Generalized factorization method for the overlap problem in a matrix model with complex action

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Collaboration with K.N. Anagnostopoulos and J. Nishimura

Contents

1 Introduction

Matrix models as a constructive definition of superstring theory

IKKT model (IIB matrix model)

⇒ **Promising candidate for the constructive definition of superstring theory.**
Ishibashi, Kawai, Kitazawa and Tsuchiya, hep-th/9612115.

$$\boxed{S = N\left(-\frac{1}{4}\mathrm{tr}\,[A_\mu, A_\nu]^2 + \frac{1}{2}\mathrm{tr}\,\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]\right).}$$

- A_μ (10d vector) and ψ_α (10d Majorana-Weyl spinor) ⇒ $N \times N$ matrices .
- Evidences for spontaneous breakdown of SO(10) symmetry to SO(4).
  Nishimura and Sugino, hep-th/0111102, Kawai, et. al. hep-th/0204240,0211272,0602044,0603146.
- Complex fermion determinant:
  - * Crucial for rotational symmetry breaking.
    Nishimura and Vernizzi, hep-th/0003223.
  - * Difficulty of Monte Carlo simulation.

2 Simplified IKKT model

Simplified model with spontaneous rotational symmetry breakdown
Nishimura, hep-th/0108070.

$$\boxed{S = \underbrace{\frac{N}{2}\mathrm{tr}\, A_\mu^2}_{=S_b} \underbrace{- \bar{\psi}^f_\alpha (\Gamma_\mu)_{\alpha\beta} A_\mu \psi^f_\beta}_{=S_f}}$$

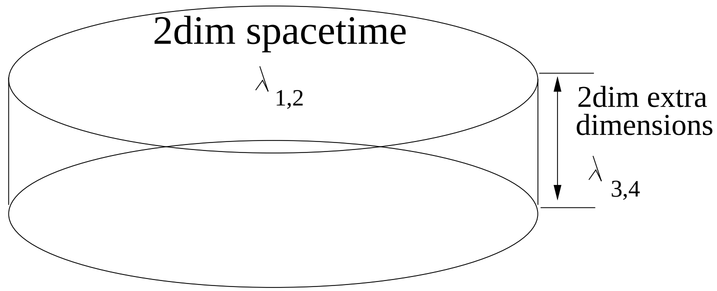
- A_μ: $N \times N$ hermitian matrices $(\mu = 1, \cdots, 4)$
  $\bar{\psi}^f_\alpha, \psi^f_\alpha$: N-dim vector $(\alpha = 1, 2, f = 1, \cdots, N_f)$
  N_f = (number of flavors)
- SO(4) rotational symmetry.
- No supersymmetry.
- Gaussian expansion analysis up to 9th order: Okubo, Nishimura and Sugino, hep-th/0412194.

Observable for probing dimensionality :

$$T_{\mu\nu} = \frac{1}{N}\mathrm{tr}\,(A_\mu A_\nu).$$

λ_n $(n = 1, 2, 3, 4)$: eigenvalues of $T_{\mu\nu}$
$(\lambda_1 \geqq \lambda_2 \geqq \lambda_3 \geqq \lambda_4)$

Spontaneous breakdown of SO(4) to SO(2) at finite $r \left(= \dfrac{N_f}{N} \right)$.



3 Monte Carlo simulation

Factorization method

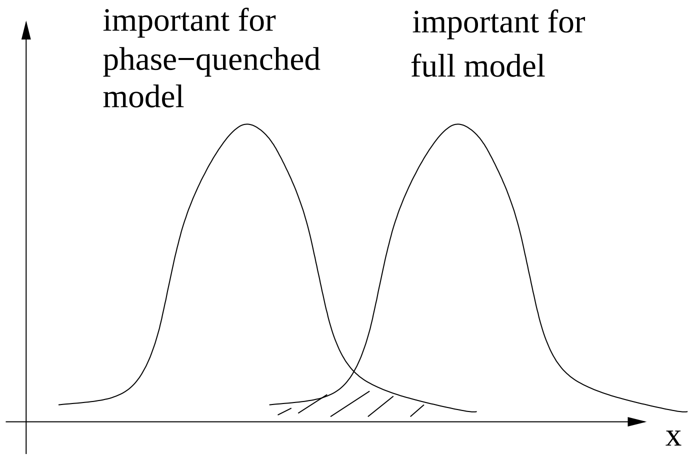
An approach to the complex action problem in Monte Carlo simulation.
Anagnostopoulos and Nishimura, hep-th/0108041,

