Monte Carlo studies of the rotational symmetry breaking in dimensionally reduced super Yang-Mills models

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with K.N. Anagnostopoulos and J. Nishimura

Contents

1 Introduction and the model

Matrix models as a constructive definition of superstring theory

IKKT model (IIB matrix model)

⇒ Promising candidate for the constructive definition of superstring theory.

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115

$$S = \underbrace{-\frac{N}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2}_{=S_B} + \underbrace{\frac{N}{2}\mathrm{tr}\,\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]}_{=S_F}.$$

- The original IKKT model is ten-dimensional ($\mu, \nu = 1, 2, \cdots, D = 10$).

  ⇒ In the following, we study the simplified six-dimensional model ($\mu, \nu = 1, 2, \cdots, D = 6$).

- SO(6) rotational symmetry and SU(N) gauge symmetry.

- Presence of $\mathcal{N} = 2$ supersymmetry.

- $Z = \int dA d\psi d\bar{\psi} e^{-S} = \int dA e^{-S_B} \underbrace{(\det \mathcal{M})}_{=\int d\psi d\bar{\psi} e^{-S_F}} = \int dA e^{-S_0} e^{i\Gamma}.$

  $Z_0 = \int dA e^{-S_B} |\det \mathcal{M}| = \int dA e^{-S_0}$: phase-quenched model, Γ: complex phase

  $\det \mathcal{M}$ is complex → Complex phase is important in SO(6) breakdown.

Results of Gaussian Expansion Method (GEM)

T. Aoyama, J. Nishimura and T. Okubo, arXiv:1007.0883, J. Nishimura, T. Okubo and F. Sugino arXiv:1108.1293

Observable for probing dimensionality : $T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu)$.

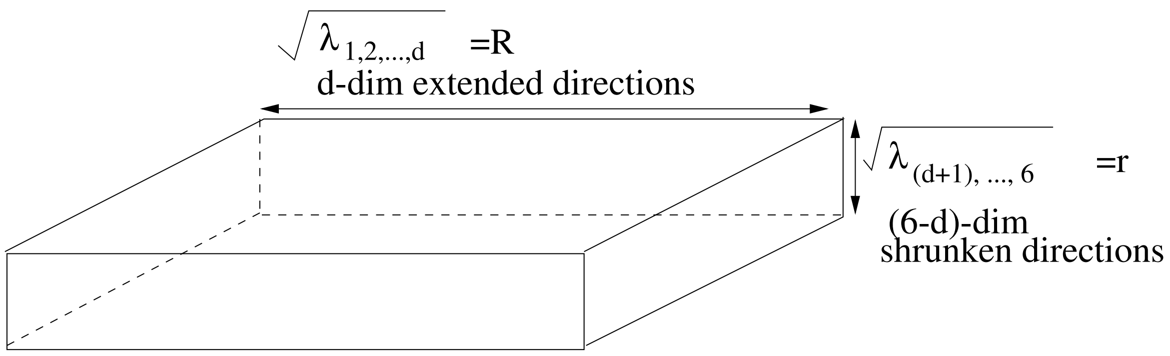
λ_n $(n = 1, \cdots, 6)$: eigenvalues of $T_{\mu\nu}$ $(\lambda_1 \geqq \lambda_2 \geqq \cdots \geqq \lambda_6)$

Extended d dim $(\lambda_1 = \cdots = \lambda_d = R^2)$ and shrunken $(6-d)$ dim $(\lambda_{d+1} = \cdots = \lambda_6 = r^2)$:

- Universal "compactification" scale: $r^2 \simeq 0.223$, independent of d.
- Constant volume property: The volume is independent of d: $V = R^d \times r^{6-d} = l^6$

