Supermatrix Models

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1 Introduction

$\boxed{\text{Constructive definition of superstring theory}}$

Large N reduced models are the most powerful candidate for the constructive definition of superstring theory.

$\boxed{\text{IKKT model}}$

N.Ishibashi, H.Kawai, Y.Kitazawa and A.Tsuchiya, hep-th/9612115.

For a review, hep-th/9908038

Dimensional reduction of $\mathcal{N} = 1$ 10-dimensional SYM theory to 0 dimension.

Matrix regularization of Green-Schwarz action of type IIB superstring theory.

This theory possesses chiral $\mathcal{N} = 2$ SUSY in 10 dimensions which is identical to that of type IIB superstring theory.

$$\boxed{S = -\frac{1}{g^2} Tr_{N\times N}\left(\frac{1}{4}\sum_{i,j=0}^{9}[A_i, A_j]^2 + \frac{1}{2}\bar{\psi}\sum_{i=0}^{9}\Gamma^i[A_i, \psi]\right).}$$

- A_i and ψ are $N \times N$ Hermitian matrices.
  - * A_i: 10-dimensional vectors
  - * ψ: 10-dimensional Majorana-Weyl (i.e. 16-component) spinors
- This model possesses $SU(N)$ gauge symmetry and $SO(9,1)$ Lorentz symmetry.
- $\mathcal{N} = 2$ SUSY: This theory must contain spin-2 gravitons if it contains massless particles.
  - * homogeneous : $\delta_\epsilon^{(1)} A_i = i\bar{\epsilon}\Gamma_i\psi, \quad \delta_\epsilon^{(1)}\psi = \frac{i}{2}\Gamma^{ij}[A_i, A_j]\epsilon.$
  - * inhomogeneous : $\delta_\xi^{(2)} A_i = 0, \quad \delta_\xi^{(2)}\psi = \xi.$
  - * We obtain the following commutation relations:

$$\begin{aligned}
&(1) \quad [\delta_{\epsilon_1}^{(1)}, \delta_{\epsilon_2}^{(1)}]A_i = 0, \ \ [\delta_{\epsilon_1}^{(1)}, \delta_{\epsilon_2}^{(1)}]\psi = 0, \\
&(2) \quad [\delta_{\xi_1}^{(2)}, \delta_{\xi_2}^{(2)}]A_i = 0, \ \ [\delta_{\xi_1}^{(2)}, \delta_{\xi_2}^{(2)}] = 0, \\
&(3) \quad [\delta_{\epsilon}^{(1)}, \delta_{\xi}^{(2)}]A_i = -i\bar{\epsilon}\Gamma_i\xi, \ \ [\delta_{\epsilon}^{(1)}, \delta_{\xi}^{(2)}]\psi = 0.
\end{aligned}$$

- The matrices describe the many-body system.
- No free parameter: $A_i \to g^{\frac{1}{2}} A_i$, $\psi \to g^{\frac{3}{4}}\psi$.

(♣) Proof of the commutation relations

1. $[\delta^{(1)}_{\epsilon_1}, \delta^{(1)}_{\epsilon_2}]A_i = 0$, $[\delta^{(1)}_{\epsilon_1}, \delta^{(1)}_{\epsilon_2}]\psi = 0$.

The commutation relation for the bosons is obtained by comparing the following two paths:

$$
\begin{aligned}
A_i &\overset{\delta^{(1)}_{\epsilon_2}}{\to} A_i + i\epsilon_2\Gamma_i\psi \overset{\delta^{(1)}_{\epsilon_1}}{\to} A_i + i(\bar{\epsilon}_1+\bar{\epsilon}_2)\Gamma_i\psi - \frac{1}{2}\bar{\epsilon}_2\Gamma_i[A_j, A_k]\Gamma^{jk}\epsilon_1, \\
A_i &\overset{\delta^{(1)}_{\epsilon_1}}{\to} A_i + i\epsilon_1\Gamma_i\psi \overset{\delta^{(1)}_{\epsilon_2}}{\to} A_i + i(\bar{\epsilon}_1+\bar{\epsilon}_2)\Gamma_i\psi - \frac{1}{2}\bar{\epsilon}_1\Gamma_i[A_j, A_k]\Gamma^{jk}\epsilon_2.
\end{aligned}
$$

Then, the commutator is

$$
\begin{aligned}
[\delta^{(1)}_{\epsilon_1}, \delta^{(1)}_{\epsilon_2}]A_i &= -\frac{1}{2}\bar{\epsilon}_2\Gamma_i[A_j, A_k]\Gamma^{jk}\epsilon_1 + \frac{1}{2}\bar{\epsilon}_1\Gamma_i[A_j, A_k]\Gamma^{jk}\epsilon_2 \\
&= [A_i, 2\bar{\epsilon}_1\Gamma^k\epsilon_2 A_k].
\end{aligned}
$$

On the other hand, the commutation relation for the fermions is obtained by

$$
\begin{aligned}
\psi &\overset{\delta^{(1)}_{\epsilon_2}}{\to} \psi + \frac{i}{2}[A_i, A_j]\Gamma^{ij}\epsilon_2 \overset{\delta^{(1)}_{\epsilon_1}}{\to} \psi + \frac{i}{2}[A_i, A_j]\Gamma^{ij}(\epsilon_1+\epsilon_2) - [A_i, \bar{\epsilon}_1\Gamma_j\psi]\Gamma^{ij}\epsilon_2, \\
\psi &\overset{\delta^{(1)}_{\epsilon_1}}{\to} \psi + \frac{i}{2}[A_i, A_j]\Gamma^{ij}\epsilon_1 \overset{\delta^{(1)}_{\epsilon_2}}{\to} \psi + \frac{i}{2}[A_i, A_j]\Gamma^{ij}(\epsilon_1+\epsilon_2) - [A_i, \bar{\epsilon}_2\Gamma_j\psi]\Gamma^{ij}\epsilon_1.
\end{aligned}
$$

By using the formula of Fierz transformation

$$
\begin{aligned}
\bar{\epsilon}_1\Gamma_j\psi\Gamma^{ij}\epsilon_2 &= (\bar{\epsilon}_1\Gamma^i\epsilon_2)\psi - \frac{7}{16}(\bar{\epsilon}_1\Gamma^k\epsilon_2)\Gamma_k\Gamma^i\psi \\
&- \frac{1}{16\times 5!}(\bar{\epsilon}_1\Gamma^{k_1\cdots k_5}\epsilon_2)\Gamma_{k_1\cdots k_5}\Gamma^i\psi,
\end{aligned}
$$

and the equation of motion

$$
\frac{dS}{d\bar{\psi}} = -\frac{1}{g^2}\Gamma^i[A_i, \psi] = 0,
$$

the commutator is computed on shell to be

$$
[\delta^{(1)}_{\epsilon_1}, \delta^{(1)}_{\epsilon_2}]\psi = [\psi, 2\bar{\epsilon}_1\Gamma^k\epsilon_2 A_k].
$$

These commutators are set to be zero by the gauge transformation.

2. $[\delta^{(2)}_{\xi_1}, \delta^{(2)}_{\xi_2}]A_i = 0$, $[\delta^{(2)}_{\xi_1}, \delta^{(2)}_{\xi_2}]\psi = 0$.

This is trivial because the inhomogeneous SUSY transformation is merely a translation of the fermions.

3. $[\delta^{(1)}_{\epsilon}, \delta^{(2)}_{\xi}]A_i = -i\bar{\epsilon}\Gamma_i\xi$, $[\delta^{(1)}_{\epsilon}, \delta^{(2)}_{\xi}]\psi = 0$.

