YITP Workshop on Quantum Information Physics
August 7th 2013

# ~ Implementation of quantum energy teleportation via the Quantum Hall System ~

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# Collaborators

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(Condensed matter)

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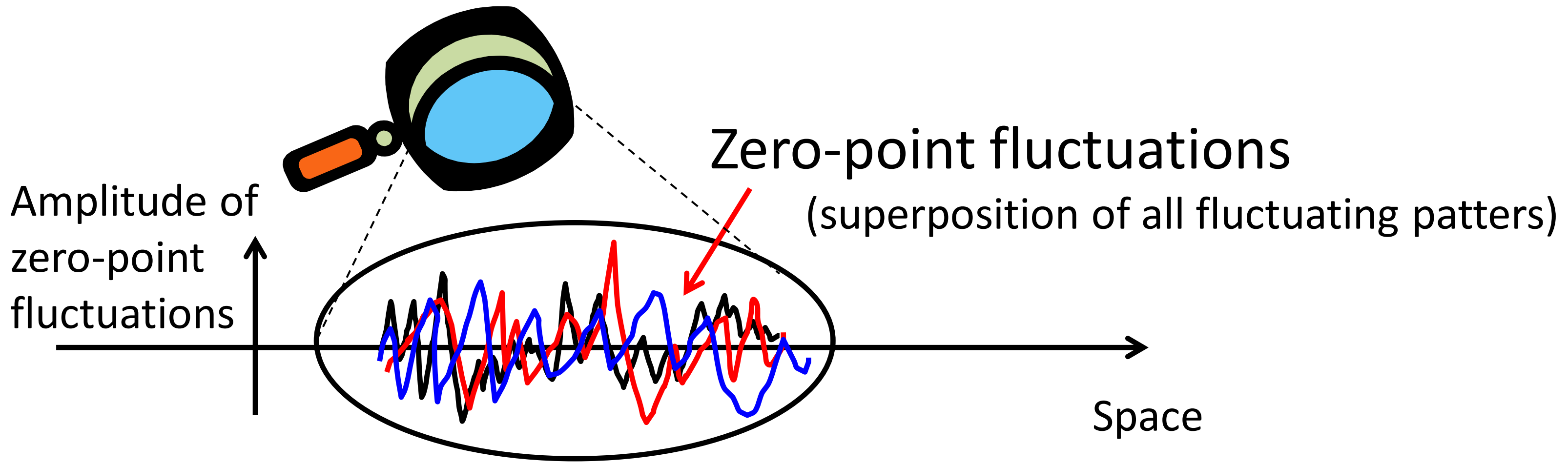
**Takaaki Noma**

} Material science (provides high quality semiconductor wafer)

# Can zero-point energy be extracted from a vacuum state?

Many-body system in the vacuum state/ground state



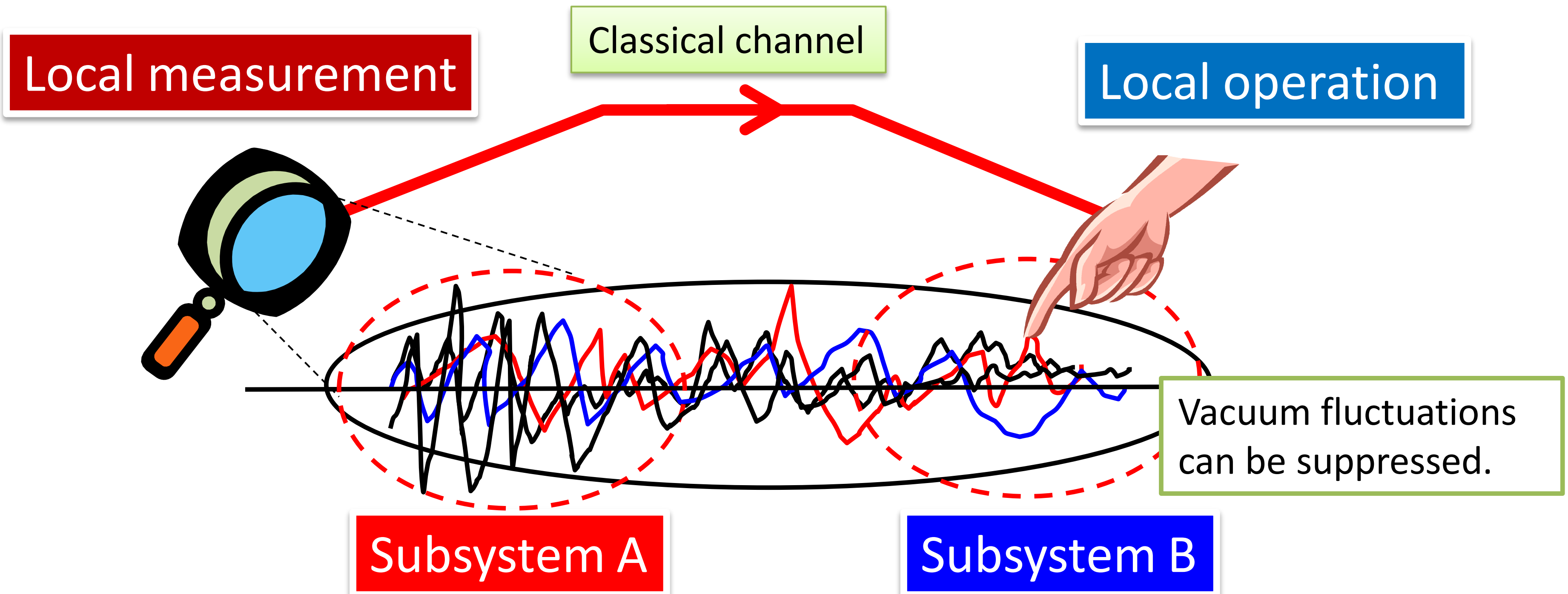

A many-body system in the vacuum state possesses a zero-point energy, however, it is <u>not possible to extract the zero-point energy</u> from the vacuum state by any operations (as it would require a state to exist with lower energy than vacuum).

**Measurement (operation) always injects energy into the vacuum.**
**~ The passivity of a vacuum state ~**

W. Pusz and S. L. Woronowicz, Commun. Math. Phys. **58**, 273 (1978)

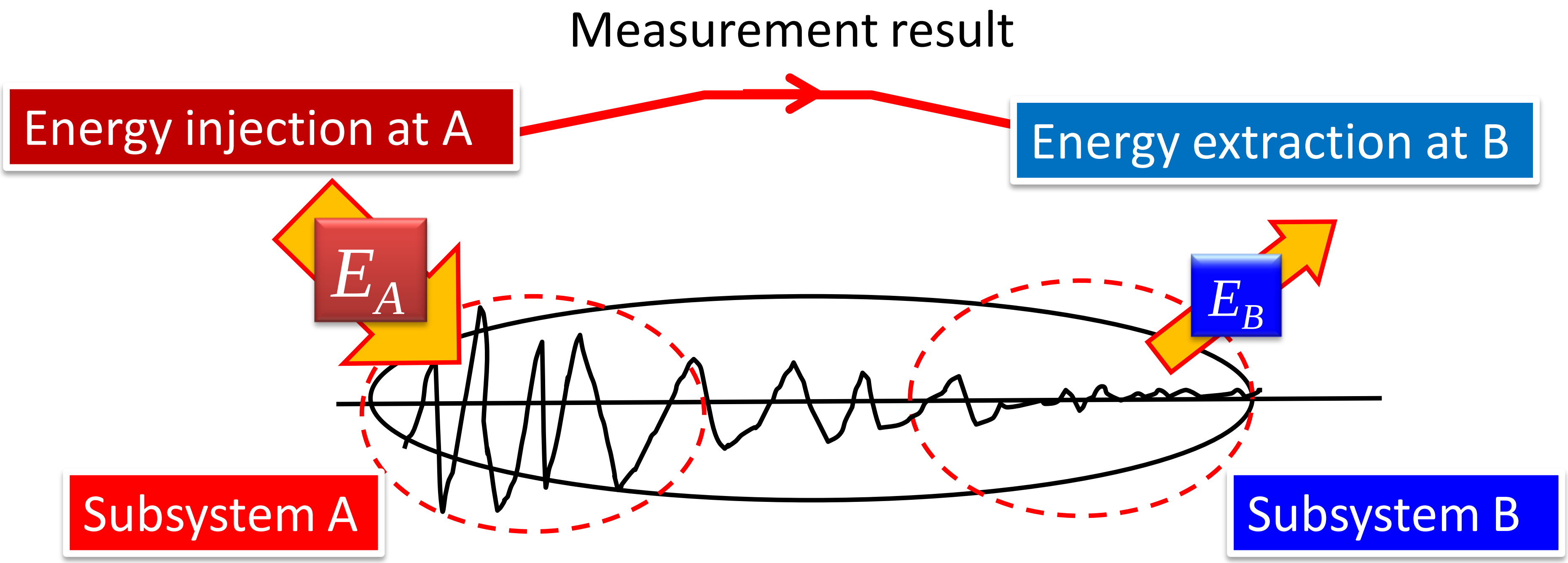
# Passivity of a *local* vacuum state can be broken

M. Hotta, Phys. Lett. A 372, 5671 (2008), Phys. Rev. D 78, 045006 (2008)




Feedback transmitted from A to B due to the local measurement can break local passivity.

# From the “Energy” viewpoint




No entity to transport energy

“Quantum energy teleportation (QET)”

M. Hotta, Phys. Lett. A 372, 5671 (2008), Phys. Rev. D 78, 045006 (2008)

The total energy of the system is positive because $E_B < E_A$

# Status of QET study

The validity of this protocol has been confirmed *mathematically*, but its *physical significance* remains unclear.

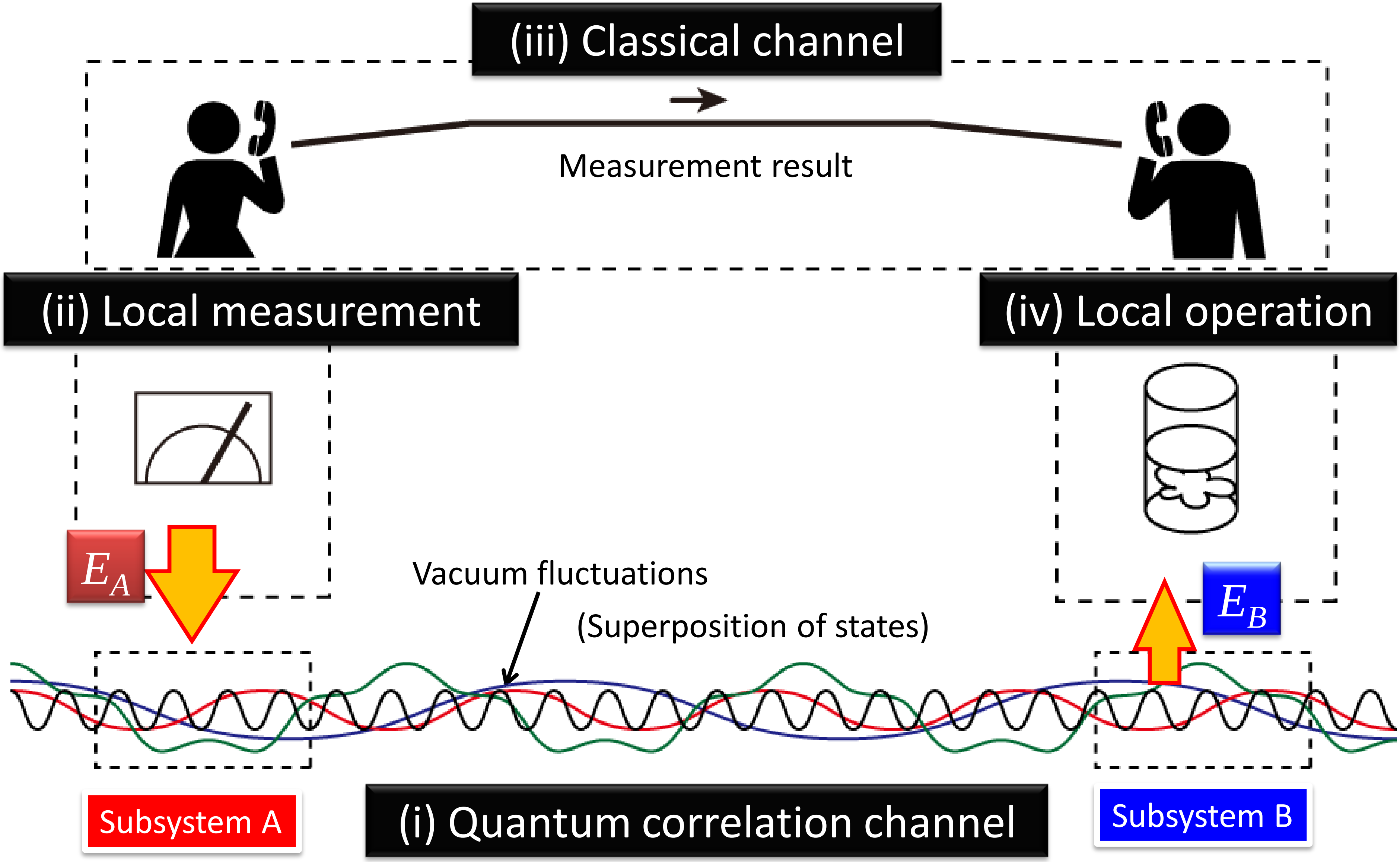
- What type of physical system is necessary for implementing this protocol?
- What is the composition of the quantum correlation channel?
- Can significant amounts of energy be "teleported" to a significant remote place?

# In this talk.....

・Discuss a more realistic possible implementation and theoretically estimate the order of "teleported" energy using reasonable experimental parameters.

・Report recent experimental progress.

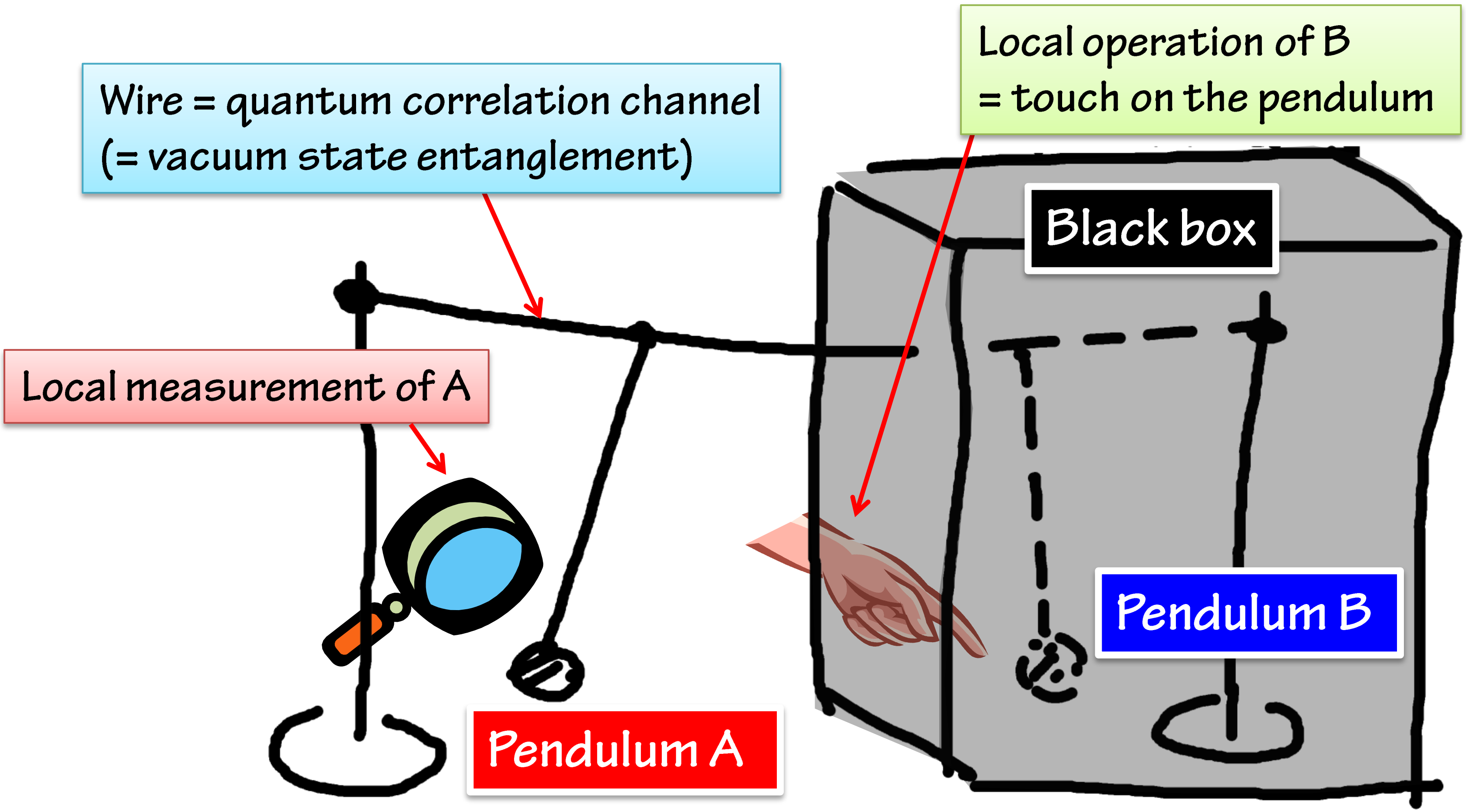
# Four key ingredients for QET



# Classical analogue to QET

## ~ coupled pendulums ~

Can you reduce the amplitude of the oscillations of Pendulum B and gain energy from it without seeing it?




Oscillations of pendulums = zero-point fluctuations

# Requirements to verify the quantum energy teleportation (QET)

(1) **Dissipationless** system

(2) **Quantum correlation channel** with macroscopic correlation length

(3) **Detection** and **operation schemes** for well-defined fluctuations in the vacuum state

(4) Suitable implementation of local operations and classical communication **(LOCC)**

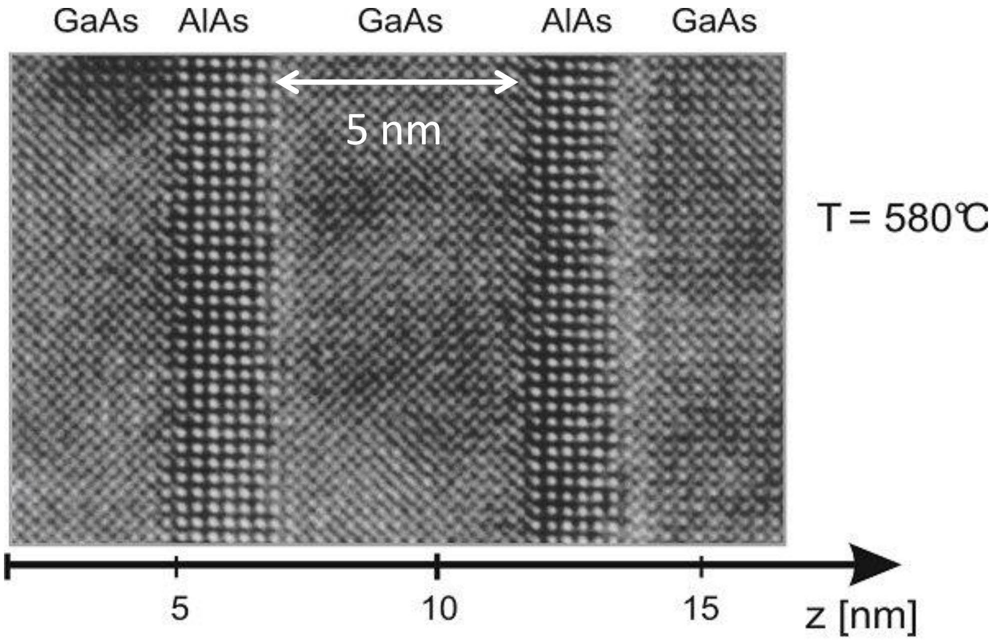
A quantum Hall system meets all these criteria.