Partition function:

$$Z = \int dA e^{-S_B} (\det \mathcal{D})^{N_f} = \int dA e^{-S_0} e^{i\Gamma}, \text{ where } \mathcal{D} = \Gamma_\mu A_\mu,$$

$$Z_0 = \int dA e^{-S_0} = \int dA e^{-S_B} |\det \mathcal{D}|^{N_f}.$$

Overlap problem: discrepancy of important configurations between phase-quenched partition function Z_0 and full partition function Z.



Factorization method: force the simulation to sample important comfigurations for the full partition function.

- **"Old" factorization method: constrain one eigenvalue with partition function $Z_{n,x}$ ($n = 1, 2, 3, 4$, $\tilde{\lambda}_n = \frac{\lambda_n}{\langle \lambda_n \rangle_0}$, $\langle \cdots \rangle_0 =$ (V.E.V. for phase-quenched partition function Z_0))**

$$Z_{n,x} = \int dA\, e^{-S_0} \delta(x - \tilde{\lambda}_n)$$

- **Generalized factorization method: constrain all eigenvalues with partition function Z_{x_1,x_2,x_3,x_4}:**

$$Z_{x_1,x_2,x_3,x_4} = \int dA\, e^{-S_0} \prod_{k=1}^{4} \delta(x_k - \tilde{\lambda}_k),$$

Generalized distribution function:

$$\rho(x_1, x_2, x_3, x_4) \;=\; \left\langle \prod_{k=1}^{4} \delta(x_k - \tilde{\lambda}_k). \right\rangle$$

$\rho(x_1, x_2, x_3, x_4)$ **factorizes** as

$$\rho(x_1, x_2, x_3, x_4) = \frac{1}{C} \rho^{(0)}(x_1, x_2, x_3, x_4)\, w(x_1, x_2, x_3, x_4) \;, \text{ where}$$

$$\rho^{(0)}(x_1, x_2, x_3, x_4) = \left\langle \prod_k \delta(x_k - \tilde{\lambda}_k) \right\rangle_0,$$

$$w(x_1, x_2, x_3, x_4) \stackrel{\text{def}}{=} \langle e^{i\Gamma} \rangle_{x_1,x_2,x_3,x_4} = \langle \cos\Gamma \rangle_{x_1,x_2,x_3,x_4}.$$

$\langle \cdots \rangle_{x_1,x_2,x_3,x_4}$ = (V.E.V. for partition function Z_{x_1,x_2,x_3,x_4}).

Monte Carlo evaluation of $\langle \tilde{\lambda}_n \rangle$:

Minimum of the free energy density $\mathcal{F}(x) = -\dfrac{1}{N^2} \log \rho(x_1, x_2, x_3, x_4)$

