Dynamical generation of gauge groups in the massive Yang-Mills-Chern-Simons matrix model

(hep-th/0504217)

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Contents

1 Introduction

Large-N reduced models

⇒ promising candidates for the constructive definition of superstring theory.

The IIB matrix model

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115

$$S = -\frac{1}{g^2}\mathrm{tr}\left(\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right).$$

Relation with the type IIB superstring theory:

- **Matrix regularization of the Green-Schwarz action of type IIB superstring theory.**
- **D-brane interaction.**
- **Derivation of the string field theory.**

Matrix models on a homogeneous space

Motivations of fuzzy manifold studies:

- **Relation between the non-commutative field theory and the superstring.**
- **Novel regularization scheme alternative to lattice regularization.**
- **Prototype of the curved-space background in the large-N reduced models.**

Matrix models on a homogeneous space G/H:

G= (a Lie group), H = (a closed subgroup of G).

$\mathrm{S}^2 = \mathrm{SU}(2)/\mathrm{U}(1)$, $\mathrm{S}^2 \times \mathrm{S}^2$, $\mathrm{S}^4 = \mathrm{SO}(5)/\mathrm{U}(2)$, $\mathrm{CP}^2 = \mathrm{SU}(3)/\mathrm{U}(2)$, $\cdots$.

Fuzzy spheres are compact $\Rightarrow$ realized by finite matrices.

The Yang-Mills-Chern-Simons (YMCS) model $\Rightarrow$ fuzzy sphere background.

2 The model and its classical solution

3d massive Yang-Mills-Chern-Simons model

⇒ a toy model with fuzzy sphere solutions:

$$\boxed{S[A] = N\alpha^4 \mathrm{tr}\left(-\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{2i}{3}\epsilon_{\mu\nu\rho}A_\mu A_\nu A_\rho + \frac{\rho^2}{2}A_\mu^2\right).}$$

- **Defined in the 3-dimensional Euclidean space $(\mu, \nu, \rho = 1, 2, 3)$.**
- **Convergence of the path integral** P. Austing and J. F. Wheater, hep-th/0310170.
  (mass term just suppresses the path integral)
- **Classical equation of motion:**

$$-[A_\nu, [A_\mu, A_\nu]] + i\,\epsilon_{\mu\nu\lambda}[A_\nu, A_\lambda] + \rho^2 A_\mu = 0.$$

Fuzzy sphere classical solution:

$$A_\mu = X_\mu = \chi \begin{pmatrix} L_\mu^{(n_1)} \otimes 1_{k_1} & & \\ & \ddots & \\ & & L_\mu^{(n_s)} \otimes 1_{k_s} \end{pmatrix}, \text{ where } \begin{array}{l} \chi = \frac{1}{2}(1+\sqrt{1-2\rho^2}), \\ \sum_{i=1}^{s} n_i k_i = N, \\ 0 < \rho < \frac{1}{\sqrt{2}}. \end{array}$$

- $L_\mu^{(n)}$: $n \times n$ representation of SU(2) Lie algebra:

$$[L_\mu^{(n)}, L_\nu^{(n)}] = i\epsilon_{\mu\nu\rho} L_\rho^{(n)}, \quad (L_\mu^{(n)})^2 = \frac{n^2-1}{4}.$$

- Collection of k_i coincident fuzzy spheres.
- Expansion around this solution $\Rightarrow$ $U(k_1) \times U(k_2) \times \cdots \times U(k_s)$ gauge theory.
- $A_\mu = 0$ solution $\Rightarrow$ $s = 1,\ n_1 = 1,\ k_1 = N$.
- Classical free energy:

$$F_{\rm cl} = S_{\rm cl} = \frac{N\alpha^4}{24} f(\chi) \left(\sum_{i=1}^{s} k_i (n_i^3 - n_i) \right), \text{ where } f(\chi) = \frac{\chi^4}{2} - \frac{2\chi^3}{3} + \frac{\rho^2 \chi^2}{2} (\lesseqqgtr 0 \text{ for } \rho \lesseqqgtr \frac{2}{3}).$$

3 Review of the massless ($\rho = 0$) case

T. Azuma, S. Bal, K. Nagao and J. Nishimura, hep-th/0401038.

First-order phase transition

Monte Carlo simulation launched from fuzzy sphere classical solution:

Critical point at $\alpha_{\rm cr} \simeq \frac{2.1}{\sqrt{N}}$.

