Dynamical generation of gauge groups in the massive Yang-Mills-Chern-Simons matrix model

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Contents

1 Introduction

Matrix models on a homogeneous space

Motivations of fuzzy manifold studies:

- Relation between the non-commutative field theory and the superstring.
- Novel regularization scheme alternative to lattice regularization.
- Prototype of the curved-space background in the large-N reduced models.

Matrix models on a homogeneous space G/H:

$G=$ (a Lie group), $H =$ (a closed subgroup of G).

$S^2 = SU(2)/U(1)$, $S^2 \times S^2$, $S^4 = SO(5)/U(2)$, $CP^2 = SU(3)/U(2)$, $\cdots$.

Fuzzy spheres are compact $\Rightarrow$ realized by finite matrices.

The Yang-Mills-Chern-Simons (YMCS) model $\Rightarrow$ fuzzy sphere background.

2 The model and its classical solution

3d massive Yang-Mills-Chern-Simons model

⇒ a toy model with fuzzy sphere solutions:

$$\boxed{S[A] = N\alpha^4 \mathrm{tr}\left(-\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{2i}{3}\epsilon_{\mu\nu\rho}A_\mu A_\nu A_\rho + \frac{\rho^2}{2}A_\mu^2\right).}$$

- **Defined in the 3-dimensional Euclidean space $(\mu, \nu, \rho = 1, 2, 3)$.**
- **Convergence of the path integral** P. Austing and J. F. Wheater, hep-th/0310170.
  (mass term just suppresses the path integral)
- **Classical equation of motion:**

$$-[A_\nu, [A_\mu, A_\nu]] + i\,\epsilon_{\mu\nu\lambda}[A_\nu, A_\lambda] + \rho^2 A_\mu = 0.$$

Fuzzy sphere classical solution:

$$A_\mu = X_\mu = \chi \begin{pmatrix} L_\mu^{(n_1)} \otimes 1_{k_1} & & \\ & \ddots & \\ & & L_\mu^{(n_s)} \otimes 1_{k_s} \end{pmatrix}, \quad \text{where} \quad \begin{array}{l} \chi = \frac{1}{2}(1+\sqrt{1-2\rho^2}), \\ \sum_{i=1}^{s} n_i k_i = N, \\ 0 < \rho < \frac{1}{\sqrt{2}}. \end{array}$$

- $L_\mu^{(n)}$: $n \times n$ representation of SU(2) Lie algebra:

$$[L_\mu^{(n)}, L_\nu^{(n)}] = i\epsilon_{\mu\nu\rho} L_\rho^{(n)}, \quad (L_\mu^{(n)})^2 = \frac{n^2-1}{4}.$$

- Collection of k_i coincident fuzzy spheres.
- Expansion around this solution $\Rightarrow$ $U(k_1) \times U(k_2) \times \cdots \times U(k_s)$ gauge theory.
- $A_\mu = 0$ solution $\Rightarrow$ $s = 1$, $n_1 = 1$, $k_1 = N$.
- Classical free energy:

$$F_{\rm cl} = S_{\rm cl} = \frac{N\alpha^4}{24} f(\chi) \left(\sum_{i=1}^{s} k_i (n_i^3 - n_i) \right), \quad \text{where } f(\chi) = \frac{\chi^4}{2} - \frac{2\chi^3}{3} + \frac{\rho^2\chi^2}{2} \left(\lesseqqgtr 0 \text{ for } \rho \lesseqqgtr \frac{2}{3}\right).$$

3 Dynamical generation of gauge group

Evaluation of free energy at one-loop level.

$\boxed{\rho < \frac{2}{3}\ \text{regime}}$

Single $(s=1, n_1 = N, k_1 = 1)$ fuzzy sphere $A_\mu = \chi L_\mu^{(N)}$ $\Rightarrow F_{\text{cl}} = -\mathrm{O}(\alpha^4 N^4) < 0$.

$\Rightarrow$ **Coincident fuzzy spheres cannot be the true vacuum.**

Free energy for single fuzzy sphere:

$$\frac{F_{\text{FS}}}{N^2} = \frac{N^2\alpha^4}{4} f(\chi) + \frac{5}{2}\log N + 4\log\alpha - \delta(\rho).$$

Free energy in the Yang-Mills phase (based on Gaussian expansion)

J. Nishimura, T. Okubo and F. Sugino, hep-th/0205253.

$$\frac{F_{\text{YM}}}{N^2} = \frac{3}{2}\log N + 3\log\alpha + \underbrace{\gamma}_{\simeq -4.5}.$$

Critical point: $\alpha_{\text{cr}} = \frac{1}{\sqrt{N}}\left(\frac{2\log N}{|f(\chi)|}\right)^{1/4}$