This can be proven by taking the difference of these two transformations:

$$
\begin{aligned}
&A_i \xrightarrow{\delta^{(2)}_{\xi}} A_i \xrightarrow{\delta^{(1)}_{\epsilon}} A_i + i\bar{\epsilon}\Gamma_i\psi \\
&A_i \xrightarrow{\delta^{(1)}_{\epsilon}} A_i + i\bar{\epsilon}\Gamma_i\psi \xrightarrow{\delta^{(2)}_{\xi}} A_i + i\bar{\epsilon}\Gamma_i(\psi + \xi), \\
&\psi \xrightarrow{\delta^{(2)}_{\xi}} \psi + \xi \xrightarrow{\delta^{(1)}_{\epsilon}} \psi + \xi + \frac{i}{2}\Gamma^{ij}[A_i, A_j]\epsilon \\
&\psi \xrightarrow{\delta^{(1)}_{\epsilon}} \psi + \frac{i}{2}\Gamma^{ij}[A_i, A_j]\epsilon \xrightarrow{\delta^{(2)}_{\xi}} \psi + \xi + \frac{i}{2}\Gamma^{ij}[A_i, A_j]\epsilon.
\end{aligned}
$$

We take the following linear combination

$$\tilde{\delta}^{(1)} = \delta^{(1)} + \delta^{(2)}, \ \tilde{\delta}^{(2)} = i(\delta^{(1)} - \delta^{(2)}).$$

This gives a shift of the bosonic variables

$$
\boxed{
\begin{aligned}
&[\tilde{\delta}^{(a)}_{\epsilon}, \tilde{\delta}^{(b)}_{\xi}]\psi = 0, \\
&[\tilde{\delta}^{(a)}_{\epsilon}, \tilde{\delta}^{(b)}_{\xi}]A_i = -2i\delta^{ab}\bar{\epsilon}\Gamma_i\xi.
\end{aligned}
}
$$

We investigate a matrix model based on super Lie algebra $osp(1|32, R)$, as a candidate of the matrix model which naturally reproduces IKKT model.

L. Smolin, hep-th/0002009

T. Azuma, S. Iso, H. Kawai and Y. Ohwashi, hep-th/0102168

- $osp(1|32, R)$ was first mentioned on 11-dimensional supergravity.

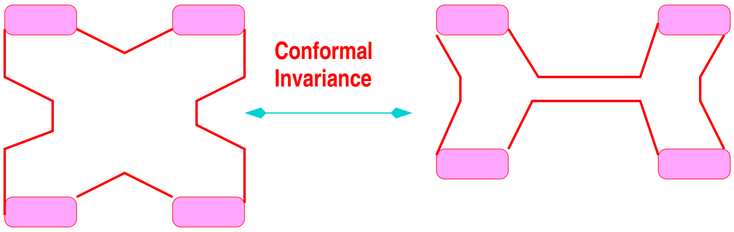
  E. Cremmer, B. Julia, J. Scherk Phys.Lett. B76 ,409 (1978)

  $\Rightarrow$ This has attracted a new attention as the unified super Lie algebra for M-theory.

- $osp(1|32, R)$ may describe the curved 10-dimensional spacetime.

- The theory is described by a cubic action :

  * The cubic interaction is the most fundamental one in string theory.



  * Chern Simons Theory is exactly solvable by means of Jones polynomial.

    E. Witten, *Commun. Math. Phys.* 121 (1989) 351

    The non-perturbative analysis may be exactly performed.

2 Notations on the supermatrices

The vectors and supermatrices are defined by

$$v = \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_m \\ b_1 \\ \vdots \\ b_n \end{pmatrix}, \quad \begin{pmatrix} \{\eta_i\} : & \text{fermions} \\ \{b_j\} : & \text{bosons} \end{pmatrix},$$

$$M = \begin{pmatrix} a & \beta \\ \gamma & d \end{pmatrix}, \quad \begin{pmatrix} a(d) : & m \times m(n \times n) \\ & \text{bosonic matrices} \\ \beta(\gamma) : & m \times n(n \times m) \\ & \text{fermionic matrices} \end{pmatrix}.$$

Transpose

- The transpose of the vector is defined by

$${}^T v = {}^T \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_m \\ b_1 \\ \vdots \\ b_n \end{pmatrix} = (\eta_1, \cdots, \eta_m, b_1, \cdots, b_n).$$

- **The transpose of the supermatrix is defined so that $^T M$ satisfies $^T(Mv) = {}^T v\, {}^T M$.**

$$\Leftrightarrow {}^T M = {}^T \begin{pmatrix} a & \beta \\ \gamma & d \end{pmatrix} = \begin{pmatrix} {}^T a & -{}^T\gamma \\ {}^T\beta & {}^T d \end{pmatrix}.$$

(Proof) We verify that this is well-defined by going back to the guiding principle $^T(Mv) = {}^T v\, {}^T M$.

$$\begin{aligned}
\text{(L.H.S.)} &= {}^T(Mv) = {}^T \begin{pmatrix} a\eta + \beta b \\ \gamma\eta + db \end{pmatrix} = ({}^T\eta\, {}^T a + {}^T b\, {}^T\beta, -{}^T\eta\, {}^T\gamma + {}^T b\, {}^T d), \\
\text{(R.H.S.)} &= ({}^T\eta, {}^T b) \begin{pmatrix} {}^T a & -{}^T\gamma \\ {}^T\beta & {}^T d \end{pmatrix} = ({}^T\eta\, {}^T a + {}^T b\, {}^T\beta, -{}^T\eta\, {}^T\gamma + {}^T b\, {}^T d).
\end{aligned}$$

- **The transpose of the transverse vector $y = ({}^T\eta, {}^T b)$ is defined so that $^T(yM) = {}^T M\, {}^T y$:**

$$\Leftrightarrow {}^T y = {}^T({}^T\eta, {}^T b) = \begin{pmatrix} -\eta \\ b \end{pmatrix}.$$

(Proof) This can be again confirmed by comparing the both hand sides:

$$\begin{aligned}
\text{(L.H.S.)} &= {}^T(yM) = {}^T({}^T\eta a + {}^T b\gamma, {}^T\eta\beta + {}^T bd) = \begin{pmatrix} -{}^T({}^T\eta a) - {}^T({}^T b\gamma) \\ {}^T({}^T\eta\beta) + {}^T({}^T bd) \end{pmatrix} \\
&= \begin{pmatrix} -{}^T a\eta - {}^T\gamma b \\ -{}^T\beta\eta + {}^T db \end{pmatrix}, \\
\text{(R.H.S.)} &= {}^T M\, {}^T y = \begin{pmatrix} {}^T a & -{}^T\gamma \\ {}^T\beta & {}^T d \end{pmatrix} \begin{pmatrix} -\eta \\ b \end{pmatrix} = \begin{pmatrix} -{}^T a\eta - {}^T\gamma b \\ -{}^T\beta\eta + {}^T\gamma b \end{pmatrix}.
\end{aligned}$$

[Remark]: **The transpose of the transpose of the vector or supermatrix does not go back to the original one:**

$$\begin{aligned}
&{}^T({}^T \begin{pmatrix} a & \beta \\ \gamma & d \end{pmatrix}) = {}^T \begin{pmatrix} {}^T a & -{}^T\gamma \\ {}^T\beta & {}^T d \end{pmatrix} = \begin{pmatrix} a & -\beta \\ -\gamma & d \end{pmatrix}, \\
&{}^T({}^T \begin{pmatrix} \eta \\ b \end{pmatrix}) = {}^T({}^T\eta, {}^T b) = \begin{pmatrix} -\eta \\ b \end{pmatrix}.
\end{aligned}$$

Hermitian Conjugate

We settle the complex conjugate of the fermionic numbers α and β as

$$(\alpha\beta)^\dagger = (\beta)^\dagger(\alpha)^\dagger.$$

- We first define the Hermitian conjugate of the vector as

$$v^\dagger = \begin{pmatrix} \eta \\ b \end{pmatrix}^\dagger = (\eta^\dagger, b^\dagger).$$

