Entanglement renormalization on the triangular lattice

October 28, 2010

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Introduction
S=1/2 anti-ferromagnetic Heisenberg model on a triangular lattice with spatial anisotropy

- **Spin hamiltonian**

\[ H = J \sum_{\langle ij \rangle} S_i \cdot S_j + J' \sum_{\langle\langle ij \rangle\rangle} S_i \cdot S_j \]

- **Related materials**
  - Cs\(_2\)CuCl\(_4\)
  - k-(ET)\(_2\)Cu\(_2\)(CN)\(_3\)
  - EtMe\(_3\)Sb[Pd (dmit)\(_2\)]\(_2\)
  - k-(ET)\(_2\)Cu\(_2\)[N(CN)\(_2\)]Cl
Numerical methods for the anisotropic triangular lattice model

- Exact diagonalization (up to 36 sites)
- Variational methods
    - BCS (or extended) + Gutzwiller projector + Long-range spin Jastrow factor
    - Up to 24x24 sites
    - up to 8x18 sites
Ground state phase diagram

High temperature series expansion


Spiral order remain in $J \leq J'$ regime?

1D 2D SF AFMLO

S. Yunoki, et al., PRB 74 (2006) 014408

D. Heidarin, et al., PRB 80 (2009) 012404

Spiral order remain in $J \leq J'$ regime?
Entanglement renormalization on a triangular lattice
● Probability amplitude is defined by the contraction of small tensors

\[ |\Psi\rangle = \sum_{\{i_1, \ldots, i_n\}} T_{i_1 \ldots i_n} |i_1, \ldots, i_n\rangle \]
Entanglement renormalization (Vidal, 2005)


- Unitary tensor for removing entanglements (Disentangler)
  - For 2 sites, \( |\phi_{kl}\rangle \equiv U_{kl,ij} |\psi_{ij}\rangle \)

- Multi-scale entanglement renormalization ansatz (MERA)

\[
\begin{array}{c}
\text{Area law of entanglement entropy is satisfied}
\end{array}
\]
ER on a triangular lattice (1)

- **Tensors**
  - Disentangler
  - Disentangler + isometry
  - Isometry

- **Entanglement renormalization of 19 sites**
Entanglement renormalization of 19 sites

1) Triangular disentangler
2) Parallelogram disentangler
   (6 sites to 2 sites)
3) Hexagonal isometry

Features

- Three-body interaction
  Hamiltonian converts to three-body one
- Width of causal cone is 6

Memory size: $O(\chi^{12})$
Computational cost: $O(\chi^{14})$

$\chi$: dimension of a tensor’s leg
Numerical results
1-step ER calculations

- ER: 19 sites to a top site
- Top sites: 2 x 3 parallelogram
- The number of sites: \((2 \times 3) \times 19 = 114\)
- Periodic boundary condition
- All tensors are different: 36 tensors
  - The number of causal cones: 228

◊ Calculations on the super computer system B at ISSP, Univ. of Tokyo
Isotropic case : $J = J'$
Spin-Spin correlations on bonds

\[ \langle S_i \cdot S_j \rangle \]

2:2:2:2 → 2:2:4:4

2:2:8:4 → 2:2:8:8

2010年10月28日木曜日
Spin-Spin auto-correlation

\[ G(0, i) \equiv \frac{(\langle S_0 \cdot S_i \rangle - \langle S_0 \rangle \cdot \langle S_i \rangle)}{(3/4 - |S_0|^2)} \]

The finite-correlation MERA for an infinite lattice can be used

Radius = 0.2 + |G(0,i)|
Magnetization of 3 sublattice

● Definition

\[ M_A = \left\langle \sum_{i \in A} S_i \right\rangle, \quad C_{AB} \equiv M_A \cdot M_B \]
\[ M_A \equiv \sqrt{C_{AA}} = |M_A|, \quad \chi_{AB} = \frac{C_{AB}}{\sqrt{C_{AA}C_{BB}}} = \cos(\theta_{AB}) \]

● For 2:2:8:8

\[ M_3 = 0.316, \quad \cos(\theta_3) = -0.497 \]

3-sublattice 120° magnetization

Angle between the origin and a site

\[ \theta_{0i} \]

180°

0
Isotropic case: $J' = J$

For $2:2:8:8$, $E/N = -0.5418$, $M = 0.314$
Anisotropic case : $J \leq J'$
Magnetization and angles

\[ \chi = 2 : 2 : 8 : 6 \]
Effect of periodic boundary conditions

Angle between the origin and a site

\[ J/J' = 0.8, \quad \chi = 2:2:8:8 \]
2-step ER calculations

- ER : 19 sites to a top site
- Top sites : 2 x 3 parallelogram
- The number of sites : \((2 \times 3) \times 19 \times 19 = 2166\)
- Periodic boundary condition
- All tensors are different : 720 tensors
  - The number of causal cones : 4332
    - Calculations on the super computer system B at ISSP, Univ. of Tokyo
Magnetization and angles

\[ \theta \]

\[ \theta_1 \]

\[ \theta_2 \]

\[ \theta_3 \]

\[ J / J' \]

2-steps : \( \chi = 2 : 2 : 4 : 4 : 4 : 4 \)
Summary

● MERA scheme for the triangular model
  ▪ Entanglement renormalization for 19 sites

● S=1/2 antiferromagnetic Heisenberg models on a triangular lattice
  ▪ Isotropic case: 120° magnetization order
  ▪ Anisotropic case
    ◇ $1 \geq J/J' \geq 0.7$: spiral long-range order from 2-steps ER calculations, no spin liquid

(Note: the dimension is small now)