Monte Carlo studies of the phase transition of finite-temperature large-N gauge theory

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Collaboration with Shingo Takeuchi and Takeshi Morita

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1 Introduction

Finite-temperature matrix quantum mechanics:

$$Z = \int dX_i dA e^{-S_{\rm YM}}, \quad \text{where}$$
$$S_{\rm YM} = N \int_0^{\frac{1}{T}} dt \left\{ \frac{1}{2} \text{tr} \sum_{i=1}^{D} (D_t X_i(t))^2 - \frac{1}{4} \text{tr} \sum_{i,j=1}^{D} [X_i(t), X_j(t)]^2 \right\}.$$

- Dimensional reduction of $(1+D)$ Yang-Mills theory
- This model may capture some universal features of large-N Yang-Mills theories through Eguchi-Kawai reduction.
- This model is useful in many contexts:
  - * Blackstring/Blackhole phase transition via gauge/gravity correspondence.
  - * Phase structure of Yang-Mills theory on the torus.
  - * Multi-baryon system in the Sakai-Sugimoto model.

    K. Hashimoto and T. Morita, arXiv:1103.5688

Previous studies of this matrix quantum mechanics:

- **Discovery of confinement/deconfinement phase transition.**
  O. Aharony, J. Marsano, S. Minwalla and T. Wiseman hep-th/0406210,
  O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas, M. Van Raamsdonk and T. Wiseman, hep-th/0508077
- **Topology change of the D-brane bound state.**
  T. Azeyanagi, M. Hanada, T. Hirata and H. Shimada arXiv:0901.4073
- **Phase transition and high-temperature expansion.**
  $\Rightarrow$ **details near the critical point is not fully understood.**
  N. Kawahara, J. Nishimura and S. Takeuchi arXiv:0706.3517, 0710.2188

So far, difficult to understand analytically.

$\Rightarrow$ **Calculation by $1/D$ expansion was proposed.**

G. Mandal, M. Mahato and T. Morita, arXiv:0910.4526, G. Mandal and T. Morita, arXiv:1103.1558

$1/D$ expansion $\Rightarrow$ approximation at $D \to +\infty$, finite N (N=matrix size).

How reliable is $1/D$ expansion?

- **Behavior of small D $\Rightarrow$ is it explained by $1/D$ expansion?**
- **Dependence of the order of phase transition and critical temperature on D.**

2 Phase transition and critical temperature

Observable for confinement/decofinenment phase transition:

$$u_n = \frac{1}{N}\mathrm{tr}\, U^n = \frac{1}{N}\sum_{a=1}^{N}\exp(in\alpha_a), \text{ where } U = \mathcal{P}\exp\left(i\int_0^{\frac{1}{T}} dt A(t)\right) = \mathrm{diag}(e^{i\alpha_1}, \cdots, e^{i\alpha_N}).$$

Calculation by the effective action derived by the $1/D$ expansion.

$$S_{\mathrm{eff}} = DN^2\left\{-\frac{\Delta^4}{8TD^{\frac{1}{3}}} + \frac{\Delta}{2T} + \sum_{n=1}^{+\infty}\frac{1}{n}\left(\frac{1}{D} - \exp\left(-\frac{n\Delta}{T}\right)\right)|u_n|^2\right\}$$

Especially when u_1 is small (i.e. except for high temperature)

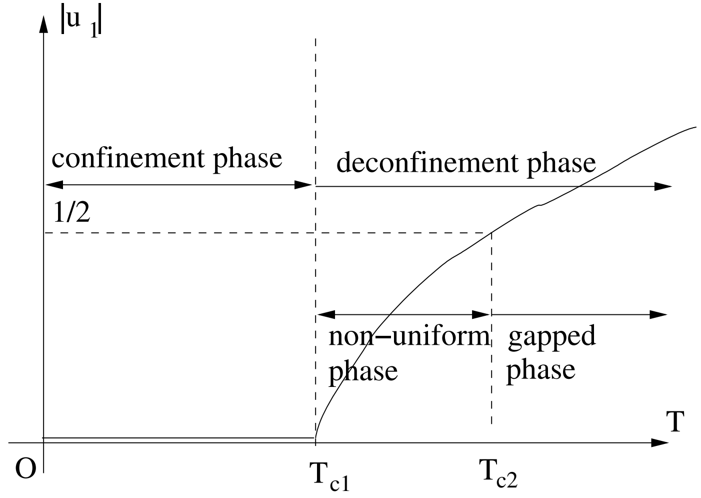
$\Rightarrow$ plugging $\Delta = D^{\frac{1}{3}}\left(1 + \frac{2}{3}\sum_{n=1}^{+\infty}|u_n|^2\exp\left(-\frac{nD^{\frac{1}{3}}}{T}\right)\right)$, we obtain the Landau-Ginzburg (LG) effective action:

$$S_{\mathrm{LG}} = DN^2\left\{\frac{3D^{\frac{1}{3}}}{8T} + b_1|u_1|^4 + \sum_{n=1}^{+\infty}a_n|u_n|^2\right\},$$

$$a_n = \frac{1}{n}\left(\frac{1}{D} - \exp\left(-\frac{nD^{\frac{1}{3}}}{T}\right)\right), \quad b_1 = \frac{D^{\frac{1}{3}}}{3T}\exp\left(-\frac{2D^{\frac{1}{3}}}{T}\right),$$

$$u_n = \frac{1}{N}\mathrm{tr}\, U^n = \frac{1}{N}\sum_{a=1}^{N}\exp(in\alpha_a)$$

Two critical temperatures T_{c1}, T_{c2}.



- $\dfrac{1}{T_{c1}} \sim \dfrac{\log D}{D^{\frac{1}{3}}}\left(1 + \dfrac{0.523}{D}\right)$: onset of nonuniformity in the eigevnvalue distribution function

  $\rho(\theta) = \dfrac{1}{N}\sum_{a=1}^{N}\langle\delta(\theta - \alpha_a)\rangle$.

- $\dfrac{1}{T_{c2}} \sim \dfrac{1}{T_{c1}} - \dfrac{\log D}{D^{\frac{4}{3}}}\left(\dfrac{1}{6} + \dfrac{0.137\log D + 0.293}{D}\right)$: emergence of gap in $\rho(\theta)$.

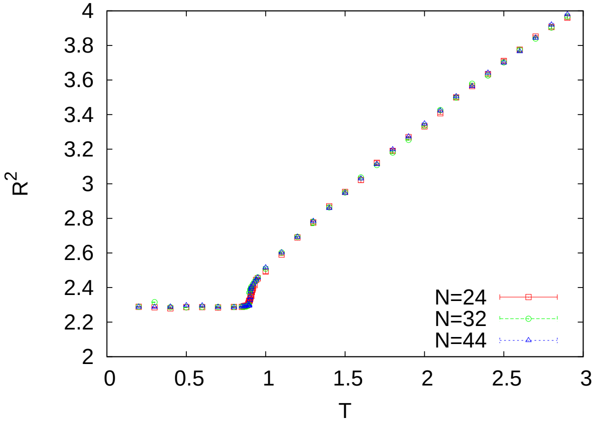
3 Validity of the $1/D$ expansion

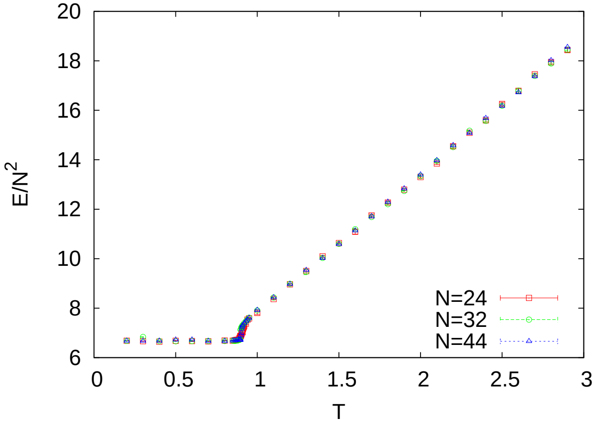
Behavior at confinement phase ($T < T_{c1}$)

