The way to the direct test of AdS/CFT correspondence 2

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collaborated with Hikaru Kawai

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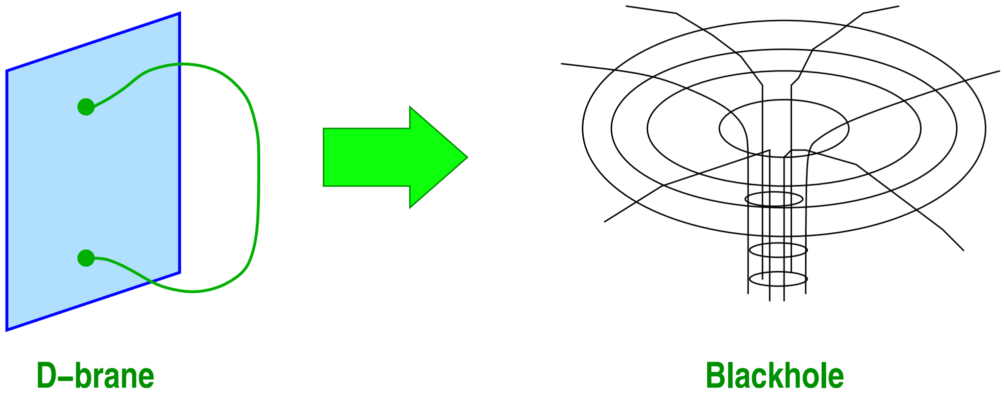
1 Introduction

AdS/CFT correspondence

AdS/CFT correspondence is a duality between the gauge theory and the supergravity.

J. Maldacena, hep-th/9711200



Especially, for the D3 brane,

$\mathcal{N} = 4$ **4-dimensional Super-Yang-Mills (SYM) theory**

$\Updownarrow$

type IIB superstring theory on the $AdS_5 \times S_5$ **space**

N **extremal black D3-brane solution**

$$ds^2 = \frac{1}{\sqrt{H(r)}}(-dt^2 + \sum_{\mu=1}^{3} dx_\mu^2) + \sqrt{H(r)}r^2 \sum_{i=4}^{9} \theta_i \theta^i.$$

- $H(r) = 1 + \frac{R^4}{r^4}$.
- **Schwarzschild radius:** $R^4 = 4\pi g_s N \alpha'^2 = \lambda \alpha'^2$.
- **Yang-Mills coupling:** $2\pi g_s = \frac{G^2}{2}, \quad \lambda = G^2 N$.

2 Expectation value of the Wilson loop

calculation via the minimal surface

The AdS/CFT correspondence enables one to calculate $\langle W[C]\rangle$ in the strong coupling region.

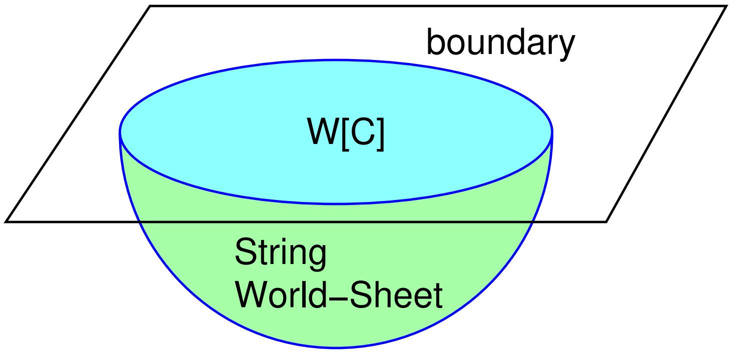
J. Maldacena, hep-th/9803002, S. Rey and J. Yee, hep-th/9803001

In the following, we consider the Wilson loop *in the Euclidean space*.

$$W[C] = \frac{1}{N} Tr P \exp[\oint_C du\{iA_\mu(y(u))\frac{dy^\mu(u)}{du} + \theta_i\phi^i(y(u))\}].$$

- The loop is parameterized by the arc length parameter u, so that $|\frac{dy_\mu(u)}{du}| = 1$.
- θ_i is set to be $\sum_{i=4}^{9}\theta_i\theta^i = 1$ and independent of the parameter u.

$\langle W[C]\rangle$ is calculated via the area of the world-sheet terminating on the Wilson loop in the AdS space.