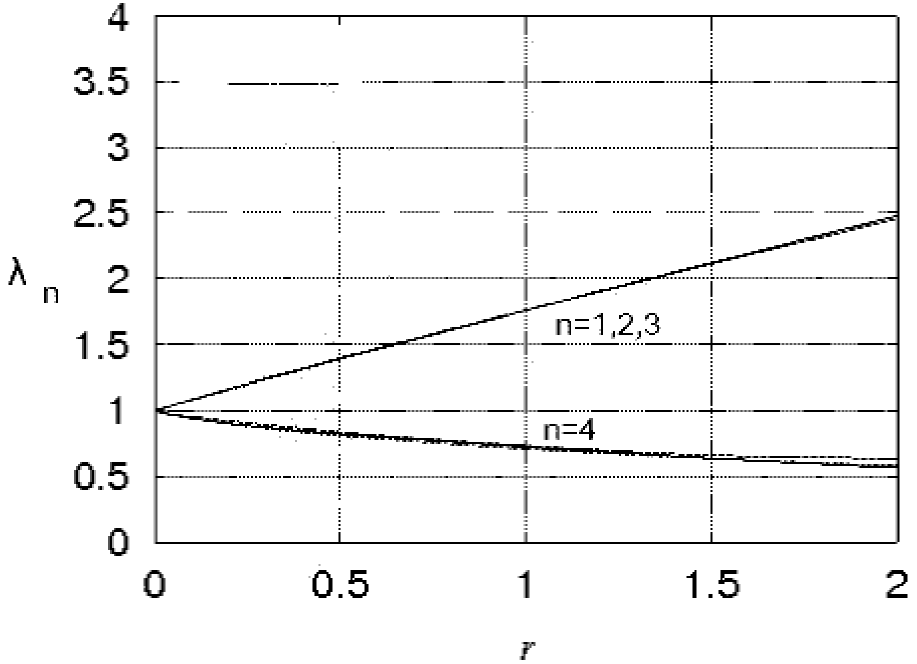
$$\frac{\partial}{\partial x_n} \mathcal{F}(x) = 0 \Rightarrow \frac{\partial}{\partial x_n} \log \rho^{(0)}(x_1, x_2, x_3, x_4) = -\frac{\partial}{\partial x_n} \log w(x_1, x_2, x_3, x_4)$$

SO(3) vacuum

Solutions which satisfy $x_1 = x_2 = x_3 > 1 > x_4$.

Result of Gaussian Expansion Method (GEM) up to 9th order at large N:

Okubo, Nishimura and Sugino, hep-th/0412194.



SO(3) ansatz

$$\tilde{\lambda}_n = \frac{\lambda_n}{\langle \lambda_n \rangle_0},$$

$$\langle \lambda_n \rangle_0 = 1 + \frac{r}{2} = 1.5 (\text{for } r = 1)$$

for $n = 1, 2, 3, 4$.

At $r = 1$, $\langle \tilde{\lambda}_{1,2,3} \rangle = 1.17$, $\langle \tilde{\lambda}_4 \rangle = 0.50$.

Minimum of the free energy density $\mathcal{F}(x)$

$$\frac{\partial}{\partial x_n} \rho^{(0)}_{3\mathrm{d}}(x_3, x_4) = -\frac{\partial}{\partial x_n} w_{3\mathrm{d}}(x_3, x_4)$$