- $M^\dagger$ is defined so that this satisfies $(Mv)^\dagger = v^\dagger M^\dagger$:

$$M^\dagger = \begin{pmatrix} a & \beta \\ \gamma & d \end{pmatrix}^\dagger = \begin{pmatrix} a^\dagger & \gamma^\dagger \\ \beta^\dagger & d^\dagger \end{pmatrix}.$$

- $y^\dagger = ({}^T\eta, {}^Tb)^\dagger$ is defined so that $(yM)^\dagger = M^\dagger y^\dagger$:

$$y^\dagger = ({}^T\eta, {}^Tb)^\dagger = \begin{pmatrix} ({}^T\eta)^\dagger \\ ({}^Tb)^\dagger \end{pmatrix}.$$

Complex Conjugate

The complex conjugate is defined so that the supermatrices and the vectors satisfy $(Mv)^* = M^* v^*$:

$$
\begin{aligned}
v^* &= ({}^T v)^\dagger = \begin{pmatrix} \eta \\ b \end{pmatrix}^* = \begin{pmatrix} \eta^* \\ b^* \end{pmatrix}, \\
M^* &= ({}^T M)^\dagger = \begin{pmatrix} a & \beta \\ \gamma & d \end{pmatrix}^* = \begin{pmatrix} a^* & \beta^* \\ -\gamma^* & d^* \end{pmatrix}, \\
y^* &= ({}^T y)^\dagger = (\eta, b)^* = (-\eta^*, b^*).
\end{aligned}
$$

[Prop] (1) ${}^T M = (M^*)^\dagger$, (2) $M^\dagger = {}^T(M^*)$, (3) $(M^*)^* = M$.

A supermatrix M is real if

M is a mapping from a real vector to a real vector.

i.e. M satisfies $M^* = M$:

$$a^* = a,\ \beta^* = \beta,\ d^* = d,\ \gamma^* = -\gamma.$$

3 $osp(1|32,R)$ (nongauged) cubic matrix model

$osp(1|32,R)$ super Lie algebra

- $M \in osp(1|32,R) \Rightarrow {}^TMG + GM = 0$, where $G = \begin{pmatrix} \Gamma^0 & 0 \\ 0 & i \end{pmatrix}$.

  The reality of M constrains the matrix G to be $G^\dagger \propto G$:

$$0 = ({}^TMG + GM)^\dagger = G^\dagger M^* + M^\dagger G^\dagger = G^\dagger M + {}^TMG^\dagger.$$

  Now, $G^\dagger = \begin{pmatrix} \Gamma^0 & 0 \\ 0 & i \end{pmatrix}^\dagger = \begin{pmatrix} {}^T(\Gamma^0) & 0 \\ 0 & -i \end{pmatrix} = \begin{pmatrix} -\Gamma^0 & 0 \\ 0 & -i \end{pmatrix} = -G$.

- $M = \begin{pmatrix} m & \psi \\ i\bar{\psi} & 0 \end{pmatrix}$, where ${}^Tm\Gamma^0 + \Gamma^0 m = 0$ and $m = u_{\mu_1}\Gamma^{\mu_1} + \frac{1}{2!}u_{\mu_1\mu_2}\Gamma^{\mu_1\mu_2} + \frac{1}{5!}u_{\mu_1\cdots\mu_5}\Gamma^{\mu_1\cdots\mu_5}$.

  (Proof) Let M be of the general form $M = \begin{pmatrix} m & \psi \\ i\bar{\phi} & v \end{pmatrix}$. The definition constrains these elements to be

$$\begin{aligned} 0 &= {}^TMG + GM = \begin{pmatrix} {}^Tm & -i{}^T\bar{\phi} \\ {}^T\psi & {}^Tv \end{pmatrix}\begin{pmatrix} \Gamma^0 & 0 \\ 0 & i \end{pmatrix} + \begin{pmatrix} \Gamma^0 & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} m & \psi \\ i\bar{\phi} & v \end{pmatrix} \\ &= \begin{pmatrix} {}^Tm\Gamma^0 + \Gamma^0 m & {}^T\bar{\phi} + \Gamma^0\psi \\ {}^T\psi\Gamma^0 - \bar{\phi} & 2iv \end{pmatrix}. \end{aligned}$$

  It follows that $v = 0$, $\psi = \phi$ and that ${}^Tm\Gamma^0 + \Gamma^0 m = 0$ (i.e. $m \in sp(32)$).

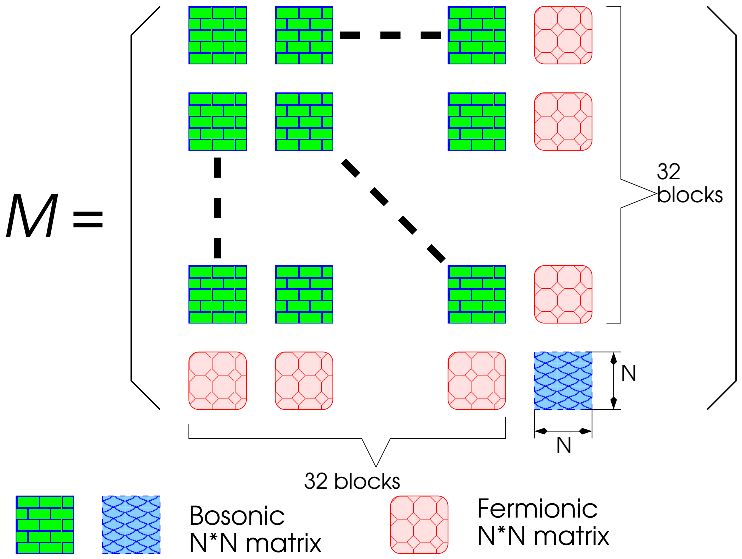
  m is determined by noting that $m = -(\Gamma^0)^{-1}({}^Tm)\Gamma^0 = \Gamma^0({}^Tm)\Gamma^0$ and that

$$\begin{aligned} \Gamma^0({}^T\Gamma^{\mu_1\cdots\mu_k})\Gamma^0 &= (-1)^{k-1}(\Gamma^0({}^T\Gamma^{\mu_k})\Gamma^0)\cdots(\Gamma^0({}^T\Gamma^{\mu_1})\Gamma^0) = (-1)^{k-1}\Gamma^{\mu_k\cdots\mu_1} \\ &= (-1)^{k-1}(-1)^{\frac{k(k-1)}{2}}\Gamma^{\mu_1\cdots\mu_k} = (-1)^{\frac{(k+2)(k-1)}{2}}\Gamma^{\mu_1\cdots\mu_k} \\ &= \begin{cases} \Gamma^{\mu_1\cdots\mu_k} & (k = 1,2,5), \\ -\Gamma^{\mu_1\cdots\mu_k} & (k = 0,3,4). \end{cases} \end{aligned}$$

Action of the cubic model

$$\begin{aligned} I &= \frac{i}{g^2} Tr_{N\times N} \sum_{Q,R=1}^{33} [(\sum_{p=1}^{32} M_p{}^Q [M_Q{}^R, M_R{}^p]) - M_{33}{}^Q [M_Q{}^R, M_R{}^{33}]] \\ &= -\frac{f^{abc}}{2g^2} \sum_{a,b,c=1}^{N^2} Str_{33\times 33}(M_a M_b M_c) \\ &= \frac{i}{g^2} Tr_{N\times N} [m_p{}^q [m_q{}^r, m_r{}^p] - 3i\bar{\psi}^p [m_p{}^q, \psi^q]]. \end{aligned}$$

- Each component of the 33×33 supermatrices is promoted to a large N hermitian matrix.



- This action is defined to be real.
- No free parameter: $M \to g^{\frac{2}{3}} M$.
- $OSp(1|32, R) \times U(N)$ gauge symmetry.
  - $M \to M + [M, (S \otimes 1_{N\times N})]$ for $S \in osp(1|32, R)$,
  - $M \to M + [M, (1_{33\times 33} \otimes U)]$ for $U \in u(N)$.