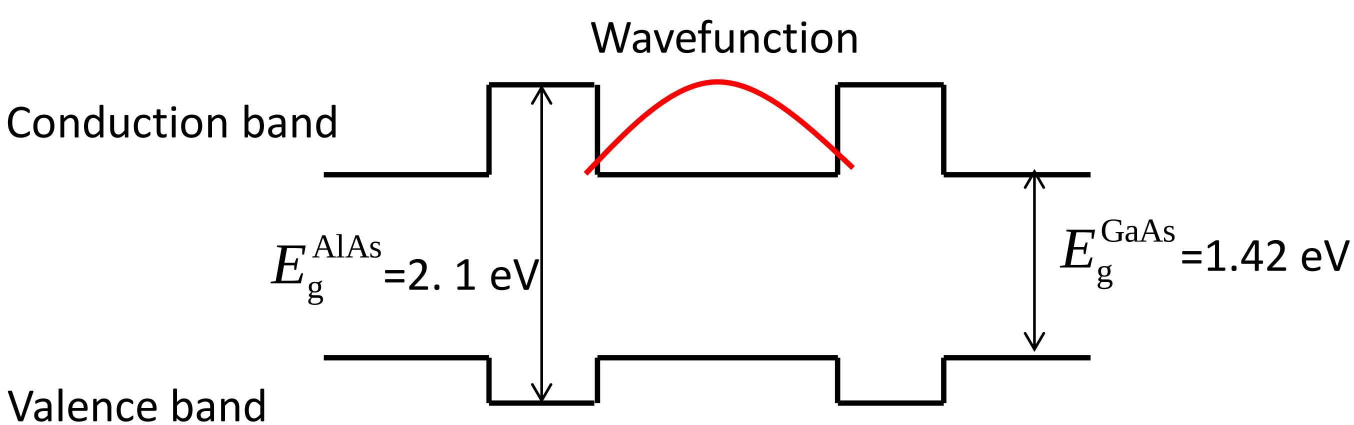
# What is a quantum Hall system?

Two-dimensional electron system
in a strong magnetic fild.

# Cross-sectional view of 5-nm semiconductor quantum well obtained by electron microscopy

Atomic image

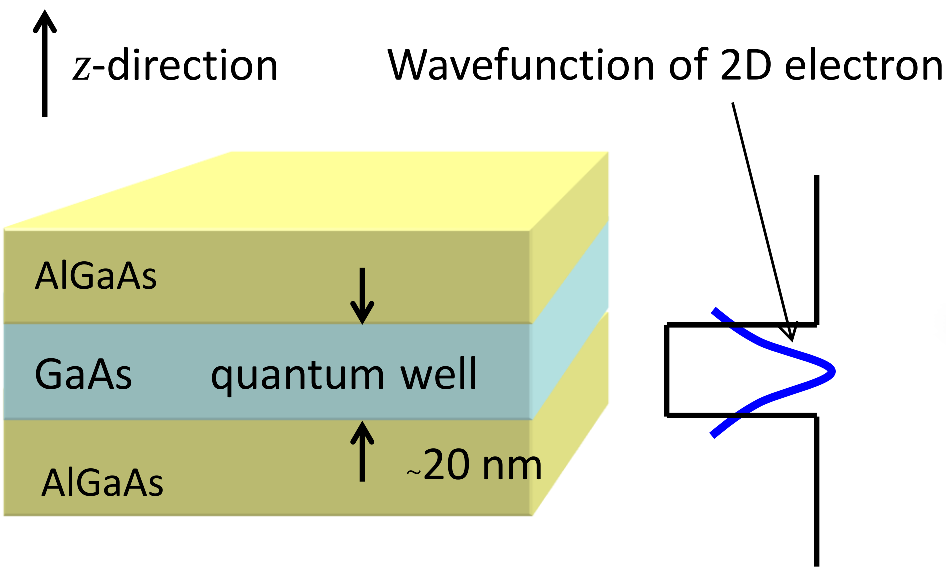


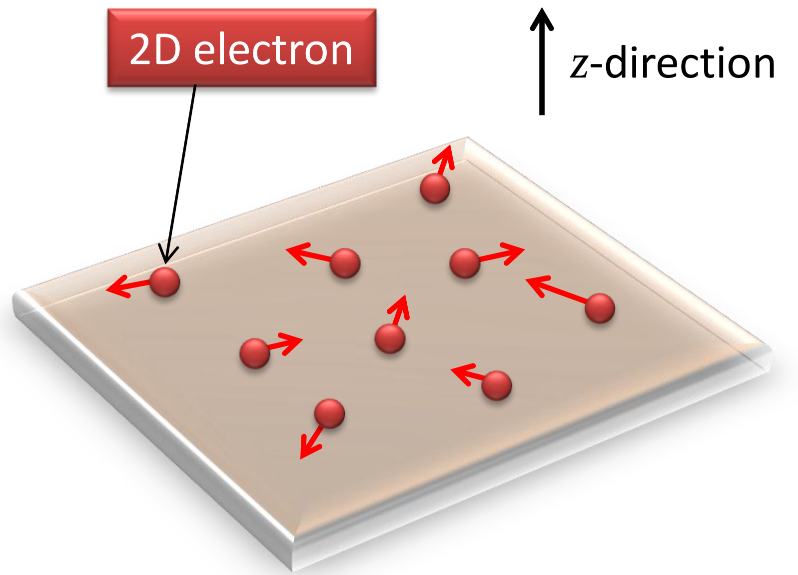

Potential diagram

# What is a quantum Hall system?

## Two-dimensional electron system



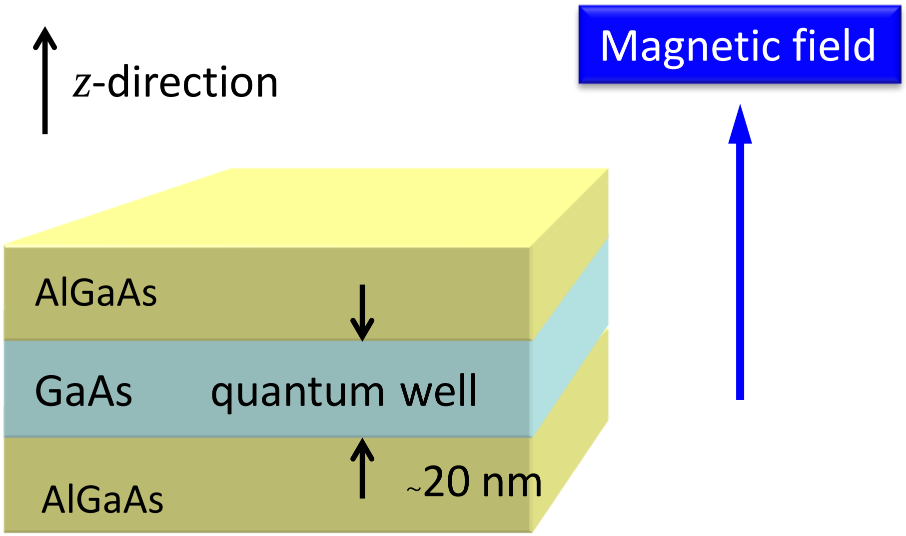

Electrons are confined in the quantum well



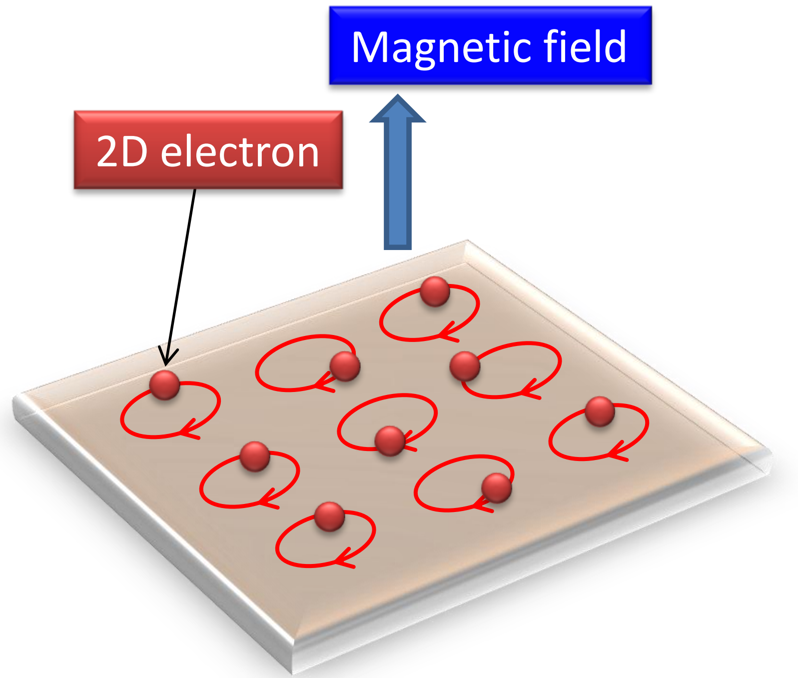

Momentum of the $z$-direction is quantized
(Electrons move freely in the $xy$-plane)

# What is a quantum Hall system?

## 2D electrons in a magnetic field




Electrons are confined in the quantum well

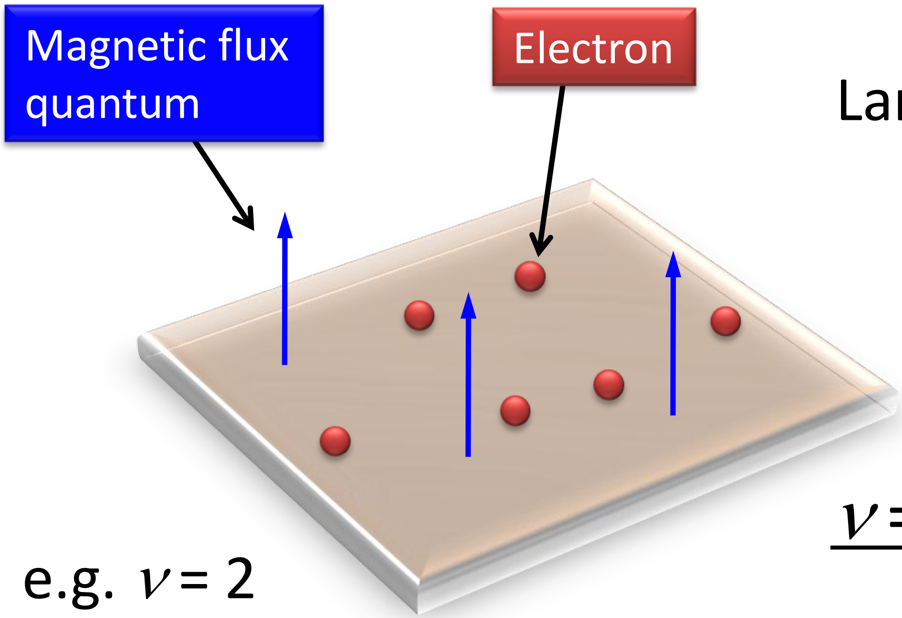



Landau quantization

Cyclotron motion

$$H_0 = \frac{1}{2m^*}(\mathbf{p} + e\mathbf{A})^2 \qquad \nabla \times \mathbf{A} = \mathbf{B}$$

# Landau level filling factor $\nu$




e.g. $\nu = 2$

Landau level filling factor

$$\nu = \frac{n_e}{n_\phi}$$

$n_e$ ← **Density of Electrons**

$n_\phi$ ← **Density of flux quanta**

<u>$\nu$ = integer:</u>

Integer quantum Hall effect

~ Landau quantization of electrons

<u>$\nu$ = rational fraction:</u>

Fractional quantum Hall effect

~ Strong Coulomb interaction

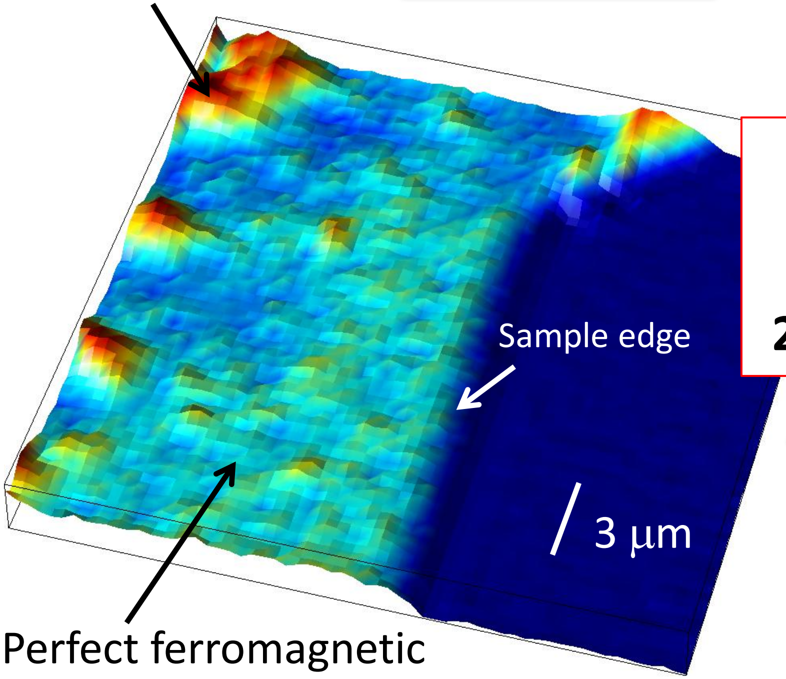
~ Incompressible quantum liquid

# Real-space imaging of fractional quantum Hall states

(Image of electron spin polarization)

Nonmagnetic quantum liquid

$\nu = 0.6659$



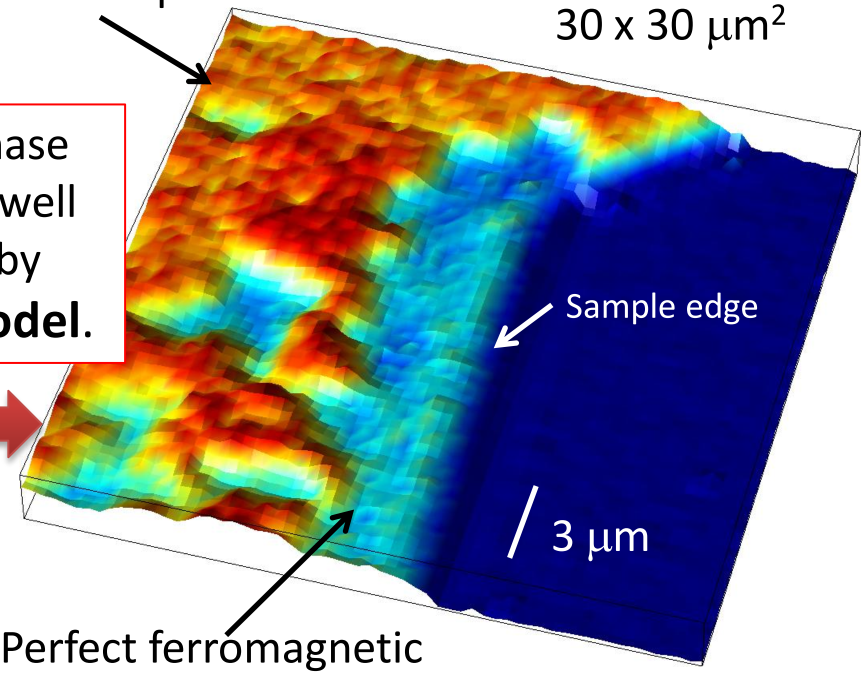

Perfect ferromagnetic quantum liquid

1st order phase transtion is well discribed by **2D Ising model**.

Nonmagnetic quantum liquid

$\nu = 0.6678$

30 x 30 $\mu m^2$



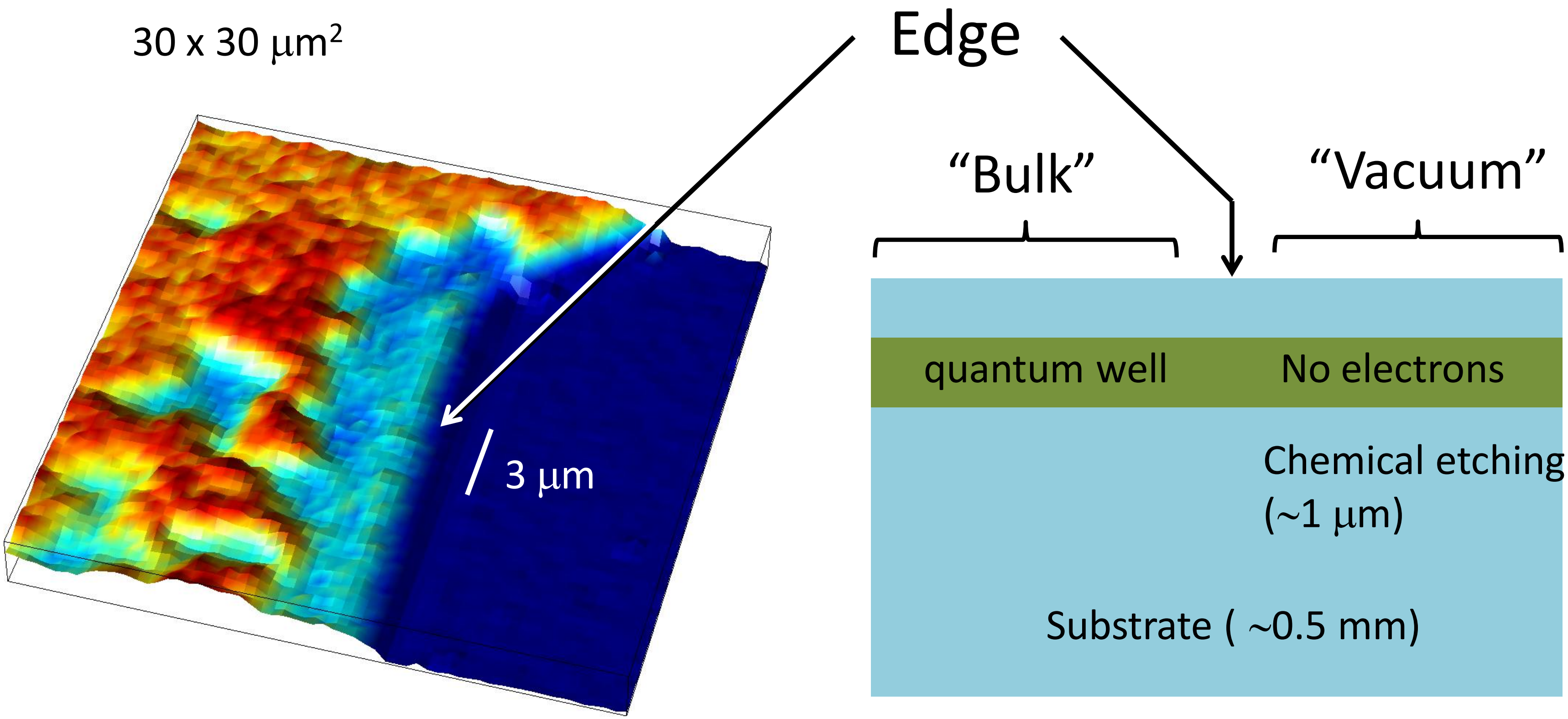

Perfect ferromagnetic quantum liquid

Conditions
Filling factor : $\nu \sim 2/3$
Temperature: 70 mK
Magnetic field: 7.5 T

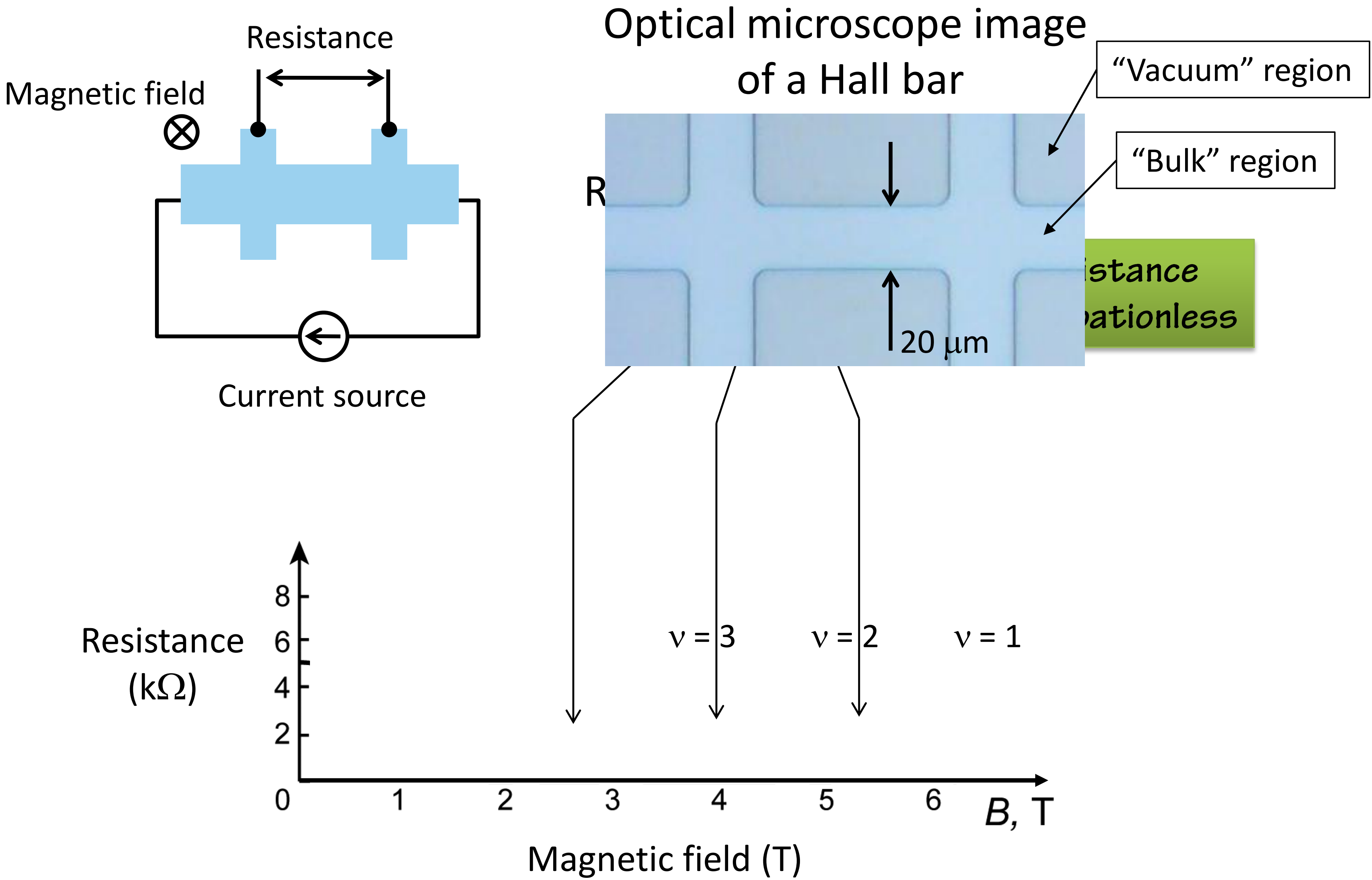
Hayakawa et al., Nature Nano. (2013)