for $n = 3, 4$,

where

$$\rho^{(0)}_{3\mathrm{d}}(x_3, x_4) = \rho^{(0)}(x_3, x_3, x_3, x_4) \ ,$$

$$w_{3\mathrm{d}}(x_3, x_4) = w(x_3, x_3, x_3, x_4)$$

Calculation of $\langle\tilde{\lambda}_{n=3}\rangle$ at $r = 1$

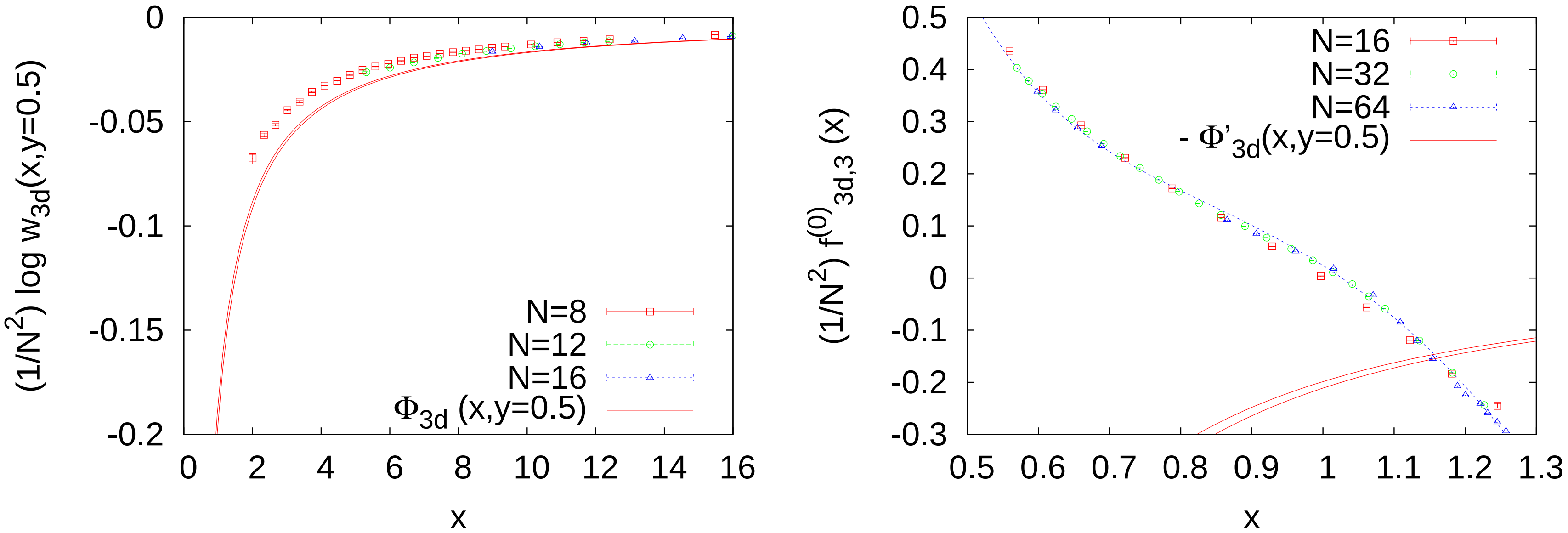
Calculation of $\langle\tilde{\lambda}_{n=3}\rangle$ for fixed $\langle\tilde{\lambda}_{n=4}\rangle = 0.5$.

$$\frac{1}{N^2} f^{(0)}_{3d,3}(x) = -\frac{\partial}{\partial x}\Phi_{3d}(x, y = 0.5),$$

where $f^{(0)}_{3d,3}(x) = \dfrac{\partial}{\partial x}\log\rho^{(0)}_{3d}(x, y = 0.5)$ **and** $\Phi_{3d}(x, y) = \dfrac{1}{N^2}\log w_{3d}(x, y)$.

Scaling behavior of the phase:

$$\Phi_{3d}(x, y = 0.5) \sim -\tilde{d}_1 x^{-1} + \tilde{d}_2 x^{-1.5}$$



Numerical Result: $\langle\tilde{\lambda}_{n=3}\rangle = 1.161 \pm 0.002$, **(GEM result** $\langle\tilde{\lambda}_{n=3}\rangle_{\mathrm{GEM}} = 1.17$**).**

Calculation of $\langle\tilde{\lambda}_{n=4}\rangle$ at $r=1$

Calculation of $\langle\tilde{\lambda}_{n=4}\rangle$ for fixed $\langle\tilde{\lambda}_{n=3}\rangle = 1.17$.