Supersymmetry

The SUSY transformation of the $osp(1|32,R)$ is identified with that of IKKT model.

- homogeneous SUSY:
  The SUSY transformation by the supercharge $Q = \begin{pmatrix} 0 & \chi \\ i\bar{\chi} & 0 \end{pmatrix}$.

$$\delta_\chi^{(1)} M = [Q, M] = \begin{pmatrix} i(\chi\bar{\psi} - \psi\bar{\chi}) & -m\chi \\ i\bar{\chi}m & 0 \end{pmatrix}.$$

- inhomogeneous SUSY:
  The translation of the fermionic field $\delta_\epsilon^{(2)}\psi = \epsilon$.

In order to see the correspondence of the fields with IKKT model, we express the bosonic 32×32 matrices in terms of the 10-dimensional indices ($i = 0, \cdots 9$, $\sharp = 10$).

$$\begin{aligned} m &= W\Gamma^\sharp + \frac{1}{2}[A_i^{(+)}\Gamma^i(1+\Gamma^\sharp) + A_i^{(-)}\Gamma^i(1-\Gamma^\sharp)] + \frac{1}{2!}C_{i_1 i_2}\Gamma^{i_1 i_2} + \\ &\quad + \frac{1}{4!}H_{i_1\cdots i_4}\Gamma^{i_1\cdots i_4\sharp} + \frac{1}{5!}[I^{(+)}_{i_1\cdots i_5}\Gamma^{i_1\cdots i_5}(1+\Gamma^\sharp) + I^{(-)}_{i_1\cdots i_5}\Gamma^{i_1\cdots i_5}(1-\Gamma^\sharp)]. \end{aligned}$$

Identification of the fields

$$\begin{aligned} \delta_\chi^{(1)} A_i^{(+)} &= \frac{1}{32}tr((\delta_\chi^{(1)} m)\Gamma_i) + \frac{-1}{32}tr((\delta_\chi^{(1)} m)\Gamma_{i\sharp}) \\ &= \frac{i}{32}tr[(\chi\bar{\psi} - \psi\bar{\chi})\Gamma_i(1-\Gamma_\sharp)] \\ &= \frac{i}{16}\bar{\chi}\Gamma_i(1-\Gamma_\sharp)\psi = \frac{i}{8}\bar{\chi}_R\Gamma_i\psi_R, \\ \delta_\chi^{(1)} A_i^{(-)} &= \frac{i}{16}\bar{\chi}\Gamma_i(1+\Gamma_\sharp)\psi = \frac{i}{8}\bar{\chi}_L\Gamma_i\psi_L, \\ \delta_\chi^{(1)}\psi &= -m\psi. \end{aligned}$$

Commutation relations

- $[\delta^{(1)}_\chi, \delta^{(2)}_\epsilon]m = -i(\chi\bar{\epsilon} - \epsilon\bar{\chi}), \quad [\delta^{(1)}_\chi, \delta^{(2)}_\epsilon]\psi = 0.$

(Proof) We compare the following two paths:

* $m \xrightarrow{\delta^{(2)}_\epsilon} m \xrightarrow{\delta^{(1)}_\chi} m + i(\chi\bar{\psi} - \psi\bar{\chi})$, whereas
$m \xrightarrow{\delta^{(1)}_\chi} m + i(\chi\bar{\psi} - \psi\bar{\chi}) \xrightarrow{\delta^{(2)}_\epsilon} m + i\chi(\bar{\psi} + \bar{\epsilon}) - i(\psi + \epsilon)\bar{\chi}$.

* $\psi \xrightarrow{\delta^{(2)}_\epsilon} \psi + \epsilon \xrightarrow{\delta^{(1)}_\chi} \psi + \epsilon - m\chi$, whereas $\psi \xrightarrow{\delta^{(1)}_\chi} \psi - m\chi \xrightarrow{\delta^{(2)}_\epsilon} \psi + \epsilon - m\chi$.

$$
\begin{aligned}
&[\delta^{(1)}_{\chi_R}, \delta^{(2)}_{\epsilon_R}]A^{(+)}_i \\
&\quad = \frac{1}{32} tr(([\delta^{(1)}_{\chi_R}, \delta^{(2)}_{\epsilon_R}]m)\Gamma_i) + \frac{-1}{32} tr(([\delta^{(1)}_{\chi_R}, \delta^{(2)}_{\epsilon_R}]m)\Gamma_{i\sharp}) \\
&\quad = \frac{i}{32} tr((\epsilon_R\bar{\chi}_R - \chi_R\bar{\epsilon}_R)\Gamma_i(1 - \Gamma_\sharp)) = \frac{i}{8}\bar{\epsilon}_R\Gamma_i\chi_R, \\
&[\delta^{(1)}_{\chi_L}, \delta^{(2)}_{\epsilon_L}]A^{(+)}_i = 0, \\
&[\delta^{(1)}_{\chi_R}, \delta^{(2)}_{\epsilon_R}]A^{(-)}_i = 0, \quad [\delta^{(1)}_{\chi_L}, \delta^{(2)}_{\epsilon_L}]A^{(-)}_i = \frac{i}{8}\bar{\epsilon}_L\Gamma_i\chi_L, \\
&[\delta^{(1)}_{\chi_L}, \delta^{(2)}_{\epsilon_R}]A^{(\pm)}_i = [\delta^{(1)}_{\chi_R}, \delta^{(2)}_{\epsilon_L}]A^{(\pm)}_i = 0.
\end{aligned}
$$

- $[\delta^{(2)}_\chi, \delta^{(2)}_\epsilon]m = [\delta^{(2)}_\chi, \delta^{(2)}_\epsilon]\psi = 0$ **is trivial.**

- $[\delta^{(1)}_\chi, \delta^{(1)}_\epsilon] m = i[\chi\bar{\epsilon} - \epsilon\bar{\chi}, m]$, $[\delta^{(1)}_\chi, \delta^{(1)}_\epsilon]\psi = i(\chi\bar{\epsilon} - \epsilon\bar{\chi})\psi$.

(Proof) This is verified by noting the following identity:

$$[\delta^{(1)}_\chi, \delta^{(1)}_\epsilon] M = [Q_\chi, [Q_\epsilon, M]] - [Q_\epsilon, [Q_\chi, M]] = [[Q_\chi, Q_\epsilon], M]$$
$$= [\begin{pmatrix} i(\chi\bar{\epsilon} - \epsilon\bar{\chi}) & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} m & \psi \\ i\bar{\psi} & 0 \end{pmatrix}] = \begin{pmatrix} [i(\chi\bar{\epsilon} - \epsilon\bar{\chi}), m] & i(\chi\bar{\epsilon} - \epsilon\bar{\chi})\psi \\ -i\bar{\psi}(\chi\bar{\epsilon} - \epsilon\bar{\chi}) & 0 \end{pmatrix}.$$

* $[\delta^{(1)}_{\chi_R}, \delta^{(1)}_{\epsilon_R}] A^{(+)}_i = \frac{i}{8}\bar{\chi}_R [m, \Gamma_i]\epsilon_R$.
In the (r.h.s.), the fields W, $C_{i_1 i_2}$ and $H_{i_1 \cdots i_4}$ survive.
$\rightarrow$ these fields are integrated out.

* $[\delta^{(1)}_{\chi_L}, \delta^{(1)}_{\epsilon_R}] A^{(+)}_i = -\frac{i}{8}\bar{\chi}_L A^{(+)}_j \Gamma_i{}^j \epsilon_R + \cdots$.
The fields $A^{(\pm)}_i$ itself remains in the commutator!

$\boxed{\text{Summary}}$

The $osp(1|32, R)$ cubic matrix model possesses a two-fold structure of the SUSY of IKKT model.