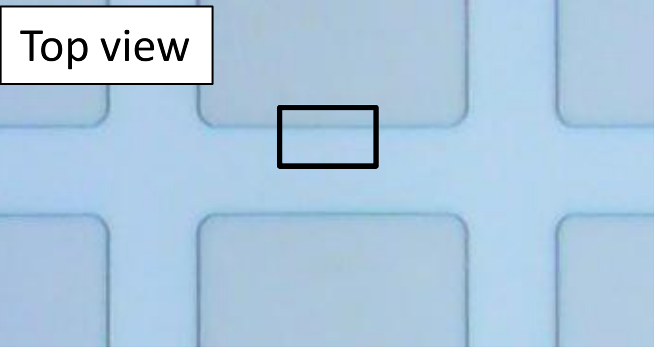
# Mesa structure and sample edge

30 x 30 μm²

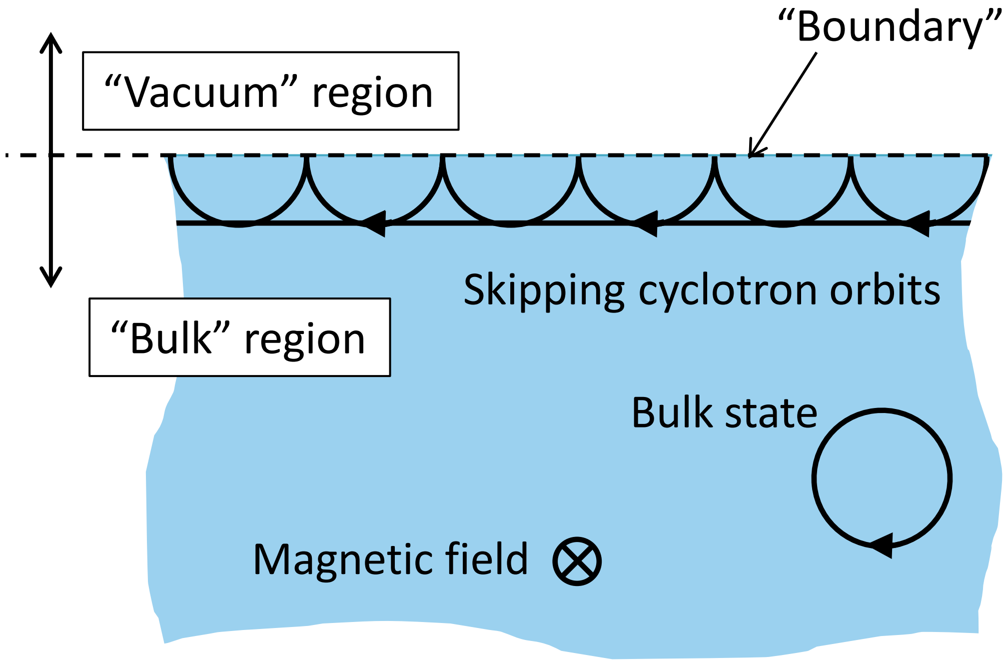


# Quantum Hall effect





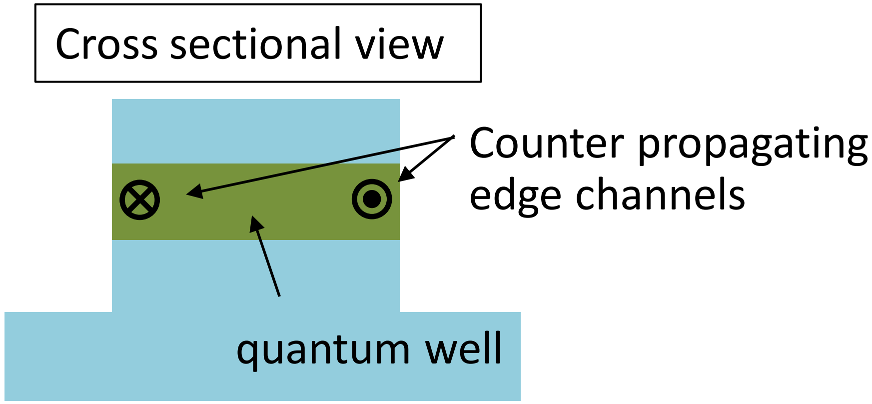

# Edge state

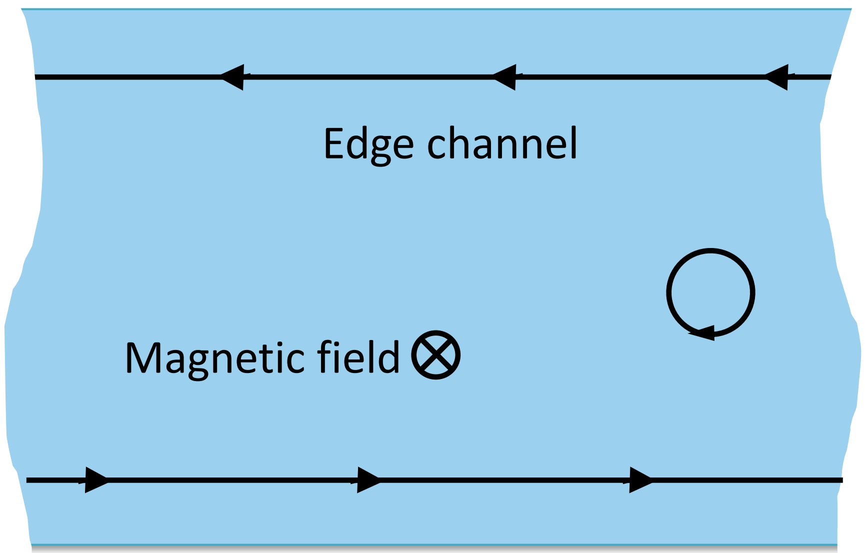



Magnetic length $l_B$ = 8 nm
@ magnetic field B = 10 T

Skipping cyclotron orbits can be seen as quasi-one-dimensional channel. Such an edge channel behaves as a Chiral Luttinger liquid, along which current flows unidirectionally.

# Edge state




“Moving walkways”


Unidirectional

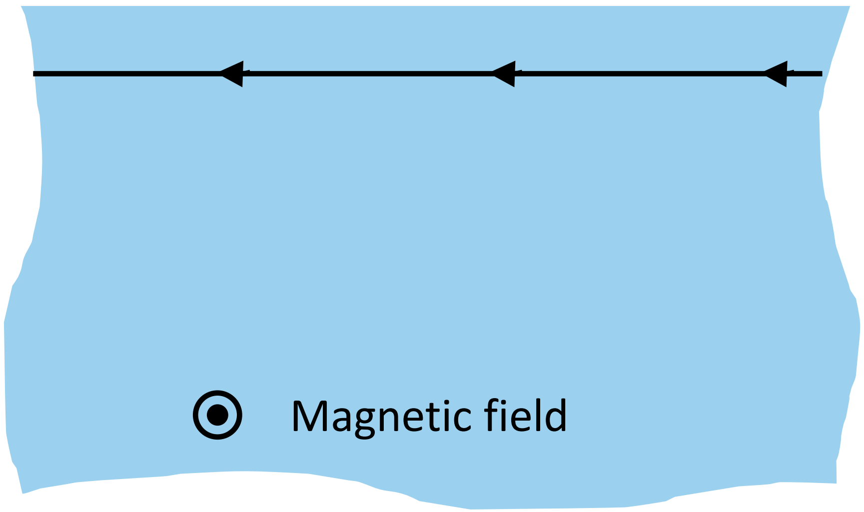
The chiral field operator in terms of density field $\varrho_S(x)$

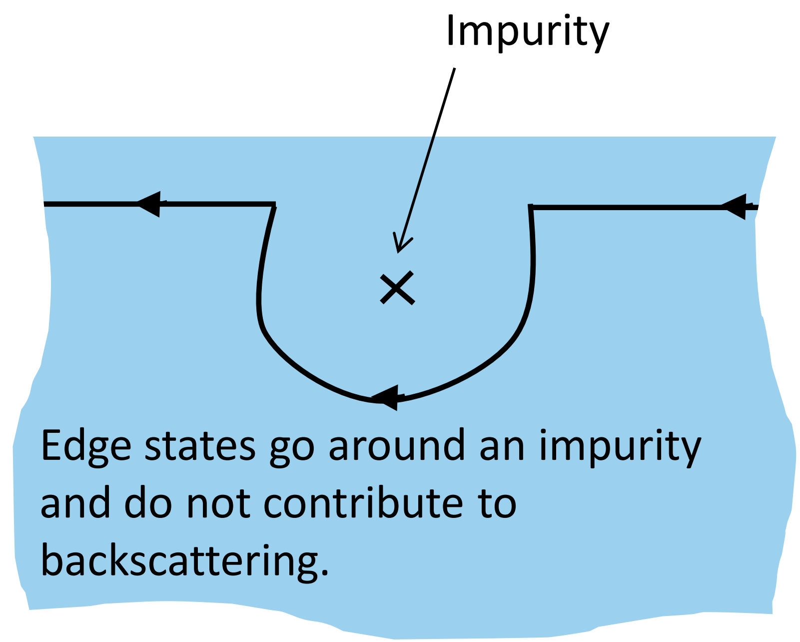
Commutation relation

$$[\varrho_S(x),\ \varrho_S(x')] = i\frac{\nu}{2\pi}\partial_x\delta(x-x')$$

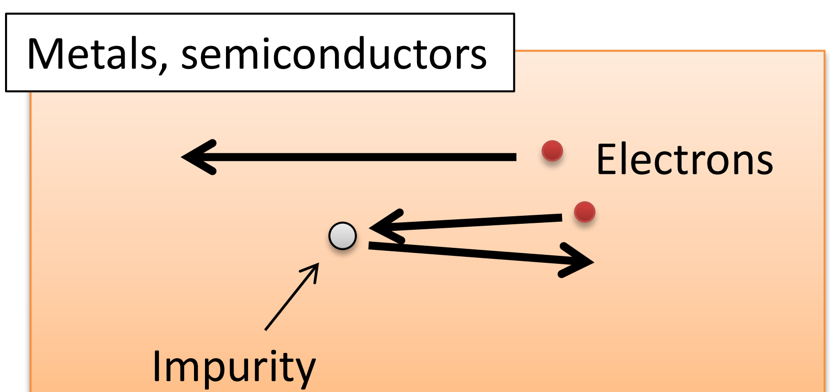
Chiral (unidirectional) massless field in one dimension

# Edge state




No backscattering
=> Zero-resistance



# Requirements to verify QET

(1) **Dissipationless** system

Zero-resistance

(2) **Quantum correlation channel** with macroscopic correlation length

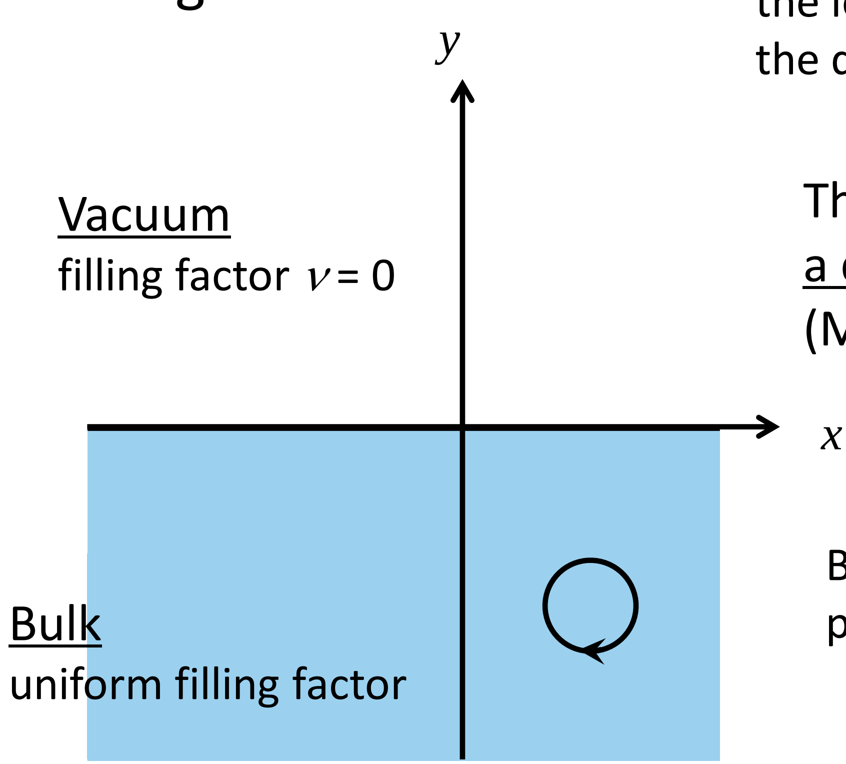
(3) **Detection** and **operation schemes** for well-defined fluctuations in the vacuum state

(4) Suitable implementation of local operations and classical communication **(LOCC)**

# The ground state and excitation of the edge state

Classical image of the edge

Quantum mechanical image
of the edge




Since the quantum Hall state is incompressible, the low-energy excitations are restricted to the deformation of the boundary.

The excitation of the edge channel is <u>a deformation of the boundary</u>. (Magnetoplasmon)

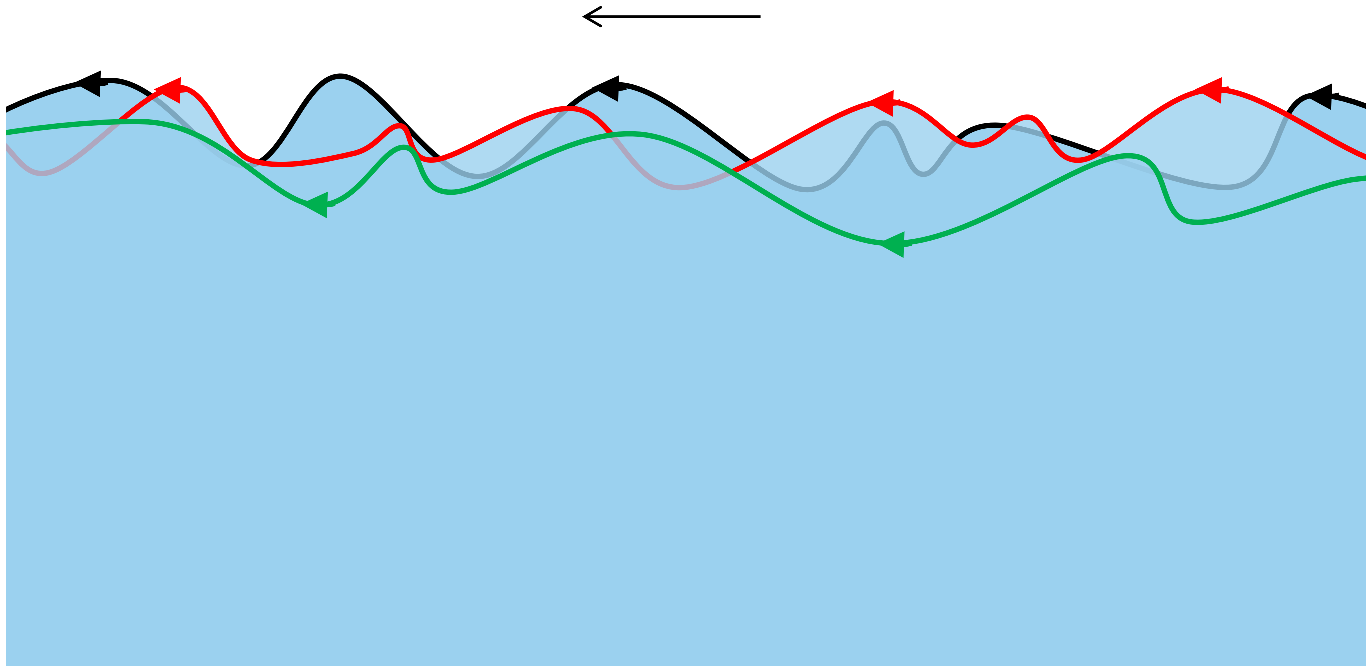
By the Lorentz force a magnetoplasmon propagates with velocity

$$v_g = \frac{E}{B}$$

$E$: the confining electric field

# The ground state of the edge state

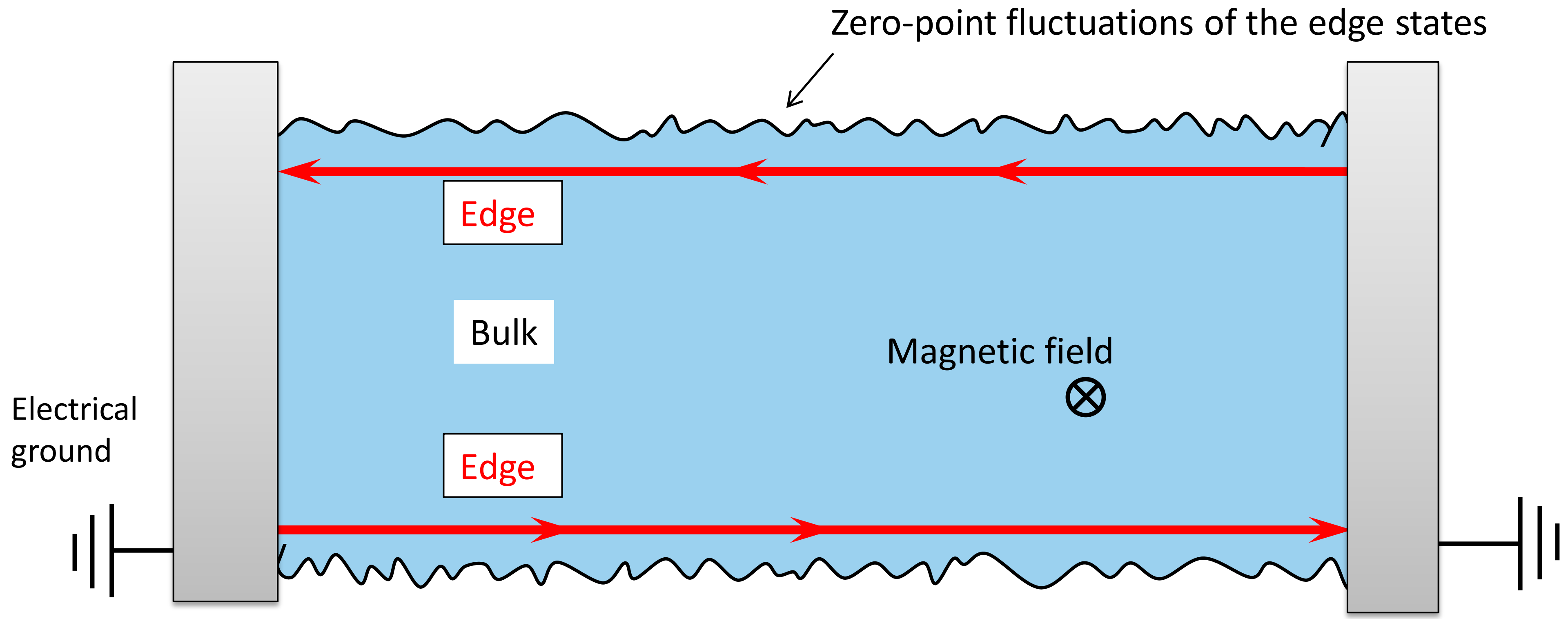
Zero-point fluctuations are superpositions of all possible patterns of deformed boundaries.