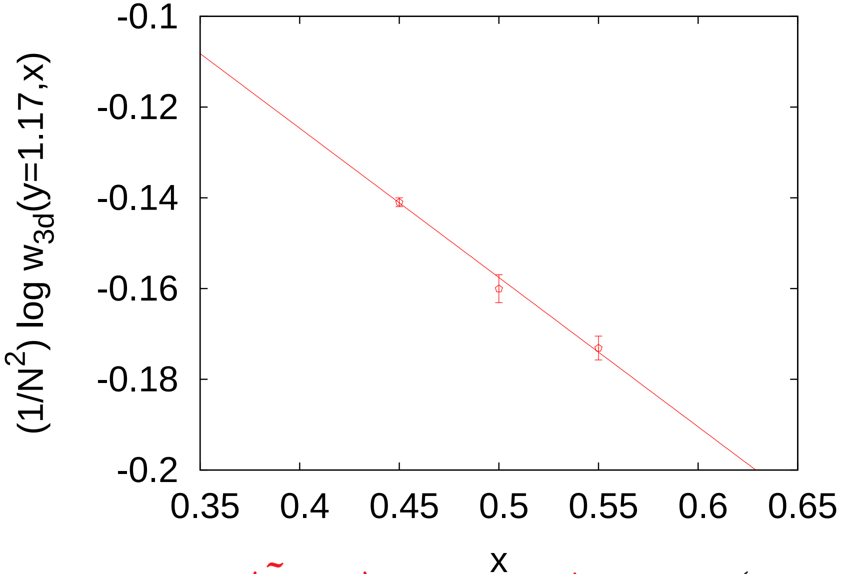
$$\frac{1}{N^2} f^{(0)}_{3\mathrm{d},4}(x) = -\frac{\partial}{\partial x}\Phi_{3\mathrm{d}}(y=1.17, x),$$

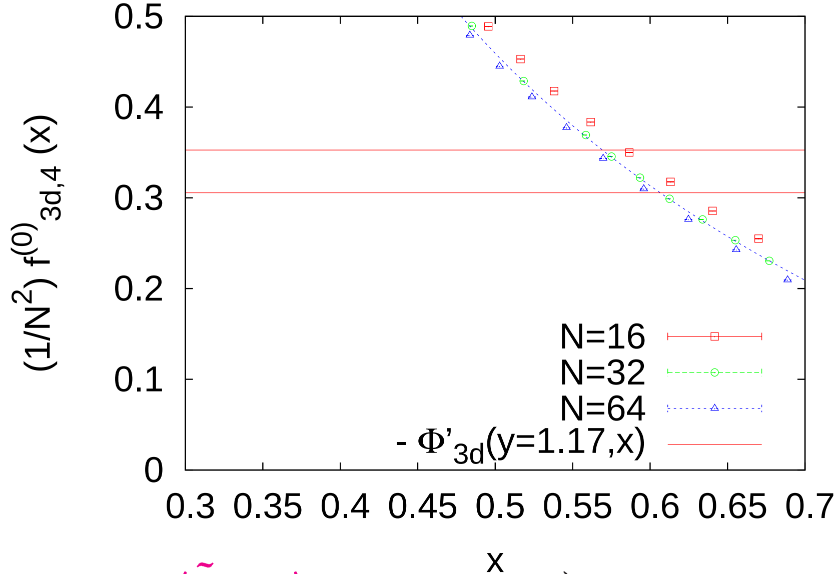
where $f^{(0)}_{3\mathrm{d},4}(x) = \frac{\partial}{\partial x}\log \rho^{(0)}_{3\mathrm{d}}(y=1.17,x)$.

$\Phi_{3\mathrm{d}}(y=1.17,x)$ **suffers finite-N effect at** $x=0.50 \Rightarrow$

Calculate $\Phi_{3\mathrm{d}}(y,x=0.45)$, $\underbrace{\Phi_{3\mathrm{d}}(y,x=0.50)}_{\text{done in calculating } \langle\tilde{\lambda}_{n=3}\rangle}$ **, and** $\Phi_{3\mathrm{d}}(y,x=0.55)$ **at** $y=1.17$,

and obtain $-\frac{\partial}{\partial x}\Phi_{3\mathrm{d}}(y=1.17,x)$.