Especially, for the circular Wilson loop,

$$\begin{aligned} S &= \frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{\det g_{MN}\partial_\alpha X^M\partial_\beta X^N} \\ &= \frac{R^2}{2\pi\alpha'}\int_0^{2\pi} d\varphi \int_\varepsilon^a \frac{adz}{z^2} = \frac{R^2}{\alpha'}(\frac{a}{\epsilon} - 1), \\ \langle W[C_{circ}]\rangle &= e^{-S} \sim e^{\frac{R^2}{\alpha}} = e^{\sqrt{\lambda}}. \end{aligned}$$

Perturbative calculation in $\mathcal{N}=4$ SYM

Straight Wilson line

For a straight Wilson line,

$$\langle W[C_{straight}]\rangle = 1.$$

N. Drukker, D. J. Gross and H. Ooguri, hep-th/9904191

(Proof) The straight line is now parameterized as $y_\mu(u) = n_\mu u$, where n_μ is a *constant* vector satisfying $|n_\mu| = 1$.
The SUSY transformation of the $\mathcal{N}=4$ 4-dimensional SYM is

$$\delta A_\mu = i\bar{\epsilon}\Gamma_\mu\psi, \quad \delta\phi_i = i\bar{\epsilon}\Gamma_i\psi.$$

The SUSY transformation of the Wilson loop is thus

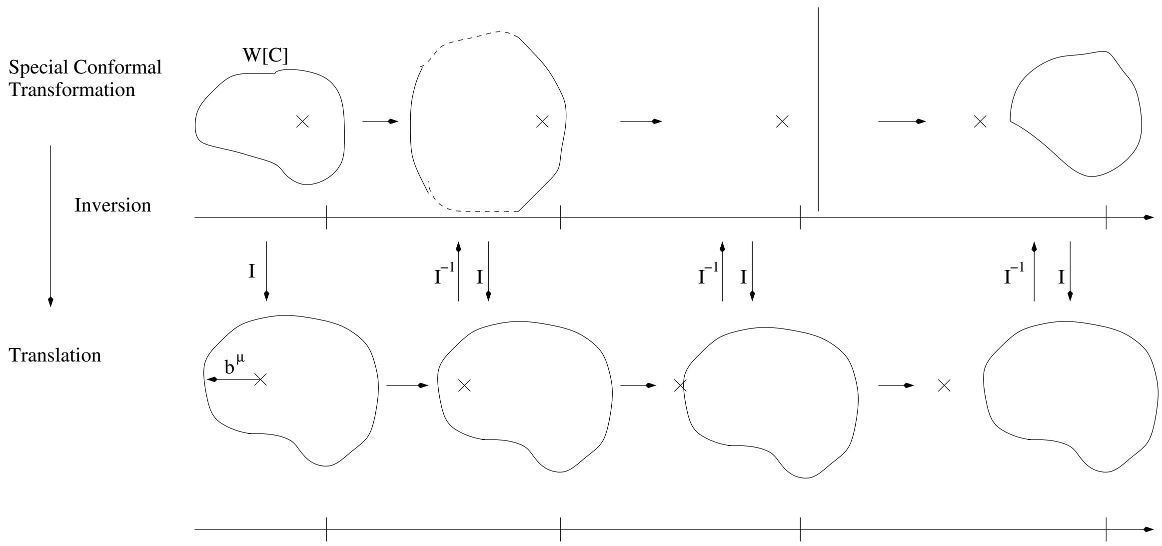
$$\begin{aligned}\delta W[C_{straight}] &= \frac{1}{N}\int_{-\infty}^{+\infty} du Tr P w_{+\infty,u}[\bar{\epsilon}(-\Gamma^\mu n_\mu + i\Gamma_i\theta^i)\psi]w_{u,-\infty}, \\ w_{b,a} &= \exp[\int_a^b du\{iA_\mu(y(u))\frac{dy^\mu(u)}{du} + \theta^i(u)\phi_i(y(u))\}].\end{aligned}$$

One can easily verify that

$$(-\Gamma^\mu n_\mu + i\Gamma_i\theta^i)^2 = -|n_\mu|^2 + |\theta_i|^2 = 0,$$

and hence that $(-\Gamma^\mu n_\mu + i\Gamma_i\theta^i)$ is nilpotent . And since n_μ and θ_i are constants, there exist zero-modes independent of the parameter u. Therefore, a straight line is a BPS saturated object, and hence does not depend on the coupling constant. (Q.E.D.)

