Monte Carlo studies of the 4d supersymmetric reduced model with the Chern-Simons term

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Contents

1 Introduction

Motivation of the fuzzy-sphere studies:

- **Relation between the non-commutative field theory and the superstring theory.**
- **Novel regularization scheme alternative to the lattice regularization.**
- **Prototype of the curved-space background in the large-N reduced model.**

Matrix models on the homogeneous space

hep-th/0101102,0103192,0204256,0207115,0209057,0301055,0303120,0307007,0309264,0312241,0403242,0405096,0405277

2 Supersymmetric YMCS model

4d supersymmetric Yang-Mills-Chern-Simons (YMCS) model:

$$\boxed{S = N\mathrm{tr}\left(-\frac{1}{4}\sum_{\mu,\nu=1}^{4}[A_\mu, A_\nu]^2 + \frac{2}{3}i\,\alpha\sum_{\mu,\nu,\rho=1}^{3}\epsilon_{\mu\nu\rho}A_\mu A_\nu A_\rho + \bar{\psi}_\alpha\sum_{\mu=1}^{4}(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]\right)},$$

A_μ (ψ_α): $N \times N$ traceless hermitian (complex) matrices.

$$\Gamma_1 = i\sigma_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix},\ \Gamma_2 = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ \Gamma_3 = i\sigma_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix},\ \Gamma_4 = \sigma_4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Classical equation of motion:

$$[A_\nu, [A_\nu, A_\mu]] + i\alpha\epsilon_{\mu\nu\rho}[A_\nu, A_\rho] = 0.$$

Fuzzy S^2 classical solution:

$$A_\mu^{(\mathrm{S}^2)} = \begin{cases} \alpha L_\mu^{(N)}, & (\text{for } \mu = 1,2,3), \\ 0, & (\text{for } \mu = 4), \end{cases}$$

Numerical studies via the hybrid Monte Carlo (HMC) algorithm up to $N = 16$.