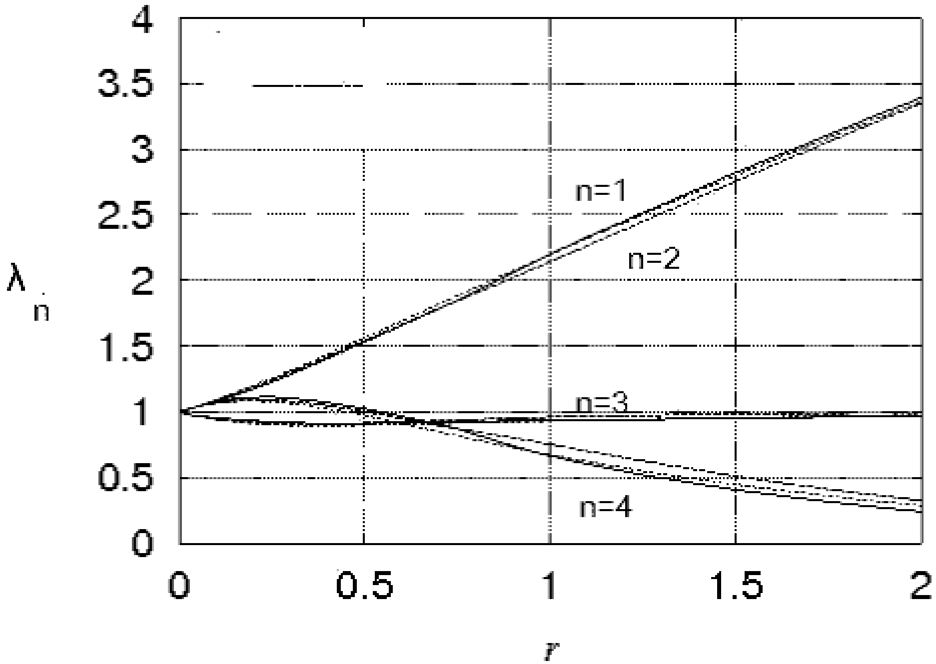
Numerical Result: $\langle\tilde{\lambda}_{n=4}\rangle = 0.59 \pm 0.02$, **(GEM result** $\langle\tilde{\lambda}_{n=4}\rangle_{\mathrm{GEM}} = 0.50$**).**

SO(2) vacuum

Solutions which satisfy $x_1 = x_2 > 1 > x_3 > x_4$.

Result of Gaussian Expansion Method (GEM) up to 9th order at large N:

Okubo, Nishimura and Sugino, hep-th/0412194.



SO(2) ansatz

$$\tilde{\lambda}_n = \frac{\lambda_n}{\langle\lambda_n\rangle_0},$$

$$\langle\lambda_n\rangle_0 = 1 + \frac{r}{2} = 1.5(\text{for } r = 1)$$

for $n = 1, 2, 3, 4$.

At $r = 1$, $\langle\tilde{\lambda}_{1,2}\rangle = 1.4$, $\langle\tilde{\lambda}_3\rangle = 0.7$, $\langle\tilde{\lambda}_4\rangle = 0.5$.

Minimum of the free energy density $\mathcal{F}(x)$

$$\frac{\partial}{\partial x_n}\rho^{(0)}_{2\text{d}}(x_2, x_3, x_4) = -\frac{\partial}{\partial x_n}w_{2\text{d}}(x_2, x_3, x_4)$$

