Monte Carlo studies of the spontaneous rotational symmetry breaking in a matrix model with the complex action

Takehiro Azuma

Department of Mathematics and Physics, Setsunan University

JPS meeting 2009 at Rikkyo University, Mar. 30th 2009, 10:45 ∼ 11:00

Collaboration with K.N. Anagnostopoulos and J. Nishimura

Contents

1 Introduction

Matrix models as a constructive definition of superstring theory

IKKT model (IIB matrix model)

⇒ **Promising candidate for the constructive definition of superstring theory.**
Ishibashi, Kawai, Kitazawa and Tsuchiya, hep-th/9612115.

$$S = N\left(-\frac{1}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2 + \frac{1}{2}\mathrm{tr}\,\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]\right).$$

- A_μ (10d vector) and ψ_α (10d Majorana-Weyl spinor) ⇒ $N \times N$ matrices .
- Evidences for spontaneous breakdown of SO(10) symmetry to SO(4).
  Nishimura and Sugino, hep-th/0111102, Kawai, et. al. hep-th/0204240,0211272,0602044,0603146.
- Complex fermion determinant:
  * Crucial for rotational symmetry breaking.
    Nishimura and Vernizzi, hep-th/0003223.
  * Difficulty of Monte Carlo simulation.

2 Simplified IKKT model

Simplified model with spontaneous rotational symmetry breakdown

Nishimura, hep-th/0108070.

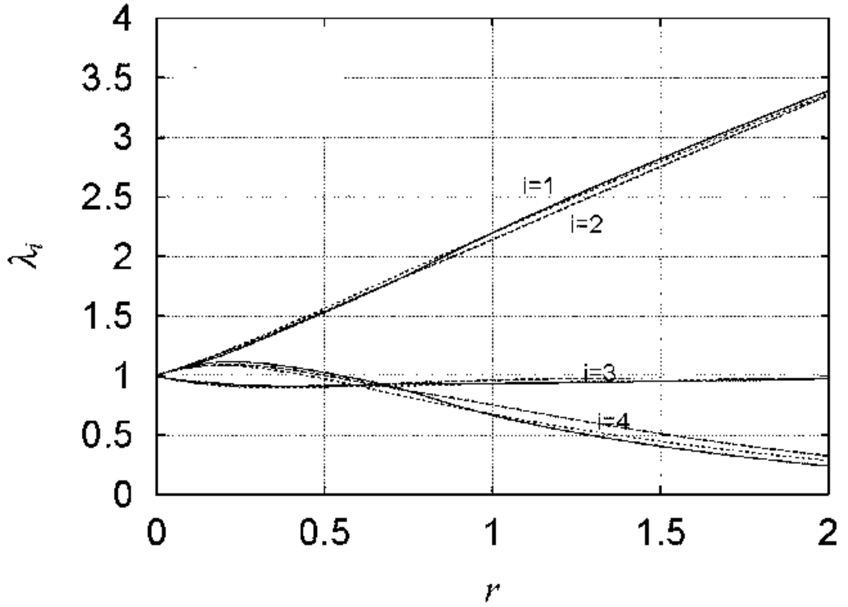
$$\boxed{S = \underbrace{\frac{N}{2}\mathrm{tr}\, A_\mu^2}_{=S_b} \underbrace{- \bar{\psi}_\alpha^f (\Gamma_\mu)_{\alpha\beta} A_\mu \psi_\beta^f}_{=S_f}}$$

- A_μ: $N \times N$ hermitian matrices ($\mu = 1, \cdots, 4$)
  $\bar{\psi}_\alpha^f, \psi_\alpha^f$: N-dim vector ($\alpha = 1, 2, f = 1, \cdots, N_f$)
  $N_f =$ (number of flavors)
- SO(4) rotational symmetry.
- No supersymmetry.

Gaussian expansion analysis up to 9th order:
Okubo, Nishimura and Sugino, hep-th/0412194.

Observable for probing dimensionality : $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$.

λ_i $(i = 1, 2, 3, 4)$ **: eigenvalues of** $T_{\mu\nu}$ $(\lambda_1 \geqq \lambda_2 \geqq \lambda_3 \geqq \lambda_4)$



Spontaneous breakdown of SO(4) to SO(2) at finite $r\left(=\frac{N_f}{N}\right)$.