IKKT model	bosons A_i	fermions ψ	SUSY parameters
SUSY I	$A^{(+)}_i$	ψ_R	χ_R, ϵ_R
SUSY II	$A^{(-)}_i$	ψ_L	χ_L, ϵ_L

Action of IKKT model

♣ We expand the action around the classical solution $A_0^{(+)} = p_1, A_1^{(+)} = q_1, \cdots, A_8^{(+)} = p_5, A_9^{(+)} = q_5$.
$([p_g, q_h] = -i\delta_{g,h}\ (g, h = 1, \cdots, 5))$.

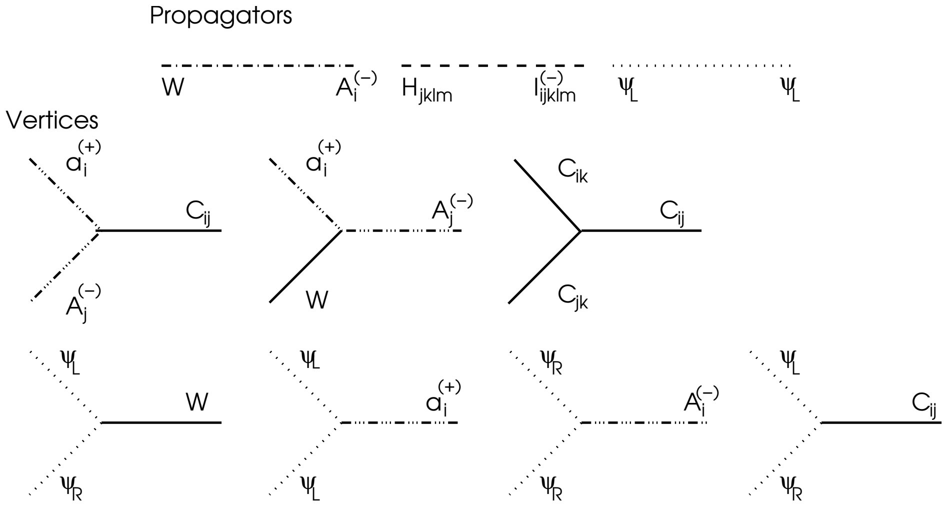
$$
\begin{aligned}
I_b &= \frac{1}{g^2} Tr_{N\times N}(-96(\partial_{i_1} A^{(-)}_{i_2})C^{i_1 i_2} + 96(\partial_i W)A^{(-)i} + 4(\partial_{i_1} H_{i_2\cdots i_5})I^{(-)i_1\cdots i_5}) \\
&+ \frac{i}{g^2} Tr_{N\times N}(-96[a^{(+)}_{i_1}, A^{(-)}_{i_2}]C^{i_1 i_2} - 96W[a^{(+)i}, A^{(-)}_i] + \frac{4}{5}W[I^{(+)}_{i_1\cdots i_5}, I^{(-)i_1\cdots i_5}] \\
&+ 4([a^{(+)}_{i_1}, H_{i_2\cdots i_5}]I^{(-)i_1\cdots i_5} - [A^{(-)}_{i_1}, H_{i_2\cdots i_5}]I^{(+)i_1\cdots i_5}) - 8C_{i_1 i_2}[I^{(+)i_1}{}_{i_3\cdots i_6}, I^{(-)i_2\cdots i_6}] \\
&+ \frac{8}{3}H^{kl}{}_{i_1 i_2}([I^{(+)}{}_{kl i_3 i_4 i_5}, I^{(-)i_1\cdots i_5}] - [I^{(-)}{}_{kl i_3 i_4 i_5}, I^{(+)i_1\cdots i_5}]) \\
&+ 32[C^{i_1}{}_{i_2}, C_{i_1 i_3}]C^{i_2 i_3} - 16C_{i_1 i_2}[H^{i_1}{}_{i_3 i_4 i_5}, H^{i_2\cdots i_5}] + \frac{1}{27}H_{i_1\cdots i_4}[H^k{}_{i_5\cdots i_7}, H_{k i_8 i_9 i_{10}}]\epsilon^{i_1\cdots i_{10}\sharp}), \\
I_f &= \frac{1}{g^2} Tr_{N\times N}(-3i\bar{\psi}_L \Gamma^i \partial_i \psi_L) \\
&+ \frac{i}{g^2} Tr_{N\times N}(-3i(-\bar{\psi}_L[W, \psi_R] + \bar{\psi}_R[W, \psi_L]) - 3i(\bar{\psi}_L \Gamma^i [a^{(+)}_i, \psi_L] + \bar{\psi}_R \Gamma^i [A^{(-)}_i, \psi_R]) \\
&- \frac{3i}{2!}(\bar{\psi}_L \Gamma^{i_1 i_2}[C_{i_1 i_2}, \psi_R] + \bar{\psi}_R \Gamma^{i_1 i_2}[C_{i_1 i_2}, \psi_L]) \\
&- \frac{3i}{4!}(-\bar{\psi}_L \Gamma^{i_1 i_2 i_3 i_4}[H_{i_1 i_2 i_3 i_4}, \psi_R] + \bar{\psi}_R \Gamma^{i_1 i_2 i_3 i_4}[H_{i_1 i_2 i_3 i_4}, \psi_L]) \\
&- \frac{3i}{5!}(2\bar{\psi}_L \Gamma^{i_1 i_2 i_3 i_4 i_5}[I^{(+)}{}_{i_1 i_2 i_3 i_4 i_5}, \psi_L] + 2\bar{\psi}_R \Gamma^{i_1 i_2 i_3 i_4 i_5}[I^{(-)}{}_{i_1 i_2 i_3 i_4 i_5}, \psi_R])).
\end{aligned}
$$

♣ The terms to be identified with the fermionic term of IKKT model are

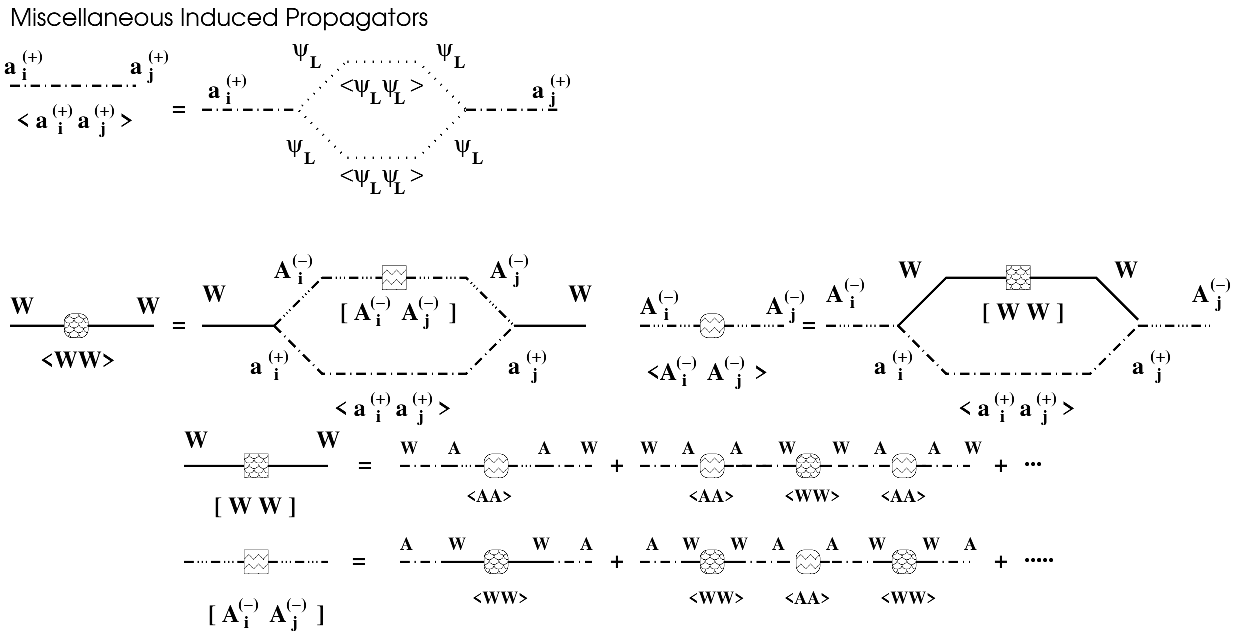
$$\bar{\psi}_R \Gamma^i A^{(+)}_i \psi_R \overset{or}{\Leftrightarrow} \bar{\psi}_L \Gamma^i A^{(-)}_i \psi_L.$$

However, these terms do not exist in this action, and we induce such terms by the multi-loop effect.