- $\alpha < \alpha_{\rm cr}$: Yang-Mills phase

  Strong quantum effects.

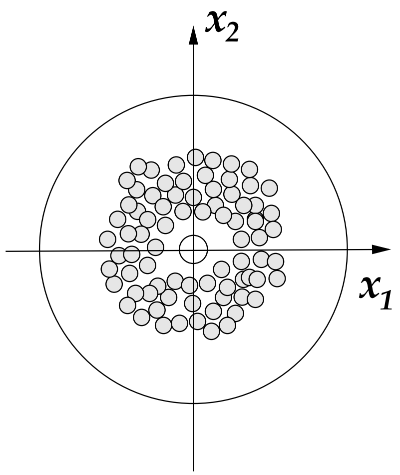
  behavior like the $\alpha = 0$ case.

  T. Hotta, J. Nishimura and A. Tsuchiya, hep-th/9811220,

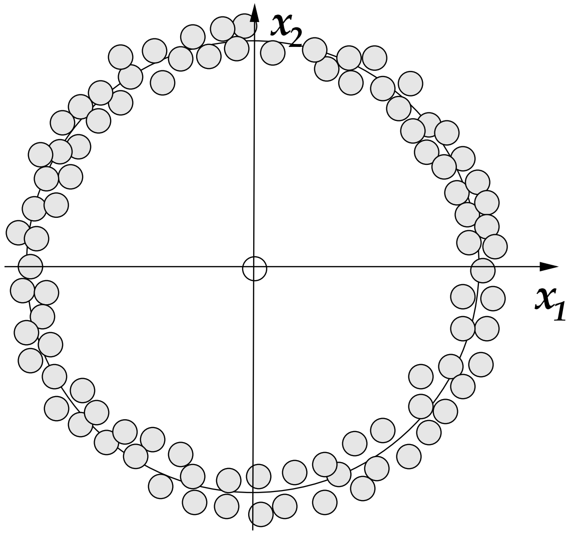
- $\alpha > \alpha_{\rm cr}$: fuzzy sphere phase.

  Fuzzy sphere configuration is stable.

Yang–Mills phase



Fuzzy sphere phase



Phase transition from the one-loop effective action

The effective action Γ is saturated at the one-loop level at large N.

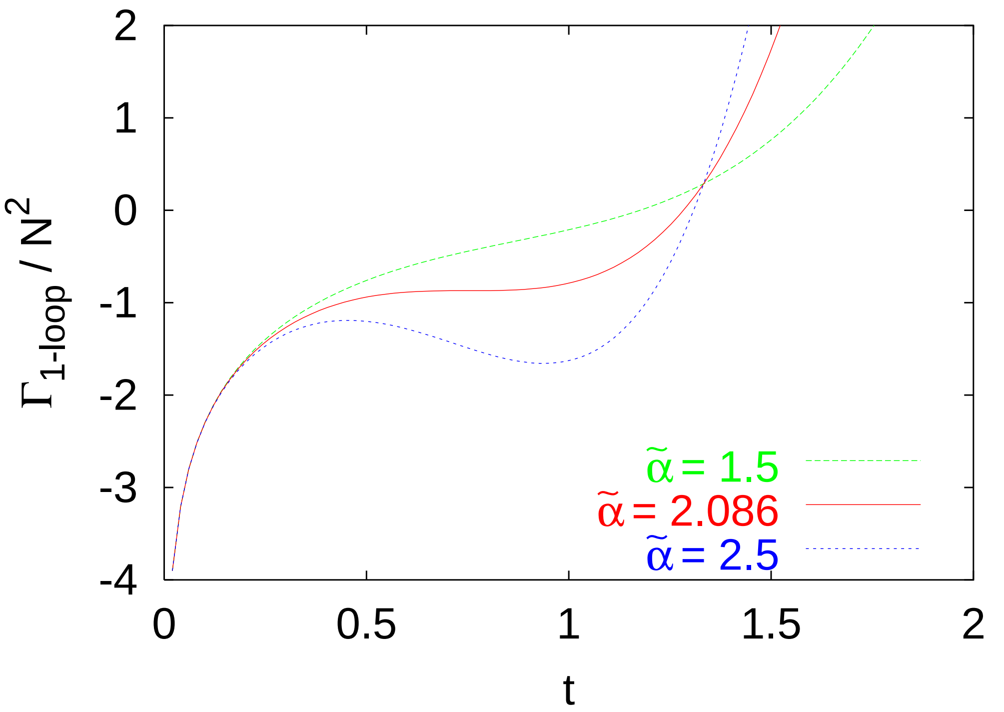
T. Imai, Y. Kitazawa, Y. Takayama and D. Tomino, hep-th/0307007.