Special conformal transformation is the translation followed and preceded by the inversion.



The conformal Killing vector $v_\mu(z)$, satisfying $\delta g_{\mu\nu} = \nabla_\mu v_\nu(z) + \nabla_\nu v_\mu(z)$, is

$$v_\mu(z) = 2z_\mu(b_\alpha z^\alpha) - b_\mu z^2.$$

The finite transformation is

$$z_\mu \Rightarrow \frac{z_\mu - b_\mu z^2}{1 - 2b_\alpha z^\alpha + b^2 z^2}.$$

(inversion) $\Rightarrow$ (translation by $-b^\mu$) $\Rightarrow$ (inversion).

$\mathcal{N} = 4$ 4-dimensional SYM is conformally invariant. A straight line is mapped to a circle by the special conformal transformation.

$\Downarrow$

Then, for the circular Wilson loop, $\langle W[C_{circ}]\rangle = 1$??

Circular Wilson loop

N. Drukker and D. J. Gross, hep-th/0010274

We calculate $\langle W[C_{circ}]\rangle$ utilizing Wick's theorem in the Feynman gauge.

$$\langle A^a_\mu(x)A^b_\nu(y)\rangle = \frac{G^2 g_{\mu\nu}\delta^{ab}}{4\pi^2(x-y)^2}, \quad \langle \phi^a_i(x)\phi^b_j(y)\rangle = \frac{G^2\delta_{ij}\delta^{ab}}{4\pi^2(x-y)^2}.$$

The circular Wilson loop is parameterized as

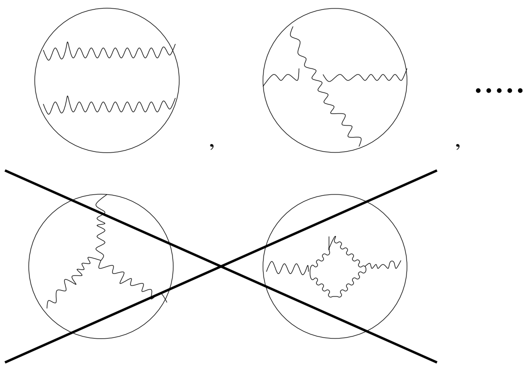
$$y_\mu(u) = (\cos u, \sin u, 0, 0), \quad 0 \le u \le 2\pi.$$

$\langle W[C_{circ}]\rangle$ is calculated to the lowest order as

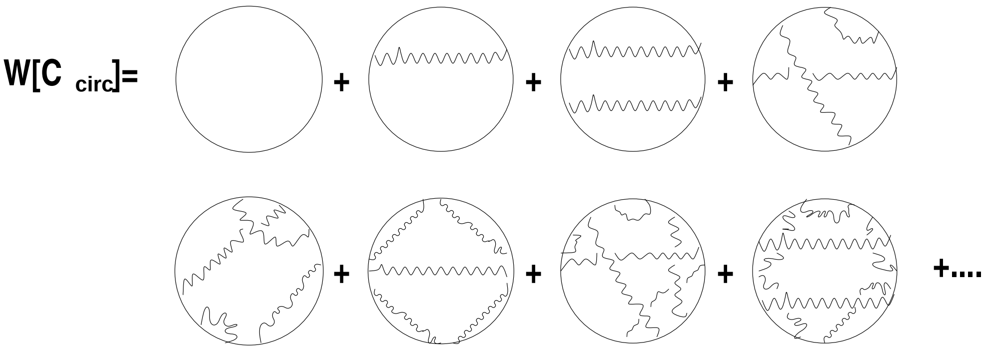
$$\begin{aligned}
\langle W[C_{circ}]\rangle &= 1 + \frac{1}{2!}\frac{1}{N}\int_0^{2\pi} du_1 du_2 \\
&\times\langle iA^a_\mu(y(u_1))\frac{dy^\mu(u_1)}{du}T_a + \theta_i(u_1)\phi^i(y(u_1))T_a\rangle \\
&\times\langle iA^b_\nu(y(u_2))\frac{dy^\nu(u_2)}{du}T_b + \theta_j(u_2)\phi^j(y(u_2))T_b\rangle + \mathcal{O}(\lambda^2) \\
&= 1 + \frac{G^2}{2N}\int_0^{2\pi} du_1 du_2 \frac{1-(\frac{dy^\mu(u_1)}{du})(\frac{dy_\mu(u_2)}{du})}{4\pi^2(y(u_1)-y(u_2))^2} Tr(T_aT^a) + \mathcal{O}(\lambda^2) \\
&= 1 + \frac{\lambda}{8} + \mathcal{O}(\lambda^2).
\end{aligned}$$

