Matrix model with manifest general coordinate invariance

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Contents

1 Introduction

Constructive definition of superstring theory

A large N reduced model has been proposed as a nonperturbative formulation of superstring theory.

IIB matrix model

N.Ishibashi, H.Kawai, Y.Kitazawa and A.Tsuchiya, hep-th/9612115.

For a review, hep-th/9908038

$$S = -\frac{1}{g^2} Tr_{N\times N}\left(\frac{1}{4}\sum_{i,j=0}^{9}[A_i, A_j]^2 + \frac{1}{2}\bar{\psi}\sum_{i=0}^{9}\Gamma^i[A_i, \psi]\right).$$

- A_i and ψ are $N \times N$ Hermitian matrices.
  - * A_i: 10-dimensional vectors
  - * ψ: 10-dimensional Majorana-Weyl (i.e. 16-component) spinors
- This model possesses $SU(N)$ gauge symmetry and $SO(9,1)$ Lorentz symmetry.
- $\mathcal{N} = 2$ SUSY: This theory must contain spin-2 gravitons if it contains massless particles.

Is it possible to build a matrix model which manifestly describe the general coordinate invariance?

2 Attempt to build a local Lorentz invariant matrix model

We identify infinitely large N matrices with differential operator.

The information of spacetime can be embedded to matrices in various ways.

- Twisted Eguchi-Kawai(TEK) model:

  A. Gonzalez-Arroyo and M. Okawa, Phys. Rev. D 27, 2397 (1983).

$$A_i \sim \partial_i + a_i.$$

  The matrices A_i represent the covariant derivative on the spacetime.

- IIB matrix model:

$$A_i \sim X_i.$$

  A_i itself represent the space-time coordinate.

IIB matrix model with noncommutative background

$$[\hat{p}_i, \hat{p}_j] = iB_{ij}, (B_{ij} = \text{c-numbers})$$

interpolates these two pictures.

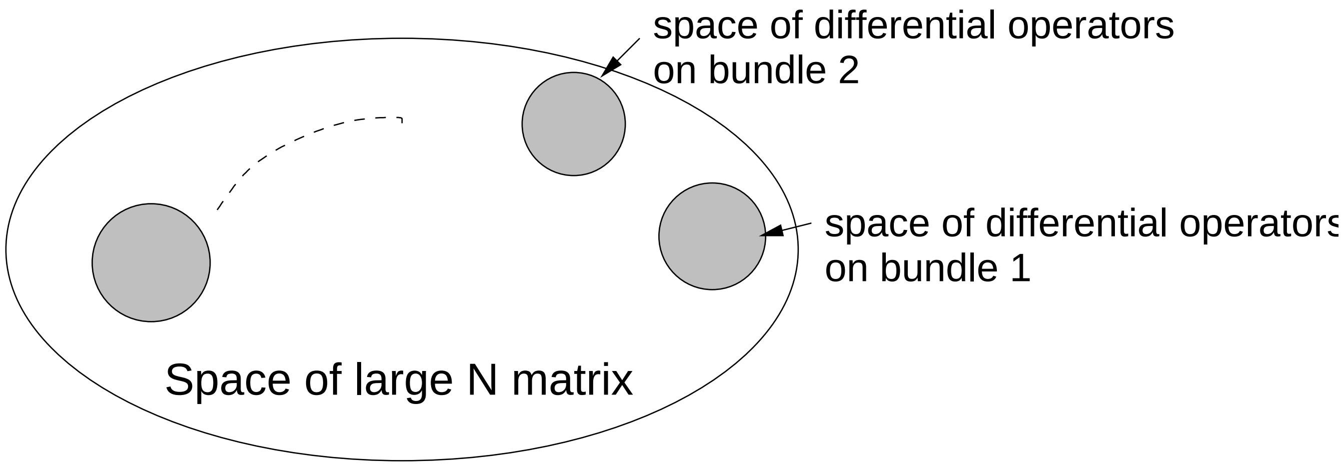
H. Aoki, N. Ishibashi, S. Iso, H. Kawai, Y. Kitazawa and T. Tada, hep-th/9908141

$Tr_{N\times N}\bar{\psi}\Gamma^i[A_i, \psi]$ reduces to the fermionic action $\int d^dx\bar{\psi}(x)\Gamma^i(\partial_i\psi(x) + [a_i(x), \psi(x)])$ in the flat space in the low-energy limit.