Effective action at one-loop around $A_\mu = tX_\mu$ (where $\tilde{\alpha} = \alpha\sqrt{N}$).

$$\frac{\Gamma_{1-\text{loop}}}{N^2} \simeq \tilde{\alpha}^4\left(\frac{t^4}{8} - \frac{t^3}{6}\right) + \log t.$$

The local minimum disappears at $\tilde{\alpha} < \tilde{\alpha}_{\text{cr}} = (\frac{8}{3})^{\frac{3}{4}} \simeq 2.086\cdots$.

Consistent with the Monte Carlo simulation.



Properties of the multi-fuzzy spheres

Expansion around k coincide fuzzy spheres $A_\mu = X_\mu + \tilde{A}_\mu$, where

$$X_\mu = L_\mu^{(n)} \otimes 1_k.$$

Quantum field theory with U(k) gauge group.

Realized by the finite $N = nk$ matrices.

Simulation from zero start $A_\mu^{(0)} = 0$ for $N = 16$, $\alpha = 2.0$.

Metastability of multi-fuzzy-sphere state.

$$\underbrace{A_\mu^{(0)} = 0}_{\text{initial state}} \to \cdots \to \underbrace{A_\mu = \begin{pmatrix} L_\mu^{(6\to5\to4\to3\to2\to1)} & 0 \\ 0 & L_\mu^{(10\to11\to12\to13\to14\to15)} \end{pmatrix}}_{\text{metastable vacuum}} \to \underbrace{A_\mu = L_\mu}_{\text{stable vacuum}}.$$

Calculation of the free energy

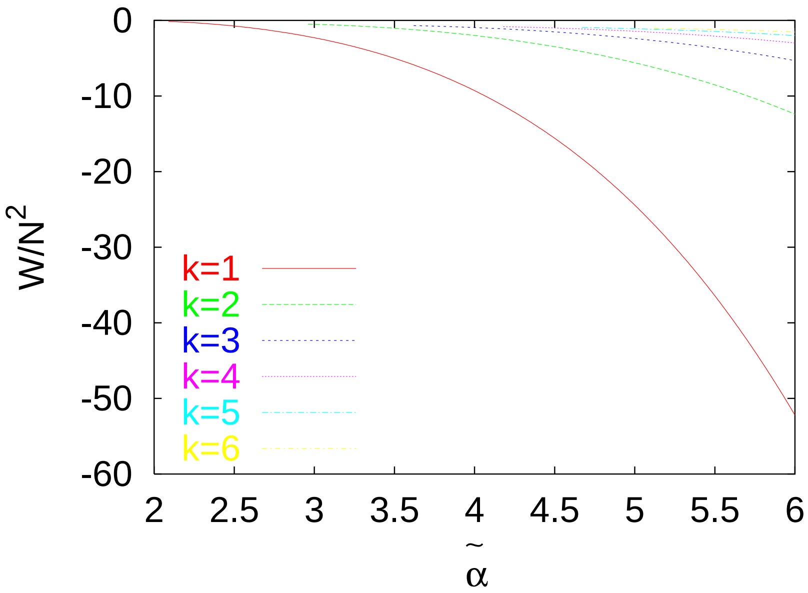
$$W = -\log\left(\int d\tilde{A} e^{-S}\right).$$