# How to prepare the “vacuum” state

- The system is cool down the extent possible.
  (The system is connected to the electrical ground.)
- No current or voltage is applied to the system.

The vacuum fluctuation of the edge state is the charge fluctuation



# Requirements for verifying QET

(1) **<u>Dissipationless</u>** system

Zero-resistance

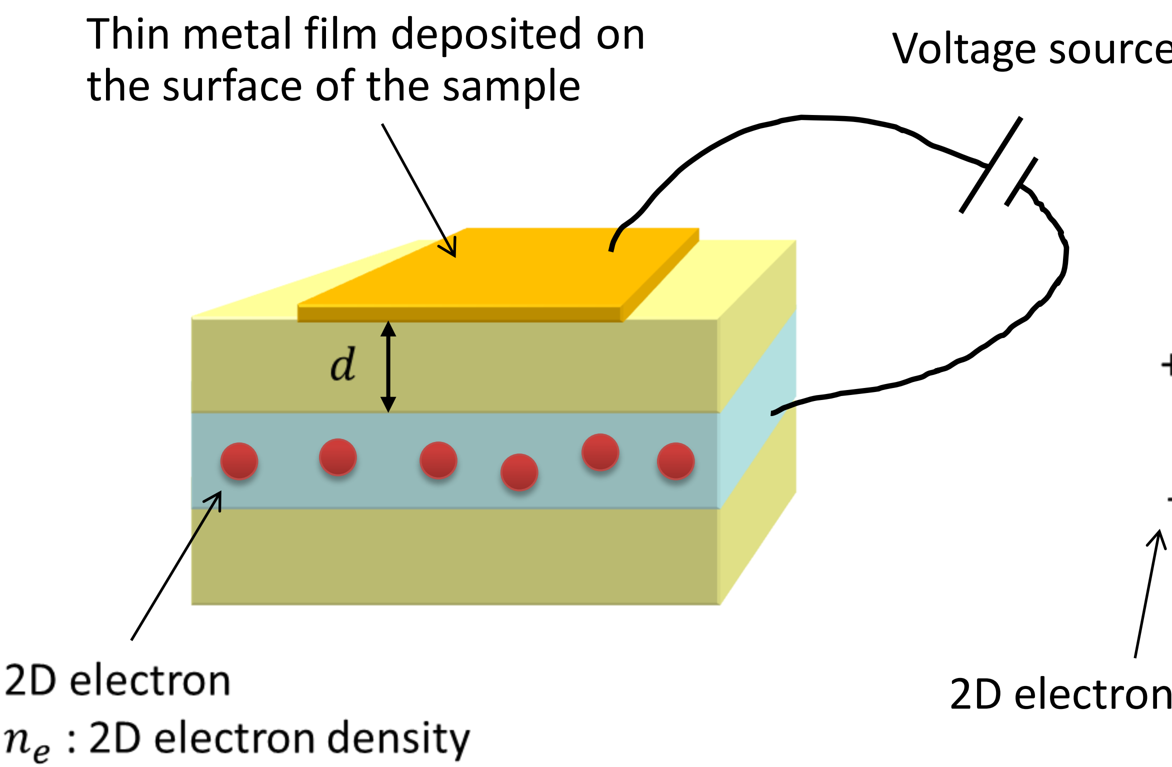
(2) **<u>Quantum correlation channel</u>** with macroscopic correlation length

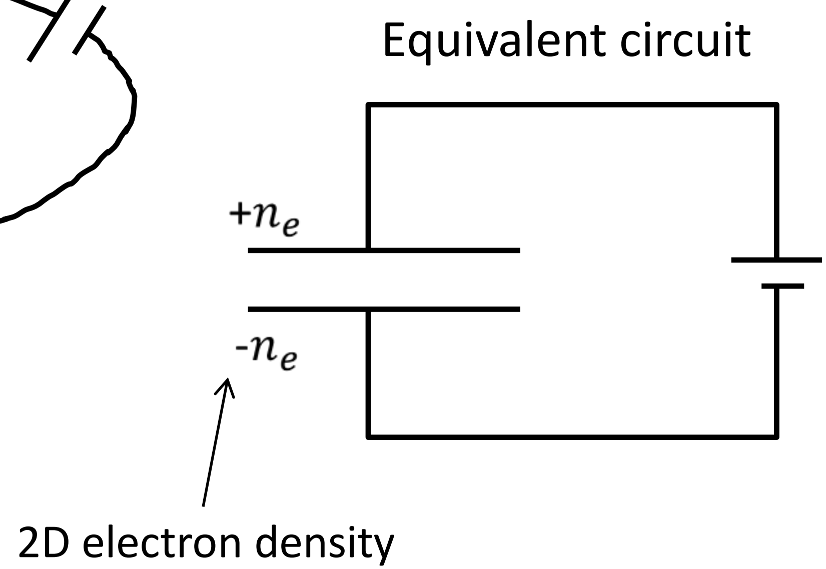
Edge channel

(3) **<u>Detection</u>** and **<u>operation schemes</u>** for well-defined fluctuations in the vacuum state

(4) Suitable implementation of local operations and classical communication **<u>(LOCC)</u>**

# How to detect charge fluctuations

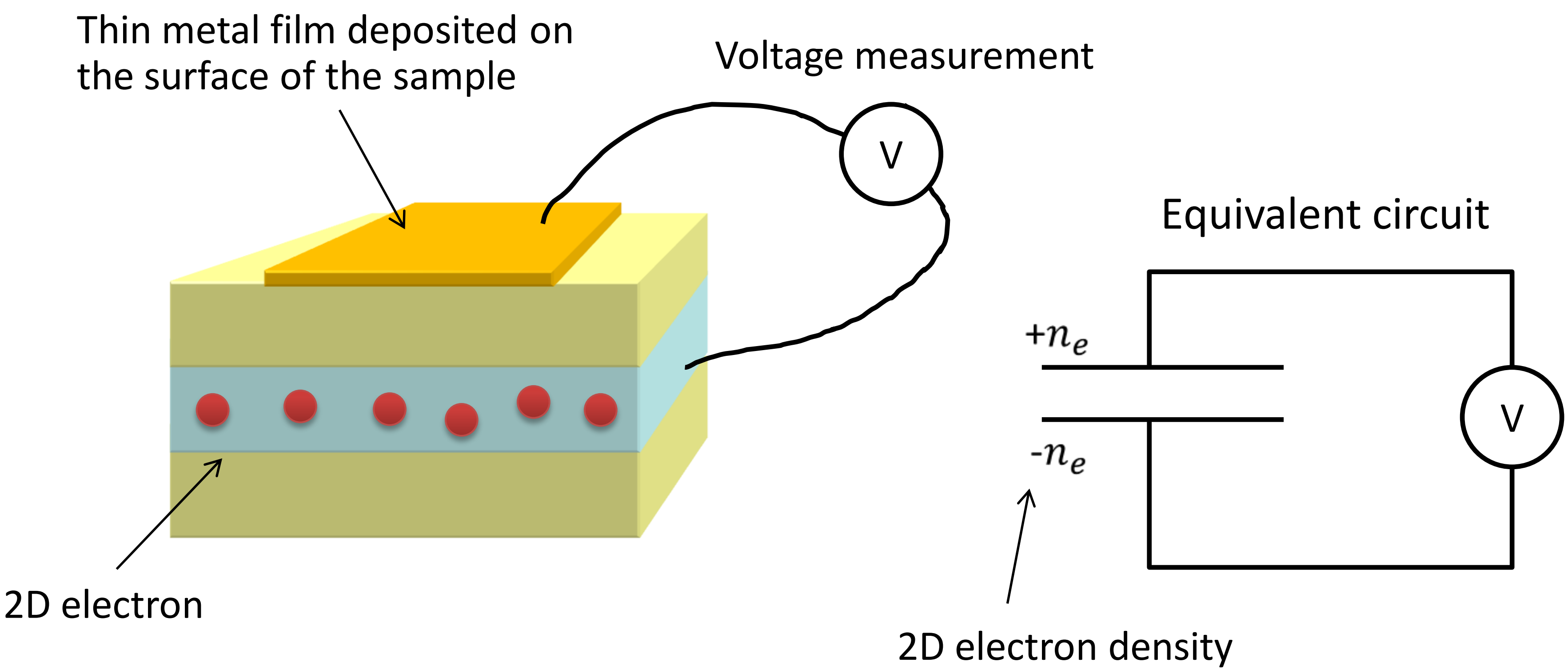



$$n_e = \frac{\varepsilon_r \varepsilon_0}{d} V$$

2D electrons + surface metal = parallel-plate capacitor

# How to detect charge fluctuations

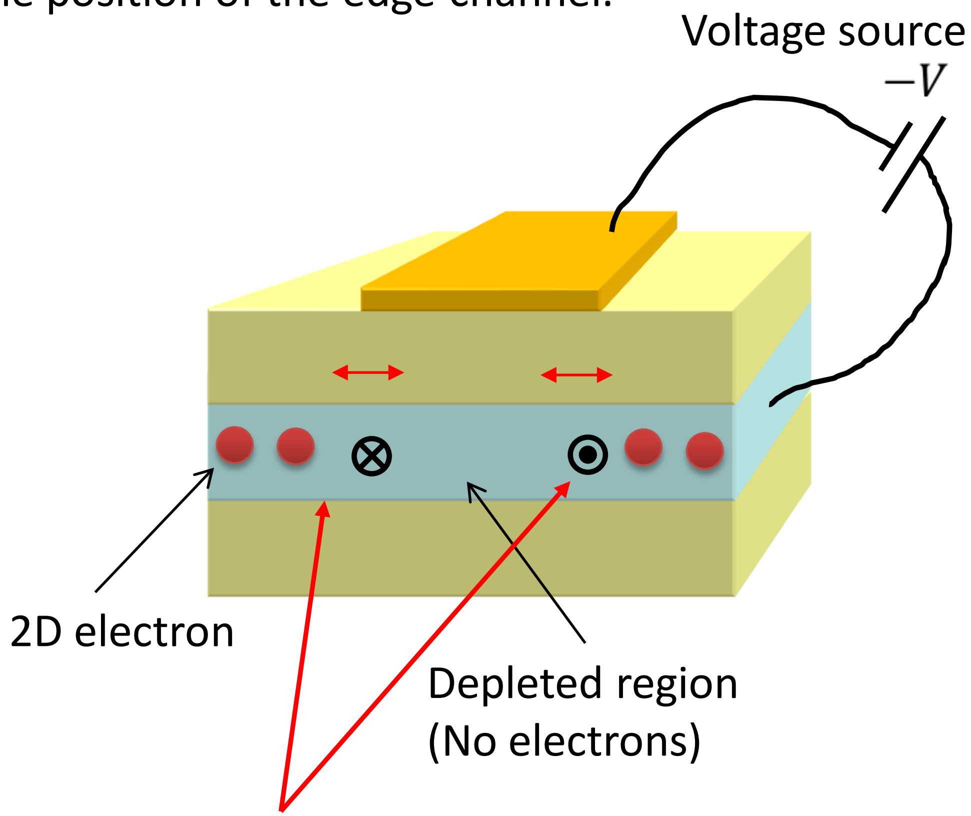



2D electrons + surface metal = parallel-plate capacitor

Measurement result = Voltage signal
An RC circuit is used as the detector.

# Electrically controlled Edge channel

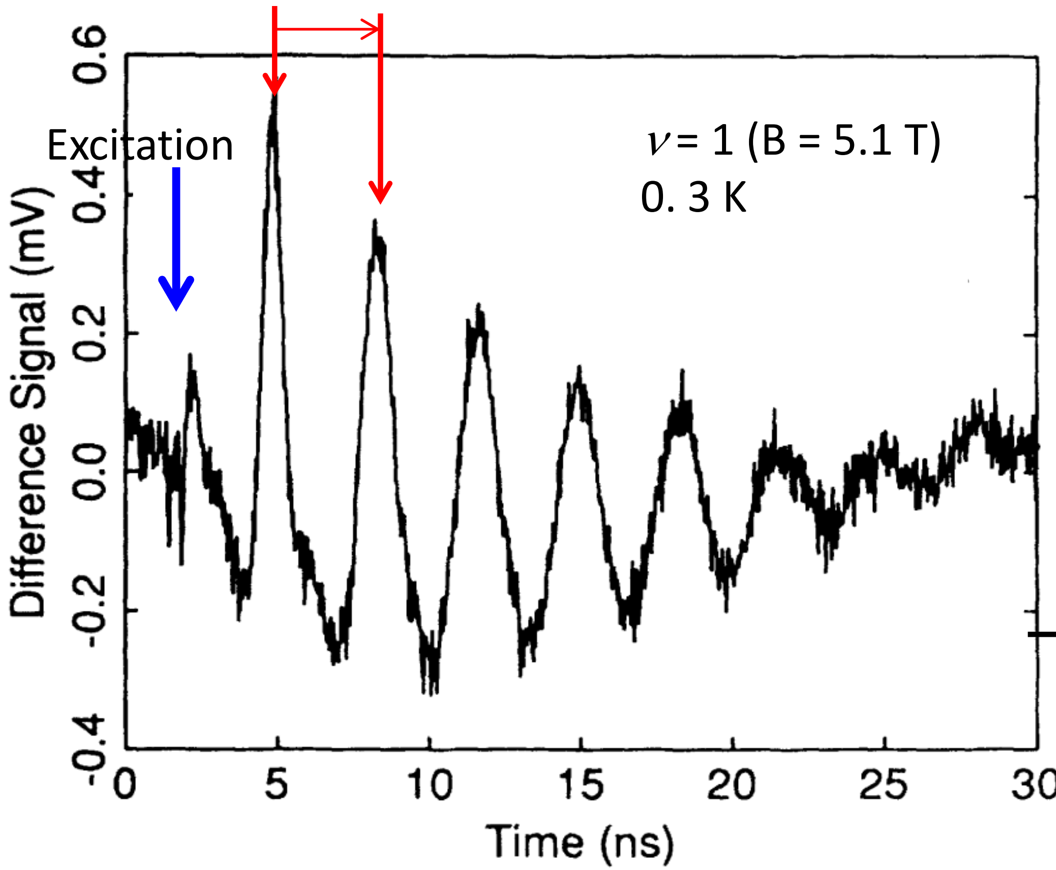
Electric field produced by the gate metal electrode can control electron density and the position of the edge channel.

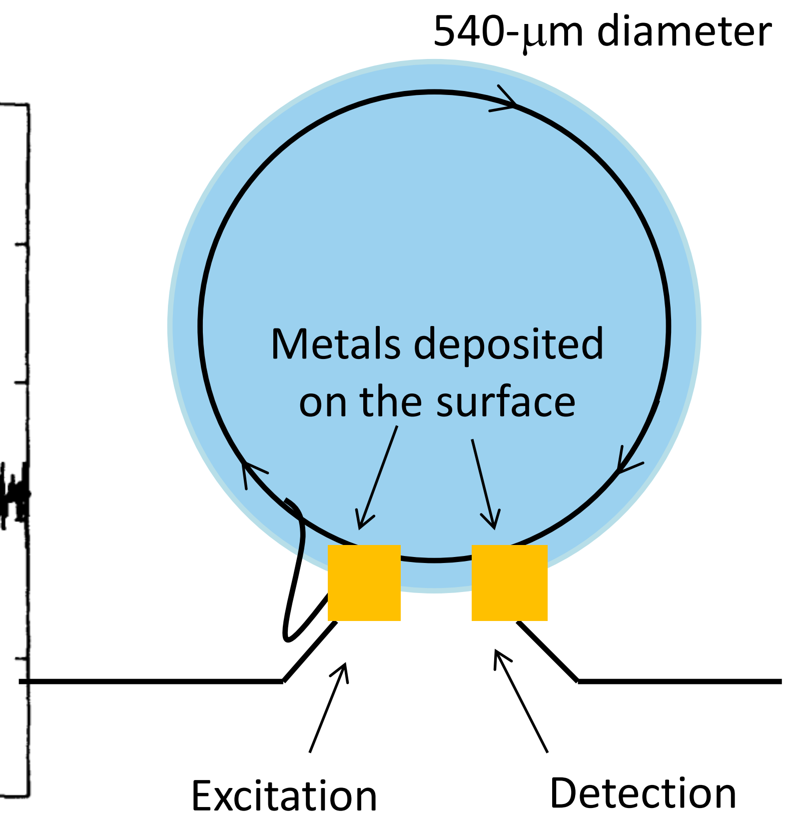



Edge channele are formed at this region and the position of the edge can be electrically controlled.

# Excitation of the edge state edge magnetoplasmon (EMP)

Time period for a wavepacket (magetoplasmon) to complete circling around the sample once.

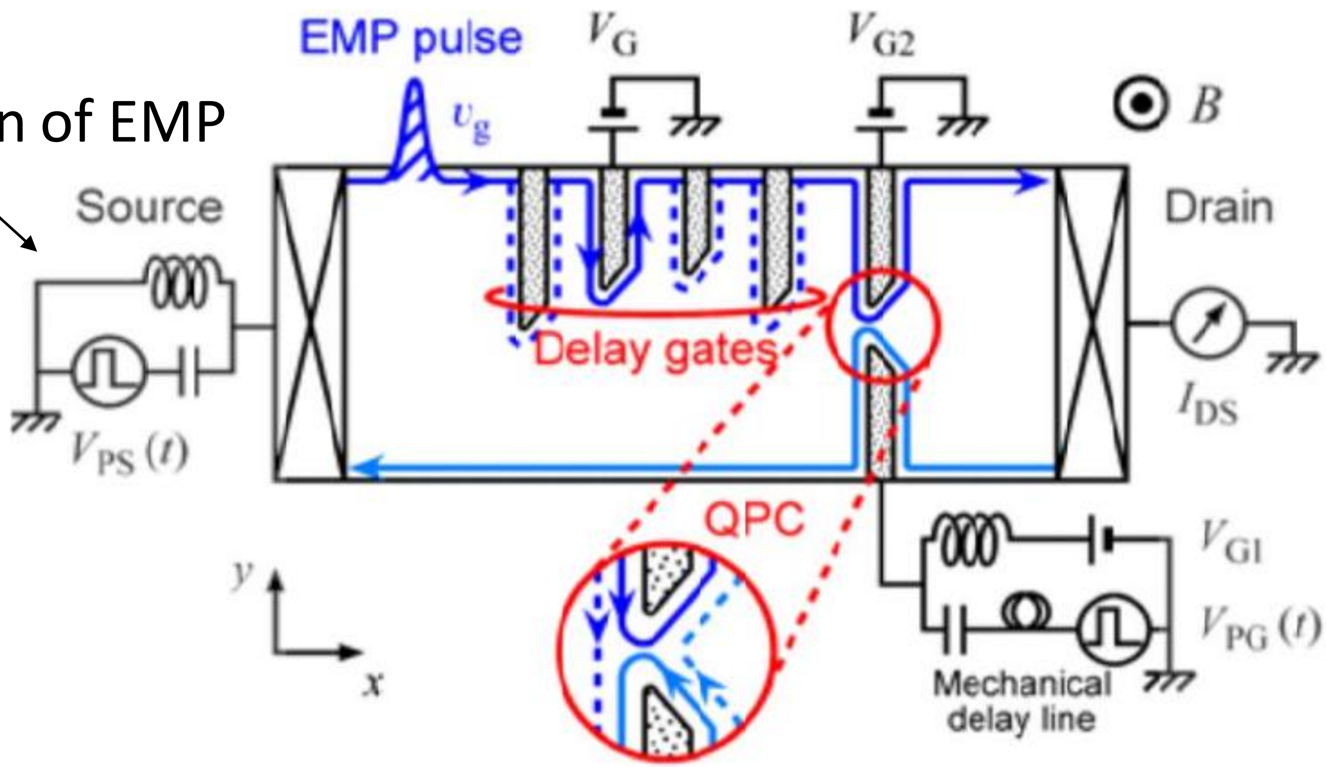



R. C. Ashoori *et. al.*, Phys. Rev. B **45**, 3894 (1992)

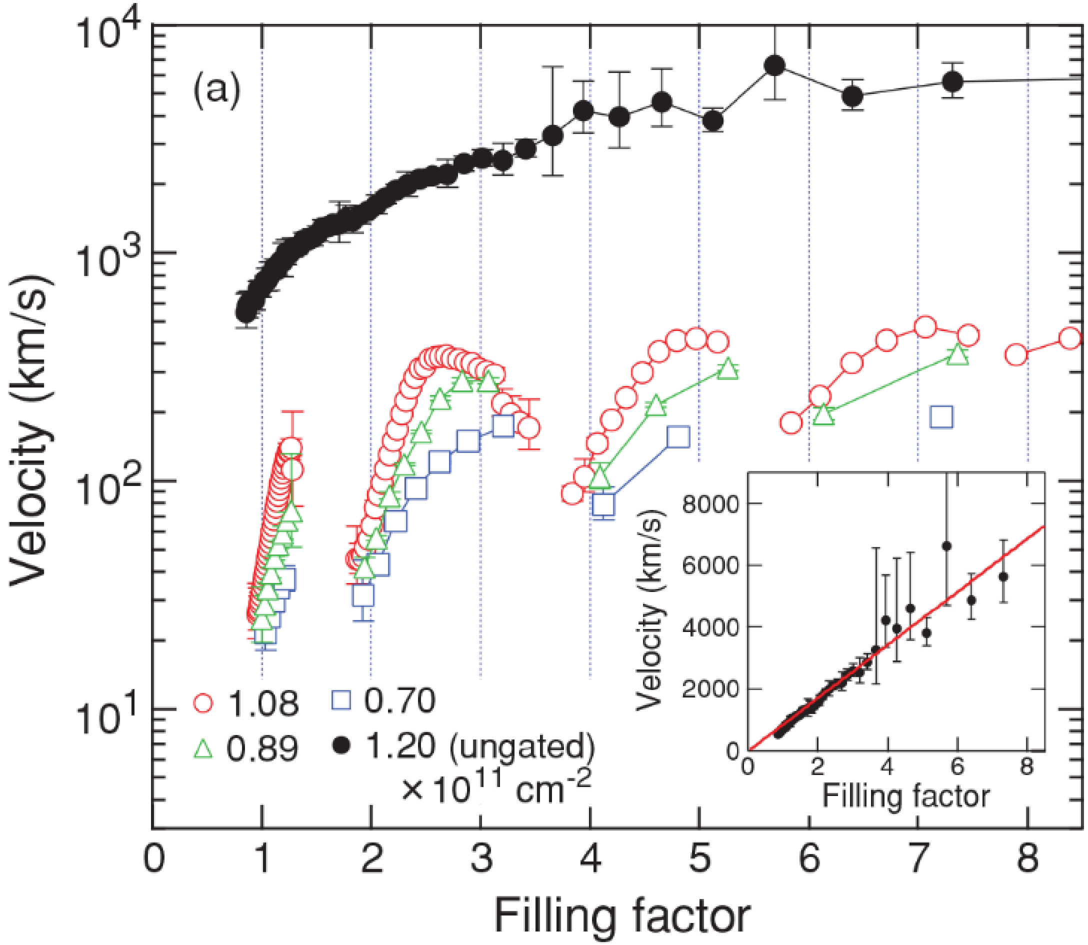
# Propagation of Edge Magnetoplasmon (EMP)