They pointed out that this discrepancy is due to an anomaly, since the special conformal transformation maps ∞ to the origin.

They approximated the contribution to the higher order by excluding the terms with internal vertices artificially.



Then, $\langle W[C_{circ}]\rangle$ is computed to all order in the $\frac{1}{N}$ expansion.



$$\langle W[C_{circ}]\rangle = \sum_{n=0}^{\infty}\frac{1}{N}\frac{1}{(2n)!}\int_0^{2\pi} du_1\cdots du_{2n}$$
$$\times Tr[\langle\prod_{k=1}^{2n}(iA_{\mu_k}(y(u_k))\frac{dy^{\mu_k}(u_k)}{du_k}+\theta_{i_k}(u_k)\phi^{i_k}(y(u_k)))\rangle]$$
$$= \sum_{n=0}^{\infty}\frac{1}{(2n)!}(\frac{\lambda}{4})^n(\ \sharp \text{ of the possible contractions})$$
$$= \sum_{n=0}^{\infty}\frac{1}{(2n)!}(\frac{\lambda}{4})^n\langle\frac{1}{N}TrM^{2n}\rangle = \langle\frac{1}{N}Tr\exp(-\frac{\lambda}{4}M)\rangle$$
$$= \frac{1}{N}L^1_{N-1}(-\frac{\lambda}{4N})\exp(\frac{\lambda}{8N})$$
$$= \frac{2}{\sqrt{\lambda}}I_1(\sqrt{\lambda})+\frac{\lambda}{48N^2}I_2(\sqrt{\lambda})+\frac{\lambda^2}{1280N^4}I_4(\sqrt{\lambda})+\cdots.$$

- **Here,** $\langle \frac{1}{N} TrM^{2n} \rangle$ **denotes the partition function of the 0-dimensional field theory.**

$$S = \frac{N}{2} TrM^2,$$
$$\langle \frac{1}{N} TrM^{2n} \rangle = \frac{1}{Z} \int [dM] \frac{1}{N} (TrM^{2n}) \exp(-S),$$
$$Z = \int [dM] \exp(-S),$$

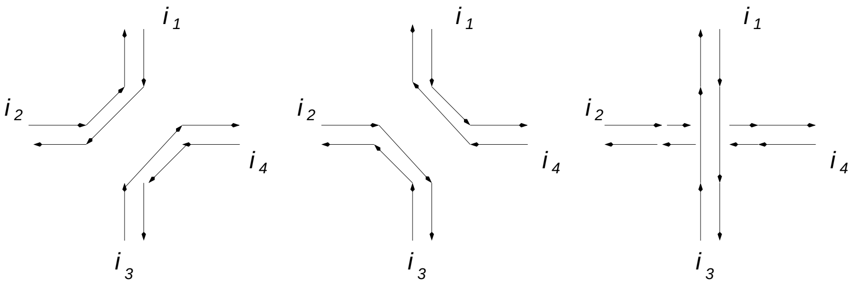
where M **is an** $N \times N$ **hermitian matrix.**

$\langle \frac{1}{N} TrM^{2n} \rangle$ **represents the number of the possible contractions.**
(Proof) We first verify the Feynman diagram of this matrix model.

$$\langle M_{ij} M_{kl} \rangle = \frac{1}{Z} \int_{-\infty}^{+\infty} [\prod_{m,n=1}^{N} dM_{mn}] M_{ij} M_{kl}$$
$$\times \exp[-\frac{N}{2} (\sum_{m=1}^{N} M_{mm} + 2 \sum_{1 \le m < n \le N} M_{mn} M_{mn}^{\dagger})] = \frac{1}{N} \delta_{il} \delta_{jk}.$$

$$\{ \frac{1}{a} = \frac{\int_{-\infty}^{+\infty} dx x^2 e^{-\frac{ax^2}{2}}}{\int_{-\infty}^{+\infty} dx e^{-\frac{ax^2}{2}}} = \frac{\int_{-\infty}^{+\infty} dx dx^{\dagger} x x^{\dagger} e^{-axx^{\dagger}}}{\int_{-\infty}^{+\infty} dx dx^{\dagger} e^{-axx^{\dagger}}} \}$$