for $n = 2, 3, 4$,

where

$$\rho^{(0)}_{2\text{d}}(x_2, x_3, x_4) = \rho^{(0)}(x_2, x_2, x_3, x_4) \ ,$$

$$w_{2\text{d}}(x_2, x_3, x_4) = w(x_2, x_2, x_3, x_4)$$

Calculation of $\langle\tilde{\lambda}_{n=2}\rangle$ at $r = 1$

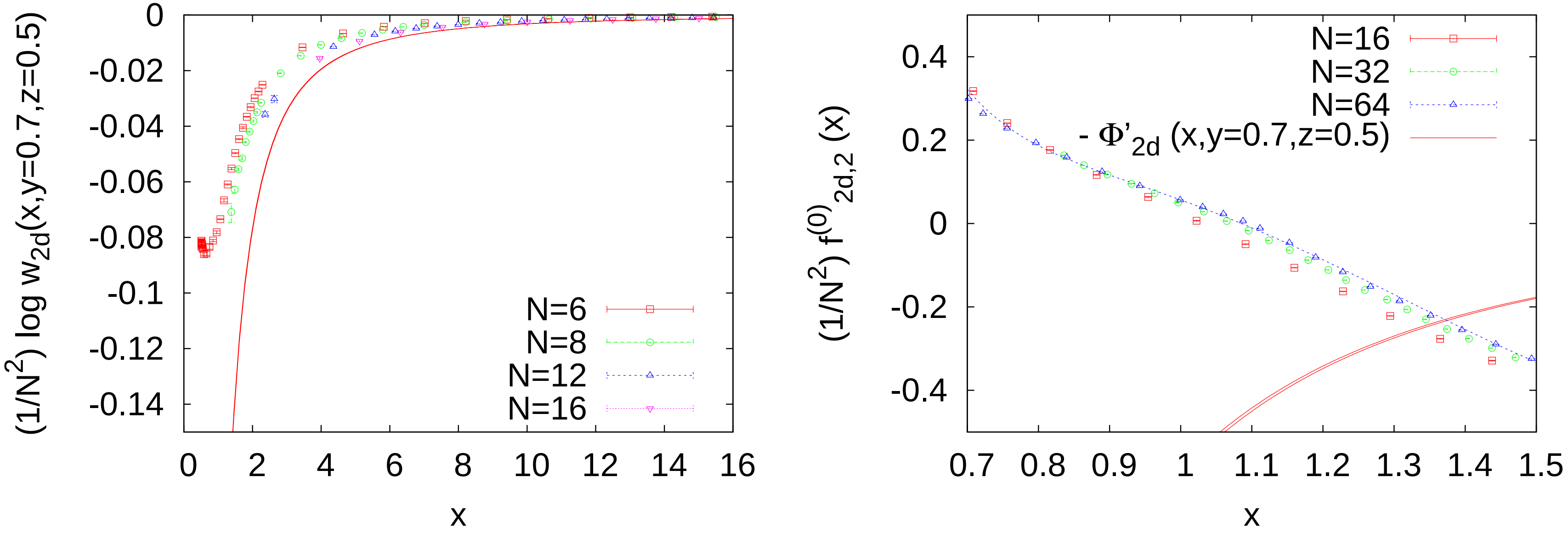
Calculation of $\langle\tilde{\lambda}_{n=2}\rangle$ for fixed $\langle\tilde{\lambda}_{n=3}\rangle = 0.7$ and $\langle\tilde{\lambda}_{n=4}\rangle = 0.5$.

$$\frac{1}{N^2} f^{(0)}_{2\mathrm{d},2}(x) = -\frac{\partial}{\partial x}\Phi_{2\mathrm{d}}(x, y = 0.7, z = 0.5),$$

where $f^{(0)}_{2\mathrm{d},2}(x) = \frac{\partial}{\partial x}\log\rho^{(0)}_{2\mathrm{d}}(x, y = 0.7, z = 0.5)$ **and** $\Phi_{2\mathrm{d}}(x, y, z) = \frac{1}{N^2}\log w_{2\mathrm{d}}(x, y, z)$.

Scaling behavior of the phase:

$$\Phi_{2\mathrm{d}}(x, y = 0.5) \sim -\tilde{d}_1 x^{-2} + \tilde{d}_2 x^{-2.5}$$



Numerical Result: $\langle\tilde{\lambda}_{n=3}\rangle = 1.372 \pm 0.002$, **(GEM result** $\langle\tilde{\lambda}_{n=3}\rangle_{\mathrm{GEM}} = 1.4$**).**