Excitation of EMP




Kamata et al. Phys. Rev. B (2010)

# Velocity of EMP




500 -5000 km/s

20 -200 km/s

Kumada et al. Phys. Rev. B (2011)

# Requirements for verifying QET

✓(1) **Dissipationless** system

Zero-resistance

✓(2) **Quantum correlation channel** with macroscopic correlation length

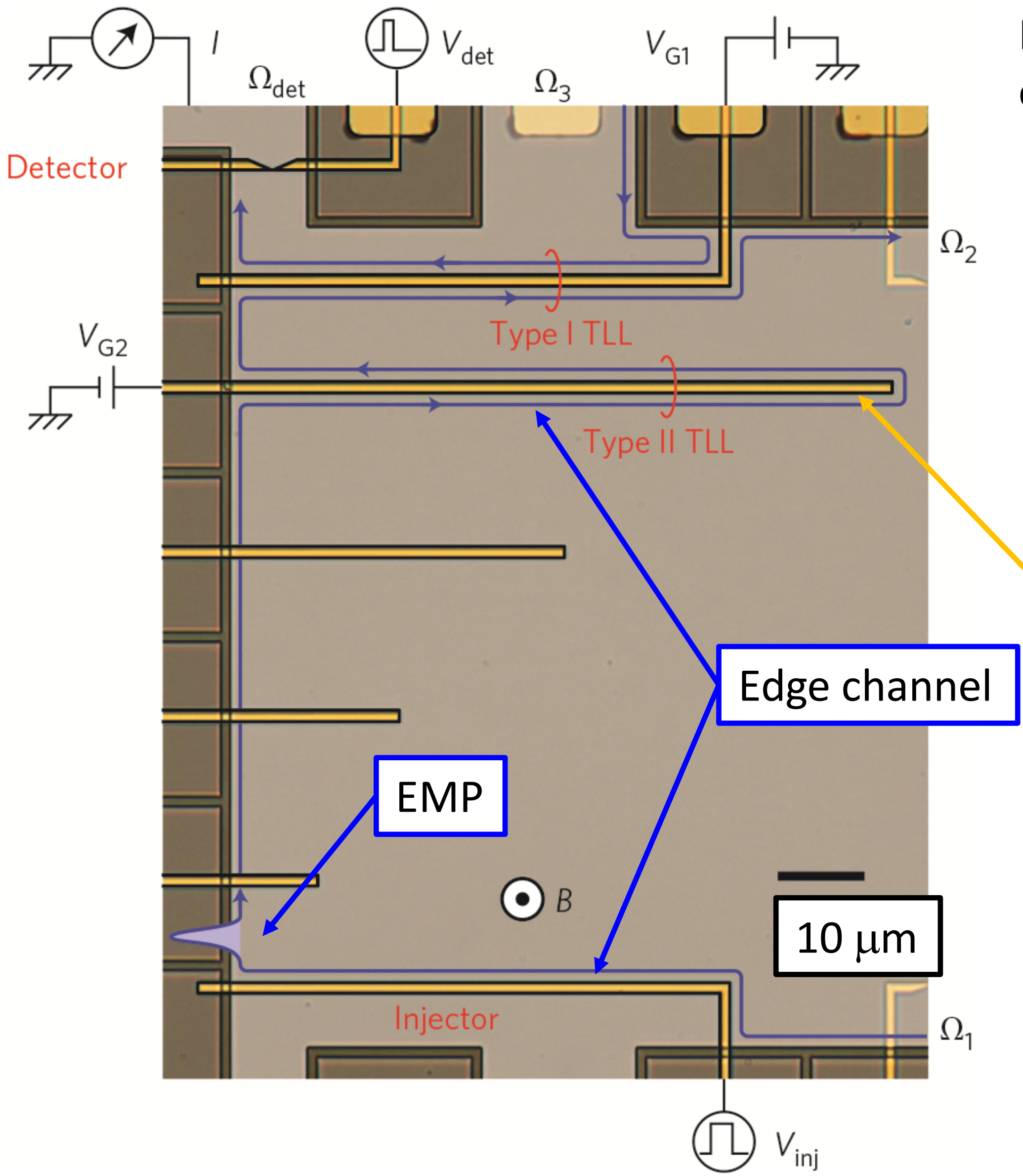
Edge channel

✓(3) **Detection** and **operation schemes** for well-defined fluctuations in the vacuum state

RC circuit for charge fluctuations

(4) Suitable implementation of local operations and classical communication **(LOCC)**

# Typical circuit to manipulate EMP




EMP (Edge magnetoplasmon) is the excitation of the edge channel.

Metal gate electrode

(To manipulate edge channels)

By using electron beam lithography < 100-nm-scale structures can be easily fabricated.

Kamata *et al*. Nature Nano. (2013)

# Requirements for Verifying QET

✔ (1) **<u>Dissipationless</u>** system — *Zero-resistance*

✔ (2) **<u>Quantum correlation channel</u>** with macroscopic correlation length — *Edge channel*

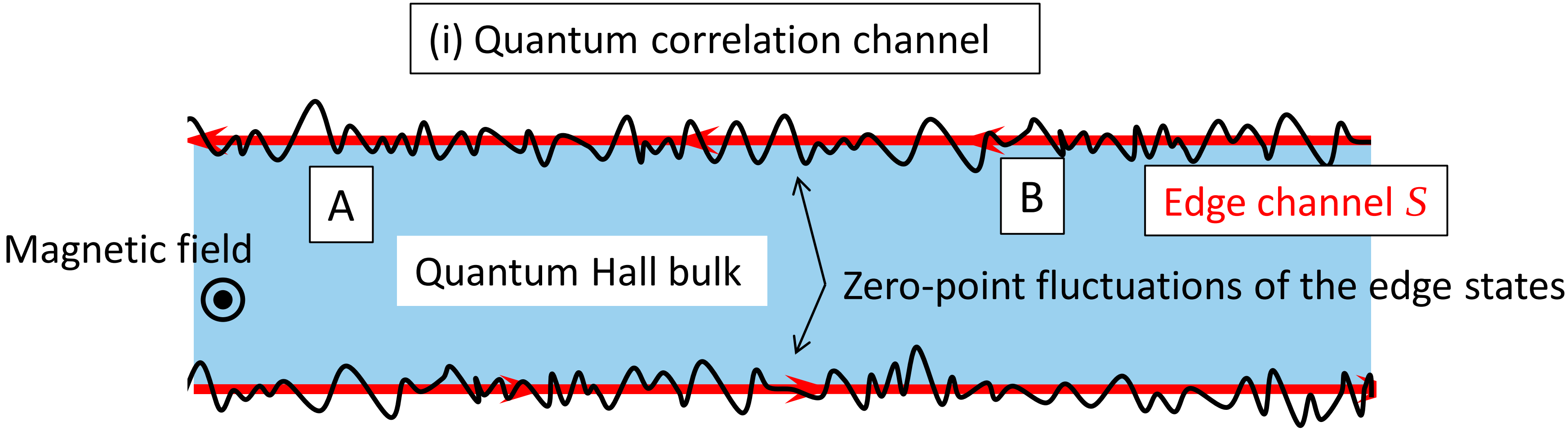
✔ (3) **<u>Detection</u>** and **<u>operation schemes</u>** for well-defined fluctuations in the vacuum state — *RC circuit for charge fluctuations*

✔ (4) Suitable implementation of local operations and classical communication **<u>(LOCC)</u>** — *Nanotechnology*

A quantum Hall system is very suitable for verifying QET protocol.

# Schematic of the quantum Hall system for verifying quantum energy teleportation

The system should be cool down to the very low temperature. (~mK)




G. Yusa, W. Izumida, M. Hotta, Phys. Rev. A 84, 032336 (2011)

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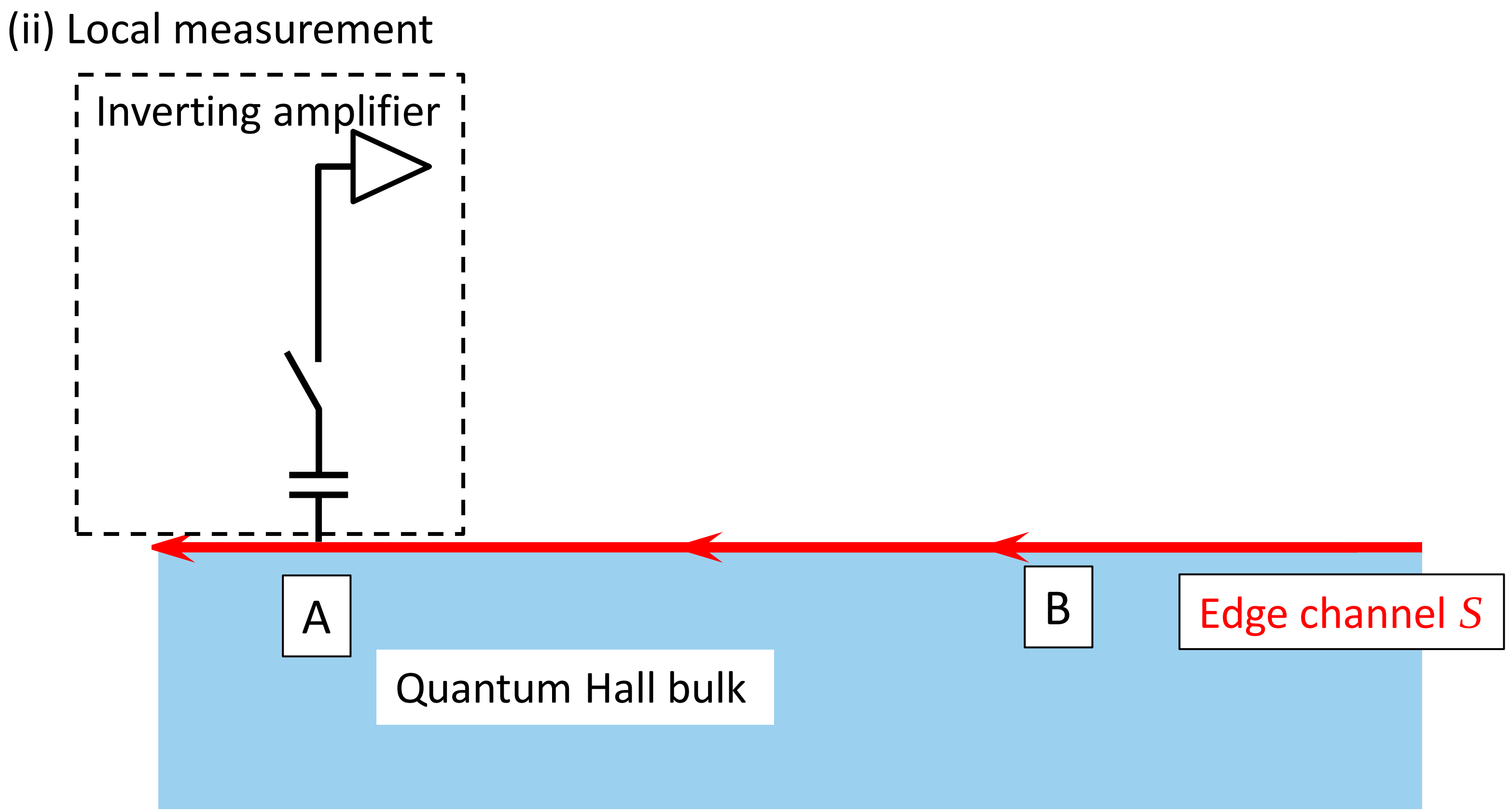


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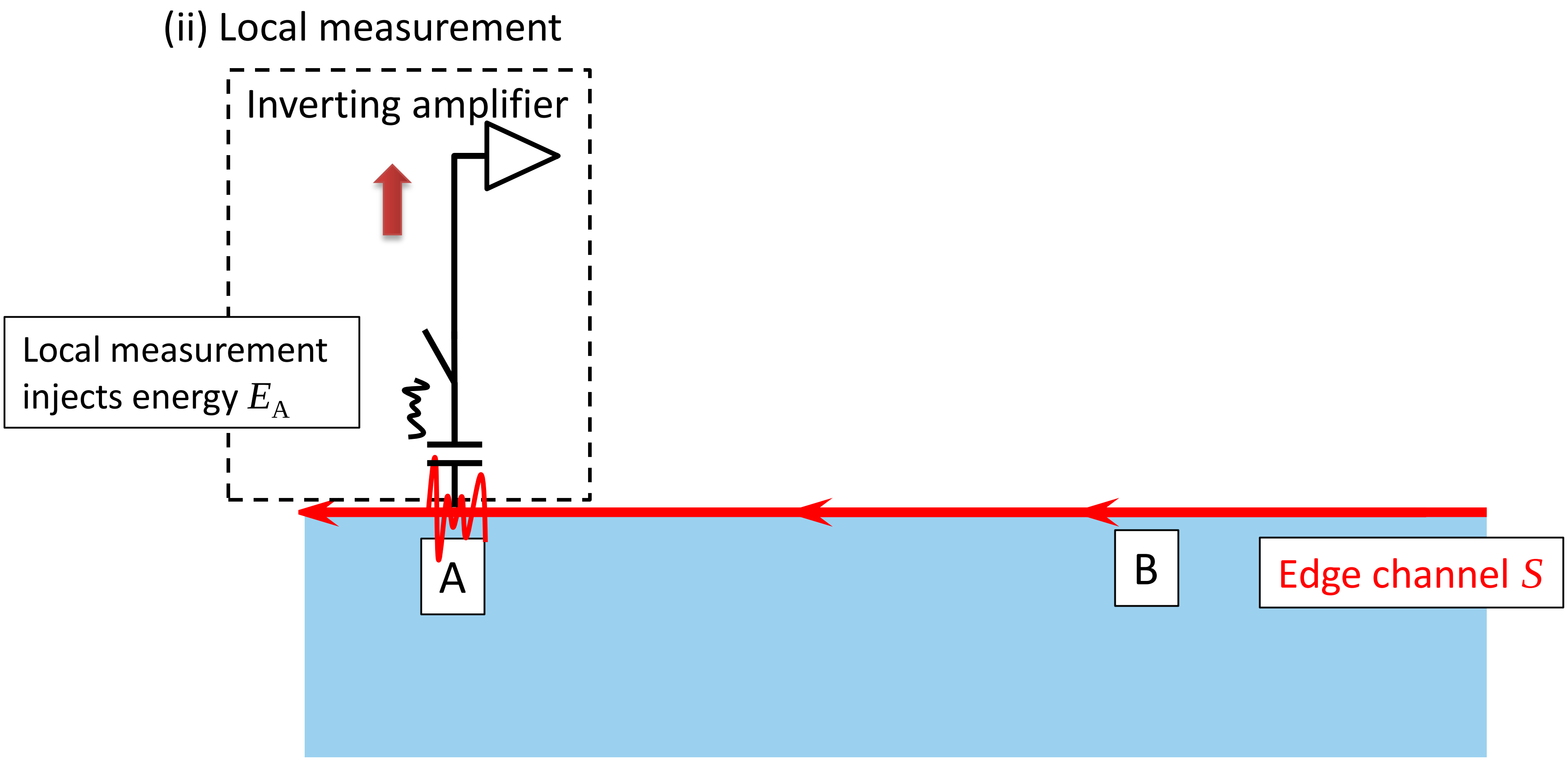


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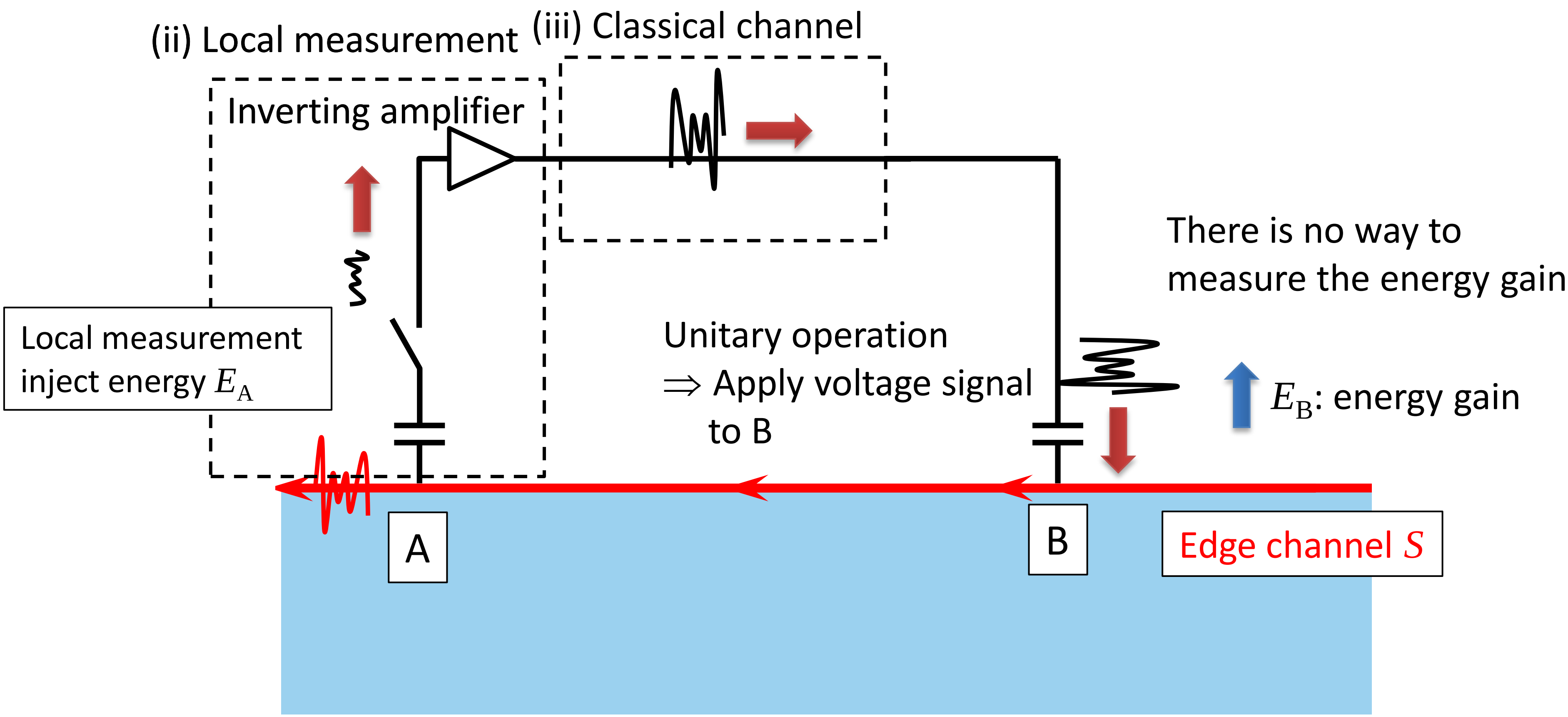


