar{\psi}(t)\rangle + \langle\psi(0)\bar{\psi}(-t)\rangle)$$
$$= c_0 + 2\sum_{n=1}^{\Lambda} \text{Re}(c_n)\cos(\omega n t)$$
$$\overset{\text{small}t}{\simeq} \cosh(m_{\text{ferm}}t), \text{ where}$$
$$b_n = \langle|\phi_n|^2\rangle, \; c_n = \langle(\mathcal{M}^{-1})_{nn}\rangle.$$



Result for $\beta = 1, m = 10, g = 100$.

- ○ (◇): mass for boson (fermion) in Fourier
- △ (▽): mass for boson (fermion) in lattice.
- □: result for lattice action with half SUSY

Supersymmetric matrix quantum mechanics

4d BFSS matrix model

$$S = \frac{1}{g^2}\int_0^\beta dt \mathrm{tr} \left(\frac{1}{2}(D_t X_i(t))^2 - \frac{1}{4}[X_i(t), X_j(t)]^2 + \bar{\psi}(t) D_t \psi(t) - \bar{\psi}(t)\sigma_i [X_i(t), \psi(t)]\right).$$

- $A(t), X_i(t)$ = ($N \times N$ Hermitian matrices, $i = 1, 2, 3$)
- $\psi(t), \bar{\psi}(t)$ = ($N \times N$ matrices with complex Gaussian entries, $\alpha = 1, 2$)
- σ_i = (2×2 Pauli matrices)
- Dimensional reduction of 4d $\mathcal{N} = 1$ U(N) SYM to 1 dimension.
- $g = \frac{1}{\sqrt{N}}$ without loss of generality.
- Boundary condition:

$$\underbrace{X_i(t+\beta) = X_i(t), \;\; A(t+\beta) = A(t)}_{\text{periodic}}, \;\; \underbrace{\psi(t+\beta) = -\psi(t), \;\; \bar{\psi}(t+\beta) = -\bar{\psi}(t).}_{\text{anti-periodic}}$$

- **Static diagonal gauge**

$$A(t) = \frac{1}{\beta}\mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N), \ \ (\alpha_a \in (-\pi, \pi], a = 1, 2, \cdots, N).$$

Faddeev-Popov ghost $S_{\rm FP} = -\sum_{a<b} 2 \log |\sin(\alpha_a - \alpha_b)/2|$.

Fourier transformation of the field $(\lambda = \Lambda - \frac{1}{2})$

$$X_i^{ab}(t) = \sum_{n=-\Lambda}^{\Lambda} X_n^{ab} e^{i\omega n t}, \ \ \psi_\alpha^{ab}(t) = \sum_{r=-\lambda}^{\lambda} \psi_r^{ab} e^{i\omega r t}, \ \ \bar{\psi}_\alpha^{ab}(t) = \sum_{r=-\lambda}^{\lambda} \bar{\psi}_r^{ab} e^{i\omega r t}.$$

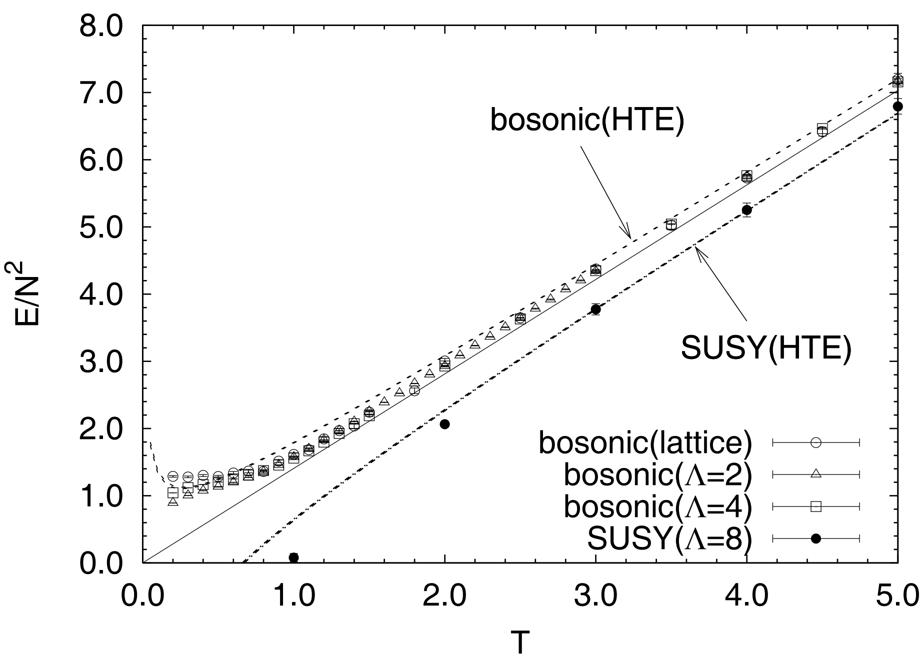
Only zero modes survives in the action:

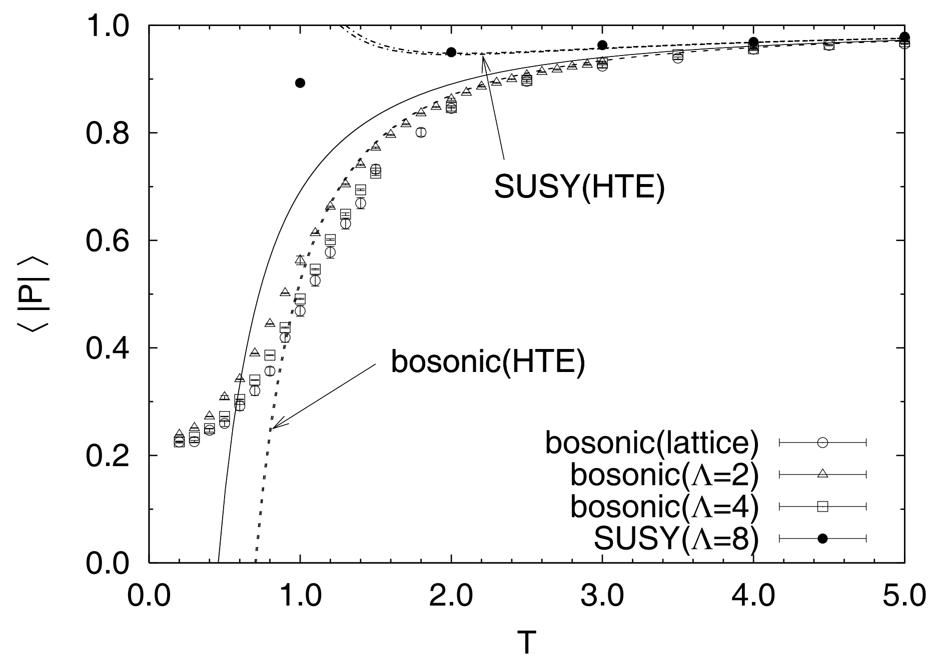
$$\begin{aligned} S \ &= \ N\beta \left[\frac{1}{2} \sum_{n=-\Lambda}^{\Lambda} \left(n\omega - \frac{\alpha_a - \alpha_b}{\beta} \right) X_{i,-n}^{ba} X_{in}^{ab} - \frac{1}{4}\mathrm{tr}\left([X_i, X_j]^2 \right)_0 \right] \\ &+ \ N\beta \left[\sum_{r=-\lambda}^{\lambda} i \left(r\omega - \frac{\alpha_a - \alpha_b}{\beta} \right) \bar{\psi}_{\alpha r}^{ba} \psi_{\alpha r}^{ab} - (\sigma_i)_{\alpha\beta}\mathrm{tr}\left(\bar{\psi}_{\alpha r} [X_i, \psi_\beta]_r \right) \right]. \end{aligned}$$

Hybrid Monte Carlo (HMC) simulation of the model ⇒ **CPU time is** $\mathrm{O}(\Lambda^3 N^6)$.

Result of the numerical simulation ($N = 4$):

- **Energy** $E = -\frac{d}{d\beta} \log Z(\beta)$.
- **Polyakov line** $P = \mathrm{tr}\, \exp\left(i \int_0^\beta dt A(t)\right) = \sum_{a=1}^{N} \exp(i\alpha_a)$.





4 Conclusion

Non-lattice simulation of the 1-dim BFSS supersymmetric matrix model.

Fourier transformation instead of lattice formulation.

Future outlooks

- **Emergence of 4-dim spacetime in 6,10-dim IKKT and BFSS matrix model.**
- **Blackhole entropy in string theory**