Then, $\langle \frac{1}{N} TrM^{2n} \rangle = \langle \frac{1}{N} M_{i_1 i_2} M_{i_2 i_3} \cdots M_{i_{2n-1} i_{2n}} M_{i_{2n} i_1} \rangle$ is the number of the ways to contract the indices by the above Feynman rule. (Q.E.D.)



The contractions of the indices when n=2.

- $L_n^m(x)$ **=(Laguerre polynomial)=** $\frac{1}{n!} e^x x^{-m} (\frac{d^n}{dx^n} e^{-x} x^{n+m})$.
- **At large** N **and large** λ**,**

$$\langle W[C_{circ}] \rangle \sim \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) \sim \sqrt{\frac{2}{\pi}} \lambda^{-\frac{3}{4}} e^{\sqrt{\lambda}}.$$

and agrees with the analysis in the AdS space.

3 Gauge Invariance

The result $\langle W[C_{circ}]\rangle = \frac{1}{N} L^1_{N-1}(-\frac{\lambda}{4N}) \exp(\frac{\lambda}{8N})$ is obtained by subtracting the interacting vertices artificially.

$$\Downarrow$$

The gauge dependence of this result is not confirmed.

We attempt to grasp its gauge invariance by interpreting the conformal anomaly in terms of the operator product expansion $T_{\mu\nu}(z)W[C]$, since $T_{\mu\nu}(z)W[C]$ is a gauge invariant quantity.

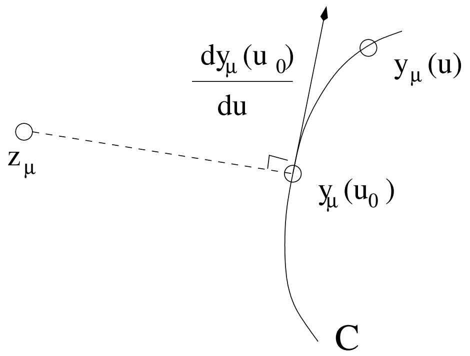
T. Azuma and H. Kawai, hep-th/0106063

The OPE in the 2-dimensional CFT

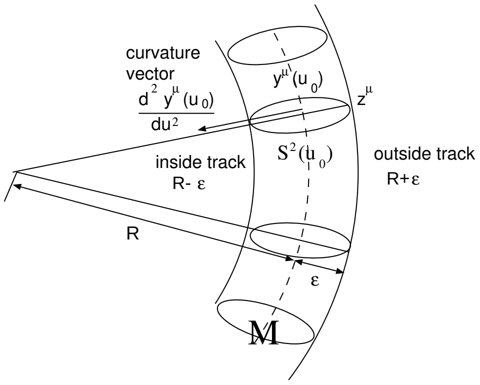
$$T(z)A(w) = \left(\begin{array}{l}\text{lower-dimensional}\\ \text{operators}\end{array}\right) + \frac{hA(w)}{(z-w)^2} + \frac{\partial A(w)}{(z-w)} + \cdots .$$

The OPE $T_{\mu\nu}(z)W[C]$ in 4-dimensional Euclidean space

$y_\mu(u_0)$: the nearest point on the loop to the point z_μ.



We consider the conformal Ward identity by wrapping the Wilson loop by the enveloping surface of the spheres $S^2(u_0)$.



$$\int_{\mathcal{M}} d^4 z \partial^\mu [T_{\mu\nu}(z) W[C] v^\nu(z)]$$
$$= \int du_0 \int_{S^2(u_0)} d\Omega [1 - (z_\alpha - y_\alpha(u_0)\frac{d^2 y^\alpha(u_0)}{du^2})] n^\mu T_{\mu\nu}(z) W[C] v^\nu(z).$$

- $S^2(u_0)$: a S^2 sphere of a fixed radius ϵ which is perpendicular to the loop and has its center at $y_\mu(u_0)$.
- $\mathcal{M}$: the region inside the enveloping surface.
- $v^\mu(z)$: the conformal Killing vector.
- $T_{\mu\nu}(z)$: energy-momentum tensor $T_{\mu\nu}(z) = \frac{2}{\sqrt{g}} \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}}$.
- The weakest singularity in the OPE than contribute to the conformal Ward identity is $\mathcal{O}(z - y(u_0))^{-2}$, since the surface area of $S^2(u_0)$ is $4\pi\epsilon^2$.

