Monte Carlo studies of the spontaneous rotational symmetry breaking in a simplified IKKT matrix model

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Collaboration with K.N. Anagnostopoulos and J. Nishimura

Contents

1 Introduction

Matrix models as a constructive definition of superstring theory

IKKT model (IIB matrix model) ⇒ Promising candidate for the constructive definition of superstring theory.

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115.

$$S = N\left(-\frac{1}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2 + \frac{1}{2}\mathrm{tr}\,\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right).$$

- **Dimensional reduction of $\mathcal{N}=1$ 10d Super-Yang-Mills (SYM) theory to 0d.**

  A_μ (10d vector) and ψ (10d Majorana-Weyl spinor) are $N \times N$ matrices .

- **Evidences for spontaneous breakdown of SO(10) symmetry to SO(4).**
  J. Nishimura and F. Sugino, hep-th/0111102, H. Kawai, et. al. hep-th/0204240,0211272,0602044,0603146.

- **Complex action is crucial for spontaneous breakdown of rotational symmetry:**
  J. Nishimura and G. Vernizzi, hep-th/0003223.

- **Difficulty of Monte Carlo simulation :**

  The determinant (from integrating out fermions) is complex.

2 Simplified IKKT model

Simplified model with spontaneous rotational symmetry breakdown,

J. Nishimura, hep-th/0108070.

$$S = \underbrace{\frac{N}{2}\mathrm{tr}\, A_\mu^2}_{=S_b} \underbrace{-\bar{\psi}_\alpha^f (\Gamma_\mu)_{\alpha\beta} A_\mu \psi_\beta^f}_{=S_f}$$

- A_μ: $N \times N$ hermitian matrices ($\mu = 1, \cdots, 4$)
  $\bar{\psi}_\alpha^f, \psi_\alpha^f$: N-dim vector ($\alpha = 1, 2, f = 1, \cdots, N_f$), N_f = (number of flavors).

$$\Gamma_1 = i\sigma_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \ \Gamma_2 = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \Gamma_3 = i\sigma_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \ \Gamma_4 = \sigma_4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

- SU(N) symmetry and SO(4) rotational symmetry.

- Partition function:

$$Z = \int dA e^{-S_B} (\det \mathcal{D})^{N_f} = \int dA e^{-S_0} e^{i\Gamma}, \text{ where}$$
$$\mathcal{D} = \Gamma_\mu A_\mu = (2N \times 2N \text{ matrices}), \quad e^{-S_0} = e^{-S_B} |\det \mathcal{D}|^{N_f}.$$

Analytical studies of the model

Solvable at $N \to \infty$ using random matrix theory (RMT) technique.

$$\langle \frac{1}{N} \mathrm{tr}\, A_\mu^2 \rangle = \begin{cases} 1 + r + \mathrm{o}(r), & (\mu = 1, 2, 3) \\ 1 - r + \mathrm{o}(r), & (\mu = 4), \end{cases}$$

for small $r = N_f/N$.

Spontaneous breakdown of SO(4) symmetry to SO(3).

For the phase-quenched partition function $Z_0 = \int dA e^{-S_0}$,

$\langle \frac{1}{N} \mathrm{tr}\, A_\mu^2 \rangle = 1 + r/2$ for $\mu = 1, 2, 3, 4$.

The phase plays a crucial role in the spontaneous rotational symmetry breakdown.

Gaussian expansion analysis up to 9th order:

T. Okubo, J. Nishimura and F. Sugino, hep-th/0412194.

Spontaneous breakdown of SO(4) to SO(2) at finite r.