- $\alpha < \alpha_{\text{cr}} \rightarrow F_{\text{YM}} < F_{\text{FS}}$: **Yang-Mills phase**
- $\alpha > \alpha_{\text{cr}} \rightarrow F_{\text{YM}} > F_{\text{FS}}$: **single fuzzy sphere**

$\boxed{\rho > \frac{2}{3} \text{ regime}}$

$\frac{F_{\rm cl}}{N^2} = \frac{1}{4}\alpha^4 f(\chi)(\sum_{i=1}^{s} r_i(n_i^3 - n_i)) > 0$ ($r_i = \frac{k_i}{N}$, so that $\sum_{i=1}^{s} n_i r_i = 1$) and
$\frac{F_{1-\rm loop}}{N^2} = \frac{3}{2}\log N + {\rm O}(1)$.

The stable fuzzy sphere should have lower free energy than $F_{A=0} = \frac{3}{2}\log N + {\rm O}(1)$.

The stable fuzzy sphere should satisfy $n_i = {\rm O}(1)$, $k_i = r_i N = {\rm O}(N)$.

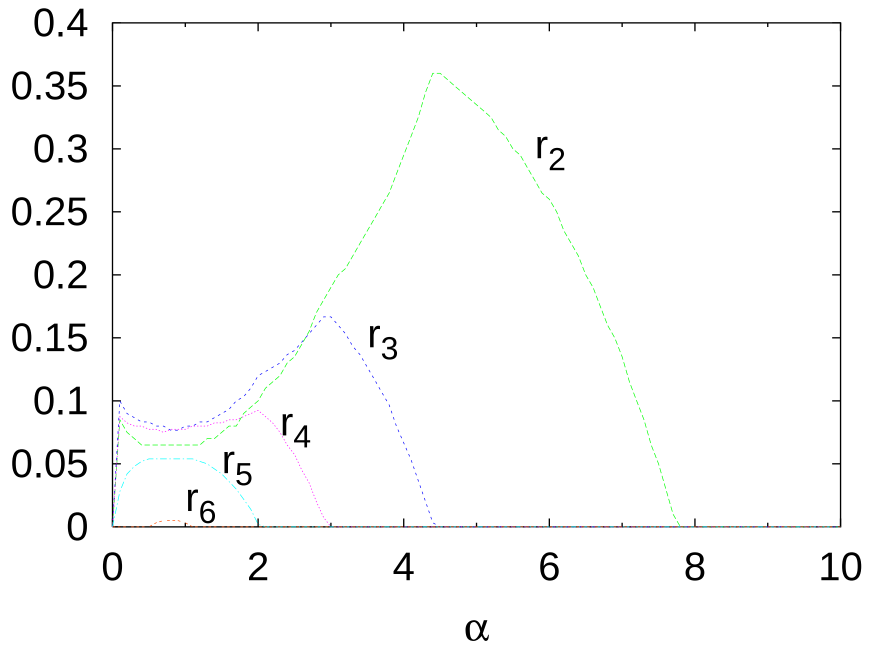
$\boxed{\text{U}(k_i) \text{ gauge group } (k_i = {\rm O}(N)) \text{ is dynamically generated.}}$

Analogous to coincident transverse 5-branes in M-theory.

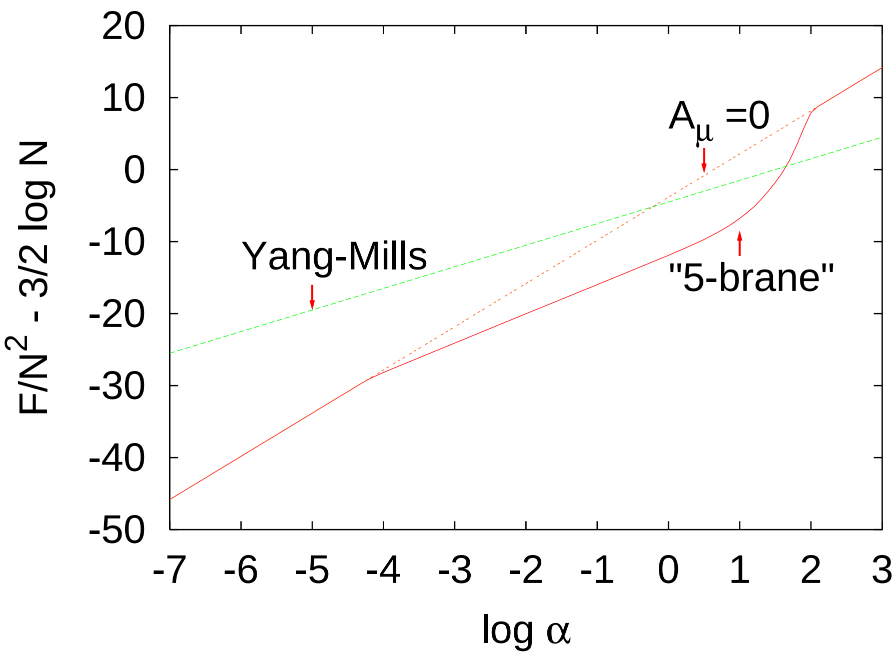
J. Maldacena, M. M. Sheikh-Jabbari and M. Van Raamsdonk, hep-th/0211139.