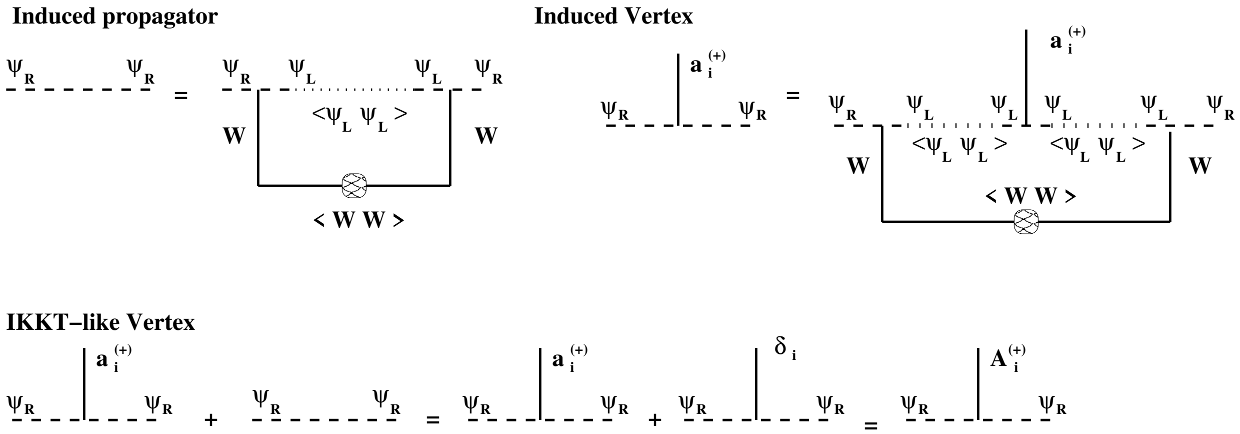
- **The Feynman rule at the tree level:**



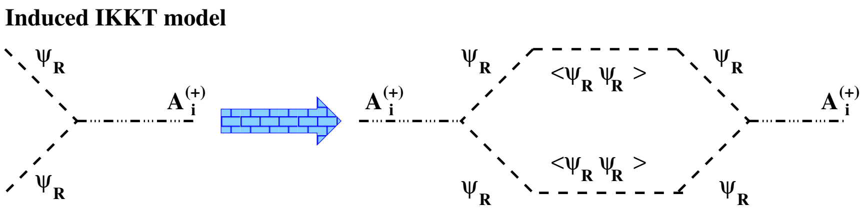
- **We induce the necessary propagators in this way:**



- The desired fermionic terms are obtained as follows:



- The bosonic term of IKKT model is induced in this way.



4 $gl(1|32, R) \otimes gl(N, R)$ gauged model

We consider the model whose gauge symmetry is enhanced by altering the direct product of the Lie algebra.

L. Smolin, hep-th/0006137

T. Azuma, S. Iso, H. Kawai and Y. Ohwashi, hep-th/0102168

(*) $\mathcal{A}, \mathcal{B}$ = [The Lie algebras whose bases are $\{a_i\}$ and $\{b_j\}$, respectively.]

- $\mathcal{A} \otimes \mathcal{B}$: The space spanned by the basis $a_i \otimes b_j$. This is not necessarily a closed Lie algebra.
- $\mathcal{A} \check{\otimes} \mathcal{B}$: The smallest Lie algebra that includes $\mathcal{A} \otimes \mathcal{B}$ as a subset.

The gauge symmetry $OSp(1|32, R) \times U(N)$ is enhanced to $osp(1|32, R) \check{\otimes} u(N)$.

- $osp(1|32, R) \otimes u(N)$ is not a closed Lie algebra.
- $osp(1|32, R) \check{\otimes} u(N) = u(1|16, 16) \otimes u(N)$.
  $u(1|16, 16)$ is the complexification of $osp(1|32, R)$.
- We consider the Lie algebra
  $gl(1|32, R) \check{\otimes} gl(N, R) = gl(1|32, R) \otimes gl(N, R)$
  as an analytical continuation of $u(1|16, 16) \otimes u(N)$.

(*) In order to grasp the intuitive image of 'Smolin's gauged theory', we consider the following simple example.

$$su(6) = su(3)\check{\otimes} su(2).$$

λ^a: basis of $su(3)$ $(a = 1, 2, \cdots 8)$.
σ^i: basis of $su(2)$ $(i = 1, 2, 3)$.

- $\lambda^a \otimes \sigma^i$ (24 dimensions): The basis of $su(3) \otimes su(2)$, which does not constitute a closed Lie algebra.
- $\lambda^a \otimes 1 + 1 \otimes \sigma^i$ (11 dimensions): The generators of the Lie group $SU(3) \times SU(2)$.
- $su(3)\check{\otimes} su(2) = (su(3) \otimes su(2)) \oplus (SU(3) \times SU(2))_{algebra}$ This is a closed 35-dimensional Lie algebra.

$SU(3) \times SU(2)$ is a 11-dimensional Lie group,
while $su(3)\check{\otimes} su(2)$ is a 35-dimensional Lie algebra.

$u(1|16,16)$ super Lie algebra

- $M \in u(1|16,16) \Rightarrow M^\dagger G + GM = 0$,
  where $G = \begin{pmatrix} \Gamma^0 & 0 \\ 0 & i \end{pmatrix}$.

  * This is the complexification of $osp(1|32, R)$, in that M is not necessarily a real supermatrix and that the transpose is replaced by Hermitian conjugate.

  * We can confirm that $G^\dagger \propto G$ in the same way as in $osp(1|32, R)$:

  $$0 = (M^\dagger G + GM)^\dagger = G^\dagger M + M^\dagger G^\dagger.$$

- $M = \begin{pmatrix} m & \psi \\ i\bar{\psi} & v \end{pmatrix}$, where $m^\dagger \Gamma^0 + \Gamma^0 m = 0$.

  * $m = u1 + u_{\mu_1}\Gamma^{\mu_1} + \frac{1}{2!}u_{\mu_1\mu_2}\Gamma^{\mu_1\mu_2} + \frac{1}{3!}u_{\mu_1\mu_2\mu_3}\Gamma^{\mu_1\mu_2\mu_3}$
  $+\frac{1}{4!}u_{\mu_1\cdots\mu_4}\Gamma^{\mu_1\cdots\mu_4} + \frac{1}{5!}u_{\mu_1\cdots\mu_5}\Gamma^{\mu_1\cdots\mu_5}$.

  * $\begin{cases} u_{\mu_1}, u_{\mu_1\mu_2}, u_{\mu_1\cdots\mu_5} & \Rightarrow \text{real number} \\ v, u, u_{\mu_1\mu_2\mu_3}, u_{\mu_1\cdots u_{\mu_4}} & \Rightarrow \text{pure imaginary} \end{cases}$

$u(1|16,16)$ is the direct sum of the two different representations of $osp(1|32,R)$.