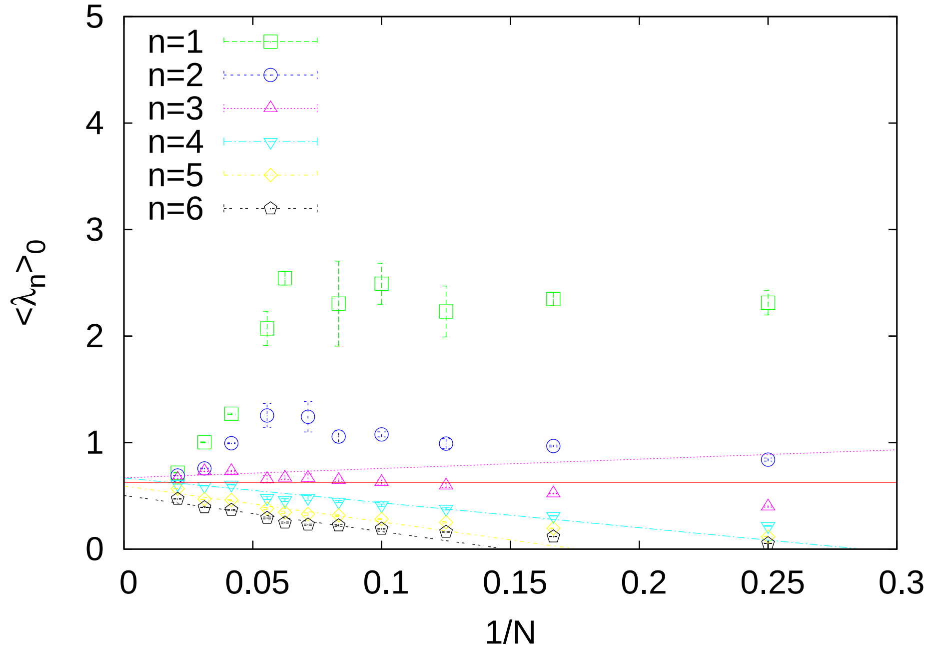
  The extent of phase-quenched model Z_0 is $l^2 \simeq 0.627$.
- The free energy takes the minimum at $d = 3$

  $\rightarrow$ Spontaneous Symmetry Breakdown (SSB) of SO(6) to SO(3).



2 Monte Carlo simulation

Simulation of the phase-quenched model



Large-N limit $\rightarrow \langle \boldsymbol{\lambda}_1 \rangle_0 = \langle \boldsymbol{\lambda}_2 \rangle_0 = \cdots = \langle \boldsymbol{\lambda}_6 \rangle_0 \simeq 0.6$.

$\langle * \rangle_0 =$(V.E.V. of the phase-quenched model Z_0).

No SO(6) breakdown in the phase-quenched model.

Consistent with the GEM result $\langle \boldsymbol{\lambda}_{1,\cdots,6} \rangle_0 = l^2 \simeq 0.627$.

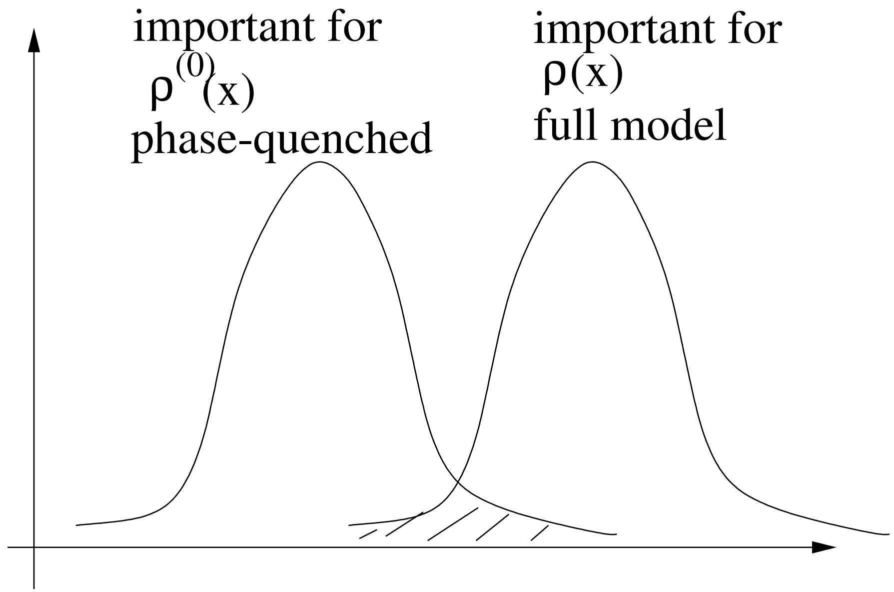
Factorization method

An approach to overcome the overlap problem in Monte Carlo simulation.

