Supermatrix Models

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Takehiro Azuma
Department of Physics, Kyoto University

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collaborated with S. Iso, H. Kawai and Y. Ohwashi

Contents

1 Introduction

$\boxed{\text{constructive definition of superstring theory}}$

Large N reduced models are the most powerful candidate for the constructive definition of superstring theory.

$\boxed{\text{IKKT model}}$

N.Ishibashi, H.Kawai, Y.Kitazawa and A.Tsuchiya, hep-th/9612115.

For a review, hep-th/9908038

Dimensional reduction of $\mathcal{N}=1$ 10-dimensional SYM theory to 0 dimension.

Matrix regularization of Green-Schwarz action of type IIB superstring theory.

$$S = -\frac{1}{g^2} Tr_{N\times N}\left(\frac{1}{4}\sum_{i,j=0}^{9}[A_i, A_j]^2 + \frac{1}{2}\bar{\psi}\sum_{i=0}^{9}\Gamma^i[A_i, \psi]\right).$$

- $SO(10)\times SU(N)$ gauge symmetry.
- $\mathcal{N}=2$ SUSY.
  - * homogeneous : $\delta_\epsilon^{(1)} A_i = i\bar{\epsilon}\Gamma_i\psi, \quad \delta_\epsilon^{(1)}\psi = \frac{i}{2}\Gamma^{ij}[A_i, A_j]\epsilon.$
  - * inhomogeneous : $\delta_\xi^{(2)} A_i = 0, \quad \delta_\xi^{(2)}\psi = \xi.$
  - * $[\delta_\epsilon^{(1)}, \delta_\xi^{(2)}]A_i = -i\bar{\epsilon}\Gamma_i\xi, \quad [\delta_\epsilon^{(1)}, \delta_\xi^{(2)}]\psi = 0.$
- The matrices describe the many-body system.
- No free parameter: $A_\mu \to g^{\frac{1}{2}}A_\mu$, $\psi \to g^{\frac{3}{4}}\psi$.

2 $osp(1|32,R)$ cubic matrix model

We investigate a matrix model based on super Lie algebra $osp(1|32,R)$.

L. Smolin, hep-th/0002009

T. Azuma, S. Iso, H. Kawai and Y. Ohwashi, hep-th/0102168

$osp(1|32,R)$ super Lie algebra

- $M \in osp(1|32,R) \Rightarrow {}^T MG + GM = 0$,
  where $G = \begin{pmatrix} \Gamma^0 & 0 \\ 0 & i \end{pmatrix}$.
- $M = \begin{pmatrix} m & \psi \\ i\bar{\psi} & 0 \end{pmatrix}$, where $m\Gamma^0 + \Gamma^0 m = 0$ $(m \in sp(32))$.
- $m = u_{\mu_1}\Gamma^{\mu_1} + \frac{1}{2!}u_{\mu_1\mu_2}\Gamma^{\mu_1\mu_2} + \frac{1}{5!}u_{\mu_1\cdots\mu_5}\Gamma^{\mu_1\cdots\mu_5}$.

action of the cubic model

$$\begin{aligned}
I &= \frac{i}{g^2} Tr_{N\times N} \sum_{Q,R=1}^{33} [(\sum_{p=1}^{32} {M_p}^Q[{M_Q}^R, {M_R}^p]) - {M_{33}}^Q[{M_Q}^R, {M_R}^{33}]] \\
&= -\frac{f^{abc}}{2g^2} \sum_{a,b,c=1}^{N^2} Str_{33\times 33}(M_a M_b M_c) \\
&= \frac{i}{g^2} Tr_{N\times N}[{m_p}^q[{m_q}^r, {m_r}^p] - 3i\bar{\psi}^p[{m_p}^q, \psi^q]].
\end{aligned}$$

- Each component of the 33×33 supermatrices is promoted to a large N hermitian matrix.
- No free parameter: $M \to g^{\frac{2}{3}} M$.
- $OSp(1|32,R) \times U(N)$ gauge symmetry.
  - $M \to M + [M, (S \otimes 1_{N\times N})]$ for $S \in osp(1|32,R)$,
  - $M \to M + [M, (1_{33\times 33} \otimes U)]$ for $U \in u(N)$.