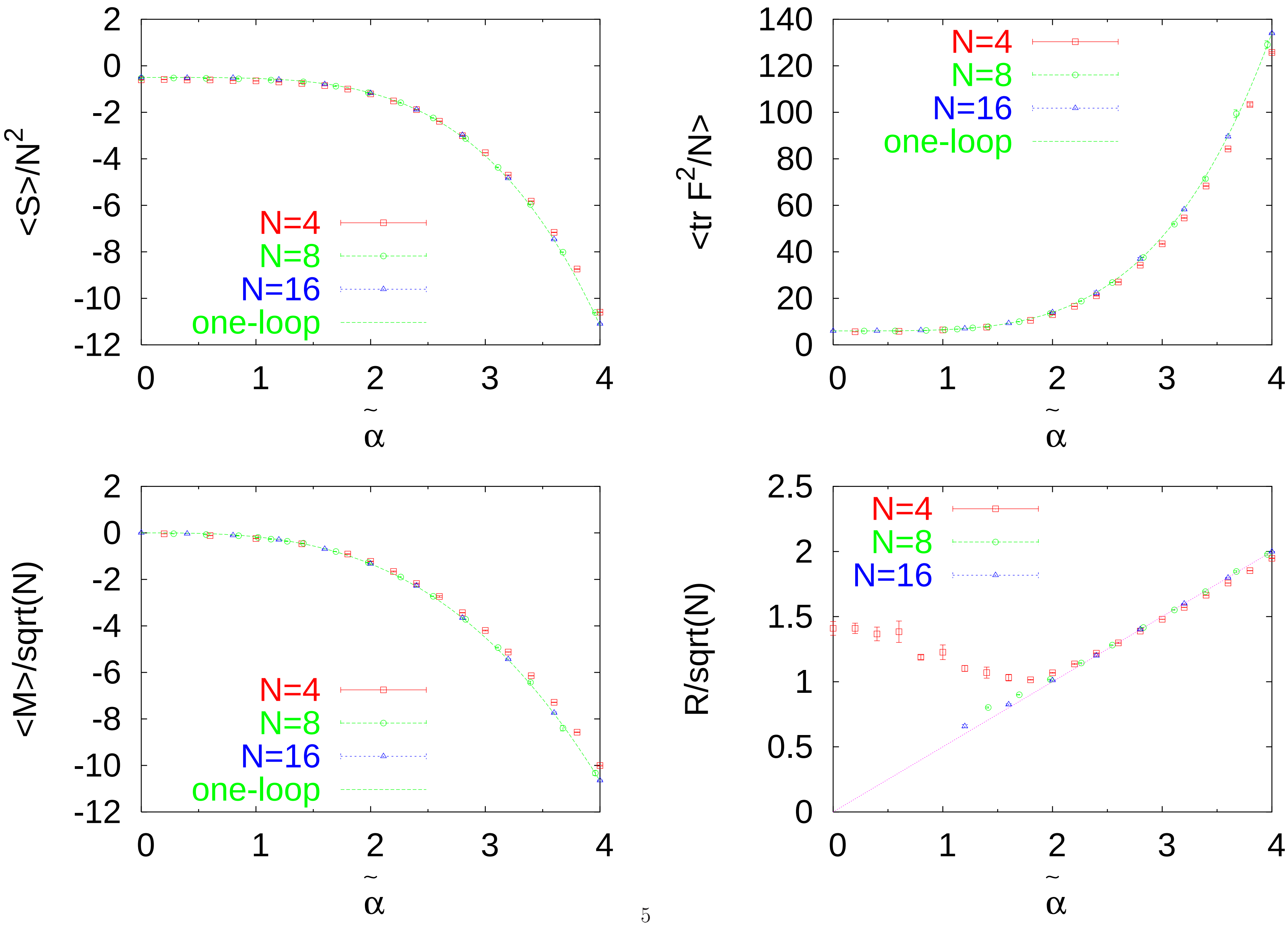
3 Result of the numerical studies

Fuzzy-sphere initial condition of the Monte Carlo simulation:

$$A_\mu^{(0)} = A_\mu^{(\mathrm{S}^2)}.$$

The observables to study ($\tilde{\alpha} = \alpha\sqrt{N}$):

$$\begin{aligned}
\frac{1}{N^2}\langle S\rangle &\simeq -\frac{\tilde{\alpha}^4}{24} \underbrace{-\frac{1}{2}}_{\mathrm{one-loop}}, \\
\langle \frac{1}{N}\mathrm{tr}\,(F_{\mu\nu}^2)\rangle &\simeq \frac{\tilde{\alpha}^4}{2} \underbrace{+6}_{\mathrm{one-loop}}, \\
\frac{1}{\sqrt{N}}\langle M\rangle &\simeq -\frac{\tilde{\alpha}^3}{6} \underbrace{+0}_{\mathrm{one-loop}}, \quad \left(M = \frac{2i}{3N}\sum_{i,j,k=1}^{3} \epsilon_{ijk}\mathrm{tr}\, A_i A_j A_k\right), \\
R &= \langle\sqrt{\frac{1}{N}\sum_{\mu=1}^{4}\mathrm{tr}\, A_\mu^2}\rangle.
\end{aligned}$$



<S>/N^2
N=4
N=8
N=16
one-loop
α̃
<tr F^2/N>
N=4
N=8
N=16
one-loop
α̃
<M>/sqrt(N)
N=4
N=8
N=16
one-loop
α̃
R/sqrt(N)
N=4
N=8
N=16
α̃

Transition point $\alpha_{\rm tr}$:

$$\alpha_{\rm tr} \simeq \begin{cases} 1.1 \ \ (N=4), \\ 0.5 \ \ (N=8), \\ 0.3 \ \ (N=16). \end{cases} \quad \Rightarrow \alpha_{\rm tr} \simeq \mathrm{O}(\frac{1}{N}).$$

Behavior of the eigenvalue distribution function $f(x)$ of the Casimir operator

$$Q = A_1^2 + A_2^2 + A_3^2 + A_4^2, \quad \langle \frac{1}{N} \mathrm{tr} Q \rangle \simeq \frac{15}{4}\alpha^2 + \underbrace{\frac{35}{48\alpha^2}}_{\rm one-loop} \quad (\text{for } N=4).$$

- $\alpha < \alpha_{\rm tr}$

  Spikes in the history of $\frac{1}{N}\mathrm{tr}\, A_\mu^2$.

  Power-law tail behavior $f(x) \simeq x^{-2}$.

  W. Krauth and M. Staudacher, Phys. Lett. B 453, 253 (1999), [hep-th/9902113].

- $\alpha > \alpha_{\rm tr}$

  No spikes in the history of $\frac{1}{N}\mathrm{tr}\, A_\mu^2$.

  Peaks of $f(x)$ around the classical radius $Q = \frac{N^2-1}{4}\alpha^2$.

  Absence of the power-law behavior.