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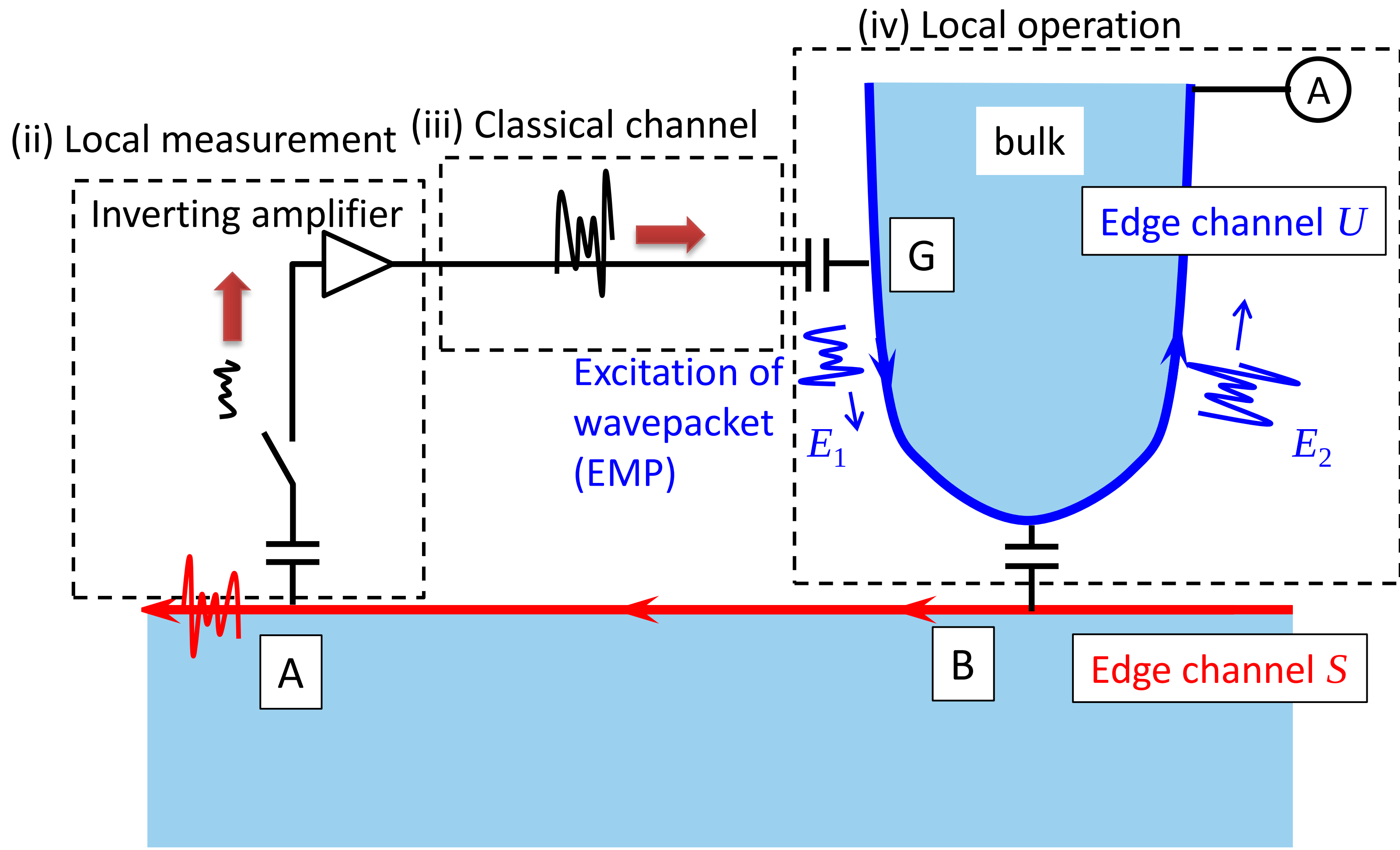


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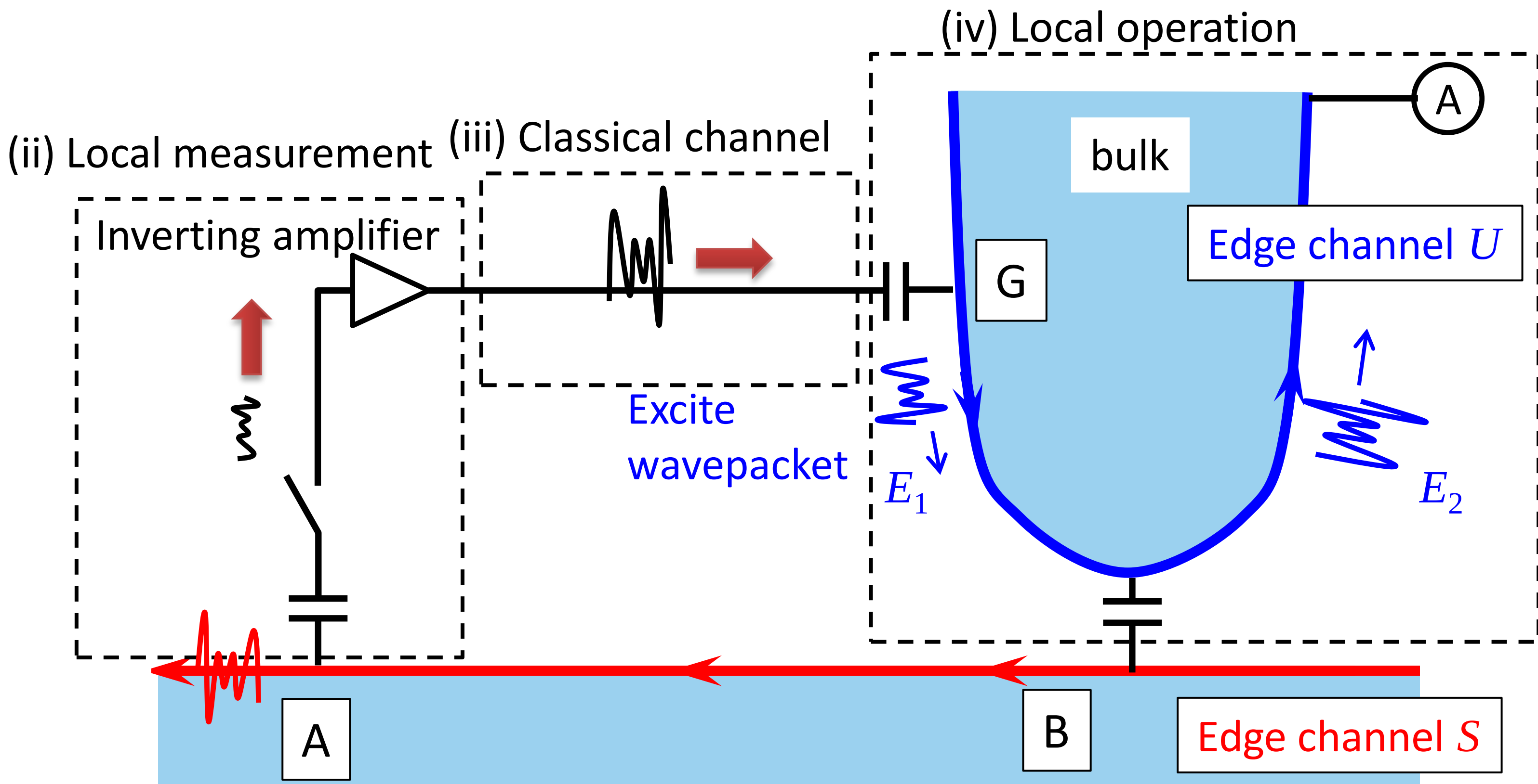
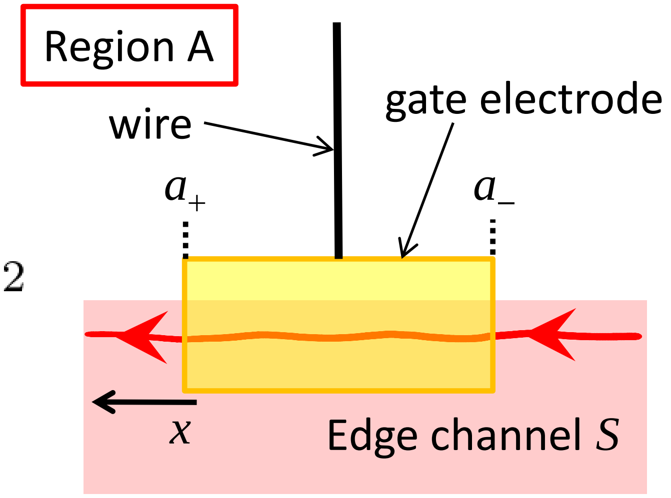



1. The wavepacket (=EMP) is created at G by the voltage signal (measurement result) and propagates with energy $E_1$.
2. The wave packet interacts with Edge channel $S$ at B.
3. The energy carried by the wavepacket changes from $E_1$ to $E_2$.

If $E_2 - E_1$ (= $E_B$) is positive, energy $E_B$ is extracted from the local vacuum state!

# $E_\mathrm{A}$ : Energy infused by the local measurement




$$E_A = \frac{\hbar v_g \nu_S}{4\pi}\left(\frac{e v_g R}{2\Delta V}\right)^2 \int_{-\infty}^{\infty} dx\, \left(\partial_x^2 w_A(x)\right)^2$$

$\nu_S$ : Filling factor at edge channel $S$ (= 3)

$v_g$ : group velocity of the edge channel, $v_g \sim 10^6\ \mathrm{m/s}$

$R$ : resistance 10 kΩ

$\Delta V$ : amplitude of the quantum noise of the voltage signal

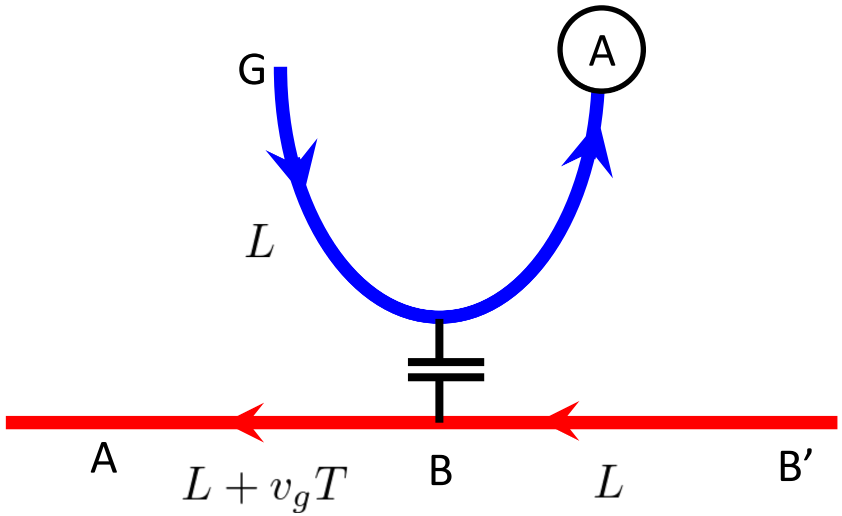
$$\Delta V \sim \sqrt{\frac{\hbar}{RC^2}} \sim 10\ \mu\mathrm{V}$$

$w_A(x)$: a window function which equal to ~1 at A $x \in [a_-,\ a_+]$

$E_A \sim 1$ meV

# $E_B$: The extracted energy at B

$$E_B \sim \frac{e^2\lambda_B}{4\pi\epsilon l}\frac{ev_gR}{l\Delta V}\left(\frac{l}{L}\right)^5$$




$L$ : ~the distance between A and B

$l$ : the size of the gate (10 μm)

$\epsilon$ : dielectric constant of the host semiconductor

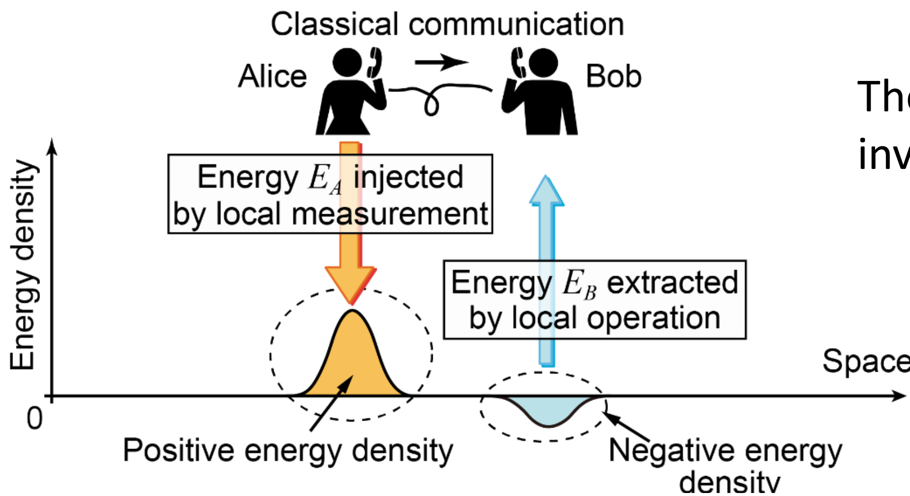
$\lambda_B$ : a window function related to the total number of excited electrons and quasi holes from the vacuum state (= 10)

$T$ : the time period for the classical signal to travel from A to G

- All positive parameters guarantee positive $E_B$.
- $E_B \sim 100$ μeV for $L \sim 2l$ (>> thermal energy ~ 1 μeV at 10 mK)
  This means that energy is extracted at a distance of ~ $2L$ (~ $4l$ = 40 μm).
- Extension of $L$ rapidly degrades magnitude of $E_B$.
  e.g., $E_B \sim 1$ μeV for $L \sim 4l$
- Since $E_B$ is not maximized, much more energy gain can be expected.

# Long distance QET

(a) Vacuum state QET




The upper bound of the energy being inversely proportional to the distance.

$$E_B \leq \frac{1}{12\pi L}$$

$L$ : distance

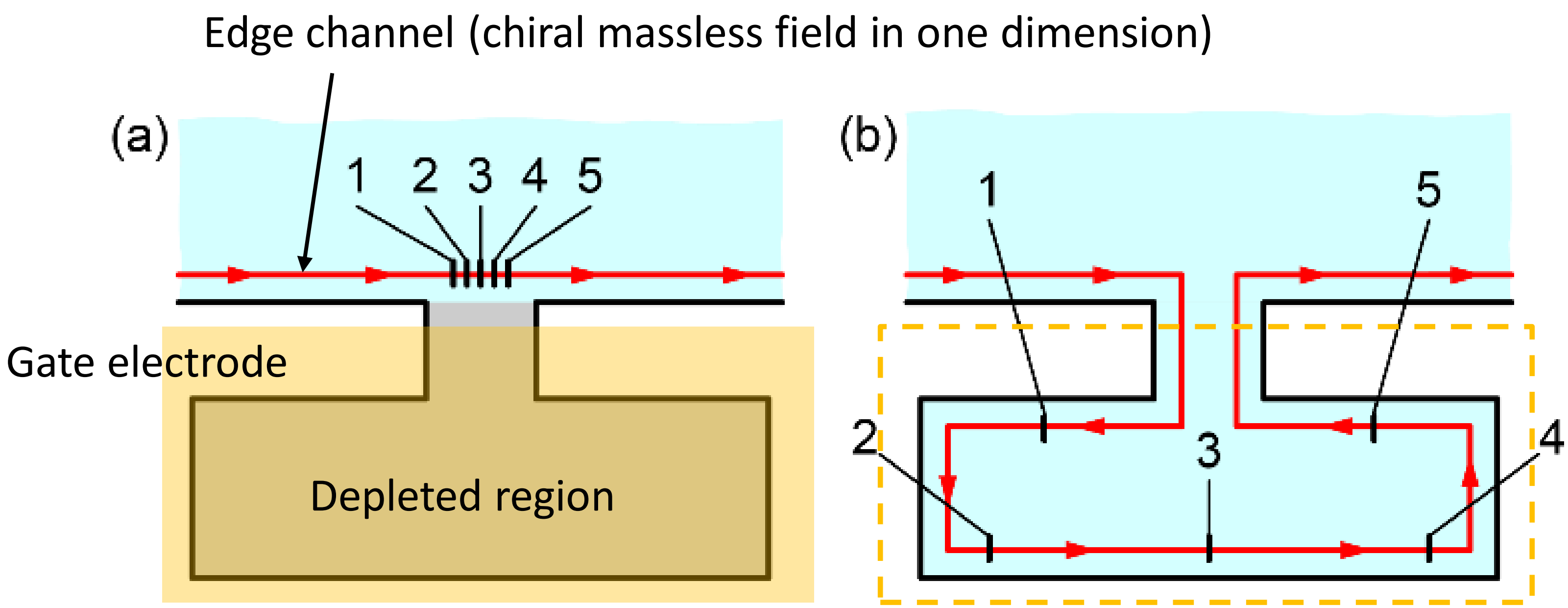
The natural unit ($C = \hbar = 1$)

Introducing squeezed vacuum states with local vacuum regions between A and B overcomes the limitation of distance.

Hotta et al. Phys. Rev. A 89 012311 (2014)

# Spatial expansion method

A local extrusion of bulk electrons toward the outside can be experimentally obtained by dynamically controlling the electron density in the depleted region.

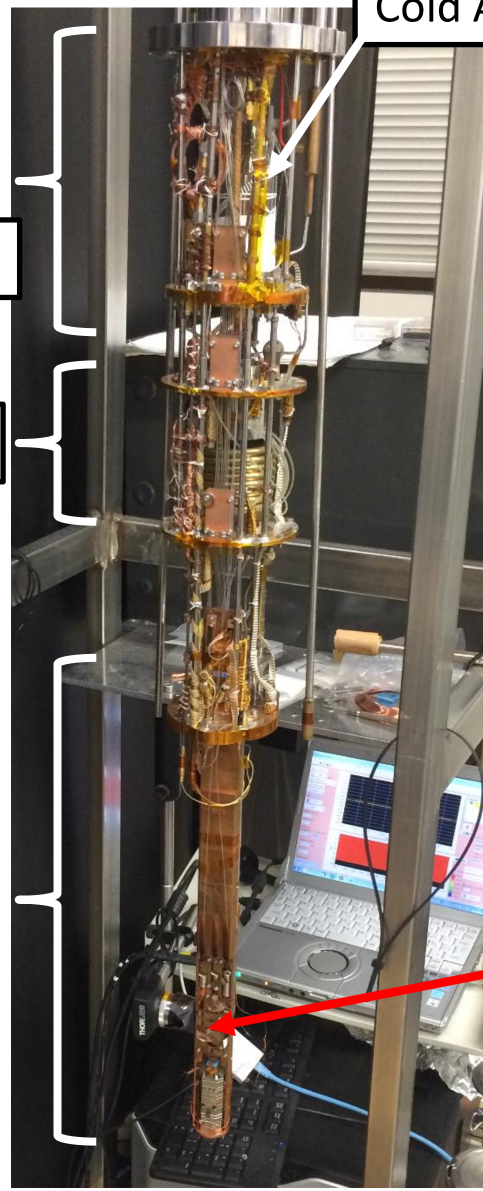


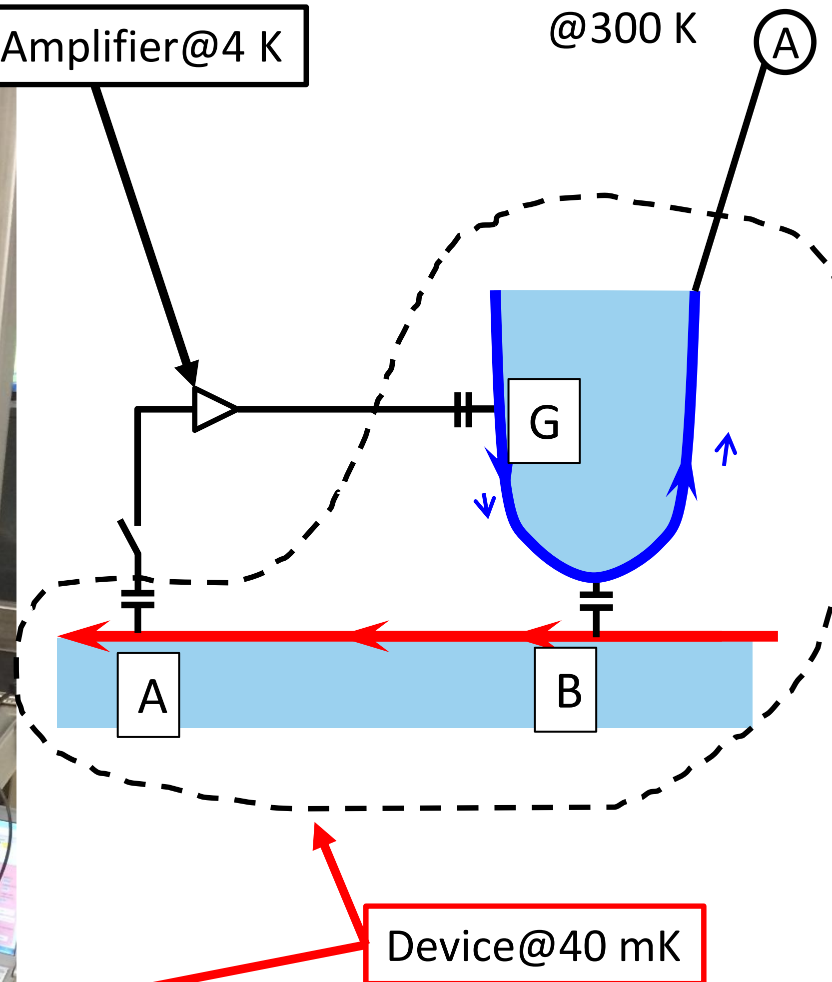

Hotta et al. Phys. Rev. A 89 012311 (2014)

# Experimental progress

## Dilution refrigerator

provides continuous cooling down to mK region using $He^3$ and $He^4$ isotopes.