Space of matrices and differential operator

We can describe the differential operators in an arbitrary spin bundle in an arbitrary manifold in the continuum limit simultaneously:

They are all embedded in the space of a large N matrix.



Naive promotion to the matrix model

The local Lorentz invariant action of the fermion in the curved space:

$$
\begin{aligned}
S_F &= \int d^dx e(x)\bar{\psi}(x)\Gamma^a e_a{}^i(x)\left(i\partial_i\psi(x) + [A_i(x), \psi(x)] + \frac{i}{4}\Gamma^{bc}\omega_{ibc}(x)\psi(x)\right) \\
&\quad \Downarrow \text{(absorb the determinant } e(x) \text{ into the definition} \\
&\qquad \text{of } \Psi(x) \text{ as } \Psi(x) = e^{\frac{1}{2}}(x)\psi(x) \text{)} \\
&= \int d^dx \bar{\Psi}(x) i\Gamma^a \Big(e_a{}^i(x)(\partial_i\Psi(x) + [A_i(x), \Psi(x)]) + \frac{1}{2}e_c{}^i(x)\omega_i{}^c{}_a(x)\Psi(x) \\
&\qquad\qquad + e_a{}^i(x)e^{\frac{1}{2}}(x)(\partial_i e^{-\frac{1}{2}}(x))\Psi(x)\Big) \\
&\quad + \frac{i}{4}\bar{\Psi}(x)\Gamma^{a_1a_2a_3}e_{[a_1}{}^i(x)\omega_{ia_2a_3]}(x)\Psi(x).
\end{aligned}
$$

The corresponding promotion to a matrix model is

$$
\begin{aligned}
S'_F &= \frac{1}{2}Tr\bar{\psi}\Gamma^a[A_a, \psi] + \frac{1}{2}Tr\bar{\psi}\Gamma^{a_1a_2a_3}\{A_{a_1a_2a_3}, \psi\} \\
&= Tr\bar{\psi}\left(\Gamma^a A_a + \Gamma^{a_1a_2a_3}A_{a_1a_2a_3}\right)\psi.
\end{aligned}
$$

(The second equality holds only when ψ is a Majorana fermion.)

$\boxed{\text{Local Lorentz transformation of the matrix model}}$

$$\delta\psi = \frac{1}{4}\Gamma^{a_1 a_2}\varepsilon_{a_1 a_2}\psi,$$

instead of $\delta\psi = \frac{1}{4}\Gamma^{a_1 a_2}\{\varepsilon_{a_1 a_2}, \psi\}$ at the cost of the hermiticity of ψ.

$$\delta S'_F = \frac{1}{4} Tr\bar{\psi}[\Gamma^a A_a + \Gamma^{a_1 a_2 a_3} A_{a_1 a_2 a_3}, \Gamma^{b_1 b_2}\varepsilon_{b_1 b_2}]\psi.$$

However, this action does not close with respect to the local Lorentz transformation:

$$\begin{aligned} &[\Gamma^{a_1 a_2 a_3} A_{a_1 a_2 a_3}, \Gamma^{b_1 b_2}\varepsilon_{b_1 b_2}] \\ = \ &\frac{1}{2}\underbrace{[\Gamma^{a_1 a_2 a_3}, \Gamma^{b_1 b_2}]}_{\text{rank } 3}\{A_{a_1 a_2 a_3}, \varepsilon_{b_1 b_2}\} + \frac{1}{2}\underbrace{\{\Gamma^{a_1 a_2 a_3}, \Gamma^{b_1 b_2}\}}_{\text{rank } 1,\ 5}[A_{a_1 a_2 a_3}, \varepsilon_{b_1 b_2}]. \end{aligned}$$

We need the terms of all odd ranks in order to formulate a local Lorentz invariant matrix model.