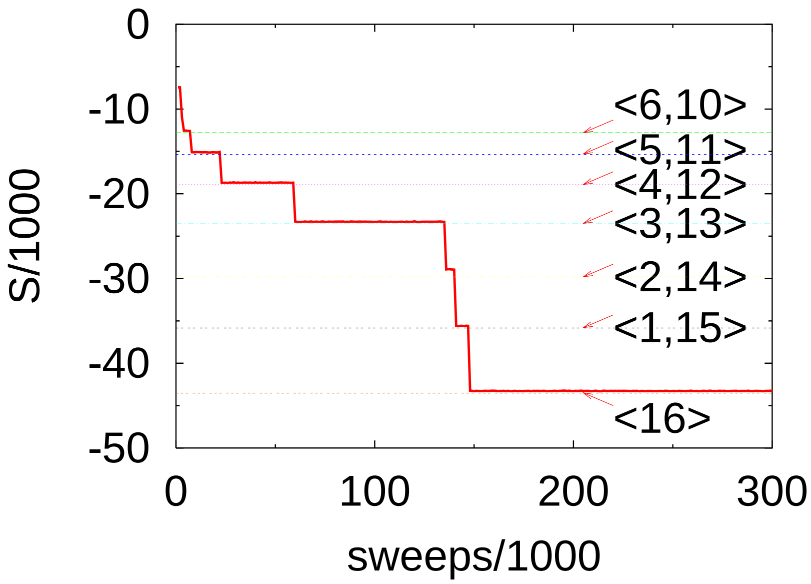
Eigenvalues of Casimir operator

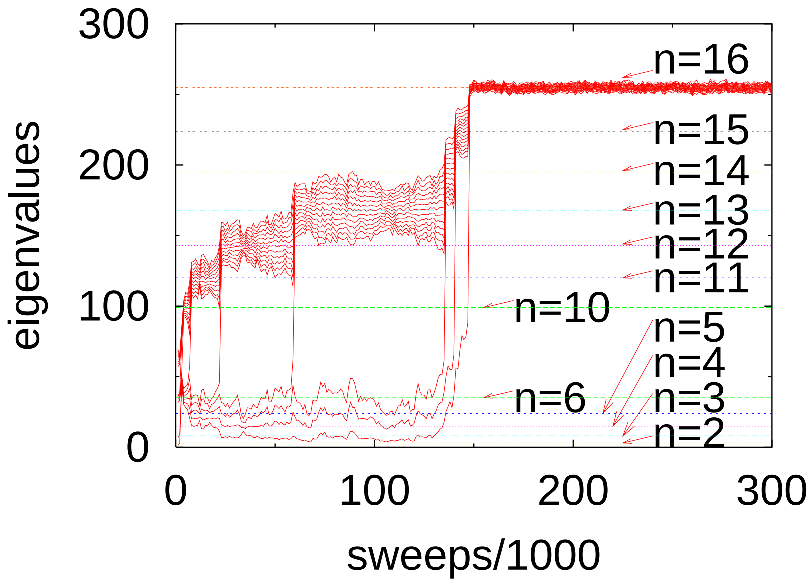
$$Q = \alpha^2(A_1^2 + A_2^2 + A_3^2).$$

$k = 1$ **has the lowest free energy to all order of perturbation.**

T. Azuma, K. Nagao and J. Nishimura, hep-th/0410263.







Attempts for other models

Extension to the models with other fuzzy sphere solution:

- **Fuzzy S^4 sphere (`hep-th/0405096`) No fuzzy S^4 phase.**
- **Fuzzy CP^2 manifold (`hep-th/0405277`).**
- **Fuzzy $S^2 \times S^2$ sphere.**
- **Supersymmetric 4d YMCS model with fuzzy S^2 solution.**

In these cases (except S^4), U(1) gauge group is favored.

4 Dynamical generation of gauge group

One-loop free energy:

$$F = F_{\rm cl} \underbrace{- \log(\mathrm{vol}(H)\mathcal{N}\Delta(X_\mu)\mathcal{Z})}_{=F_{1-\mathrm{loop}}}.$$

Perturbation around the fuzzy sphere solution $A_\mu = X_\mu + \tilde{A}_\mu$

$\Rightarrow$ **flat directions corresponding to the transformation:**

$$A_\mu \to A_\mu^g = g A_\mu g^\dagger, \text{ where } g \in H = \mathrm{U}(N)/\prod_{i=1}^{s} \mathrm{U}(k_i).$$

- $\mathrm{vol}(H)$: gauge volume for the coset space H.

$$\mathrm{vol}(H) = \frac{\mathrm{vol}(\mathrm{U}(N))}{(\prod_{i=1}^{s} \mathrm{vol}(\mathrm{U}(k_i)))}, \text{ where } \mathrm{vol}(n) = \frac{(2\pi)^{n(n+1)/2}}{(n-1)! \cdots 1!0!}.$$

- $\mathcal{N} = (\frac{N\alpha^4}{2\pi})^{\frac{1}{2}\{N^2 - \sum_{i=1}^{s}(k_i)^2\}}$.