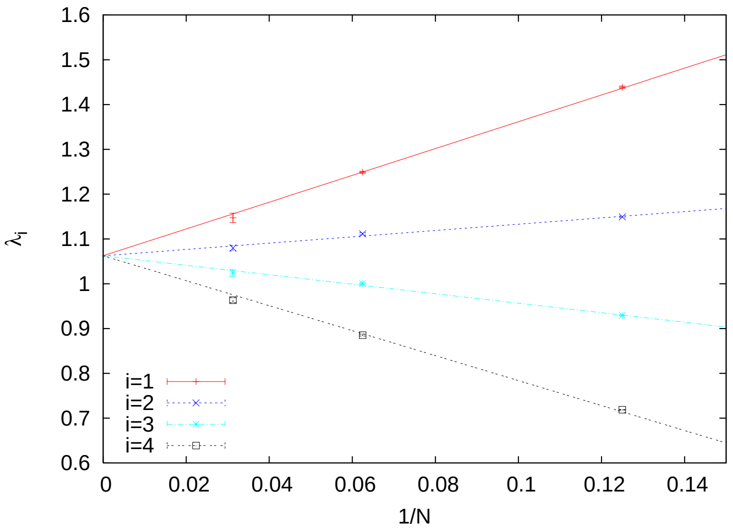
3 Monte Carlo studies of the model

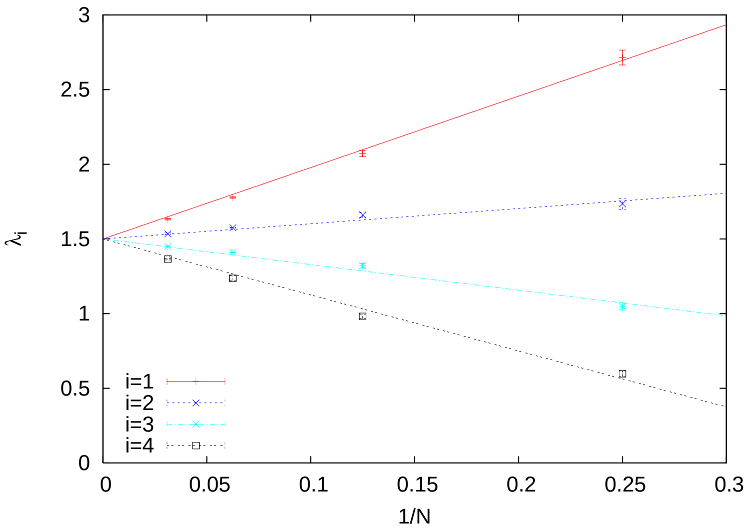
Hybrid Monte Carlo (HMC) simulation of the phase-quenched model

HMC simulation of the partition function Z_0 with the phase omitted.

Observable for probing dimensionality : $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$.

λ_i $(i = 1, 2, 3, 4)$: eigenvalues of $T_{\mu\nu}$ $(\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4)$





Results for $r = \frac{1}{8}$ (left) and $r = 1$ (right).

$$\lambda_1 = \cdots = \lambda_4 \to 1 + \frac{r}{2} \text{ (as } N \to \infty).$$

Factorization method

An approach to the complex action problem in Monte Carlo simulation.

K. N. Anagnostopoulos and J. Nishimura, hep-th/0108041,
J. Ambjorn, K. N. Anagnostopoulos, J. Nishimura and J. J. M. Verbaarschot, hep-lat/0208025.

Overlap problem: Discrepancy of a distribution function between the phase-quenched model Z_0 and the full model Z.

Force the simulation to sample the important region for the full model.

Standard reweighting method:

$$\langle \lambda_i \rangle = \frac{\langle \lambda_i \cos \Gamma \rangle_0}{\langle \cos \Gamma \rangle_0}, \text{ where } \langle * \rangle_0 = (\text{ V.E.V. for the phase-quenched model } Z_0).$$

(Number of configurations required) $\simeq e^{O(N^2)}$. $\Rightarrow$ complex-action problem.

$\tilde{\lambda}_i \stackrel{\text{def}}{=} \lambda_i / \langle \lambda_i \rangle_0$: **deviation from 1 $\Rightarrow$ effect of the phase.**

Distribution function

$$\rho_i(x) \stackrel{\text{def}}{=} \langle \delta(x - \tilde{\lambda}_i) \rangle = \frac{1}{C} \rho_i^{(0)}(x) w_i(x),$$