We expand the OPE $T_{\mu\nu}(z)W[C]$ as a power series expansion in $z_\mu - y_\mu(u_0)$.

$$T_{\mu\nu}(z)W[C] = (T_{\mu\nu}(z)W[C])_c + (T_{\mu\nu}(z)W[C])_{vec} + (T_{\mu\nu}(z)W[C])_{sca}.$$

- $(T_{\mu\nu}(z)W[C])_c$: the terms containing $W[C]$ itself without any insertion of the fields to it.
- $(T_{\mu\nu}(z)W[C])_{vec,sca}$: the insertion of the fields $A_\mu(y(u_0))$ and $\phi_i(y(u_0))$ to $W[C]$, respectively.

$T(z)A(w)$	lower$\cdots$	$\frac{hA(w)}{(z-w)^2}$	$\frac{\partial A(w)}{z-w}$
$T_{\mu\nu}(z)W[C]$	$-$	$(T_{\mu\nu}(z)W[C])_c$	$(T_{\mu\nu}(z)W[C])_{vec}$ $(T_{\mu\nu}(z)W[C])_{sca}$

The general form is obtained by means of the dimensional analysis and the properties of $T_{\mu\nu}(z)$, up to the free parameters $q, q', \alpha, \beta, \gamma$.

- $T_\mu{}^\mu(z) = 0$: conformal invariance of the action.
- $\partial^\mu T_{\mu\nu}(z) = 0$: the conservation law of the energy and the momentum.

$$\begin{aligned}
(T_{\mu\nu}(z)W[C])_c &= \frac{q}{24\pi^2}\Big[\frac{1}{|z-y(u_0)|^4}\Big(g_{\mu\nu} - 2\frac{dy_\mu(u_0)}{du}\frac{dy_\nu(u_0)}{du} \\
&\quad -2\frac{(z_\mu - y_\mu(u_0))(z_\nu - y_\nu(u_0))}{|z-y(u_0)|^2}\Big)\Big]W[C] \\
&+ \frac{1}{24\pi^2}\Big[-2(q+q')\frac{(z_\mu - y_\mu(u_0))(z_\nu - y_\nu(u_0))(z_\alpha - y_\alpha(u_0))\frac{d^2y^\alpha(u_0)}{du^2}}{|z-y(u_0)|^6} \\
&\quad +(-q+\frac{q'}{2})\frac{(z_\mu - y_\mu(u_0))\frac{d^2y_\nu(u_0)}{du^2} + (z_\nu - y_\nu(u_0))\frac{d^2y_\mu(u_0)}{du^2}}{|z-y(u_0)|^4} \\
&\quad +(-4q+q')\frac{\frac{y_\mu(u_0)}{du}\frac{y_\nu(u_0)}{du}(z_\alpha - y_\alpha(u_0))\frac{d^2y^\alpha(u_0)}{du^2}}{|z-y(u_0)|^4} \\
&\quad +2qg_{\mu\nu}\frac{(z_\alpha - y_\alpha(u_0))\frac{d^2y^\alpha(u_0)}{du^2}}{|z-y(u_0)|^4}\Big]W[C] + \mathcal{O}(z-y(u_0))^{-2},
\end{aligned}$$