- $\Delta(X_\mu)$: Faddeev-Popov determinant for the gauge fixing.

$$\Delta(X_\mu) = \prod_{i,j=1}^{s} {\prod}'^{(n_i+n_j)/2-1}_{l=|n_i-n_j|/2} [\chi^2 l(l+1)]^{k_i k_j (2l+1)}.$$

  ($\prod' \to l = 0$ is excluded).

- $\mathcal{Z}$: integration over the fluctuation $\tilde{A}_\mu$.

  Gauge fixing term: $S_{\mathrm{g.f.}} = -\frac{1}{2} N\alpha^4 \mathrm{tr}[X_\mu, A_\mu]^2$.

$$\begin{aligned} S_{\mathrm{tot}} &= S + S_{\mathrm{g.f.}} = S_{\mathrm{cl}} + \frac{1}{2} N\alpha^4 \mathrm{tr}(\tilde{A}_\mu \mathcal{Q}_{\mu\nu} \tilde{A}_\nu), \text{ where} \\ \mathcal{Q}_{\mu\nu} &= \{\chi^2 (\mathcal{L}_\lambda)^2 + \rho^2\} \delta_{\mu\nu} - i\rho \epsilon_{\mu\nu\lambda} \mathcal{L}_\lambda, \quad \mathcal{L}_\lambda M = [L_\lambda, M]. \end{aligned}$$

  Eigenvalue problem $\mathcal{Q}_{\mu\nu} \tilde{A}_\nu = \lambda \tilde{A}_\mu$.

  K. Dasgupta, M. M. Sheikh-Jabbari and M. Van Raamsdonk, hep-th/0205185.

For $n_i \times n_j$ blocks, $(\tilde{A}_\mu)_{n_i \times n_j} = \sum_{l=|n_i-n_j|/2}^{(n_i+n_j)/2-1} \sum_{m=-l}^{l} \tilde{A}_\mu^{(l,m)} |l,m\rangle$.
$(\mathcal{L}_1 \pm i\mathcal{L}_2)|l,m\rangle = \underbrace{\sqrt{l(l+1) - m(m\pm 1)}}_{=b_\pm} |l, m\pm 1\rangle$.

Eigenvalue equation for each block:

$$\begin{aligned}
&\{\Lambda - \rho^2(1-m)\}\tilde{A}_+^{(l,m)} = \rho^2 b_- \tilde{A}_3^{(l,m-1)} \ , \\
&\{\Lambda - \rho^2(1+m)\}\tilde{A}_-^{(l,m)} = -\rho^2 b_+ \tilde{A}_3^{(l,m+1)} \ , \\
&\{\Lambda - \rho^2\}\tilde{A}_3^{(l,m)} = \frac{1}{2}\rho^2 (b_+ \tilde{A}_+^{(l,m+1)} - b_- \tilde{A}_-^{(l,m-1)}) \ , \ \text{where} \\
&\tilde{A}_\pm^{(l,m)} = \tilde{A}_1^{(l,m)} \pm i\tilde{A}_2^{(l,m)}, \quad \Lambda = \lambda - \chi^2 l(l+1).
\end{aligned}$$

Three eigenvalues for $l \geq \frac{1}{2}$ ($l=0 \to \lambda = \rho^2$ with 3-fold degeneracy):

$$\underbrace{\lambda_1 = \chi^2 l(l+1)}_{2l+1 \text{ degeneracy}}, \quad \underbrace{\lambda_2 = \chi^2 l(l+1) - \rho^2 l}_{2l-1 \text{ degeneracy}}, \quad \underbrace{\lambda_3 = \chi^2 l(l+1) + \rho^2(l+1)}_{2l+3 \text{ degeneracy}}.$$