3 The model

$$S = Tr_{N\times N}\left[tr_{16\times 16}V(m^2) + \bar{\psi}m\psi\right].$$

- Tr: the trace of the infinite dimensional space of the differential operator.
  $\Rightarrow$ The parameter τ of the order $[(\mathrm{length})^2]$ serves to regularize the divergence.

  (The potential $V(m^2)$ is generically $V(m^2) \sim \exp(-(m^2)^\star)$.)
- m: hermitian differential operator

$$\begin{aligned} m &= \tau^{\frac{1}{2}}D, \text{ where} \\ D &= A_a\Gamma^a + \frac{i}{3!}A_{a_1a_2a_3}\Gamma^{a_1a_2a_3} - \frac{1}{5!}A_{a_1\cdots a_5}\Gamma^{a_1\cdots a_5} - \frac{i}{7!}A_{a_1\cdots a_7}\Gamma^{a_1\cdots a_7} \\ &\quad + \frac{1}{9!}A_{a_1\cdots a_9}\Gamma^{a_1\cdots a_9}. \end{aligned}$$

- $A_{a_1\cdots a_{2n-1}}$: hermitian differential operator expanded by the number of the derivatives:

$$A_{a_1\cdots a_{2n-1}} = a_{a_1\cdots a_{2n-1}}(x) + \sum_{k=1}^{\infty}\frac{i^k}{2}\{\partial_{i_1}\cdots\partial_{i_k}, a^{(i_1\cdots i_k)}{}_{a_1\cdots a_{2n-1}}(x)\}.$$

- ψ: a fermionic differential operator, but it is not hermitian:

$$\psi = \left(\chi(x) + \sum_{l=1}^{\infty} i^l\chi^{(i_1\cdots i_l)}(x)\partial_{i_1}\cdots\partial_{i_l}\right)\exp(-(\tau D^2)^\star).$$

Considering the correspondence with the local Lorentz invariant fermion's action, we take the coefficients to be

$$
\begin{aligned}
(a_a(x))_0 &= -\frac{i}{2}(\partial_i e_a{}^i(x)) + \frac{i}{2} e_c{}^i(x)\omega_{ica}(x) + i e_a{}^i(x) e^{\frac{1}{2}}(x)(\partial_i e^{-\frac{1}{2}}(x)), \\
(a_a{}^{(i)}(x))_0 &= e_a{}^i(x), \\
(a_{a_1a_2a_3}(x))_0 &= \frac{3}{2} e_{[a_1}{}^i(x)\omega_{ia_2a_3]}(x).
\end{aligned}
$$

- We find it natural to identify $a_a{}^i(x)$ with the vielbein of the background metric.
- The fields $a'^{(i_1\cdots i_k)}{}_{a_1\cdots a_{2n-1}}(x)$, defined as

  $$
  \begin{aligned}
  &a'_a(x) = a_a(x) - (a_a(x))_0, \quad a'_{a_1a_2a_3}(x) = a_{a_1a_2a_3} - (a_{a_1a_2a_3}(x))_0, \\
  &a'^{(i_1\cdots i_k)}{}_{a_1\cdots a_{2n-1}}(x) = a^{(i_1\cdots i_k)}{}_{a_1\cdots a_{2n-1}}(x) \ \text{(otherwise)},
  \end{aligned}
  $$

  are identified with the matter fields.

For a generic $V(m^2)$, the bosonic part of the action reduces to the Einstein gravity

$$
S \sim \int \frac{d^d x}{(2\pi\tau)^{\frac{d}{2}}}(\tau R(x) + \mathcal{O}(\tau^{\frac{3}{2}})),
$$

in the classical low-energy limit.