$n_1 = 1, n_2 = 2, \cdots, n_s = s$, **for brevity.**

r_i**'s that minimize the free energy at** $\rho = 0.7$**.**

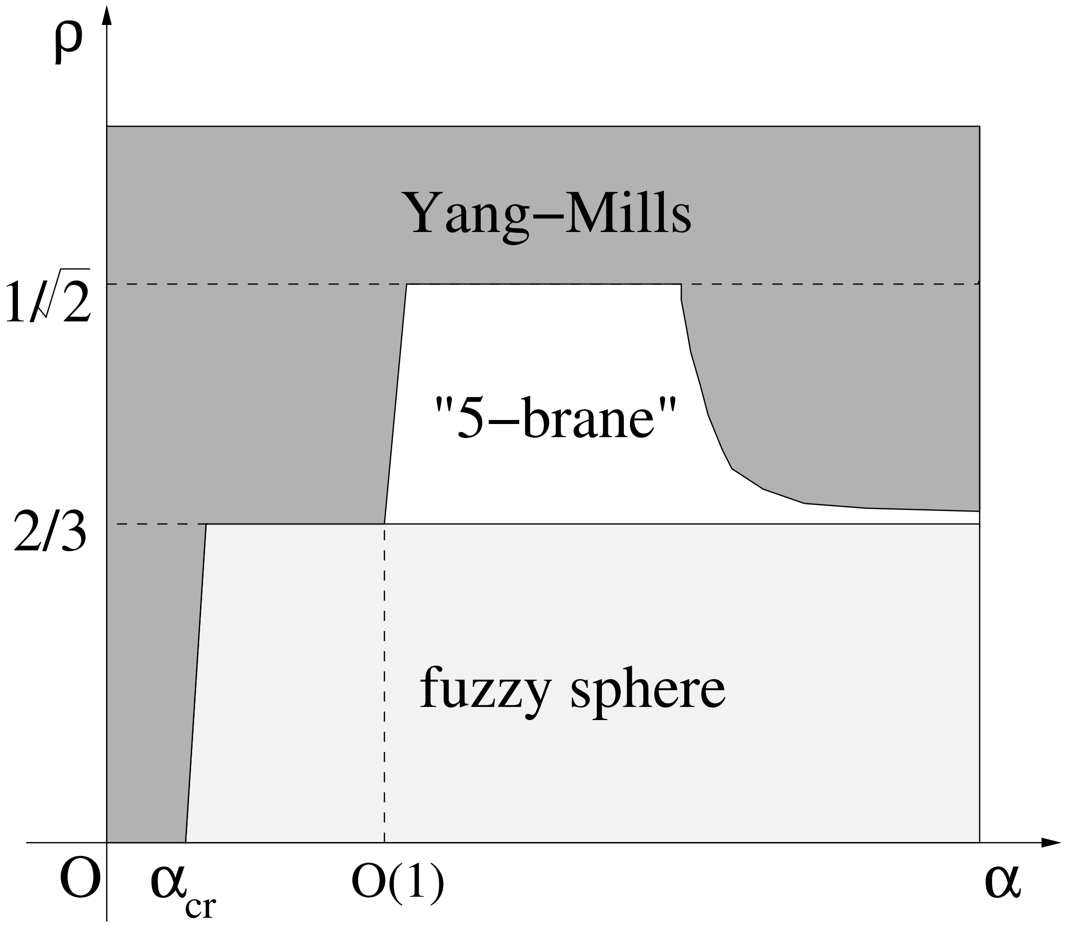


Comparison of free energy with Yang-Mills phase at $\rho = 0.7$**.**



The Yang-Mills phase takes over at $\alpha \simeq \mathrm{O}(1)$**.**

Phase diagram



4 Conclusion

Massive Yang-Mills-Chern-Simons model:

Many local minima with different gauge groups.

A nice laboratory for testing ideas and methods to study superstring theory by matrix models.

In this work, we have found

$\boxed{\text{Dynamical generation of nontrivial gauge group.}}$

The road to deriving the SU(3) × SU(2) × U(1) gauge group from the large-N reduced model?