Final result:

$$\begin{aligned}
\mathcal{Z} &= \left(\frac{2\pi}{N\alpha^4}\right)^{\frac{3}{2}N^2} \times \prod_{i=1}^{s} \rho^{-3(k_i)^2} \times \prod_{i,j=1}^{s} (q_{n_i n_j})^{k_i k_j}, \ \text{where} \\
q_{nm} &= {\prod_{l=|n-m|/2}^{(n+m)/2-1}}' \left[\lambda_1^{2l+1}\lambda_2^{2l-1}\lambda_3^{2l+3}\right]^{-\frac{1}{2}}.
\end{aligned}$$

$\rho < \frac{2}{3}$ regime

Single ($s = 1, n_1 = N, k_1 = 1$) fuzzy sphere $A_\mu = \chi L_\mu^{(N)}$ $\Rightarrow F_{\rm cl} = -\mathrm{O}(\alpha^4 N^4) < 0$.

$\Rightarrow$ **Coincident fuzzy spheres cannot be the true vacuum.**

Free energy for single fuzzy sphere:

$$\frac{F_{\rm FS}}{N^2} = \frac{N^2\alpha^4}{4} f(\chi) + \frac{5}{2}\log N + 4\log\alpha - \delta(\rho), \text{ where}$$

$$f(\chi) = \frac{\chi^4}{2} - \frac{2\chi^3}{3} + \frac{\rho^2\chi^2}{2} (\lesseqqgtr 0 \text{ for } \rho \lesseqqgtr \frac{2}{3}).$$

Free energy in the Yang-Mills phase (based on Gaussian expansion)

J. Nishimura, T. Okubo and F. Sugino, hep-th/0205253.

$$\frac{F_{\rm YM}}{N^2} = \frac{3}{2}\log N + 3\log\alpha + \underbrace{\gamma}_{\simeq -4.5}.$$

Critical point: $\alpha_{\rm cr} = \frac{1}{\sqrt{N}} \left(\frac{2\log N}{|f(\chi)|} \right)^{1/4}$

- $\alpha < \alpha_{\rm cr} \rightarrow F_{\rm YM} < F_{\rm FS}$: **Yang-Mills phase**
- $\alpha > \alpha_{\rm cr} \rightarrow F_{\rm YM} > F_{\rm FS}$: **single fuzzy sphere**

$\rho > \frac{2}{3}$ regime

$\frac{F_{\rm cl}}{N^2} = \frac{1}{4}\alpha^4 f(\chi)(\sum_{i=1}^{s} r_i(n_i^3 - n_i)) > 0$ ($r_i = \frac{k_i}{N}$, so that $\sum_{i=1}^{s} n_i r_i = 1$) and
$\frac{F_{1-{\rm loop}}}{N^2} = \frac{3}{2}\log N + {\rm O}(1)$.

The stable fuzzy sphere should have lower free energy than $F_{A=0} = \frac{3}{2}\log N + {\rm O}(1)$.

The stable fuzzy sphere should satisfy $n_i = {\rm O}(1)$, $k_i = r_i N = {\rm O}(N)$.

$U(k_i)$ gauge group ($k_i = {\rm O}(N)$) is dynamically generated.

Analogous to coincident transverse 5-branes in M-theory.

J. Maldacena, M. M. Sheikh-Jabbari and M. Van Raamsdonk, hep-th/0211139.

$\boxed{\text{Rough estimate at large } \alpha}$

$$\frac{F}{N^2} \simeq \underbrace{\frac{1}{4}\alpha^4 f(\chi)}_{=A} \sum_{i=1}^{s} r_i(n_i^3 - n_i) + \underbrace{2\log\alpha}_{=B} \sum_{i=1}^{s} (r_i)^2.$$

Minimize this free energy under $\sum_{i=1}^{s} n_i r_i = 1$.

$$r_i = \frac{1}{2B}\left[\lambda n_i - A(n_i^3 - n_i)\right], \text{ where}$$

$$\lambda = \frac{1}{\sum_{i=1}^{s} n_i^2}\left[2B + A\sum_{i=1}^{s}(n_i^4 - n_i^2)\right].$$