K. N. Anagnostopoulos and J. Nishimura, hep-th/0108041,
J. Ambjorn, K. N. Anagnostopoulos, J. Nishimura and J. J. M. Verbaarschot, hep-lat/0208025.

K. N. Anagnostopoulos, T. Azuma and J. Nishimura, arXiv:1009.4504,1108.1534.

Overlap problem: Discrepancy of a distribution function between the phase-quenched model Z_0 and the full model Z.



Factorization property of the distribution function

($\tilde{\lambda}_n \stackrel{\text{def}}{=} \lambda_n / \langle \lambda_n \rangle_0$: deviation from 1 $\Rightarrow$ effect of the phase)

$$\rho_n(x) \stackrel{\text{def}}{=} \langle \delta(x - \tilde{\lambda}_n) \rangle \stackrel{\text{reweighting}}{=} \frac{\langle \delta(x - \tilde{\lambda}_n) e^{i\Gamma} \rangle_0}{\langle e^{i\Gamma} \rangle_0}$$

$$= \underbrace{\frac{1}{\langle e^{i\Gamma} \rangle_0}}_{=\frac{1}{C}} \times \underbrace{\langle \delta(x - \tilde{\lambda}_n) \rangle_0}_{=\rho_n^{(0)}(x)} \times \underbrace{\frac{\langle \delta(x - \tilde{\lambda}_n) e^{i\Gamma} \rangle_0}{\langle \delta(x - \tilde{\lambda}_n) \rangle_0}}_{=w_n(x)} = \frac{1}{C} \rho_n^{(0)}(x) w_n(x)$$

where

$$C = \langle e^{i\Gamma} \rangle_0 = \langle \cos \Gamma \rangle_0, \quad \rho_n^{(0)}(x) = \langle \delta(x - \tilde{\lambda}_n) \rangle_0, \quad w_n(x) = \langle e^{i\Gamma} \rangle_{n,x} = \langle \cos \Gamma \rangle_{n,x},$$

$$\langle * \rangle_{n,x} = [\text{V.E.V. for the partition function } Z_{n,x} = \int dA e^{-S_0} \delta(x - \tilde{\lambda}_n)].$$

Simulation of partition function $Z_{n,x}$ $\Rightarrow$ x is trapped at $\tilde{\lambda}_n$.

The system visits the configurations important for full partition function Z.

Resolution of overlap problem.

Monte Carlo evaluation of $\langle \tilde{\lambda}_n \rangle$

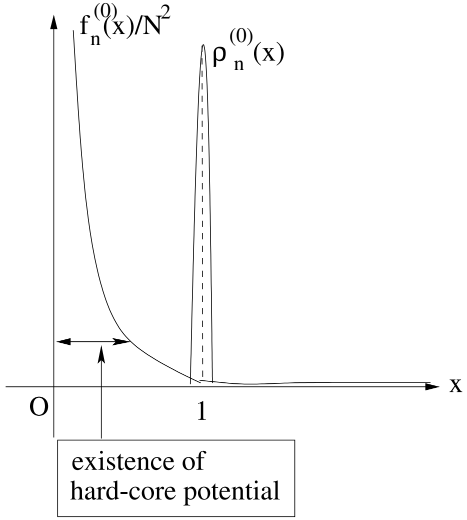
$w_n(x) > 0 \Rightarrow \langle \tilde{\lambda}_n \rangle$ is the minimum of the free energy $\mathcal{F}_n(x) = -\frac{1}{N^2} \log \rho_n(x)$:

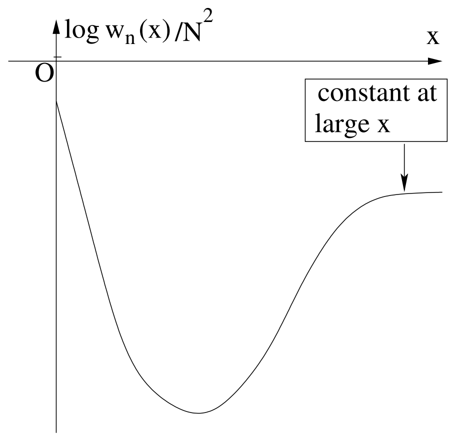
We solve $\mathcal{F}'_n(x) = 0$, namely $\frac{1}{N^2} f_n^{(0)}(x) = -\frac{d}{dx}\left\{ \frac{1}{N^2} \log w_n(x) \right\}$, where $f_n^{(0)}(x) \stackrel{\text{def}}{=} \frac{\partial}{\partial x} \log \rho_n^{(0)}(x)$.

Scaling properties in the large-N limit:

$$\frac{f_n^{(0)}(x)}{N^2} = \begin{cases} \mathrm{O}(N^0) & x < 1 \\ \mathrm{O}\left(\frac{1}{N}\right) & x > 1 \end{cases} \qquad \frac{1}{N^2} \log w_n(x) = \mathrm{O}(N^0)$$

- Existence of the hard-core potential in $\frac{1}{N^2} f_n^{(0)}(x)$ at large N.
- $\frac{1}{N^2} \log w_n(x)$ becomes almost constant at large x.





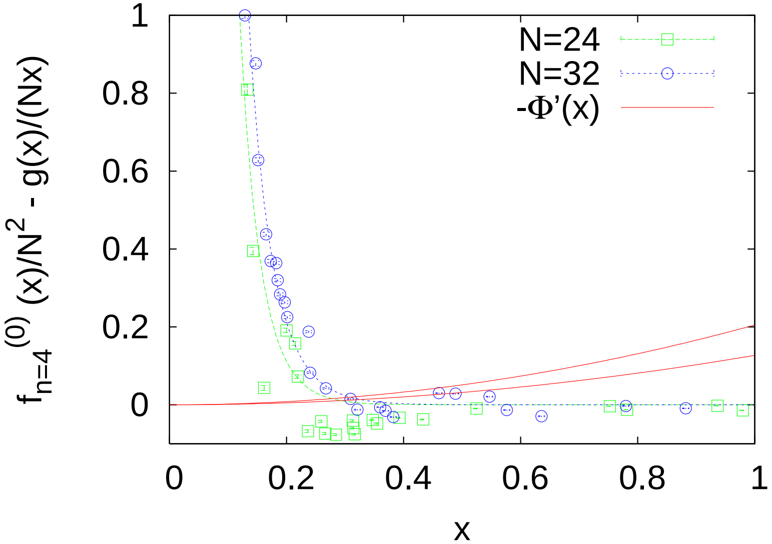
Results

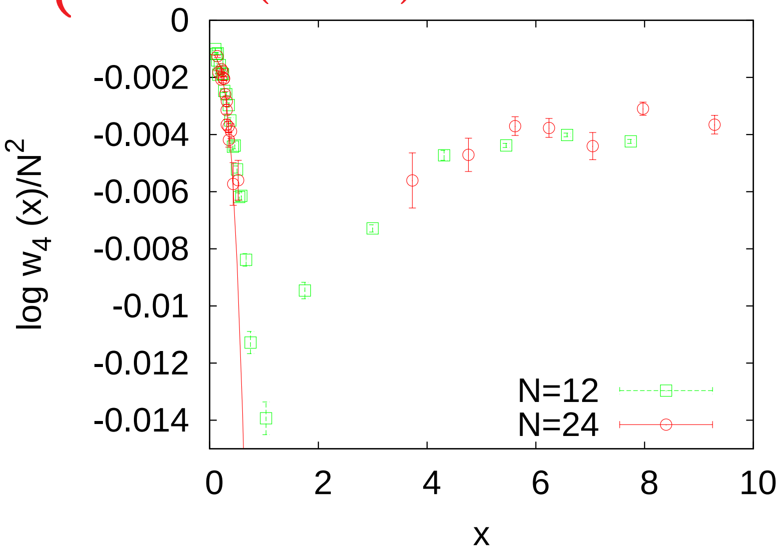
Around $x \simeq 1$, $\frac{1}{N} f_n^{(0)}(x)$ scales at large N:

Ansatz around $x \simeq 1 \to \frac{x}{N} f_n^{(0)}(x) \simeq g(x) = \{a_{1,n}(x-1) + a_{2,n}(x-1)^2 + a_{3,n}(x-1)^3\}$.