Supersymmetry

The SUSY transformation of the $osp(1|32,R)$ is identified with that of IKKT model.

- homogeneous SUSY:
  The SUSY transformation by the supercharge
  $Q = \begin{pmatrix} 0 & \chi \\ i\bar{\chi} & 0 \end{pmatrix}$.

$$\delta^{(1)}_{\chi} M = [Q, M] = \begin{pmatrix} i(\chi\bar{\psi} - \psi\bar{\chi}) & -m\chi \\ i\bar{\chi}m & 0 \end{pmatrix}.$$

- inhomogeneous SUSY:
  The translation of the fermionic field $\delta^{(2)}_{\epsilon}\psi = \epsilon$.

In order to see the correspondence of the fields with IKKT model, we express the bosonic 32×32 matrices in terms of the 10-dimensional indices ($i = 0, \cdots 9$, $\sharp = 10$).

$$\begin{aligned} m &= W\Gamma^{\sharp} + \frac{1}{2}[A^{(+)}_i \Gamma^i (1 + \Gamma^{\sharp}) + A^{(-)}_i \Gamma^i (1 - \Gamma^{\sharp})] + \frac{1}{2!} C_{i_1 i_2} \Gamma^{i_1 i_2} + \\ &\quad + \frac{1}{4!} H_{i_1 \cdots i_4} \Gamma^{i_1 \cdots i_4 \sharp} + \frac{1}{5!}[I^{(+)}_{i_1 \cdots i_5} \Gamma^{i_1 \cdots i_5}(1 + \Gamma^{\sharp}) + I^{(-)}_{i_1 \cdots i_5} \Gamma^{i_1 \cdots i_5}(1 - \Gamma^{\sharp})]. \end{aligned}$$

Identification of the fields

$$\begin{aligned} \delta^{(1)}_{\chi} A^{(+)}_i &= \frac{i}{16} \bar{\chi} \Gamma_i (1 - \Gamma_{\sharp}) \psi = \frac{i}{8} \bar{\chi}_R \Gamma_i \psi_R, \\ \delta^{(1)}_{\chi} A^{(-)}_i &= \frac{i}{16} \bar{\chi} \Gamma_i (1 + \Gamma_{\sharp}) \psi = \frac{i}{8} \bar{\chi}_L \Gamma_i \psi_L, \\ \delta^{(1)}_{\chi} \psi &= -m\psi. \end{aligned}$$

Commutation relations

- $[\delta^{(1)}_\chi, \delta^{(2)}_\epsilon] m = -i(\chi\bar{\epsilon} - \epsilon\bar{\chi}), \quad [\delta^{(1)}_\chi, \delta^{(2)}_\epsilon]\psi = 0.$

$$[\delta^{(1)}_{\chi_R}, \delta^{(2)}_{\epsilon_R}] A^{(+)}_i = \frac{i}{8}\bar{\epsilon}_R \Gamma_i \chi_R, \quad [\delta^{(1)}_{\chi_L}, \delta^{(2)}_{\epsilon_L}] A^{(+)}_i = 0,$$

$$[\delta^{(1)}_{\chi_R}, \delta^{(2)}_{\epsilon_R}] A^{(-)}_i = 0, \quad [\delta^{(1)}_{\chi_L}, \delta^{(2)}_{\epsilon_L}] A^{(-)}_i = \frac{i}{8}\bar{\epsilon}_L \Gamma_i \chi_L,$$

$$[\delta^{(1)}_{\chi_L}, \delta^{(2)}_{\epsilon_R}] A^{(\pm)}_i = [\delta^{(1)}_{\chi_R}, \delta^{(2)}_{\epsilon_L}] A^{(\pm)}_i = 0.$$