3 Monte Carlo simulation

Factorization method

An approach to the complex action problem in Monte Carlo simulation.
Anagnostopoulos and Nishimura, hep-th/0108041,

Partition function:

$$Z = \int dA e^{-S_B} (\det \mathcal{D})^{N_f} = \int dA e^{-S_0} e^{i\Gamma}, \text{ where } \mathcal{D} = \Gamma_\mu A_\mu,$$

$$Z_0 = \int dA e^{-S_0} = \int dA e^{-S_B} |\det \mathcal{D}|^{N_f}.$$

Distribution function

$$\rho_i(x) \stackrel{\text{def}}{=} \langle \delta(x - \tilde{\lambda}_i) \rangle = \frac{1}{C} \rho_i^{(0)}(x) w_i(x), \text{ where}$$

$$\tilde{\lambda}_i = \lambda_i / \langle \lambda_i \rangle_0, \quad C = \langle \cos \Gamma \rangle_0,$$

$$\rho_i^{(0)}(x) = \langle \delta(x - \tilde{\lambda}_i) \rangle_0, \quad w_i(x) = \langle \cos \Gamma \rangle_{i,x},$$

$$\langle * \rangle_0 = [\text{V.E.V. for the phase-quenched partition function } Z_0]$$

$$\langle * \rangle_{i,x} = [\text{V.E.V. for the partition function } Z_{i,x} = \int dA e^{-S_0} \delta(x - \tilde{\lambda}_i)].$$

The position of the peak x_p for the distribution function $\rho_{i,V}(x)$:

$$0 = \frac{\partial}{\partial x} \log \rho_{i,V}(x) = f_i^{(0)}(x) - \langle \lambda_i \rangle_0 V'(\langle \lambda_i \rangle_0 x), \text{ where } f_i^{(0)}(x) \stackrel{\text{def}}{=} \frac{\partial}{\partial x} \log \rho_i^{(0)}(x).$$

Monte Carlo evaluation of $\langle\tilde{\lambda}_i\rangle$

$w_i(x) > 0 \Rightarrow \langle\tilde{\lambda}_i\rangle$ is the minimum of $\mathcal{F}_i(x)$:

$$\mathcal{F}_i(x) = (\text{free energy density}) = -\frac{1}{N^2}\log\rho_i(x).$$

We solve $\mathcal{F}_i'(x) = 0$, namely $\frac{1}{N^2}f_i^{(0)}(x) = -\frac{d}{dx}\left\{\frac{1}{N^2}\log w_i(x)\right\}$.

Both $\frac{1}{N^2}\log w_i(x)$ and $\frac{1}{N^2}f_i^{(0)}(x)$ scale at large N as

$$\frac{1}{N^2}\log w_i(x) \to \Phi_i(x), \quad \frac{1}{N^2}f_i^{(0)}(x) \to F_i(x).$$

Behavior of $\Phi_i(x)$

Asymptotic behavior of $\Phi_i(x) = \frac{1}{N^2}\log w_i(x)$ at $x \ll 1$ and $x \gg 1$.

When we fix the i-th largest eigenvalue $\to$

- $x \ll 1$ $(i = 2,3,4)$: $(5-i)$ directions are shrunk $\Rightarrow$ $(i-1)$-dim. configuration
- $x \gg 1$ $(i = 1,2,3)$: $(4-i)$ directions are shrunk $\Rightarrow$ i-dim. configuration