$$
\begin{aligned}
(T_{\mu\nu}(z)W[C])_{vec} = & \frac{1}{N} Tr P w_{L,u_0} \frac{i}{4\pi|z-y(u_0)|^3} \\
& \times[-(z_\mu - y_\mu(u_0))F_{\nu\alpha}(y(u_0))\frac{dy^\alpha(u_0)}{du} - (z_\nu - y_\nu(u_0))F_{\mu\alpha}(y(u_0))\frac{dy^\alpha(u_0)}{du} \\
& \quad + g_{\mu\nu}(z^\alpha - y^\alpha(u_0))F_{\alpha\beta}(y(u_0))\frac{dy^\beta(u_0)}{du} \\
& \quad + (z^\alpha - y^\alpha(u_0))[F_{\mu\alpha}(y(u_0))\frac{dy_\nu(u_0)}{du} + F_{\nu\alpha}(y(u_0))\frac{dy_\mu(u_0)}{du}]]w_{u_0,0} \\
& + \mathcal{O}(z-y(u_0))^{-1}, \\
(T_{\mu\nu}(z)W[C])_{sca} = & \frac{1}{N} Tr P w_{L,u_0} [\frac{\theta_i(u_0)\phi^i(y(u_0))}{24\pi|z-y(u_0)|^3}(2 - \frac{\alpha}{10} + \frac{\beta}{2}) \\
& \quad \times[g_{\mu\nu} - 3\frac{(z_\mu - y_\mu(u_0))(z_\nu - y_\nu(u_0))}{|z-y(u_0)|^2} - \frac{dy_\mu(u_0)}{du}\frac{dy_\nu(u_0)}{du}] \\
& + \frac{\theta_i(u_0)\phi^i(y(u_0))}{24\pi|z-y(u_0)|^3}[(1 + \frac{\alpha}{10} - \frac{\beta}{2} + \frac{\gamma}{5})((z_\mu - y_\mu(u_0))\frac{d^2y_\nu(u_0)}{du^2} + (z_\nu - y_\nu(u_0))\frac{d^2y_\mu(u_0)}{du^2}) \\
& \quad + (1 - \frac{\alpha}{5} + \beta - \frac{\gamma}{5})g_{\mu\nu}(z_\alpha - y_\alpha(u_0))\frac{dy^\alpha(u_0)}{du^2} \\
& \quad + (-3 + \frac{3\alpha}{5} - 3\beta - \frac{3\gamma}{5})\frac{(z_\mu - y_\mu(u_0))(z_\nu - y_\nu(u_0))(z_\alpha - y_\alpha(u_0))\frac{dy^\alpha(u_0)}{du^2}}{|z-y(u_0)|^3} \\
& \quad + (-3 + \gamma)\frac{dy_\mu(u_0)}{du}\frac{dy_\nu(u_0)}{du}(z_\alpha - y_\alpha(u_0))\frac{d^2y^\alpha(u_0)}{du^2}] \\
& + \frac{\frac{d\theta_i(u_0)}{du}}{24|z-y(u_0)|^3}\phi^i(y(u_0))(2 + \frac{\alpha}{5} - \beta)[(z_\mu - y_\mu(u_0))\frac{dy_\nu(u_0)}{du} + (z_\nu - y_\nu(u_0))\frac{dy_\mu(u_0)}{du}] \\
& + \frac{\theta_i(u_0)}{24\pi|z-y(u_0)|^3}[(-4 + \frac{\alpha}{5})((z_\mu - y_\mu(u_0))D_\nu\phi^i(y(u_0)) + (z_\nu - y_\nu(u_0))D_\mu\phi^i(y(u_0))) \\
& \quad + (4 - \frac{\alpha}{5})g_{\mu\nu}(z_\alpha - y_\alpha(u_0))D^\alpha\phi^i(y(u_0)) \\
& \quad + (-6 - \frac{3\alpha}{5})\frac{(z_\mu - y_\mu(u_0))(z_\nu - y_\nu(u_0))(z_\alpha - y_\alpha(u_0))D^\alpha\phi^i(y(u_0))}{|z-y(u_0)|^2} \\
& \quad + (-2 + \alpha)\frac{dy_\mu(u_0)}{du}\frac{dy_\nu(u_0)}{du}(z_\alpha - y_\alpha(u_0))D^\alpha\phi^i(y(u_0)) \\
& \quad + \beta\frac{dy_\alpha(u_0)}{du}(D^\alpha\phi^i(y(u_0)))[(z_\mu - y_\mu(u_0))\frac{dy_\nu(u_0)}{du} \\
& \qquad + (z_\nu - y_\nu(u_0))\frac{dy_\mu(u_0)}{du}]]]w_{u_0,0} + \mathcal{O}(z-y(u_0))^{-1}.
\end{aligned}
$$