$\boxed{\mathcal{N} = 2 \text{ SUSY}}$

The SUSY transformation of the model:

$$\delta\psi = 2V'(m^2)\epsilon, \quad \delta\bar{\psi} = 2\bar{\epsilon}V'(m^2),$$
$$\delta m = \epsilon\bar{\psi} + \psi\bar{\epsilon}.$$

Commutator of the SUSY transformation on shell:

$$[\delta_\epsilon, \delta_\xi]m = 2[\xi\bar{\epsilon} - \epsilon\bar{\xi}, V'(m^2)],$$
$$[\delta_\epsilon, \delta_\xi]\psi = 2\psi(\bar{\epsilon}m\frac{V'(m^2)}{m^2}\xi - \bar{\xi}m\frac{V'(m^2)}{m^2}\epsilon),$$

where we have utilized the equation of motion:

$$\frac{\partial S}{\partial\bar{\psi}} = 2m\psi = 0, \quad \frac{\partial S}{\partial\psi} = 2\bar{\psi}m = 0.$$

In order to see the structure of the $\mathcal{N}=2$ SUSY, we separate the SUSY parameters into the hermitian and the antihermitian parts as

$$\epsilon=\epsilon_1+i\epsilon_2,\ \ \xi=\xi_1+i\xi_2,$$

($\xi_1,\xi_2,\epsilon_1,\epsilon_2$ are Majorana-Weyl fermions.)

The translation is attributed to the quartic term in the Taylor expansion of $V(m)=\sum_{k=1}^{\infty}\frac{a_{2k}}{2k}m^{2k}$:

$$\begin{aligned}
[\delta_\epsilon,\delta_\xi]\psi &= 2\sum_{k=2}^{n}a_{2k}\psi(\bar{\xi}_1 m^{2k-3}\epsilon_1+\bar{\xi}_2 m^{2k-3}\epsilon_2)\\
&= 2a_4(\bar{\xi}_1\Gamma^i\epsilon_1+\bar{\xi}_2\Gamma^i\epsilon_2)\psi A_i+\cdots\\
&= -ia_4(\bar{\xi}_1\Gamma^i\epsilon_1+\bar{\xi}_2\Gamma^i\epsilon_2)(\partial_i\psi)+\cdots,\\
[\delta_\epsilon,\delta_\xi]A_a &= \frac{1}{8}\sum_{k=2}^{n}a_{2k}\left(\bar{\xi}_1[m^{2k-2},\Gamma_a]\epsilon_1+\bar{\xi}_2[m^{2k-2},\Gamma_a]\epsilon_2\right)\\
&= -\frac{a_4}{4}(\bar{\xi}_1\Gamma^i\epsilon_1+\bar{\xi}_2\Gamma^i\epsilon_2)[A_i,A_a]+\cdots\\
&= -\frac{ia_4}{4}(\bar{\xi}_1\Gamma^i\epsilon_1+\bar{\xi}_2\Gamma^i\epsilon_2)(\partial_i A_a)+\cdots.
\end{aligned}$$

There is a discrepancy between the coefficients of the fermions and the bosonic vectors.

It is a future problem to overcome this difficulty.

$\boxed{\text{Type IIB Supergravity in the low-energy limit}}$

If this matrix model is to be reduced to type IIB supergravity in the low-energy limit,

$$\Downarrow$$

the following field should be massless, while the other fields should be massive.

- The antisymmetric even-rank tensors $a'^{(i)}{}_{ia_1 \cdots a_{2n}}(x)$.
- Dilatino $\chi(x)$, gravitino $\chi^{(i)}(x)$.

$V(m^2)$ is determined so that

- The fields $a'^{(i)}{}_{ia_1 \cdots a_{2n}}(x)$ become massless.
- The cosmological constant cancels.
- The differential operator in the flat space $i\Gamma^a \partial_a$ becomes a classical solution.

We surmise that the desired fermionic fields will be massless due to supersymmetry.

If we find such a model, this will be an extension of IIB matrix model, with the gravity encoded more manifestly!!