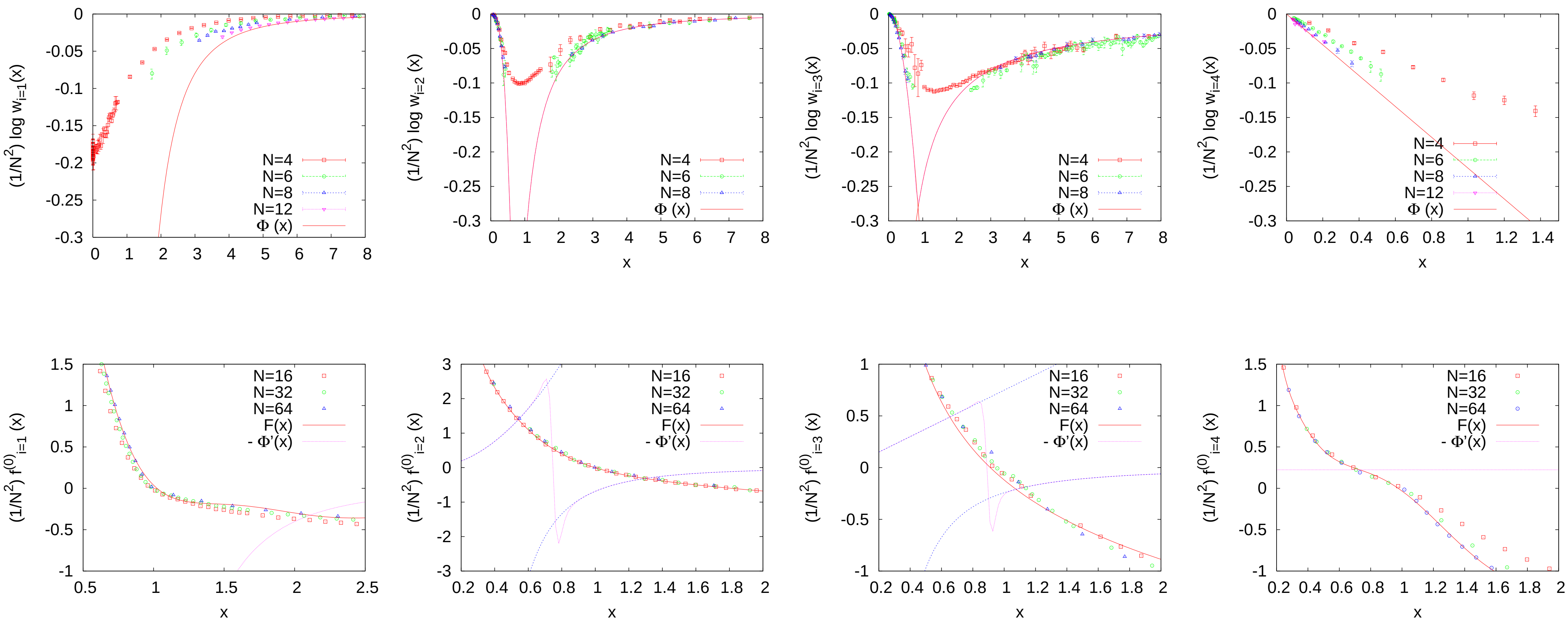
Expected power behaviors:

$$\Phi_i(x) \propto \begin{cases} c_{i,0}x^{5-i} + \cdots & (x \ll 1, i = 2,3,4) \\ \frac{d_{i,0}}{x^{4-i}} + \cdots & (x \gg 1, i = 1,2,3) \end{cases}$$

Simulation for $r = 1$

Evaluation of $\langle \tilde{\lambda}_i \rangle$ at the leading order.

$i = 1$ $i = 2$ $i = 3$ $i = 4$



$\langle \tilde{\lambda}_{i=2} \rangle = 1.3$, $\langle \tilde{\lambda}_{i=3} \rangle = 0.7 \Rightarrow$ **Rotational symmetry breaking SO(4) → SO(2).**

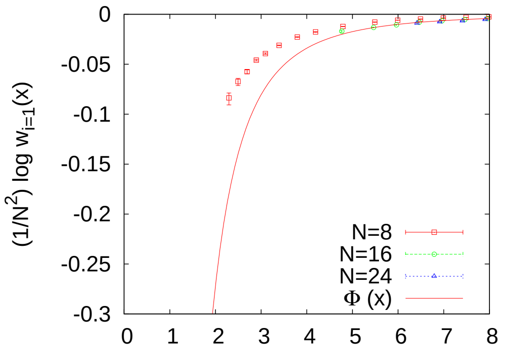
Result of 9th-order Gaussian expansion:

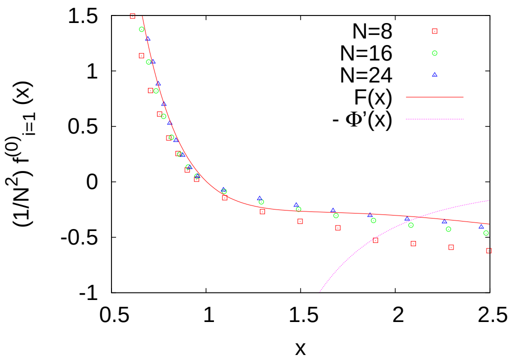
$$\tilde{\lambda}_{i=1} \simeq 1.4,\ \tilde{\lambda}_{i=2} \simeq 1.4,\ \tilde{\lambda}_{i=3} \simeq 0.7,\ \tilde{\lambda}_{i=4} \simeq 0.5.$$

Simulation for $r = 2$

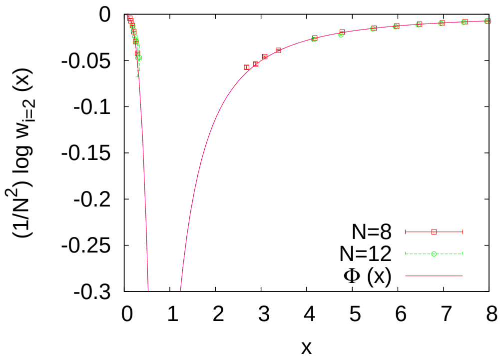
Evaluation of $\langle \tilde{\lambda}_i \rangle$ at the leading order.

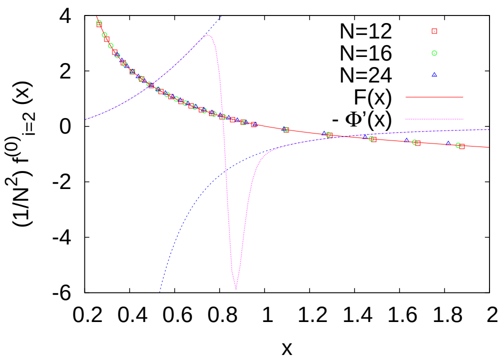
$i = 1$



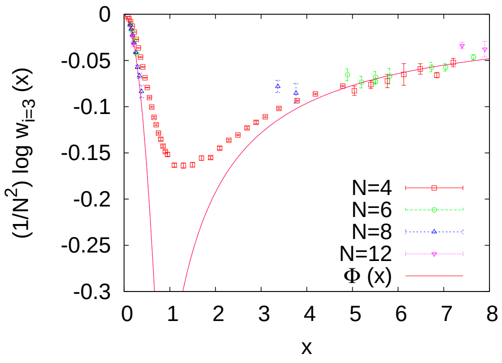


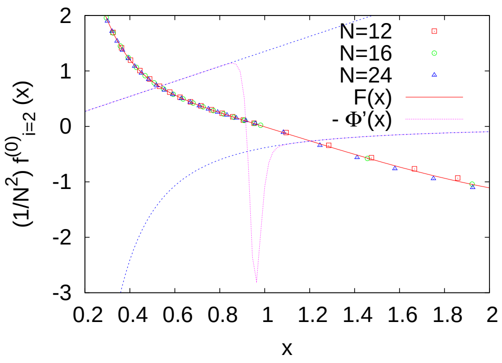
$i = 2$



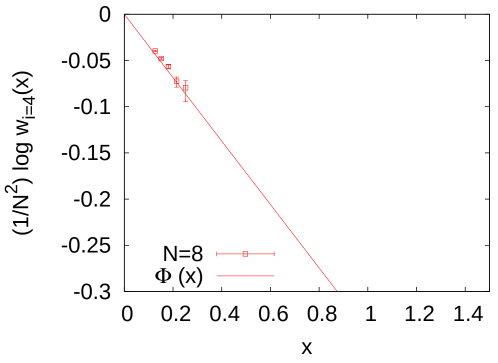


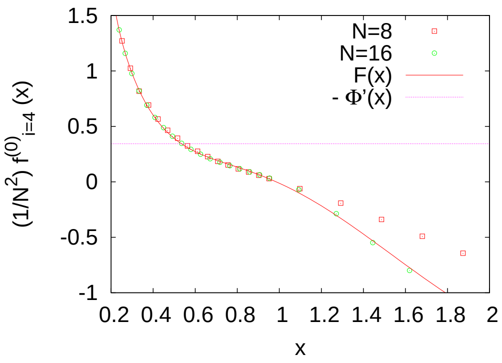
$i = 3$





$i = 4$





$\langle \tilde{\lambda}_{i=2} \rangle = 1.4$, $\langle \tilde{\lambda}_{i=3} \rangle = 0.5 \Rightarrow$ **Rotational symmetry breaking SO(4) → SO(2).**