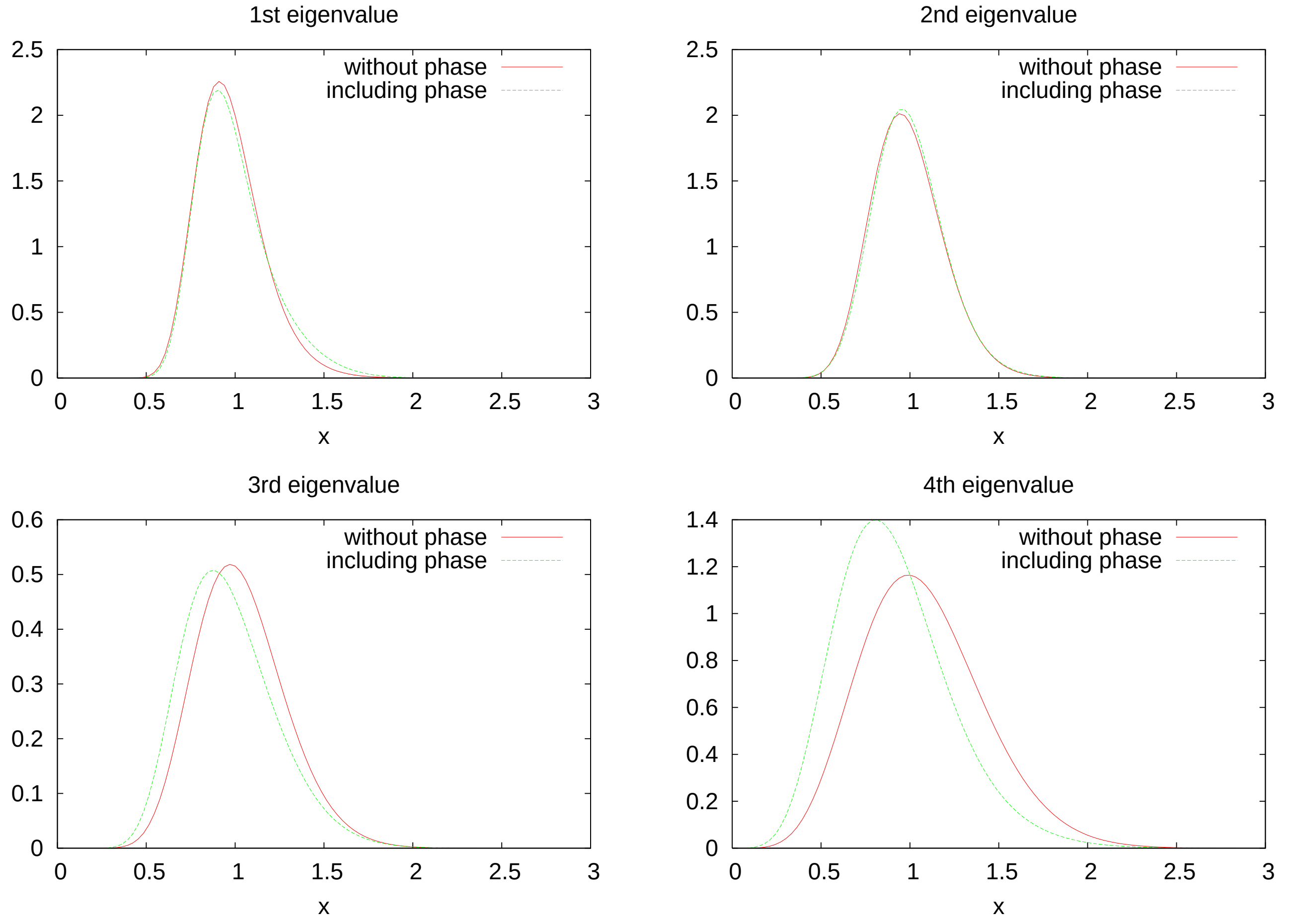
where

$$C = \langle \cos \Gamma \rangle_0, \quad \rho_i^{(0)}(x) = \langle \delta(x - \tilde{\lambda}_i) \rangle_0, \quad w_i(x) = \langle \cos \Gamma \rangle_{i,x},$$

$$\langle * \rangle_{i,x} = [\text{V.E.V. for the partition function } Z_{i,x} = \int dA e^{-S_0} \delta(x - \tilde{\lambda}_i)].$$

Resolution of the overlap problem: The system is forced to visit the configurations where $\rho_i(x)$ is important.

Eigenvalue distribution $\rho_i^{(0)}(x)$ (without phase) and $\rho_i(x)$ (including phase) $(i = 1, 2, 3, 4)$ for $N = 4, N_f = 4$:



(*) Gaussian expansion predicts the symmetry breakdown SO(4) $\rightarrow$ SO(2).

4 Conclusion

Monte Carlo simulation of the simplified IKKT model via factorization method.

Simulation of the $N = 4, N_f = 4$ case → symmetry breakdown of SO(4) to SO(2).

Future problems

- **Application of the multi-canonical method to matrix models.**

  B. A. Berg and T. Neuhaus, hep-lat/9202004 .

- **Simulation of the 6,10-dimensional IKKT model**

In practice, we approximate the partition function $Z_{i,x}$ by

$$Z_{i,V} = \int dA e^{-S_0} e^{-V(\lambda_i)}, \text{ where } V(x) = \frac{\gamma}{2}(x-\xi)^2, \quad \gamma, \xi = (\text{parameters}).$$

Monte Carlo evaluation of $\rho_i^{(0)}(x)$ and $w_i(x)$:

$$\rho_{i,V}(x) \stackrel{\text{def}}{=} \langle \delta(x - \tilde{\lambda}_i) \rangle_{i,V} \propto \rho_i^{(0)}(x) \exp(-V(\langle \lambda_i \rangle_0 x)).$$

The position of the peak x_p for the distribution function $\rho_{i,V}(x)$:

$$0 = \frac{\partial}{\partial x} \log \rho_{i,V}(x) = f_i^{(0)}(x) - \langle \lambda_i \rangle_0 V'(\langle \lambda_i \rangle_0 x), \text{ where } f_i^{(0)}(x) \stackrel{\text{def}}{=} \frac{\partial}{\partial x} \log \rho_i^{(0)}(x).$$

- Determination of x_p: $\rho_{i,V}(x)$ has a sharp peak for large γ

  $\Rightarrow$ x_p is approximated as $x_p \simeq \langle \tilde{\lambda}_i \rangle_{i,V}$.

- Determination of $\rho_i^{(0)}(x)$: Vary ξ, and calculate $f_i^{(0)}(x_p)$ for different x_p.

  Then, evaluate $\rho_i^{(0)}(x) = \exp[\int_0^x dz f_i^{(0)}(z) + \text{const.}]$.

$\boxed{\text{Multi-canonical method}}$ B. A. Berg and T. Neuhaus, hep-lat/9202004 .

Simulation of various ξ all at once.