♣ $u(1|16,16) = \mathcal{H} \oplus \mathcal{A}'$, where

$$\mathcal{H} = \{M = \begin{pmatrix} m_h & \psi_h \\ i\bar{\psi}_h & 0 \end{pmatrix} |^T HG + GH = 0,$$
$$m_h = u_{\mu_1}\Gamma^{\mu_1} + \frac{1}{2!}u_{\mu_1\mu_2}\Gamma^{\mu_1\mu_2} + \frac{1}{5!}u_{\mu_1\cdots\mu_5}\Gamma^{\mu_1\cdots\mu_5},$$
$$u_{\mu_1}, u_{\mu_1\mu_2}, u_{\mu_1\cdots\mu_5}, \psi_h \in \mathcal{R}\},$$
$$\mathcal{A}' = \{M = \begin{pmatrix} m_a & i\psi_a \\ \bar{\psi}_a & v \end{pmatrix} |^T AG - GA = 0,$$
$$m_a = u + \frac{1}{3!}u_{\mu_1\mu_2\mu_3}\Gamma^{\mu_1\mu_2\mu_3} + \frac{1}{4!}u_{\mu_1\cdots\mu_4}\Gamma^{\mu_1\cdots\mu_4},$$
$$u, u_{\mu_1\mu_2\mu_3}, u_{\mu_1\cdots\mu_4}, i\psi_a, v \in \text{(pure imaginary)}\}.$$

Commutation relations of these super Lie subalgebra:

(1)$[H_1, H_2] \in \mathcal{H}$, (2)$[H, A'] \in \mathcal{A}'$, (3)$[A'_1, A'_2] \in \mathcal{H}$,
(4)$\{H_1, H_2\} \in \mathcal{A}'$, (5)$\{H, A'\} \in \mathcal{H}$, (6)$\{A'_1, A'_2\} \in \mathcal{A}'$.

where $H, H_1, H_2 \in \mathcal{H}$ and $A', A'_1, A'_2 \in \mathcal{A}'$.

(Proof) These properties can be verified by taking the transpose:

1. ${}^T[H_1, H_2]G = {}^TH_2{}^TH_1G - {}^TH_1{}^TH_2G = {}^TH_2(-GH_1) - {}^TH_1(-GH_2)$
$= GH_2H_1 - GH_1H_2 = -G[H_1, H_2]$,
2. ${}^T[H, A']G = {}^TA'{}^THG - {}^TH{}^TA'G = {}^TA'(-GH) - {}^TH(GA')$
$= -GA'H + GHA' = G[H, A']$,
3. ${}^T[A'_1, A'_2]G = {}^TA'_2{}^TA'_1G - {}^TA'_1{}^TA'_2G = {}^TA'_2(GA'_1) - {}^TA'_1(GA'_2)$
$= GA'_2A'_1 - GA'_1A'_2 = -G[A'_1, A'_2]$,
4. ${}^T\{H_1, H_2\}G = {}^TH_2{}^TH_1G + {}^TH_1{}^TH_2G = {}^TH_2(-GH_1) + {}^TH_1(-GH_2)$
$= GH_2H_1 + GH_1H_2 = G\{H_1, H_2\}$,
5. ${}^T\{H_1, A'_1\}G = {}^TA'_1{}^TH_1G + {}^TH_1{}^TA'_1G = {}^TA'_1(-GH_1) + {}^TH_1(GA'_1)$
$= -GA'_1H_1 - GH_1A'_1 = -G\{H_1, A'_1\}$,
6. ${}^T\{A'_1, A'_2\}G = {}^TA'_2{}^TA'_1G + {}^TA'_1{}^TA'_2G = {}^TA'_2GA'_1 + {}^TA'_1GA'_2$
$= GA'_2A'_1 + GA'_1A'_2 = G\{A'_1, A'_2\}$.

Promotion to large N matrices

Commutation relations of (anti)-Hermitian matrices:

$$(1)[h_1, h_2] \in \mathrm{A}, \quad (2)[h, a] \in \mathrm{H}, \quad (3)[a_1, a_2] \in \mathrm{A},$$
$$(4)\{h_1, h_2\} \in \mathrm{H}, \quad (5)\{h, a\} \in \mathrm{A}, \quad (6)\{a_1, a_2\} \in \mathrm{H}.$$

- Hermitian matrices: $\mathrm{H} = \{M \in M_{N\times N}(\mathrm{C}) | M^\dagger = M\}$.
  $h, h_1 h_2$ belong to H.
- Anti-hermitian matrices : $\mathrm{A} = \{M \in M_{N\times N}(\mathrm{C}) | M^\dagger = -M\}$.
  a, a_1, a_2 belong to A.

$osp(1|32, R) \otimes u(N)$ is not a closed Lie algebra.

(Proof) The tensor product of the Lie algebra must close with respect to the commutator

$$[A \otimes B, C \otimes D] = \frac{1}{2}(\{A, C\} \otimes [B, D]) + \frac{1}{2}([A, C] \otimes \{B, D\}).$$

$osp(1|32, R) \otimes u(N)$ does not close because

$$\begin{aligned}[(\mathcal{H} \otimes \mathrm{H}), (\mathcal{H} \otimes \mathrm{H})] &= (\{\mathcal{H}, \mathcal{H}\} \otimes [\mathrm{H}, \mathrm{H}]) \oplus ([\mathcal{H}, \mathcal{H}] \otimes \{\mathrm{H}, \mathrm{H}\}) \\ &= (\mathcal{A}' \otimes \mathrm{A}) \oplus (\mathcal{H} \otimes \mathrm{H}).\end{aligned}$$

$osp(1|32, R) \check{\otimes} u(N) = u(1|16, 16) \otimes u(N)$ is the smallest closed Lie algebra that includes $osp(1|32, R) \otimes u(N)$:

- **$\mathcal{H}$ is promoted to Hermitian matrices.**
- **$\mathcal{A}$ is promoted to anti-Hermitian matrices.**

$gl(1|32, R)$ super Lie algebra

- $M \in gl(1|32, R) \Rightarrow M = \begin{pmatrix} m & \psi \\ i\bar{\phi} & v \end{pmatrix}$
- $$m = u1 + u_{\mu_1}\Gamma^{\mu_1} + \frac{1}{2!}u_{\mu_1\mu_2}\Gamma^{\mu_1\mu_2} + \frac{1}{3!}u_{\mu_1\mu_2\mu_3}\Gamma^{\mu_1\mu_2\mu_3} + \frac{1}{4!}u_{\mu_1\cdots\mu_4}\Gamma^{\mu_1\cdots\mu_4} + \frac{1}{5!}u_{\mu_1\cdots\mu_5}\Gamma^{\mu_1\cdots\mu_5}.$$
- $u, u_{\mu_1}, \cdots, u_{\mu_1\cdots\mu_5}, \psi, \phi, v$ are all real numbers.

$gl(1|32, R)$ is the analytical continuation of $u(1|16, 16)$:

$$gl(1|32, R) = \mathcal{H} \oplus \mathcal{A}, \quad \text{where } \mathcal{A}' = i\mathcal{A}.$$

Each element of $gl(1|32, R)$ is promoted to a real $gl(N, R)$ matrix:
$gl(1|32, R) \otimes gl(N, R)$ is trivially a closed Lie algebra.

Action of the cubic model

$$\begin{aligned} I &= \frac{1}{g^2} Tr_{N\times N} \sum_{Q,R=1}^{33} [(\sum_{p=1}^{32} M_p{}^Q M_Q{}^R M_R{}^p) - M_{33}{}^Q M_Q{}^R M_R{}^{33}] \\ &= \frac{1}{g^2} \sum_{a,b,c=1}^{N^2} Str_{33\times 33}(M_a M_b M_c) Tr_{N\times N}(T^a T^b T^c) \\ &= \frac{1}{g^2} Tr_{N\times N}[m_p{}^q m_q{}^r m_r{}^p - 3i\bar{\phi}^p m_p{}^q \psi^q - 3iv\bar{\phi}^p \psi_p - v^3]. \end{aligned}$$

- Each component of the 33×33 supermatrices is promoted to a real $gl(N, R)$ matrix.
- No free parameter: $M \to g^{\frac{2}{3}} M$.
- $gl(1|32, R) \otimes gl(N, R)$ gauge symmetry.

$$M \to M + [M, (S \otimes U)]$$
$$\text{for } S \in gl(1|32, R) \text{ and } U \in gl(N, R).$$

- This model loses invariance under the constant shift of the fields, and we introduce the space-time translation by the Wigner Inönü contraction.
- The bosonic 32×32 matrices are separated into m_e and m_o in terms of 10-dimensional indices.