Calculation of $\langle \tilde{\lambda}_{n=3} \rangle$ at $r = 1$

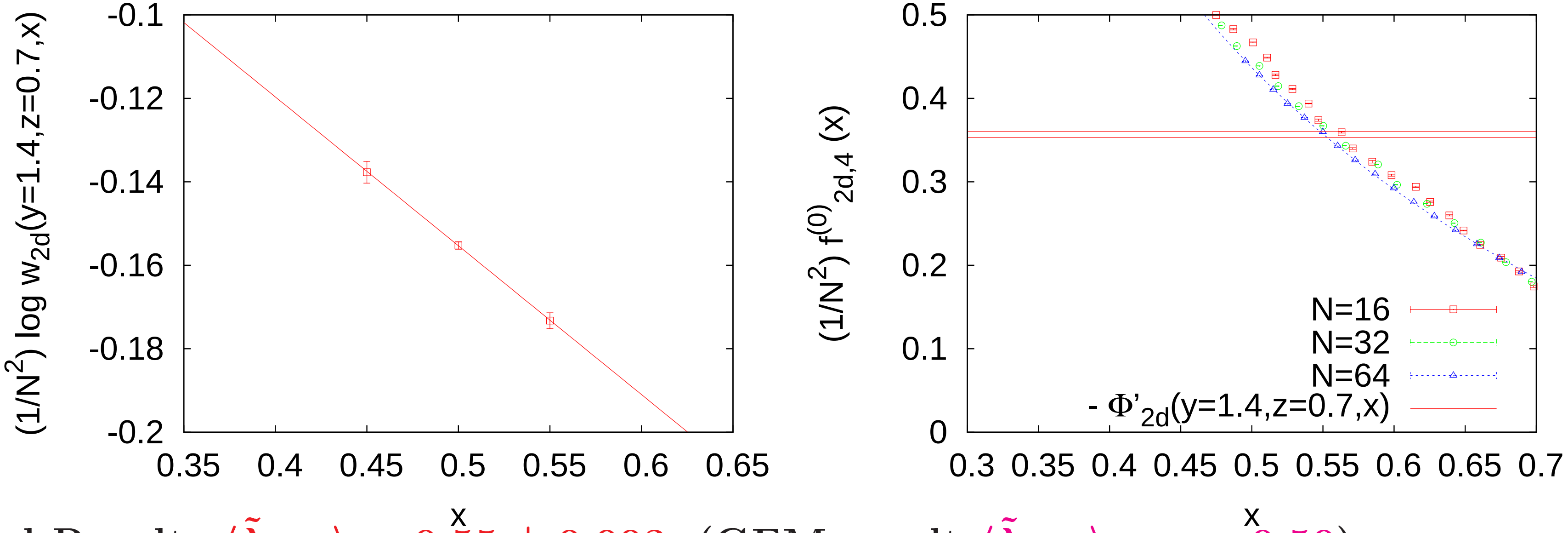
???

Calculation of $\langle\tilde{\lambda}_{n=4}\rangle$ at $r = 1$

Calculation of $\langle\tilde{\lambda}_{n=4}\rangle$ for fixed $\langle\tilde{\lambda}_{n=2}\rangle = 1.4$ and $\langle\tilde{\lambda}_{n=3}\rangle = 0.7$.

$$\frac{1}{N^2} f^{(0)}_{2\mathrm{d},4}(x) = -\frac{\partial}{\partial x}\Phi_{2\mathrm{d}}(y = 1.4, z = 0.77, x),$$

Calculate $\Phi_{2\mathrm{d}}(y, z = 0.7, x = 0.45)$, $\Phi_{2\mathrm{d}}(y, z = 0.7, x = 0.50)$, and $\Phi_{2\mathrm{d}}(y, z = 0.7, x = 0.55)$ at $y = 1.4$, and obtain $-\frac{\partial}{\partial x}\Phi_{2\mathrm{d}}(y = 1.4, z = 0.7, x)$.



Numerical Result: $\langle\tilde{\lambda}_{n=4}\rangle = 0.55 \pm 0.003$, (GEM result $\langle\tilde{\lambda}_{n=4}\rangle_{\mathrm{GEM}} = 0.50$).

4 Conclusion

Monte Carlo simulation of the simplified IKKT model.

Improvement of factorization method to overcome "overlap problem".

Better agreement with GEM result.

Future problems

Comparison of the free energy (which is favored, SO(2) or SO(3) vacuum?)

Monte Carlo Simulation of the IKKT model Anagnostopoulos, Aoyama, T. A., Hanada and Nishimura, in progress

Effect of supersymmetry on dynamical generation of spacetime.