The conformal Ward identity for the special transformation is computed to be

$$\int_{\mathcal{M}} d^4z \partial^\mu [T_{\mu\nu}(z) W[C] v^\nu(z)]$$
$$= -\int ds (\frac{\delta W[C]}{\delta y^\nu(s)})[2y^\nu(s)(b_\alpha y^\alpha(s)) - b^\nu(y(s))^2].$$

- s: general parameterization running over $0 \le s \le 2\pi$. The relationship to the arc length parameter is $\frac{du_0}{ds} = |\frac{dy_\mu(s)}{ds}|$.
- $(\frac{\delta W[C]}{\delta y^\nu(s)})$: deformation of the Wilson loop.

$$\begin{aligned}(\frac{\delta W[C]}{\delta y^\nu(s)}) &= \frac{1}{N} TrP\hat{w}_{2\pi,s}[iF_{\nu\alpha}(y(s))\frac{dy^\alpha(s)}{ds} \\ &\quad + |\frac{dy_\mu(s)}{ds}|\theta_i(s) D_\nu \phi^i(y(s))]\hat{w}_{s,0} \\ &- \frac{d}{ds}[\frac{1}{N} TrP\hat{w}_{2\pi,s}\{\theta_i(s)\phi^i(y(s))\frac{\frac{dy_\nu(s)}{ds}}{|\frac{dy_\mu(s)}{ds}|}\}\hat{w}_{s,0}], \\ \hat{w}_{b,a} &= \exp[\int_a^b ds\{iA_\mu(y(s))\frac{dy^\mu(s)}{ds} + |\frac{dy_\mu(s)}{ds}|\phi_i(y(s))\theta^i(s)\}].\end{aligned}$$

- The Wilson loop in $\mathcal{N}=4$ SYM does not possess an anomalous dimension and only the deformation of the loop occurs in the special conformal transformation.
- We conjecture that the anomaly emerging when the circle is mapped to a straight line may be related to the free parameters q, q'.

The premise of the conjecture:

In calculating the above conformal Ward identity, we utilize the following partial integral, and drop the surface term because the loop is closed. When we consider the open Wilson line, this surface term may play an essential role.

$$\begin{aligned}
&\int_{\mathcal{M}} d^4 z \partial^\mu [(T_{\mu\nu}(z) W[C])_c v^\nu(z)] \\
= \;& -\frac{q}{6\pi\epsilon} \int du_0 \frac{d^2 y_\nu(u_0)}{du^2} [2y^\nu(u_0)(b_\alpha y^\alpha(u_0)) - b^\nu(y(u_0))^2] W[C] \\
& -\frac{q}{3\pi\epsilon} \int du_0 (b_\alpha y^\alpha(u_0)) W[C] \\
= \;& -\frac{q}{3\pi\epsilon} \int du_0 (1 - \frac{dy^\nu(u_0)}{du} \frac{dy_\nu(u_0)}{du})(b_\alpha y^\alpha(u_0)) W[C] \\
& -\frac{q}{6\pi\epsilon} [\frac{dy_\nu(u_0)}{du}(2y^\nu(u_0)(b_\alpha y^\alpha(u_0)) - b^\nu(y(u_0))^2)]_{surface} \\
= \;& 0.
\end{aligned}$$

4 Conclusion

- The expectation value of the straight Wilson loop is $\langle W[C_{straight}]\rangle = 1$, while that of the circular Wilson loop is $\langle W[C_{circ}]\rangle = \frac{1}{N}L^1_{N-1}(-\frac{\lambda}{4N})\exp(\frac{\lambda}{8N})$. Gross and Drukker attributed this discrepancy to the conformal anomaly of the special conformal transformation.
- We have attempted to interpret the conformal anomaly in terms of the OPE $T_{\mu\nu}(z)W[C]$, in order to grasp the gauge invariance of the result $\langle W[C_{circ}]\rangle = \frac{1}{N}L^1_{N-1}(-\frac{\lambda}{4N})\exp(\frac{\lambda}{8N})$.
- We have constructed the general form of the OPE $T_{\mu\nu}(z)W[C]$ as a power series expansion in $z_\mu - y_\mu(u_0)$, up to the free parameters q, q', α, β, γ.
- We conjecture that this conformal anomaly may related to the free parameters q, q'.