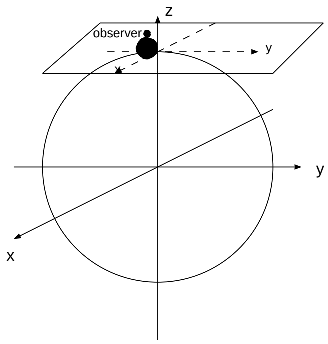
$$\begin{aligned} m_e &= Z + W\Gamma^\sharp + \frac{1}{2!}(C_{i_1 i_2}\Gamma^{i_1 i_2} + D_{i_1 i_2}\Gamma^{i_1 i_2 \sharp}) + \frac{1}{4!}(G_{i_1\cdots i_4}\Gamma^{i_1\cdots i_4} + H_{i_1\cdots i_4}\Gamma^{i_1\cdots i_4\sharp}), \\ m_o &= \frac{1}{2}(A_i^{(+)}\Gamma^i(1+\Gamma^\sharp) + A_i^{(-)}\Gamma^i(1-\Gamma^\sharp)) \\ &+ \frac{1}{2\times 3!}(E^{(+)}_{i_1 i_2 i_3}\Gamma^{i_1 i_2 i_3}(1+\Gamma^\sharp) + E^{(-)}_{i_1 i_2 i_3}\Gamma^{i_1 i_2 i_3}(1-\Gamma^\sharp)) \\ &+ \frac{1}{5!}(I^{(+)}_{i_1\cdots i_5}\Gamma^{i_1\cdots i_5}(1+\Gamma^\sharp) + I^{(-)}_{i_1\cdots i_5}\Gamma^{i_1\cdots i_5}(1-\Gamma^\sharp)). \end{aligned}$$

Wigner Inönü contraction

We consider the hyperboloid in the AdS space whose radius R is sufficiently large. The hyperboloid is approximated by the $R^{9,1}$ flat plane at the "north pole".

AdS space: $x^\mu x^\nu \eta_{\mu\nu} = -R^2$, with $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, \cdots, 1, -1)$.

(*)The intuitive image of the Wigner Inönü contraction in the 3-dimensional case.



♣ The Lorentz transformation in the 11-dimensional space $(\mu, \nu = 0, 1, \cdots, 9, \sharp)$:

$$[M_{\mu\nu}, M_{\rho\sigma}] = \eta_{\nu\rho} M_{\mu\sigma} + \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\mu\rho} M_{\nu\sigma} - \eta_{\nu\sigma} M_{\mu\rho}.$$

♣ We consider the algebra in the plane perpendicular to the $x^\sharp$ direction.

- Translation:
  $P_i =$(The translation in the direction of x_i) $= \frac{1}{R} M_{\sharp i} = \frac{1}{R} \Gamma_{\sharp i}$.
- Lorentz transformation:
  $M_{ij} =$(The Lorentz transformation on the $x_i x_j$ plane) $= \Gamma_{ij}$.

♣ The commutation relations of the translations and the Lorentz transformations:

- $[M_{ij}, M_{kl}] = \eta_{jk} M_{il} + \eta_{il} M_{jk} - \eta_{ik} M_{jl} - \eta_{jl} M_{ik}$.
- $[P_i, M_{jk}] = -\eta_{ik} P_j + \eta_{ij} P_k$.
- $[P_i, P_j] = \frac{1}{R^2} M_{ij} \to 0$. Two translations commute with each other when the radius R is large.

In order to perform the Wigner Inönü contraction, we alter the action as

$$I = \frac{1}{3}Tr(StrM_t^3) - R^2 Tr(StrM_t).$$

The EOM $\frac{\partial I}{\partial M_t} = M_t^2 - R^2 1_{33\times 33} = 0$ possesses a classical solution $\langle M \rangle = \begin{pmatrix} R\Gamma^\sharp \otimes 1_{N\times N} & 0 \\ 0 & R \otimes 1_{N\times N} \end{pmatrix}$.

$$\begin{aligned} M_t &= (\text{classical solution } \langle M \rangle) \quad + (\text{fluctuation } M) \\ &= \begin{pmatrix} R\Gamma^\sharp \otimes 1_{N\times N} & 0 \\ 0 & R \otimes 1_{N\times N} \end{pmatrix} + \begin{pmatrix} m & \psi \\ i\bar{\phi} & v \end{pmatrix}. \end{aligned}$$

The action is expressed in terms of the fluctuation as

$$\begin{aligned} I &= R(tr(m_e^2\Gamma^\sharp) - v^2 - 2i\bar{\phi}_R\psi_L) + (\frac{1}{3}m_e^3 + tr(m_e m_o^2)) \\ &- i(\bar{\phi}_R(m_e + v)\psi_L + \bar{\phi}_L(m_e + v)\psi_R + \bar{\phi}_L m_o \psi_L + \bar{\phi}_R m_o \psi_R) - \frac{1}{3}v^3. \end{aligned}$$

The fluctuation is rescaled as

- $m_t = R\Gamma^\sharp + m = R\Gamma^\sharp + R^{-\frac{1}{2}}m'_e + R^{\frac{1}{4}}m'_o$,
- $v_t = R + v = R + R^{-\frac{1}{2}}v'$,
- $\psi = \psi_L + \psi_R = R^{-\frac{1}{2}}\psi'_L + R^{\frac{1}{4}}\psi'_R$,
- $\bar{\phi} = \bar{\phi}_L + \bar{\phi}_R = R^{\frac{1}{4}}\bar{\phi}'_L + R^{-\frac{1}{2}}\bar{\phi}'_R$.

We obtain the vanishing effective action by integrating out m'_e, ψ'_L, $\bar{\phi}'_R$ and v' .

$$\begin{aligned} &e^{-W} = \int dm'_e d\psi'_L d\bar{\phi}'_R dv e^{-I}, \\ \Rightarrow\ &W = -\frac{1}{4}tr(\Gamma^\sharp\{m_o'^2 + i(\psi'_R\bar{\phi}'_L)\}^2) - \frac{1}{4}(\bar{\phi}'_L\psi_R)^2 + \frac{i}{2}(\bar{\phi}'_L m_o'^2 \psi'_R) = 0. \end{aligned}$$

This gauged model may be related to a topological matrix model.

S. Hirano and M. Kato, *Prog. Theor. Phys.* 98 (1997) 1371, hep-th/9708039

5 Conclusion

Summary

- We have investigated the (nongauged) cubic model whose gauge symmetry is the super Lie algebra $OSp(1|32, R) \times U(N)$ as a candidate of the matrix model which naturally reproduces IKKT model.
  - * $osp(1|32, R)$ cubic matrix model possesses a two-fold structure of the $\mathcal{N} = 2$ SUSY of IKKT model.
  - * IKKT model is induced from the $osp(1|32, R)$ cubic matrix model by the multi-loop effect.
- We have investigated the $gl(1|32, R) \otimes gl(N, R)$ gauged model as an extension.
  - * The space-time translation is introduced by means of the Wigner-Inönü contraction.
  - * The effective action vanishes, and this model is related to a topological matrix model.

Related problems

- The diffeomorphism invariance of matrix models.
- Cubic matrix model described by exceptional Jordan Lie algebra:

  L. Smolin hep-th/0104050