At small x, $\frac{1}{N^2} f_n^{(0)}(x) - \frac{g(x)}{Nx} \propto \frac{7-n}{2x}$.

$w_n(x)$ behaves as $\Phi_n(x) = \lim_{n \to +\infty} \frac{1}{N^2} \log w_n(x) \propto \begin{cases} x^{7-n} & (x<1) \\ \text{const.} & (x>1) \end{cases}$.





Result for $n = 4$:

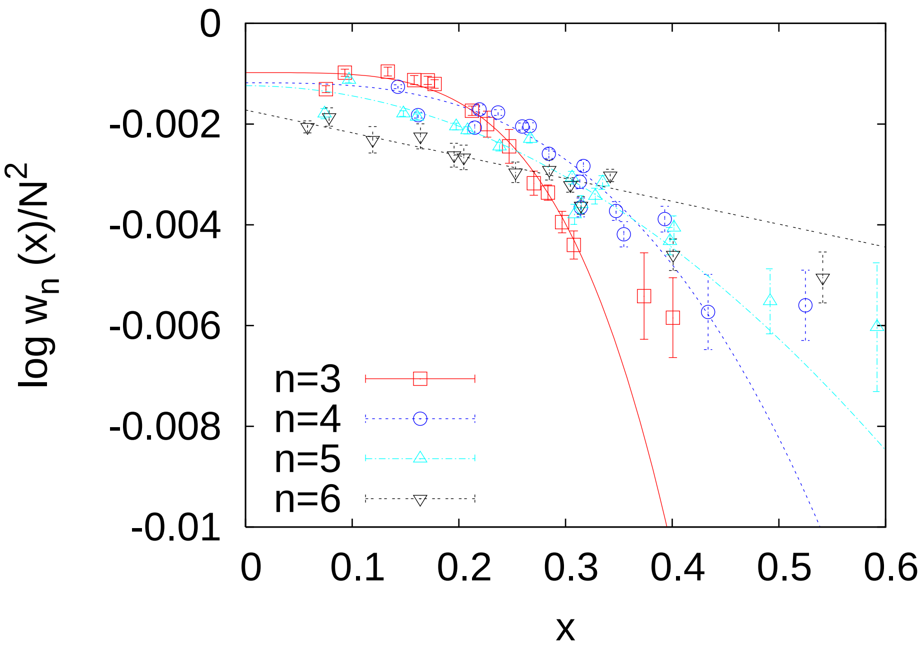
The solution of $\frac{1}{N^2} f_n^{(0)}(x) = -\frac{d}{dx}\left\{\frac{1}{N^2} \log w_n(x)\right\}$ is $x_s = 0.31(1)$.

(c.f.) GEM result for shrunken directions: $\langle \tilde{\lambda}_n \rangle = \frac{\langle \lambda_n \rangle}{\langle \lambda_n \rangle_0} = \frac{r^2}{l^2} \simeq \frac{0.223}{0.627} = 0.355 \cdots$.

Comparison of the free energy → evaluate $\Delta = -\mathcal{F}_3(x_s) + \mathcal{F}_4(x_s)$.

