Spin incoherent behavior in strongly correlated systems

Adrian Feiguin
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Outline:

- Luttinger liquids
- Spin-incoherent behavior
- The factorized wave function
- Thermofield/ancilla representation
- Finite Temperature state
- SILL behavior in the ground state of strongly correlated systems

Collaborator:
Greg Fiete (U.T. Austin)

References:
AEF and G. Fiete: arXiv:1005.4707
The excitations don’t carry the same quantum numbers as the original electron $\rightarrow$ zero quasi-particle weight
ARPES at $T=0$

1D $t$-$J$ model ($J=0.5$)
Spin incoherent behavior

See G. Fiete, RMP (07)
Model Hamiltonian

Hubbard model:

\[ H = -t \sum_{i=1,\sigma}^{L-1} \left( c_{i\sigma}^{\dagger} c_{i+1\sigma} + h.c. \right) + U \sum_{i=1}^{L} n_{i\uparrow} n_{i\downarrow} \]

- Kinetic energy
- On-site interaction
- Large U

\[ H = -t \sum_{i=1,\sigma}^{L} \left( c_{i\sigma}^{\dagger} c_{i+1\sigma} + h.c. \right) + J \sum_{i=1}^{L} \vec{S}_i \cdot \vec{S}_{i+1} \]

- t-J model (no double occupancy)
- Heisenberg
Evolution in imaginary time: single spin

We introduce and auxiliary spin (ancilla)

$$|I_0\rangle = \frac{1}{\sqrt{2}} ( |\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle)$$

We trace over ancilla:

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The density matrix corresponds to the physical spin at infinite temperature!

Evolution in imaginary time

The thermal state is equivalent to evolving the maximally mixed state in imaginary time:

\[ |\psi(\beta)\rangle = e^{-\beta H/2} |I\rangle \]

- The ancillas and the real sites do not interact!
- The global state is modified by the action of the Hamiltonian on the real sites, that are entangled with the ancillas.
- The mixed state can be written as a pure state in an enlarged Hilbert space (ladder-like).
Evolution in imaginary time:
Thermal averages

A thermal average:

\[ \langle A \rangle = Z^{-1}(\beta) \text{Tr}\{A e^{-\beta H}\}, \quad Z(\beta) = \text{Tr}\{e^{-\beta H}\}. \]

Can be obtained using a wave function instead of density matrices:

\[ \langle A \rangle = \frac{\langle \psi(\beta)|A|\psi(\beta) \rangle}{\langle \psi(\beta)|\psi(\beta) \rangle} = Z^{-1}(\beta) \sum_n \langle n|A|n \rangle e^{-\beta E_n} \]

with \( Z(\beta) = \langle \psi(\beta)|\psi(\beta) \rangle \)

AEF and S. R. White, PRB, Rapid (05)
Green’s functions

The finite temperature Green’s function can be obtained as:

$$G(x - x_0, t, \beta) = \langle \psi(\beta) | e^{iH_{t-J}t} \hat{O}^\dagger(x) e^{-iH_{t-J}t} \hat{O}(x_0) | \psi(\beta) \rangle$$

Since the thermal state is not an eigenstate, we need to evolve in time both:

$$e^{-iH_{t-J}t} | \psi(\beta) \rangle$$

$$e^{-iH_{t-J}t} \hat{O}(x_0) | \psi(\beta) \rangle$$
ARPES at $T=0$

1D $t$-$J$ model ($J=0.05$)
Results: Thermodynamics

\[ J = 0.05; \ L = 32, \ N = 24; \ n = 0.75 \]
Correlation functions

$J=0.05; L=32, N=24; n=0.75$

Spin structure factor

Momentum distribution

$\beta=0-80$

$S(k)$

$k/\pi$

$n(\mathbf{k})$

$k/\pi$
Fermi momentum

\[ J = 0.05; \ L = 32, \ N = 24; \ n = 0.75 \]
ARPES at finite $T$

$L=32, \ N=24, \ J=0.05$
SILL regime

DMRG, $\beta=10$

AEF and G. Fiete

H. Steinberg et al., PRB (06)
SI behavior in the ground state of strongly interacting models
The factorized (Ogata and Shiba) wave function in the limit $U \to \infty, J \to 0$

$$|\text{g.s.}\rangle = |\phi\rangle \otimes |\chi\rangle$$

$\epsilon(k) = -2t \cos(k)$

All configurations are degenerate

$H = H_c + H_s$
The factorized wave function (infinite spin Temperature)

\[ |O(\beta)\rangle = |\phi\rangle \otimes |\chi\rangle \]

charge
\[ \epsilon(k) = -2t \cos(k) \]

Spin-ancilla singlets
\[ S_1S_2S_3\ldots S_N \]
(I) Kondo lattice

McCulloch et al, PRB '02, K. Hallberg et al, PRL '04
Tsunetsugu, Sigrist, Ueda, RMP '97
(I) Kondo lattice

McCulloch et al, PRB ’02, K. Hallberg et al, PRL ’04
Tsunetsugu, Sigrist, Ueda, RMP ’97
The factorized wave function in the limit $J \rightarrow 0, J_K=0$

$$|\text{g.s.}\rangle = |\varphi\rangle \otimes |\chi\rangle \otimes |\sigma\rangle$$

- All spin configurations are degenerate.
- When we turn on the interactions with the impurities $J_K$:
  1. The system becomes ferromagnetic,
  2. The conduction spins and the impurities get entangled
  3. A gap opens to break a pair
The factorized wave function in the limit $J \to 0, J_K \to \infty$

$$|\text{g.s.}\rangle = |\varphi\rangle \otimes |\chi\rangle \otimes |\sigma\rangle$$

$$|\varphi^*\rangle \otimes |S\rangle \otimes |\uparrow \ldots \uparrow\rangle$$

“heavy” charge

$\epsilon(k) = -t \cos(k)$

Ogata and Shiba wave function at infinite spin $T$!!!
Ground state energy and effective “specific heat” (J=0.05)
Correlation functions
Kondo lattice at $T=0$

$L=32, N=24, J=0.05$
(II) t-J ladders

\[ J=0, J' = 0 \]
\[ |g.s.\rangle = |\varphi_1\rangle \otimes |\varphi_2\rangle \otimes |\chi_1\rangle \otimes |\chi_2\rangle \]

\[ J=0, J' \to \infty \]
\[ |g.s.\rangle = |\varphi^*\rangle \otimes |S\rangle \]

AEF and G. Fiete, in preparation
Correlation functions (J=0.05)
t-J ladder lattice at $T=0$

$L=32, N=24, J=0.05$
Conclusions

• We showed an application of the time dependent DMRG combining evolution in real-time and imaginary time.
• We studied the crossover from spin incoherent to spin coherent behavior
• We generalized the Ogata and Shiba’s factorized wave function to finite spin temperatures
• We found that the t-J ladder in some regime of parameters and the Kondo lattice exhibit SI behavior in the ground-state.

• This SI behavior is not SILL, but results indicate that it might be possible to describe it within the same framework, and may present some universal features.