Ultracold Fermi atom condensates: effects of disorder and imbalance in 1D

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[PRL 100, 110403; NJP 12, 055029]
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[PRA 82, 043613]
Contents

• “Knobs of control” in ultracold Fermi gases
  – Interaction, trap, disorder, population imbalance
• DMRG Studies of 1D two-species Fermi gases

(1) Trapped population-imbalanced gas:
Larkin-Ovchinnikov type condensate
[Tezuka and Ueda: PRL 100, 110403; NJP 12, 055029]

(2) Equal-population gas in quasiperiodic potential:
disorder enhances pairing;
“metallic” regime without quasi-long-range pair correlation
[Tezuka and García-García: PRA 82, 043613]
Superconductivity

Superfluidity

Introduction

Disorder: enhance? suppress?

Interaction: pairing force? other phase?

Population imbalance = “magnetic field” : FFLO?

Now widely controllable in cold Fermi atomic gases

- Optical lattice ← laser standing wave
  - Bichromatic lattice [Roati et al.: Nature 453, 895 (2008)]

- Holographic potential imprinting in 2D
Trapped 1D Fermi gases with popl. imbalance realized

Speckle potential for bosons

Holographic potential imprinting in 2D

Bichromatic lattice
Our motivation: what happens in 1D?

- Quantum fluctuation suppresses true long-range order (LRO) even for $T=0$
- Condensate can exist (algebraic decay of pair correlation = quasi LRO)
- Spatial distribution of order parameter? FFLO?
- Is non-localized system always superfluid at low $T$?
- Study with numerically exact method (DMRG)
(1) population-imbalanced case

Trapped 1D Fermi gases with popl. imbalance: Is condensate with pair momentum realized?

Experimental realization

Model: 1D Hubbard model (at low filling; \( n = N/L \sim 0.2 \)) with trapping potential

\[
H = -t \sum_{\langle i,j \rangle, \sigma = \uparrow, \downarrow} (\hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + \hat{c}_{j\sigma}^\dagger \hat{c}_{i\sigma}) + U \sum_i \hat{n}_{i\uparrow}\hat{n}_{i\downarrow} + \sum_{i,\sigma} \phi_i \hat{n}_{i\sigma}
\]

Calculated ground state with DMRG

M. Tezuka and M. Ueda: PRL 100, 110403 (2008); NJP 12, 055029 (2010)

\[\Delta \sim \exp(iqx)\]

Fulde-Ferrell state

\[\Delta \sim \cos(qx)\]

Larkin-Ovchinnikov state

Density difference oscillates with wave number 2q

Normalized real-space coordinate
Discretization of the space

\[ U(r) = g \delta(r) \]

Kinetic energy
\[ K(k) = \hbar^2 k^2 / 2m \]

Confining potential
\[ \phi(r) = Ar^2 \]

Site potential
\[ \phi_i = A a^2 \left( i - \frac{L-1}{2} \right)^2 \]

L-site Hubbard model
\[ H = -t \sum_{\langle i,j \rangle, \sigma = \uparrow, \downarrow} \left( \hat{c}^\dagger_{i\sigma} \hat{c}_{j\sigma} + \hat{c}^\dagger_{j\sigma} \hat{c}_{i\sigma} \right) + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + \sum_{i,\sigma} \phi_i \hat{n}_{i\sigma} \]

cf. optical lattice systems

\[ 2V_0 \]

\[ \lambda = 2a \]
Pair correlation and density distribution

Pair correlation

\[
\left\langle \psi_0^{(N)} \left| \hat{c}_{i,\downarrow} \hat{c}_{i,\uparrow} \hat{c}_{j,\uparrow} \hat{c}_{j,\downarrow} \right| \psi_0^{(N)} \right\rangle \approx \Delta(z_i) \Delta(z_j)
\]

imbalance parameter

\[
P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}
\]

M. Tezuka and M. Ueda, PRL 100, 110403 (2008)
Pair correlation and density distribution

Pair correlation

\[ \langle \psi_0^{(N)} | \hat{c}_{i,\downarrow} \hat{c}_{i,\uparrow} \hat{c}_{j,\uparrow} \hat{c}_{j,\downarrow} | \psi_0^{(N)} \rangle \approx \Delta(z_i)^* \Delta(z_j) \]

imbalance parameter

\[ P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} \]

M. Tezuka and M. Ueda, PRL 100, 110403 (2008)
Conclusion: (1)
Population imbalance + harmonic trap

- Pairing? $\Rightarrow$ LO (quasi-) condensate
- Phase separation? $\Rightarrow$ Yes (LO at center)
- Upper limit in imbalance $P$ for condensation? $\Rightarrow$ not observed

see also:
Feiguin and Heidrich-Meisner: PRB 76, 220508R (2007); PRL 102, 076403 (2009) (Ladder);
Machida, Yamada, Okumura, Ohashi, and Matsumoto: PRA 77, 053614 (2008);

M. Tezuka and M. Ueda,
PRL 100, 110403 (2008); NJP 12, 055029 (2010)
(2) Effect of disorder

\[ \hat{H} = -t \sum_{i=1,\sigma}^{L-1} (\hat{c}_{i+1,\sigma}^\dagger \hat{c}_{i,\sigma} + \text{h.c.}) + U \sum_{i=0}^{L-1} \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} + \sum_{i=0}^{L-1} V(i)\hat{n}_i, \]