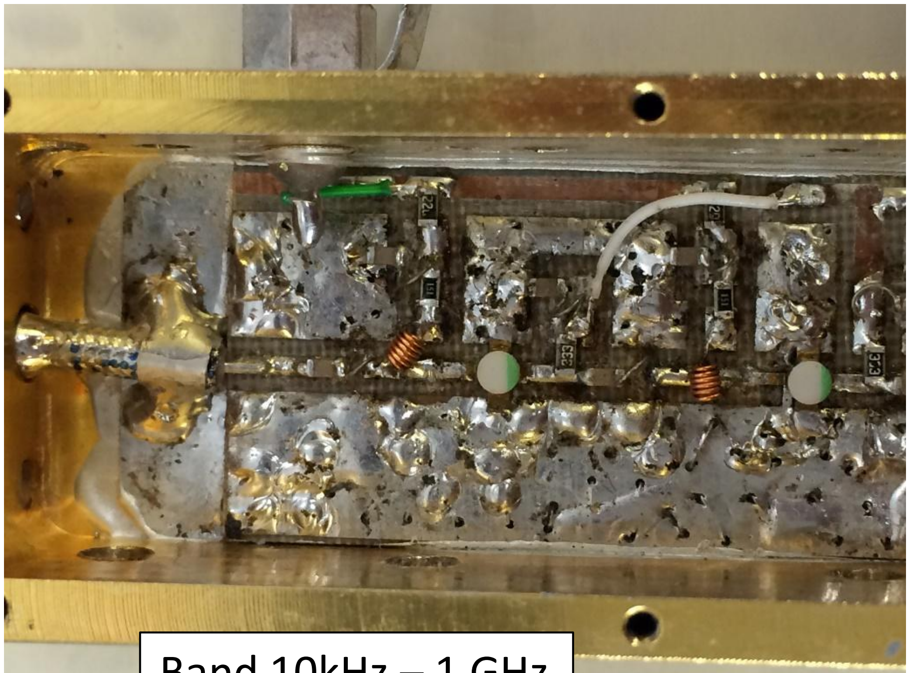




Min temperature ~ 40 mK
Max magnetic field 16 T

We sink the whole dilution unit into liquid helium bath (4 K).

# Low noise amplifier operating at 4 K cryo amplifier

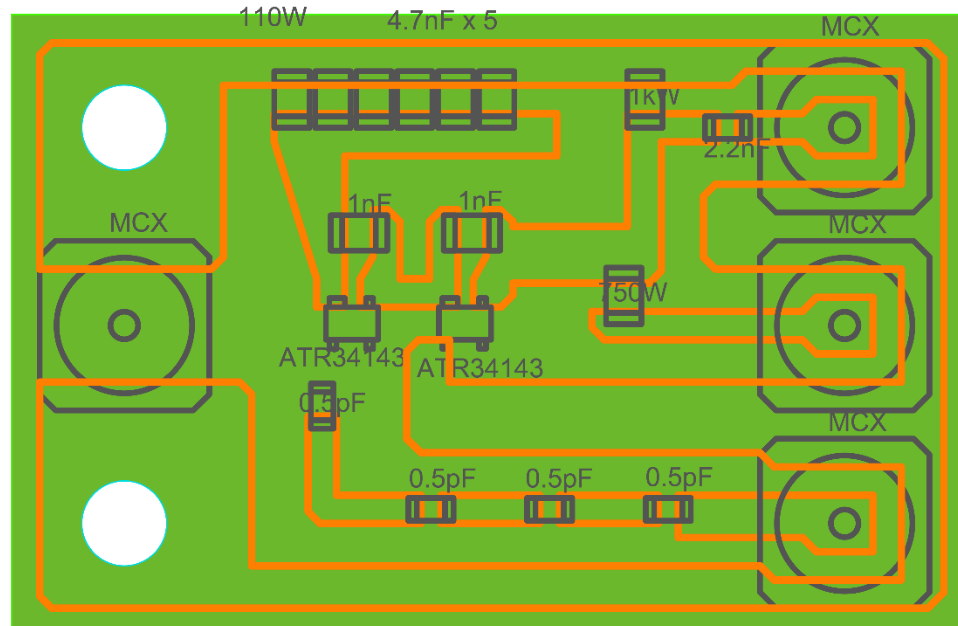
Example of cyro amp.



Band 10kHz – 1 GHz
Gain 20 (dB)
Noise Figure 1 dB

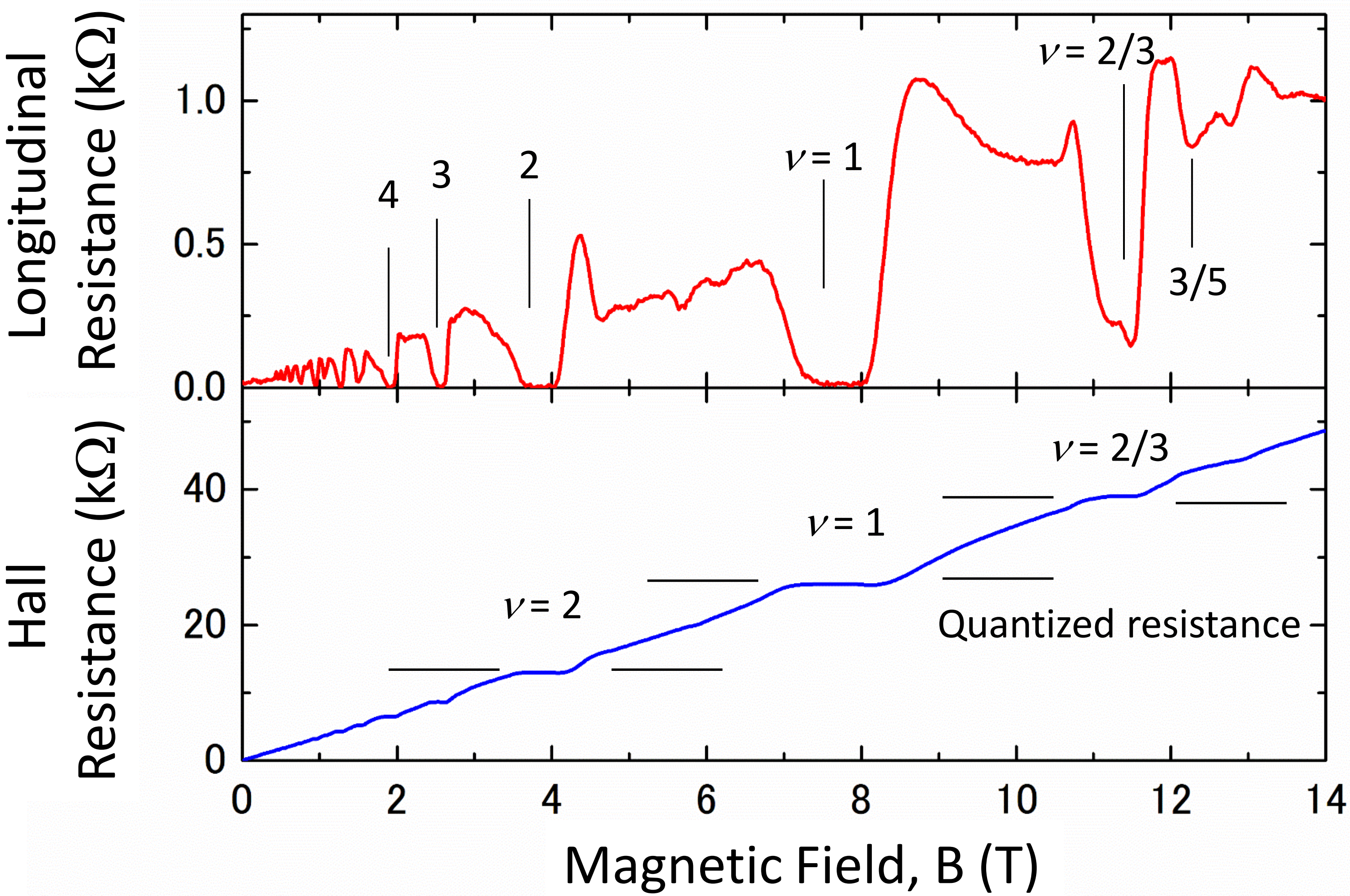
Still have some problem with heating.

Example of circuit diagram of cyro amp.



# Quality of semiconductor wafers

The wafer is processed into a Hall bar structure.
Four terminal measurement was performed using lock-in amplifier.

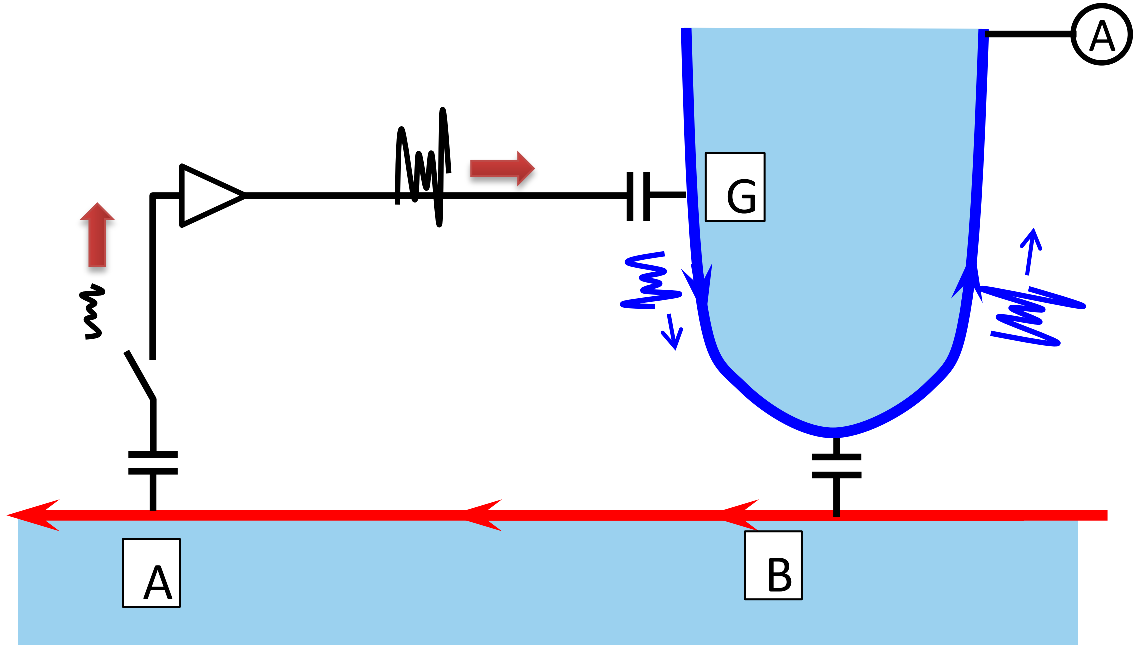



The wafer is provided from National Institute for Material Science (NIMS).

# Summary

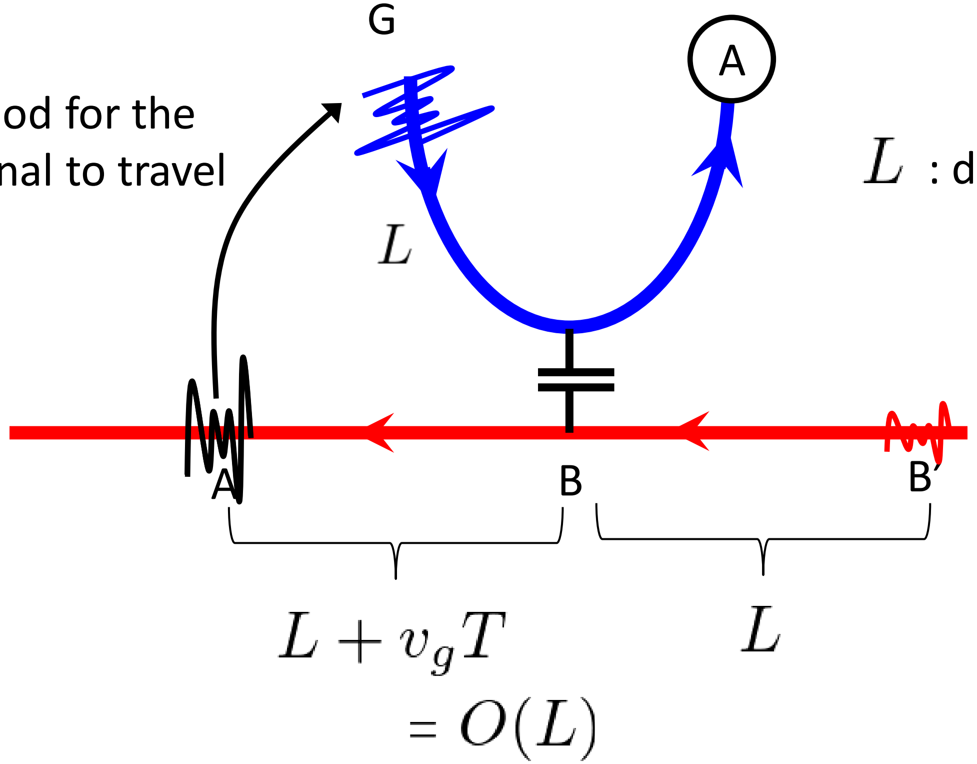
- We have theoretically implemented a quantum energy protocol (quantum energy teleportation).

- $E_B$ is positive and is estimated to be the order of $\sim$ 100 μeV at a distance of 40 μm.
  (= 4000 times of the size of an electron)



# Effective "teleported" distance

$T$: time period for the classical signal to travel from A to G

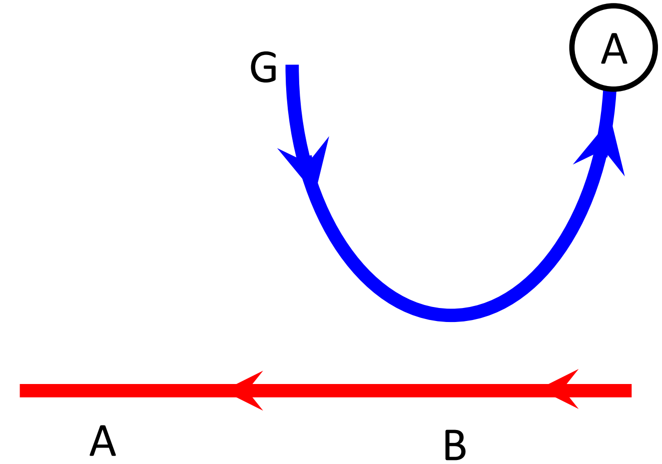



$L$ : distance from G to B

Since the wavepacket created at G interacts with the local vacuum state originally located at B', the effective "teleported" distance is not $L$ but $\sim 2L$.

# $E_B$: The extracted energy at B

$$E_B \sim \frac{e^2 \lambda_B}{4\pi\epsilon l} \frac{e v_g R}{l \Delta V} \left(\frac{l}{L}\right)^5$$




$L$ : ~the distance between A and B

$l$ : the size of the gate(10 μm)

$\epsilon$ : dielectric constant of the host semiconductor

$\lambda_B$ : a window function related to the total number of excited electrons and quasi holes from the vacuum state (= 10)

- All positive parameters guarantee positive $E_B$.
- $E_B \sim 100$ μeV for $L \sim 2l$ (>> thermal energy ~ 1 μeV at 10 mK)
- Extension of $L$ rapidly degrades magnitude of $E_B$.
  (e.g., $E_B \sim 1$ μeV for $L \sim 4l$)

The energy density

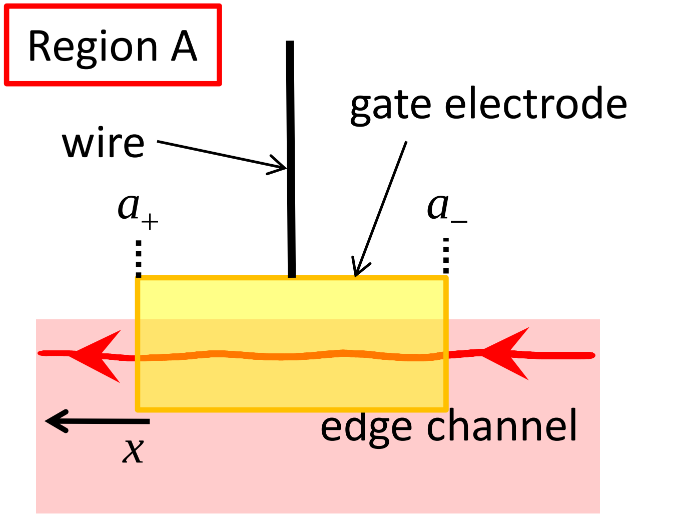
$$\varepsilon = \frac{\pi\hbar}{\nu_P e^2 v_g} j^2$$ ~ 10 μeV/μm -> current $j \sim 10$ nA

$\nu_P$: filling factor of the edge channel P

# Experimental parameters we used

- Capacitance $C \sim 10\ \mathrm{fF}$
- Input resistance $R \sim 10\ \mathrm{k\Omega}$
- Typical group velocity of a charge density wave
  $$v_g \sim 10^6\ \mathrm{m/s}$$
- Typical length scale $l \sim 10\ \mu\mathrm{m}$
  - the length of regions A, B, and G
- Temperature $\sim$mK

# Formulation of chiral edge channel *S*




The chiral field operator $\varrho_S(x)$

$$[\varrho_S(x),\ \varrho_S(x')] = i\frac{\nu_S}{2\pi}\partial_x\delta(x-x')\varrho_S(x)$$

The energy density operator of $\varrho_S(x)$

$$\varepsilon_S(x) = \frac{\pi\hbar v_g}{\nu_S} : \varrho_S\left(x\right)^2 :$$

$\nu_S$ : the Landau level filling factor of *S*

$v_g$ : the group velocity of a charge density wave

:: denotes normal ordering which makes $\langle 0_S|\varepsilon_S(x)|0_S\rangle = 0$

The free Hamiltonian of $S$:

$$H_S = \int_{-\infty}^{\infty}\varepsilon_S(x)dx \qquad H_S|0_S\rangle = 0$$

The charge fluctuation at A

$$Q_S(t) = e\int_{-\infty}^{\infty}\varrho_S\left(x+v_g t\right)w_A(x)dx \qquad (1)$$

$w_A(x)$ : window function which equals to 1 in $x \in [a_-,\ a_+]$

Quantum noise of the voltage $V(t)$

$$\hat{V} = -\sqrt{\frac{\hbar}{\pi RC^2}} \int_0^\infty d\omega \left[ \frac{\sqrt{\omega}}{\omega - \frac{1}{iRC}} a_{in}(\omega) + \frac{\sqrt{\omega}}{\omega + \frac{1}{iRC}} a_{in}^\dagger(\omega) \right] \qquad (2)$$

$a_{in}(\omega)$ $a_{in}(\omega)^\dagger$ : annihilation and creation operators of excitation of the charge density wave (magneto-plasmon)

Reference: G. Feve *et al*. Phys. Rev. B **77** 035308 (2008).