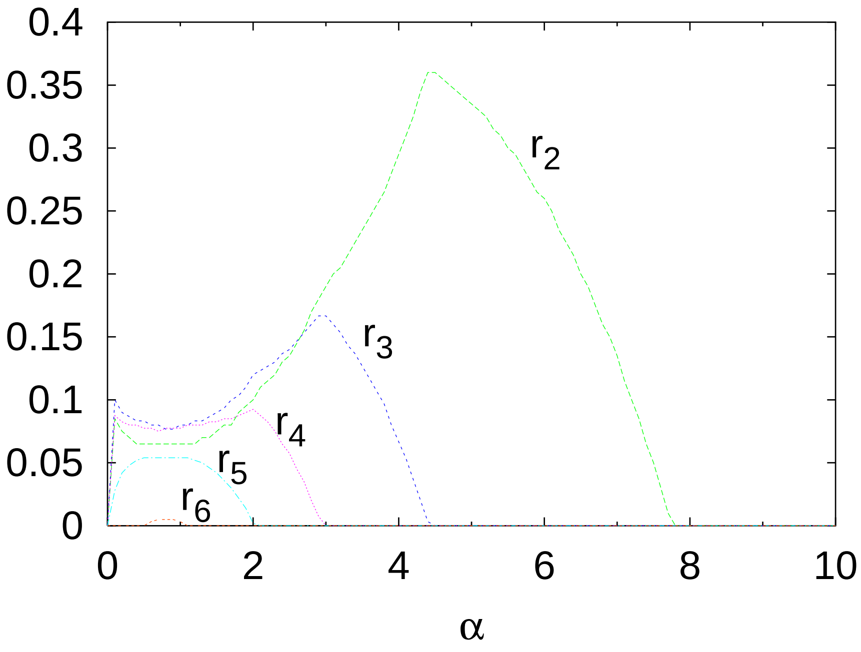
For brevity, $n_1 = 1, n_2 = 2, n_3 = 3, \cdots, n_s = s$.

Condition $r_i > 0 \Rightarrow s(s^2-1)(4s^2-1) < \frac{60B}{A} (= \frac{480\log\alpha}{\alpha^4 f(\chi)})$.

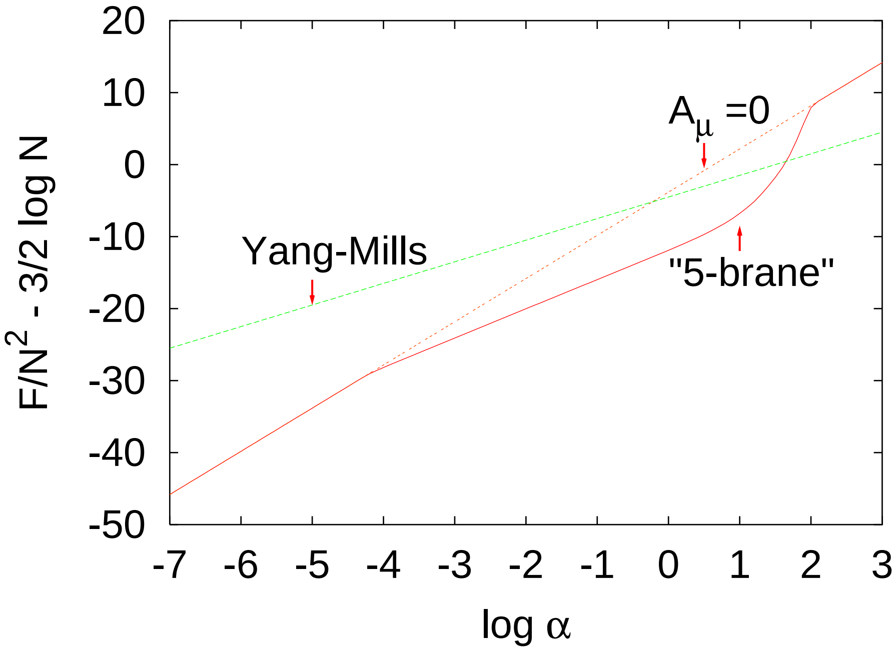
- **Large** $\alpha \Rightarrow$ **Only** $s = 1$ **is possible** $\Rightarrow$ $A_\mu = 0$ **is favored.**
- $s = 2$ **(2 × 2 spheres) appears at** $\frac{A}{B} = \frac{2}{3}$.