$\Delta < 0 \Rightarrow n = 3$ direction does not shrink → SO(3) spacetime is preferred to SO(2).

$$\Delta = \frac{\log \rho_3(x_s) - \log \rho_4(x_s)}{N^2} = \frac{\log w_3(x_s) - \log w_4(x_s)}{N^2} - \underbrace{\int_{\mathrm{SO}(2)}^{\mathrm{SO}(3)} \frac{1}{N^2}\frac{d}{dx}\log \rho^{(0)}}_{\to 0 (\text{as } N\to+\infty)}$$

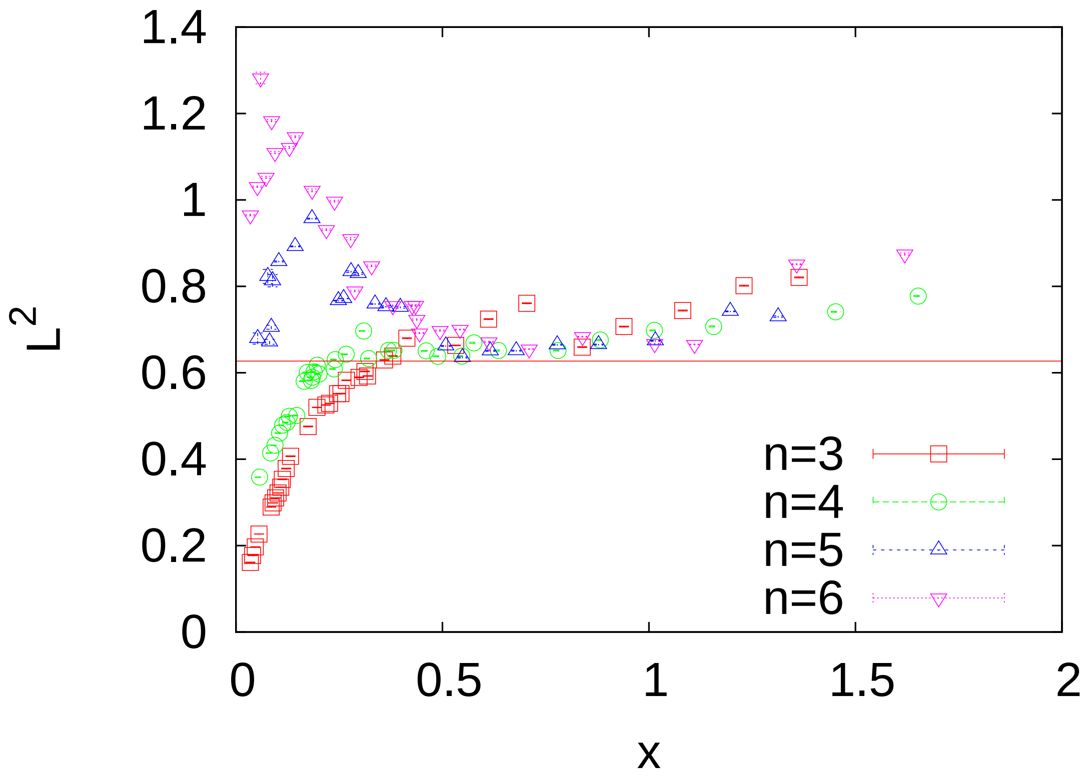


$\dfrac{1}{N^2}\log w_n(x)$ for $n = 3, 4, 5, 6,\ N = 24$:

Very subtle, but $\dfrac{1}{N^2}\log w_{n=4}(x_s)$ is the highest at $x_s \simeq 0.3$.

Constant volume property

Geometric mean $L^2 = \left(\prod_{i=1}^{6} \langle \lambda_i \rangle_{n,x}\right)^{\frac{1}{6}}$ **for** $n = 3, 4, 5, 6$:



$L^2 \simeq 0.627$ **at** $x = 1$ **(which corresponds to setting** λ_n **at the value** $\langle \lambda_n \rangle_0$**).**

Consistent with the GEM result $V = R^d \times r^{6-d} \simeq l^6$**, where** $l^2 \simeq 0.627$**.**

3 Conclusion

Monte Carlo simulation of 6d IKKT model $\Rightarrow$ spontaneous breakdown of SO(6) symmetry.

- **Numerical evidence for symmetry breakdown SO(6) $\rightarrow$ SO(3).**
- **Constant volume property.**

Future works

- **Simulation of larger N $\Rightarrow$ study the finite-N effect.**
- **Ultimately, we would like to study the 10d IKKT model.**