Voltage after the measurement

$$V(t=+0) = \hat{V} + \underline{R\dot{Q}_S(0)} \qquad (3)$$

voltage shift induced by the signal

The amplitude of $\hat{V}$

$$\Delta V = \sqrt{\langle 0_{RC} | \hat{V}^2 | 0_{RC} \rangle} \sim \sqrt{\frac{\hbar}{RC^2}} \sim 10\ \mu\text{V}$$

The root mean square value of the voltage shift

$$\sqrt{\langle 0_S | \left( R\dot{Q}_S(0) \right)^2 | 0_S \rangle} \quad \underline{\sim 100\ \mu\text{V}}$$ from Eq. (1)

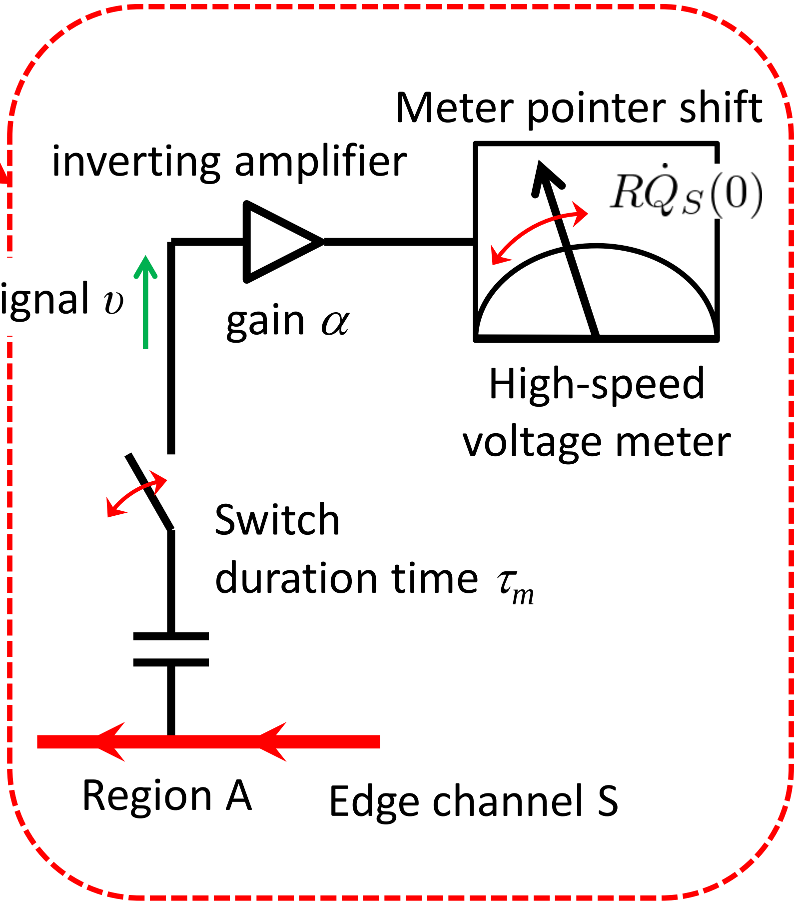
**Quantum fluctuations of the edge channel is detectable**

# Formulation of local measurement

## by pointer-basis of von Neumann

Measurement operator $M_v$ of the macroscopic system with the output value of $v$

Gedanken experiment




Measurement Hamiltonian

$$H_m(t) = \hbar\delta(t)R\dot{Q}_S(0)P_{\hat{V}}$$

$P_{\hat{V}}$ : conjugate momentum operator of $\hat{V}$

Wavefunction of the quantum pointer before the measurement (assumption):

$$\Psi_i(v) \propto \exp\left[-\frac{1}{4\Delta V^2}v^2\right]$$

Wavefunction after the measurement:

$$\Psi_f(v) \propto \exp\left[-\frac{1}{4\Delta V^2}\left(v - R\dot{Q}_S(0)\right)^2\right]$$

M. A. Nielsen and I. L. Chuang "Quantum computation and quantum information" Cambridge Univ. Press (2000)

Measurement operator

$$M_\upsilon = \left(\frac{1}{2\pi\Delta V^2}\right)^{1/4} \exp\left[-\frac{1}{4\Delta V^2}\left(\upsilon - R\dot{Q}_S(0)\right)^2\right]$$

Corresponding POVM

$$\Pi_\upsilon = M_\upsilon^\dagger M_\upsilon$$

The emergence probability density of the result being $\upsilon$:

$$p(\upsilon) = \langle 0_S|\Pi_\upsilon|0_S\rangle$$

The post-measurement state of $\varrho_S(x)$ corresponding to the result being $\upsilon$:

$$M_\upsilon|0_S\rangle$$

The average state of $\varrho_S(x)$ right after the measurement

$$\rho_1 = \int_{-\infty}^{\infty} M_\upsilon|0_S\rangle\langle 0_S|M_\upsilon^\dagger d\upsilon$$

The energy infused by the measurement:

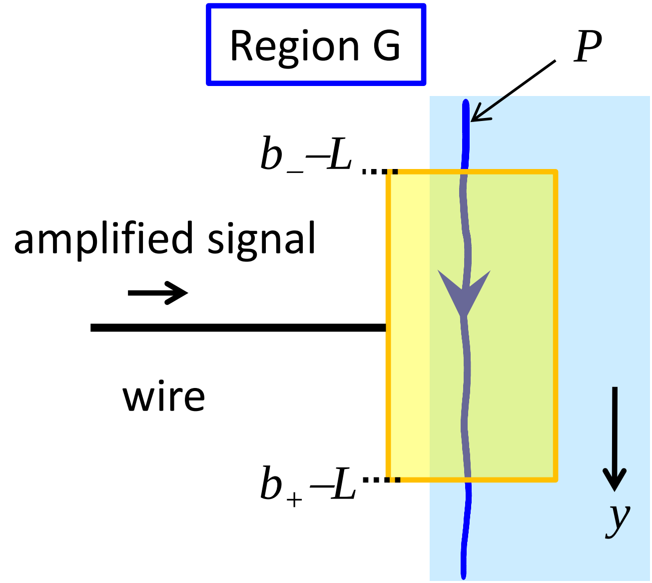
$$E_A = \int_{-\infty}^{\infty} \langle 0_S|M_\upsilon^\dagger H_S M_\upsilon|0_S\rangle d\upsilon$$

$$= \frac{\hbar v_g \nu_S}{4\pi}\left(\frac{e v_g R}{2\Delta V}\right)^2 \int_{-\infty}^{\infty} dx \left(\partial_x^2 w_A(x)\right)^2$$ ~1 meV for $\nu_s$~3

*Since the voltage meter is classical & macroscopic, the estimation of $M_\upsilon$ and $E_A$ remains unchanged even if the amplified signal is directly sent to region G without voltage meter.*

# Formulation of local operation

$\upsilon$-dependent unitary operation $U_\upsilon$ $\quad |\upsilon_P\rangle = U_\upsilon |0_P\rangle$




The interaction Hamiltonian of $\upsilon$-dependent unitary operation $U_\upsilon$

$$H_\upsilon = \int_{b_- - L}^{b_+ - L} F_\upsilon(y,t) \varrho_P(y)\, dy \qquad (4)$$

The electric potential at region G (assumption)

$$F_\upsilon(y,t) = -\frac{\pi\hbar}{\nu_P \Delta V} \lambda_B(y) \delta_{\tau_m}(t - t_o) \upsilon$$

$\delta_{\tau_m}(t - t_o)$ : A real localized function $\lim_{\tau_m \to 0} \delta_{\tau_m}(t - t_o) = \delta(t - t_o)$

$\lambda_B(y)$ : A window function related to the total number of excited electrons and quasi-holes from the vacuum state.

The gain of the amplifier:

$$\alpha = \frac{\pi\hbar}{\nu_P \Delta V \tau_m} \max_y \lambda_B(y)$$

The order of $F_\upsilon$

$$O(F_\upsilon) = \alpha O(\upsilon) = \alpha \Delta V$$

$\lambda_B(y)$ is related to the shape of the electrode at G.
We take the amplitude of $\lambda_B(y)$ to be the order of 10.

The unitary operation at G (displacement operator)

$$U_v = \exp\left(\frac{\pi i v}{\nu_P \Delta V}\int_{b_- - L}^{b_+ - L} \lambda_B(y)\varrho_P(y)\,dy\right)$$

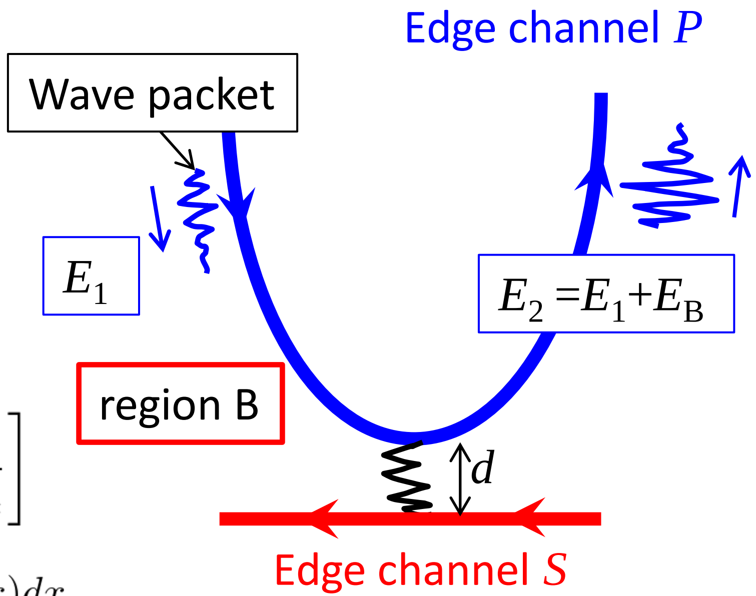
from Eq. (4)

The composite state of $S$ and $P$ at a time $T$ when a wave packet has been generated

$$\rho_{SP} = \int_{-\infty}^{\infty} dv e^{-\frac{iT}{\hbar}H_S} M_v |0_S\rangle\langle 0_S| M_v^\dagger e^{\frac{iT}{\hbar}H_S} \otimes |v_P\rangle\langle v_P|$$

The free Hamiltonian of the charge wave packet

$$H_B = \frac{\pi\hbar v}{\nu_P}\int_{-\infty}^{\infty} :\varrho_P(y)^2:\,dy$$




The energy average of the wave packet

$$E_1 = \mathrm{Tr}\left[H_B \rho_{SP}\right]$$
$$= \frac{\pi\hbar v_g}{\nu_P}\int_{-\infty}^{\infty}(\partial_y \lambda_B(y))^2\,dy\left[\langle 0_S|G_S^2|0_S\rangle + \frac{1}{4}\right]$$

$$G_S = -\frac{e v_g R}{2\Delta V}\int_{-\infty}^{\infty}\varrho_S(x)\,\partial_x w_A(x)dx$$

$E_1$ ~10 meV for $\nu_S \sim 3$ and $\nu_P \sim 6$

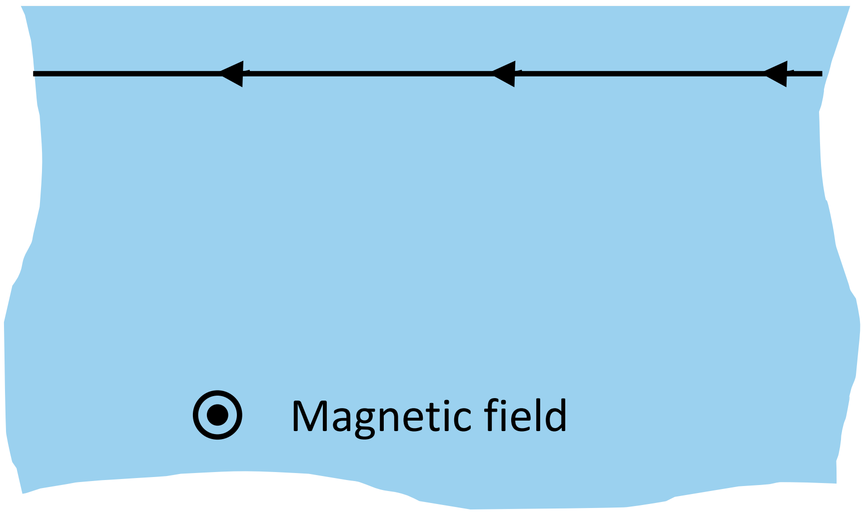
The Coulomb interaction Hamiltonian

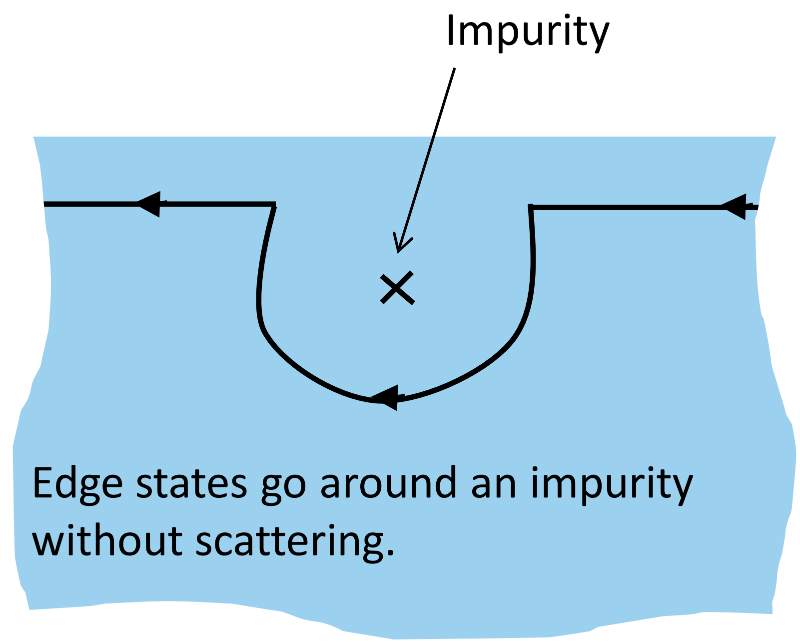
$$H_{int} = \frac{e^2}{4\pi\epsilon}\int_{b_-}^{b_+} dx \int_{b_-}^{b_+} dy \varrho_S(x) f(x,y) \varrho_P(y)$$

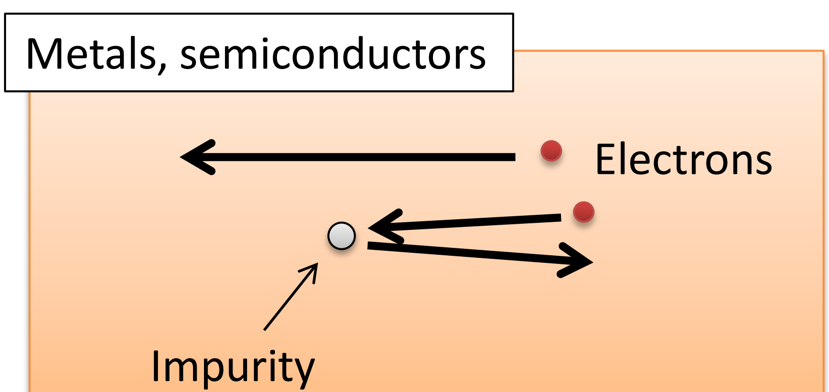
$\epsilon$: Dielectric constant of GaAs

$$f(x,y) = \frac{1}{\sqrt{(x-y)^2 + d^2}}$$

# Edge state



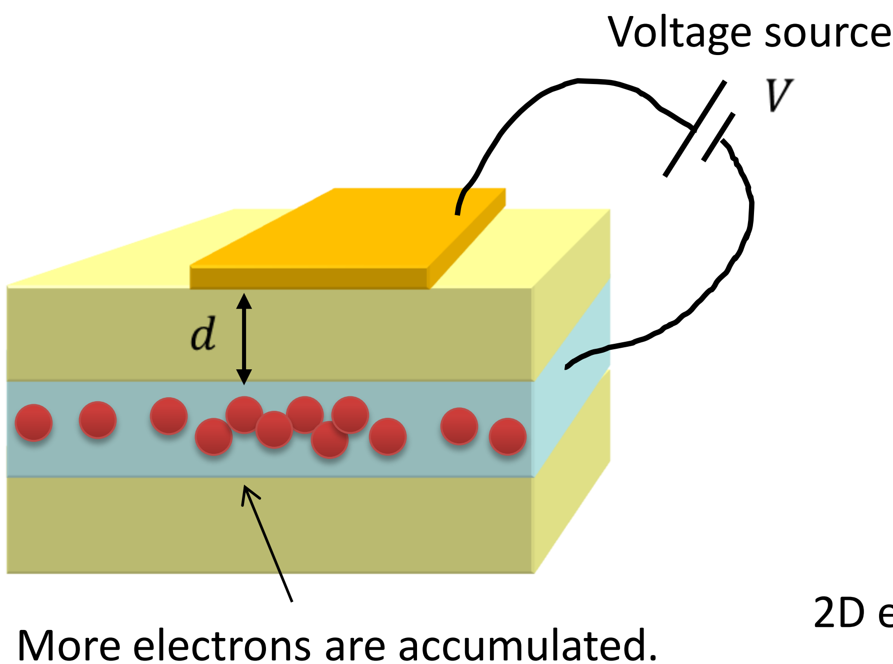



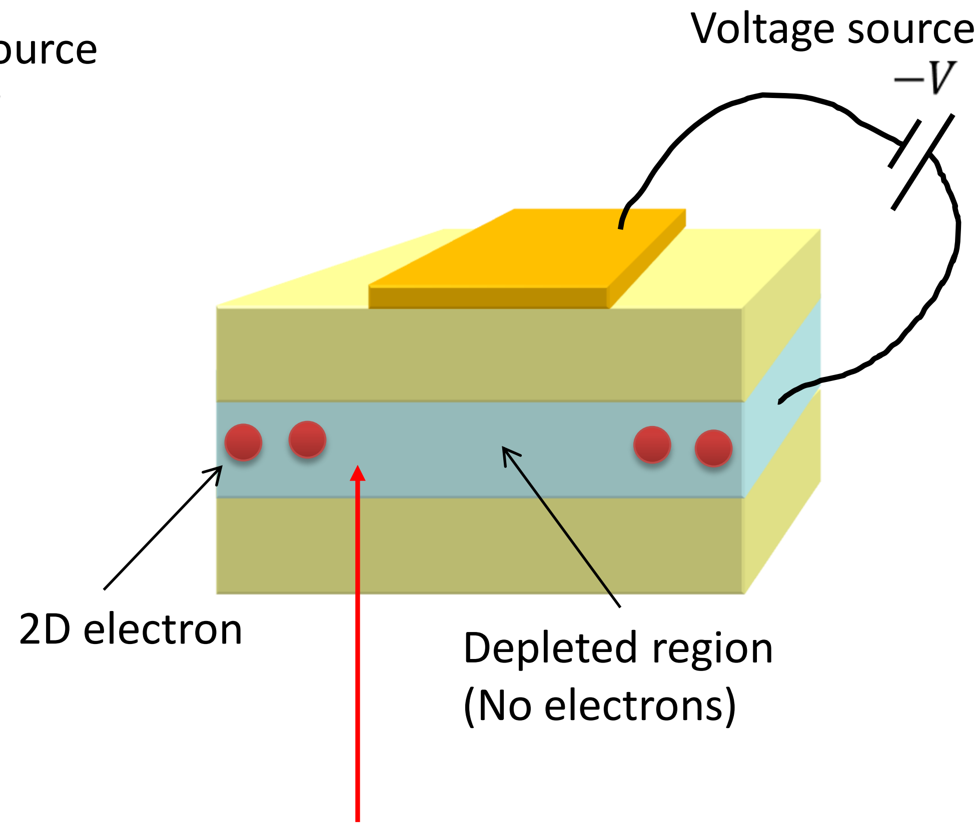




No backscattering
=> Zero-resistance

# Electrically controlled Edge channel

Electric field produced by the gate metal electrode can control electron density and the position of the edge channel.




Electron density

$$n_e = \frac{\varepsilon_r \varepsilon_0}{d} V$$

The edge state are formed at this region and the position of the edge can be electrically controlled.