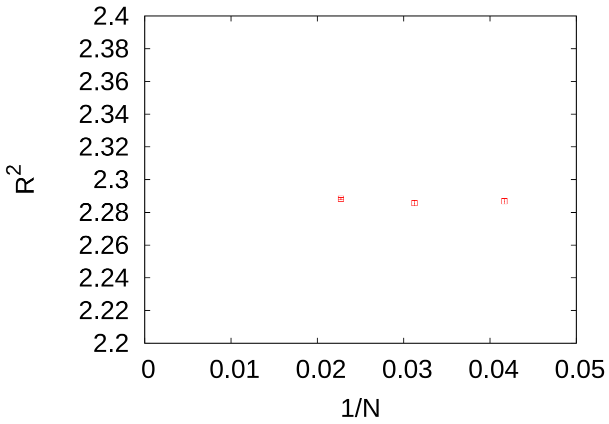
Monte Carlo (MC) simulation of the action $S_{\rm YM}$ for $D = 9$.

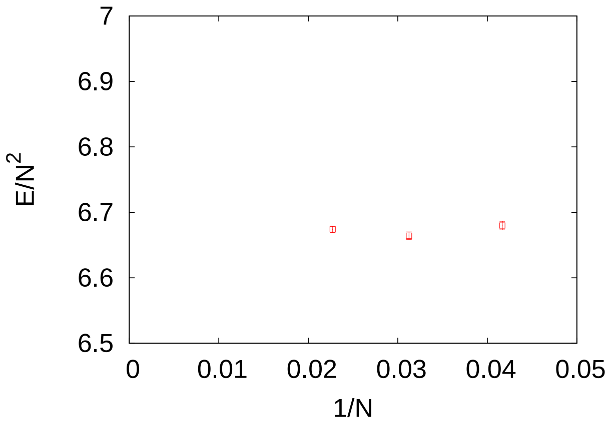
Little dependence on T and good convergence with respect to N.

(bottom: data of $T = 0.5 (< T_{c1})$).