Optical lattice with on-site interaction

Introduce disorder by modulating the site potential \( V(n) \)

- **Speckle potential**
  - All eigenstates exponentially localized
  - L. Sanchez-Palencia *et al.*: PRL 98, 210401 (2007)

- **Fibonacci potential** ABAABABA ...
  - Critical irrespective of the strength of \( \lambda/t \)

- **Bichromatic potential**
  - non-interacting: **metal-insulator transition** at \( \lambda=2t \) for any filling
  - Bose Hubbard (DMRG): Deng *et al.*: PRA 78, 013625 (2008); Roux *et al.*: PRA 78, 023628 (2008)
  - Spinless Fermions: Chaves and Satija (ED, PRB 55, 14076 (1997)); Schuster *et al.* (PBC DMRG, PRB 65, 115114 (2002))

\[ V(n) \equiv \lambda \cos(2\pi\omega n + \theta) \]
Formulation: Hubbard model + quasiperiodic site potential

\[ \hat{H} = -t \sum_{i=1,\sigma}^{L-1} (\hat{c}_{i+1,\sigma}^{\dagger} \hat{c}_{i,\sigma} + \text{h.c.}) + U \sum_{i=0}^{L-1} \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} + t=1 \ \text{(unit of energy)} \]

\[ + \sum_{i=0}^{L-1} V(i) \hat{n}_i, \quad V(n) \equiv \lambda \cos(2\pi \omega n + \theta), \quad \omega = \frac{F_k}{F_{k+1}} \approx \left(\sqrt{5} - 1\right)/2, \quad L = F_{k+1} + 1 \]

\[ (F_0 = F_1 = 1, F_{k+2} = F_{k+1} + F_k) \]

- Ratio of consecutive Fibonacci numbers \( \rightarrow \) golden ratio (=irrational number) as \( k \rightarrow \infty \)
- \((N_\sigma, L) = (10, 90), (26, 234), (42, 378) : \) filling \( n=2/9 \)

Plot of \( V(n) \) for \( L=145 \)
Population-balanced case

What should happen?

- What comes between these limits?
  - Does disorder enhance/suppress pairing?
  - Different transition points?

⇒ Let us check numerically

Strong attraction:
(soft-core) bosons
effective hopping $\sim t^2/|U|$

Weak attraction:
many-body gap;
long coherence length

Insulating transition should suppress superfluidity

Small $\lambda$: superfluid state

$\lambda/t$

2

Disorder

$|U|/t$

Attraction
How to detect pairing and delocalization?

Pairing

On-site pair correlation function: easy to calculate with DMRG
Depends on the site potentials of the site pair

Averaged equal-time pair structure factor
Sum of pair correlation for all lengths
average over sites

cf. Hurt et al.: PRB 72, 144513 (2005);
Mondaini et al.: PRB 78, 174519 (2008)

\[
\Gamma(i, r) \equiv \left\langle \hat{c}^\dagger_{i+r, \downarrow} \hat{c}^\dagger_{i, \uparrow} \hat{c}_{i, \uparrow} \hat{c}_{i+r, \downarrow} \right\rangle
\]

\[
P_s \equiv \left\langle \sum_r \Gamma(i, r) \right\rangle_i
\]
Increasing function of \( L \)
if decay of correlation is slow

Delocalization

Phase sensitivity: requires (anti-)periodic condition [see e.g. Schuster et al.: PRB 65, 115114 (2002)]
Hard to calculate within DMRG (not open BC)

Inverse participation ratio (IPR)
Add 2 atoms \( \Rightarrow \) How uniformly is the population increase distributed?

\[
I_E \equiv \left( \sum_i \left( \langle \hat{n}_i \rangle_{N+1,N+1} - \langle \hat{n}_i \rangle_{N,N} \right)^2 \right)^{-1}
\]

Compare between different system sizes
The case without disorder ($\lambda=0$)

Pair structure factor
indicator of global (quasi long-range) superfluidity

Inverse participation ratio
indicator of atom delocalization

Both increase with $|U|$, and system size $L$
Pair correlation function

\[ U = -1 \]

Bichromatic potential depth: 0.4

\[ N = 42 \rightarrow N = 43 \]

Occupancy change

\[ L = 378 \quad m = 400 \]
$U = -1$:
Quasi long-range pairing disappears ($\lambda_p \sim 0.95$) before localization ($\lambda_c \sim 1.00$)

Inverse participation ratio

Pair structure factor

Tezuka and García-García: PRA 82, 043613 (2010)
$U = -6$:
Quasi long-range pairing disappears at around localization ($\lambda_c \sim 0.30$)

Inverse participation ratio
Spin gap

Minimum energy to break a pair by spin flipping

\[ \Delta_s \equiv E_0(N+1, N-1) - E_0(N, N) \]

Continues to increase even after \( \lambda_p, \lambda_c \)

Delocalized, superconducting pairs below sc-metal transition

Localized, tightly bound pairs at large \( \lambda \)

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Conclusion: (2)

- Effect of coexisting disorder (bichromatic potential) and short-range attractive interaction
  - DMRG study for 1D Fermi atoms on optical lattice

- For strong attraction ($|U| \gg t$), pairing decreases as disorder $\lambda$ is increased, and localizes at around the insulating transition $\lambda_c$

- For weaker attraction ($|U| \sim t$), pairing has a peak as a function of disorder $\lambda$, but localizes before $\lambda_c$

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