General analysis at moderate α

r_i's that minimize the free energy at $\rho = 0.7$.

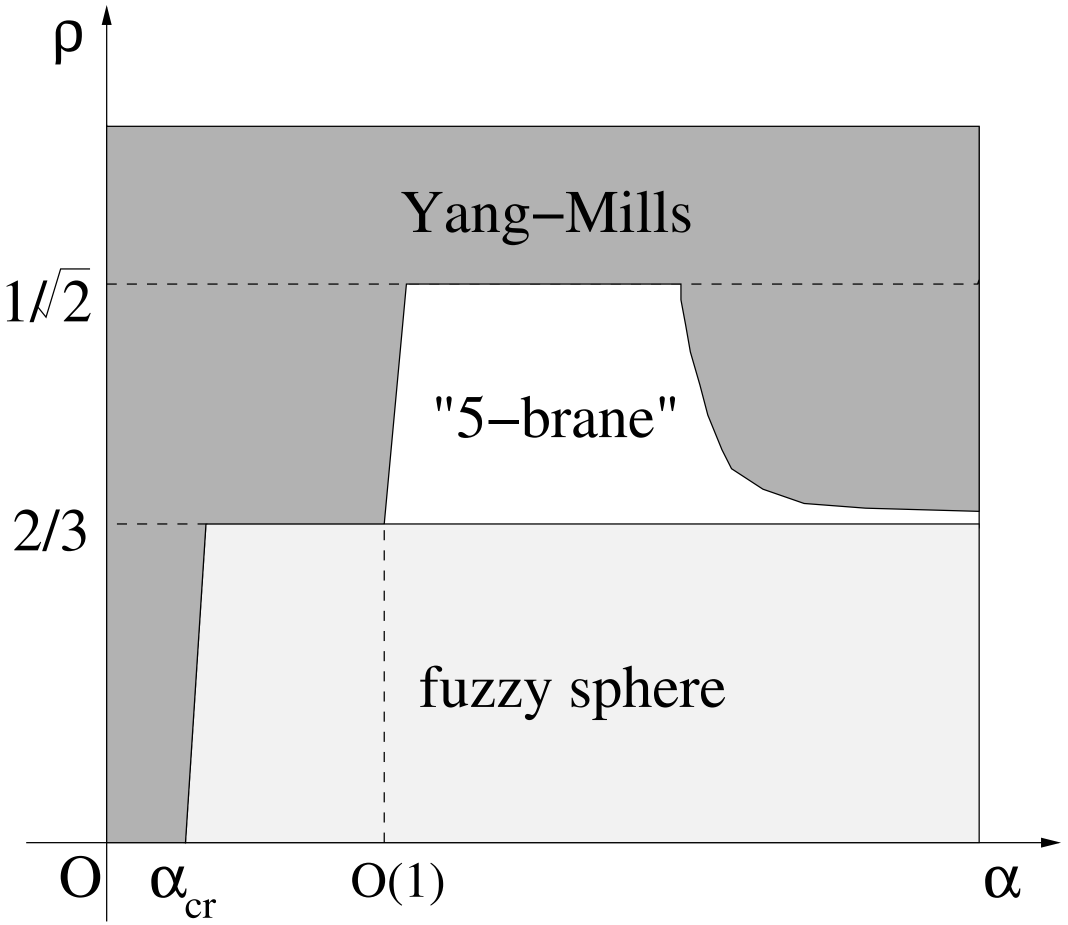


Comparison of free energy with Yang-Mills phase at $\rho = 0.7$.



The Yang-Mills phase takes over at $\alpha \simeq O(1)$.

Phase diagram



5 Conclusion

Massive Yang-Mills-Chern-Simons model:

Many local minima with different gauge groups.

A nice laboratory for testing ideas and methods to study superstring theory by matrix models.

In this work, we have found

$\boxed{\textbf{Dynamical generation of nontrivial gauge group.}}$

The road to deriving the SU(3) $\times$ SU(2) $\times$ U(1) gauge group from the large-N reduced model?