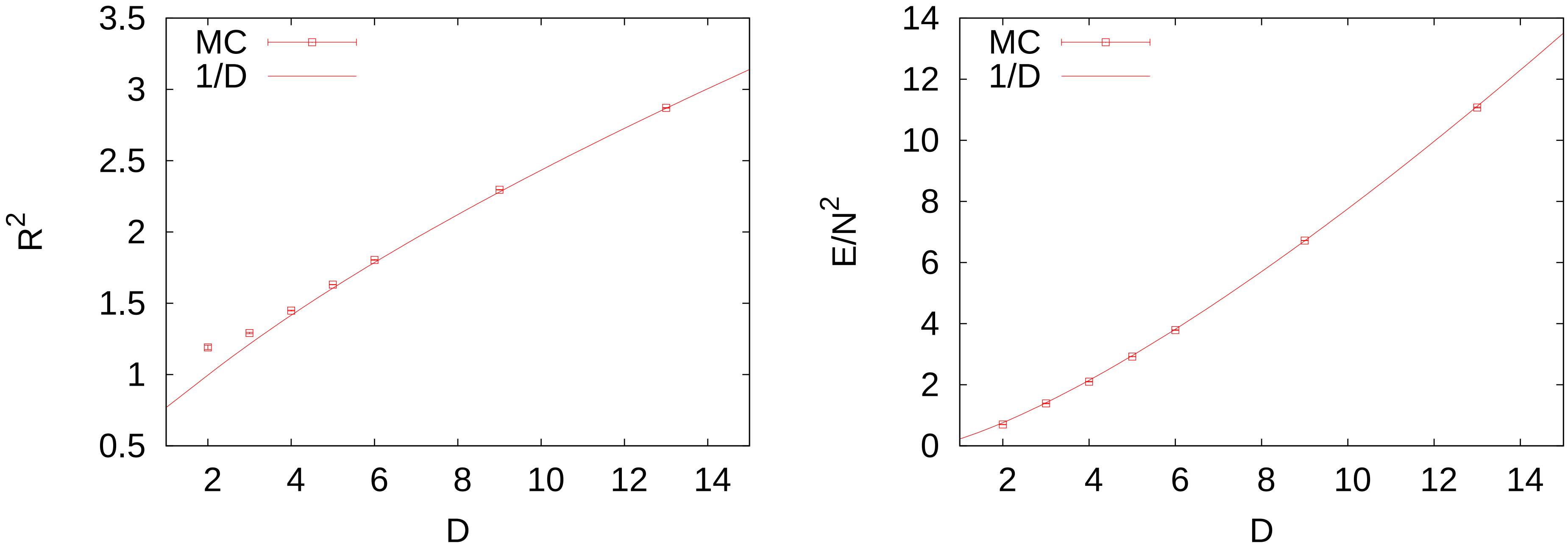




Results of $1/D$ expansion

- $R^2 = \frac{T}{N}\int_0^{\frac{1}{T}} \mathrm{tr}\, X_i^2(t)dt = \frac{D^{\frac{1}{3}}}{2}\left(1 + \frac{0.2405}{D} + \cdots\right).$
- $\frac{E}{N^2} = -\frac{3T}{4N}\int_0^{\frac{1}{T}} \mathrm{tr}\, [X_i(t), X_j(t)]^2 dt = D^{\frac{4}{3}}\left(\frac{3}{8} - \frac{0.1476}{D}\right) + \cdots$ (Free energy).

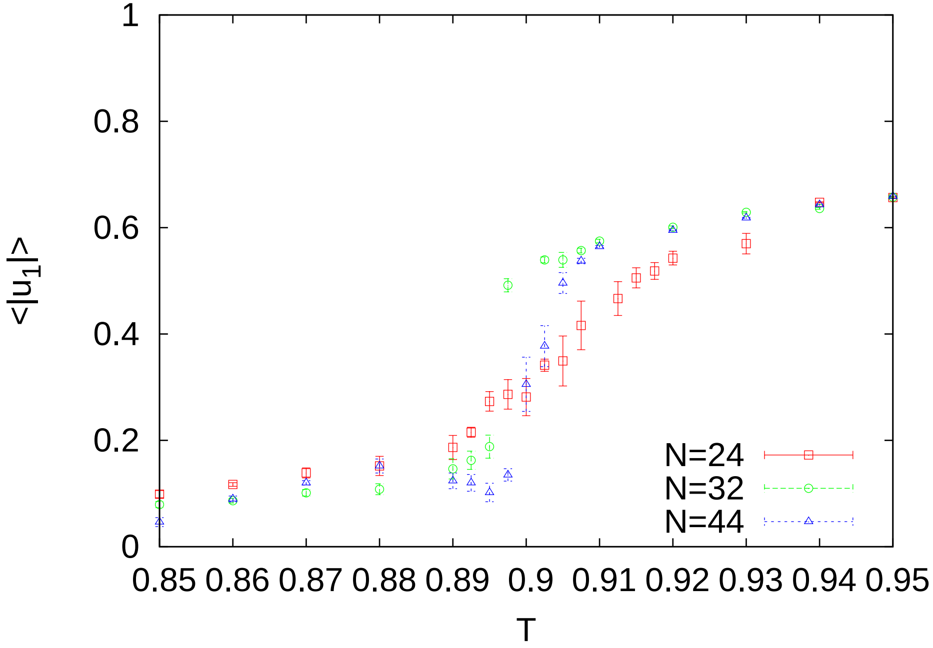
These observables (as well as $\langle |u_n| \rangle$) are constant at $T < T_{c1}$.



Monte Carlo data of $S_{\rm YM}$ for $T = 0.5 (< T_{c1})$, $N = 44$: $1/D$ expansion works well!.

Behavior around the critical point

Plots: Monte Carlo data of $S_{\rm YM}$ for $D = 9$.