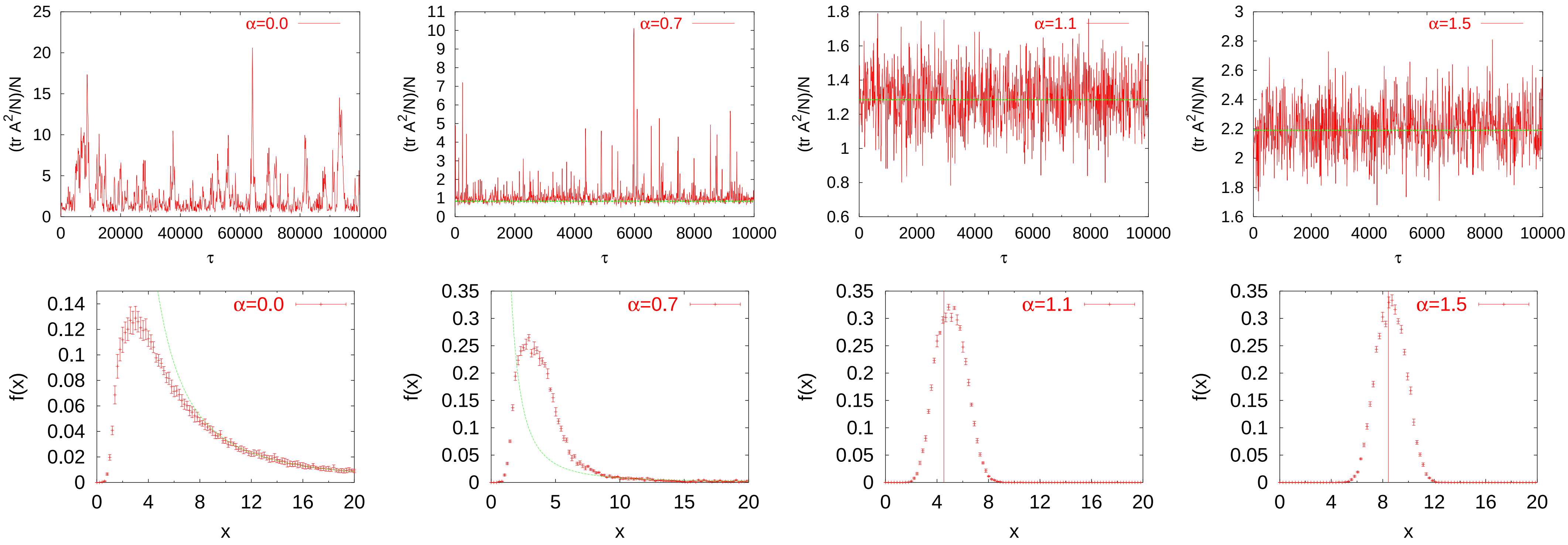


Figure 1: The history of $\frac{1}{N}\mathrm{tr}\, Q$ and the eigenvalue distribution $f(x)$ for $N = 4$.

Argument from one-loop effective action

One-loop perturbation around the fuzzy sphere solution

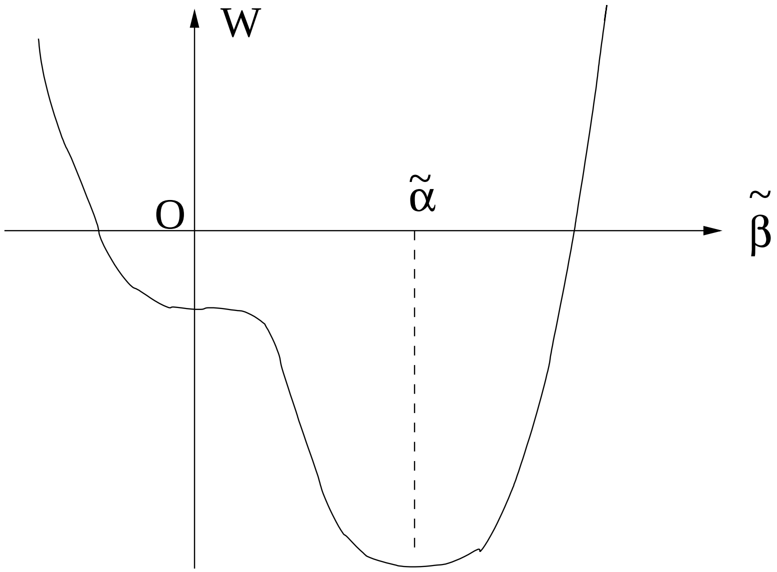
$$A_\mu = \begin{cases} \beta L_\mu^{(N)}, & (\text{for } \mu = 1, 2, 3), \\ 0, & (\text{for } \mu = 4), \end{cases}$$

One-loop effective action at large N:

$$W = N^2 \left(\frac{\tilde{\beta}^4}{8} - \frac{\tilde{\alpha}\tilde{\beta}^3}{6} \right) - N^2 \log N.$$

Minimum at $\tilde{\beta} = \tilde{\alpha}$.

The fuzzy sphere is always stable for fixed $\tilde{\alpha} (> 0)$ at large N.



4 Conclusion

Four-dimensional supersymmetric Yang-Mills-Chern-Simons (YMCS) model via the hybrid Monte Carlo (HMC) algorithm.

- The transition point $\alpha_{\rm tr} \simeq \mathrm{O}(\frac{1}{N})$.

  * $\alpha < \alpha_{\rm tr}$: Power-law behavior of eigenvalue distribution function $f(x) \simeq x^{-2}$.

  * $\alpha > \alpha_{\rm tr}$: Stability of the fuzzy sphere.

    Absence of the power-law behavior.

Future direction

- Simulation for larger N using more effecient codes.