- $[\delta^{(2)}_\chi, \delta^{(2)}_\epsilon] m = [\delta^{(2)}_\chi, \delta^{(2)}_\epsilon]\psi = 0$ is trivial.
- $[\delta^{(1)}_\chi, \delta^{(1)}_\epsilon] m = i[\chi\bar{\epsilon} - \epsilon\bar{\chi}, m], \quad [\delta^{(1)}_\chi, \delta^{(1)}_\epsilon]\psi = i(\chi\bar{\epsilon} - \epsilon\bar{\chi})\psi.$

  * $[\delta^{(1)}_{\chi_R}, \delta^{(1)}_{\epsilon_R}] A^{(+)}_i = \frac{i}{8}\bar{\chi}_R [m, \Gamma_i]\epsilon_R.$
  In the (r.h.s.), the fields W, $C_{i_1 i_2}$ and $H_{i_1 \cdots i_4}$ survive.
  $\rightarrow$ these fields are integrated out.

  * $[\delta^{(1)}_{\chi_L}, \delta^{(1)}_{\epsilon_R}] A^{(+)}_i = -\frac{i}{8}\bar{\chi}_L A^{(+)}_j \Gamma_i{}^j \epsilon_R + \cdots.$
  The fields $A^{(\pm)}_i$ itself remains in the commutator!

Summary

The $osp(1|32, R)$ cubic matrix model possesses a two-fold structure of the SUSY of IKKT model.

IKKT model	bosons A_i	fermions ψ	SUSY parameters
SUSY I	$A^{(+)}_i$	ψ_R	χ_R, ϵ_R
SUSY II	$A^{(-)}_i$	ψ_L	χ_L, ϵ_L

3 $gl(1|32, R) \otimes gl(N)$ gauged model

We consider the model whose gauge symmetry is enhanced by altering the direct product of the Lie algebra.

L. Smolin, hep-th/0006137

T. Azuma, S. Iso, H. Kawai and Y. Ohwashi, hep-th/0102168

(*) $\mathcal{A}, \mathcal{B}$ = [The Lie algebras whose bases are $\{a_i\}$ and $\{b_j\}$, respectively.]

- $\mathcal{A} \otimes \mathcal{B}$: The space spanned by the basis $a_i \otimes b_j$. This is not necessarily a closed Lie algebra.
- $\mathcal{A} \check{\otimes} \mathcal{B}$: The smallest Lie algebra that includes $\mathcal{A} \otimes \mathcal{B}$ as a subset.

The gauge symmetry $OSp(1|32, R) \times U(N)$ is enhanced to $osp(1|32, R) \check{\otimes} u(N)$.

- $osp(1|32, R) \otimes u(N)$ is not a closed Lie algebra.
- $osp(1|32, R) \check{\otimes} u(N) = u(1|16, 16) \otimes u(N)$.
  $u(1|16, 16)$ is the complexification of $osp(1|32, R)$.
- We consider the Lie algebra
  $gl(1|32, R) \check{\otimes} gl(N) = gl(1|32, R) \otimes gl(N)$
  as an analytical continuation of $u(1|16, 16) \otimes u(N)$.

4 Conclusion

- **We have investigated the cubic model whose gauge symmetry is the super Lie algebra $\boldsymbol{OSp(1|32,R)\times U(N)}$.**
- **$\boldsymbol{osp(1|32,R)}$ cubic matrix model possesses a two-fold structure of the $\boldsymbol{\mathcal{N}=2}$ SUSY of IKKT model.**
- **IKKT model is induced from the $\boldsymbol{osp(1|32,R)}$ cubic matrix model by the multi-loop effect.**
- **We have investigated the $\boldsymbol{gl(1|32,R)\otimes gl(N)}$ gauged model as an extension.**