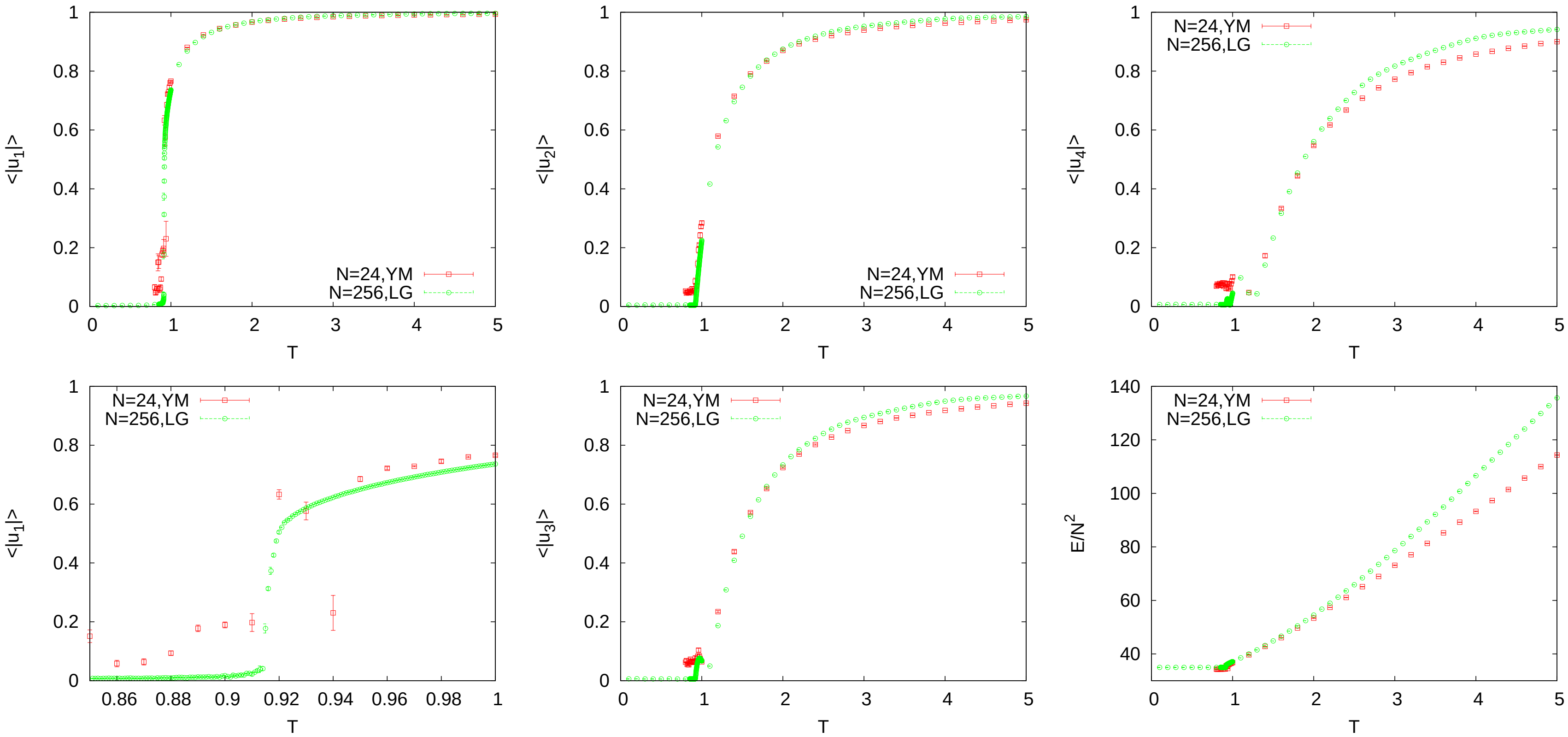
- **Monte Carlo data around the critical point ⇒ large dependence on N.**
- **Difficulty in determining the order of phase transition numerically.**

Behavior of "Middle-temperature" region

"Middle-temperature" region : $T > T_{c2}$ (but not $T \gg T_{c2}$).

$S_{\rm LG}$ is difficult to solve analytically in this region

$\Rightarrow$ Comparison of Monte Carlo data between $S_{\rm LG}$ and $S_{\rm YM}$ at $D = 30$.



$S_{\rm LG}$ agree with $S_{\rm YM}$ at $T < 2$.

4 Conclusion

Confinement/deconfinement phase transition of finite-temperature matrix quantum mechanics.

- **Comparison of S_{LG} derived by $1/D$ expansion with S_{YM}.**
- **Low temperature ($T < T_{c1}$) and Middle temperature ($T > T_{c2}$ but not $T \gg T_{c2}$):**
  $1/D$ expansion works well.

Further development

- **Determination of the order of phase transition.**
- **Effects of matter fields on the confinement/deconfinement phase transition.**

  T. Azuma, T. Morita and S. Takeuchi, in progress