Result of 9th-order Gaussian expansion:

$$\tilde{\lambda}_{i=1} \simeq 1.7,\ \tilde{\lambda}_{i=2} \simeq 1.7,\ \tilde{\lambda}_{i=3} \simeq 0.5,\ \tilde{\lambda}_{i=4} \simeq 0.1.$$

Behavior of $\frac{1}{N^2} f_i^{(0)}(x)$

Small x $(x \ll 1) \to (5-i)$ directions are shrunk.

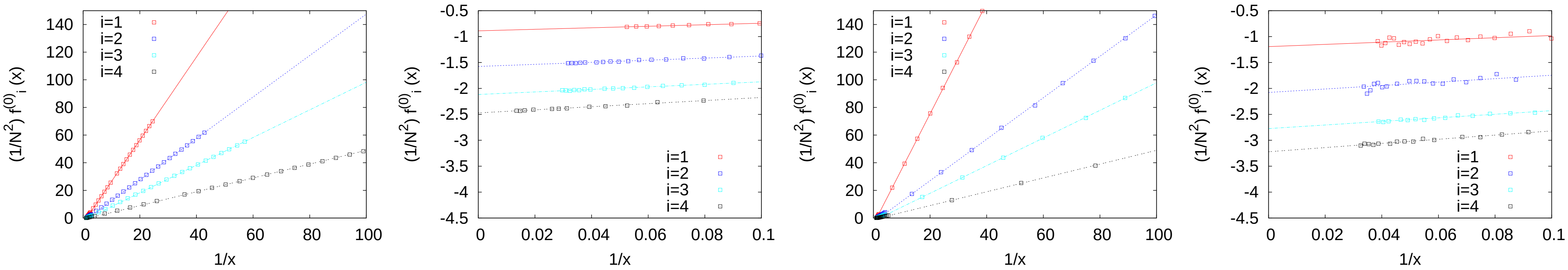
- $i = 2, 3, 4$: $\rho_i^{(0)}(x) \simeq (\sqrt{x})^{N^2(5-i)} \Rightarrow \frac{1}{N^2} f_i^{(0)}(x) = \left(\frac{5-i}{2x}\right) + e_i$
- $i = 1$: **Eigenvalues of A_μ are collapsed to zero.**

  $\Rightarrow$ **Add the effect of fermionic determinant (polynomial of A_μ with degree $2N^2 r$).**

  $\Rightarrow$ $\rho_{i=1}^{(0)}(x) \simeq (\sqrt{x})^{2N^2(1+r)} \Rightarrow \frac{1}{N^2} f_i^{(0)}(x) = \left(\frac{2+r}{x}\right) + e_{i=1}$

Large x $(x \gg 1)$: $\frac{1}{N^2} f_i^{(0)}(x) \xrightarrow{x\to+\infty} g_{i,0} + \frac{g_{i,1}}{x} + \cdots, \quad g_{i,0} = -\frac{xi}{2}\langle\lambda_0\rangle \xrightarrow{\text{large } N} -\frac{xi}{2}\left(1 + \frac{r}{2}\right)$

Simulation for $r = 1, N = 16$ **$r = 2, N = 16$**



4 Conclusion

Monte Carlo simulation of the simplified IKKT model.

Factorization method forces the simulation to visit important configurations.

Rotational symmetry breaking SO(4) $\rightarrow$ SO(2)

Future problems

Simulation of the IKKT model

Anagnostopoulos, Aoyama, T. A., Hanada and Nishimura, in progress

Supersymmetry $\Rightarrow$ Solution of $\mathcal{F}'_i(x) = 0$ at $x \ll 1$ and $x \gg 1$.

Asymptotic behaviors of $\frac{1}{N^2} \log w_i(x)$ and $\frac{1}{N^2} f_i^{(0)}(x)$ are important.