Algebraic aspects of chiral symmetry

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1. Broken (Hidden) chiral symmetry

Chiral symmetry is hidden

Ordinary symmetry (Algebraic)

Example (Isospin), πN -> Δ -> πN scattering

π: I = 1, N: I = 1/2 $\quad$ I_{tot} = 1/2 or 3/2

$$\left\langle \pi^+ p \middle| T \middle| \pi^+ p \right\rangle = \sum_I \left\langle \pi^+ p \middle| I \right\rangle \langle I | T | I \rangle \left\langle I \middle| \pi^+ p \right\rangle = \left\langle \pi^+ p \middle| 3/2 \right\rangle T(3/2) \left\langle 3/2 \middle| \pi^+ p \right\rangle = T(3/2)$$

$$\left\langle \pi^0 p \middle| T \middle| \pi^0 p \right\rangle = \left\langle \pi^0 p \middle| 3/2 \right\rangle T_{3/2} \left\langle 3/2 \middle| \pi^0 p \right\rangle = \left| \left(1\, 0\, \tfrac{1}{2}\, \tfrac{1}{2} \,|\, \tfrac{3}{2}\, \tfrac{1}{2} \right) \right|^2 T_{3/2}$$

$$\left\langle \pi^0 p \middle| T \middle| \pi^+ p \right\rangle = \left\langle \pi^0 p \middle| 3/2 \right\rangle T_{3/2} \left\langle 3/2 \middle| \pi^+ p \right\rangle = \left(1\, 0\, \tfrac{1}{2}\, \tfrac{1}{2} \,|\, \tfrac{3}{2}\, \tfrac{1}{2} \right) \left(1\, 1\, \tfrac{1}{2} - \tfrac{1}{2} \,|\, \tfrac{3}{2}\, \tfrac{1}{2} \right) T_{3/2}$$

Matrix elements are related by the CG coefficients

Mass degeneracy

Pion: $m(\pi^+) \sim m(\pi^-) \sim m(\pi^0)$, Nucleon: $m(p) \sim m(n)$

Symmetry transformation $g = e^{i\theta Q}$ Q : generator, θ: parameter

Operator $O \to gOg^\dagger$ State $|A\rangle \to g|A\rangle = |B\rangle$

$$H \to gHg^\dagger \sim H + i\theta[Q, H] + \ldots = H \qquad [Q, H] = 0$$

Mass relation

$$m_A = \langle A|H|A\rangle = \langle B|gHg^\dagger|B\rangle = \langle B|H|B\rangle = m_B$$

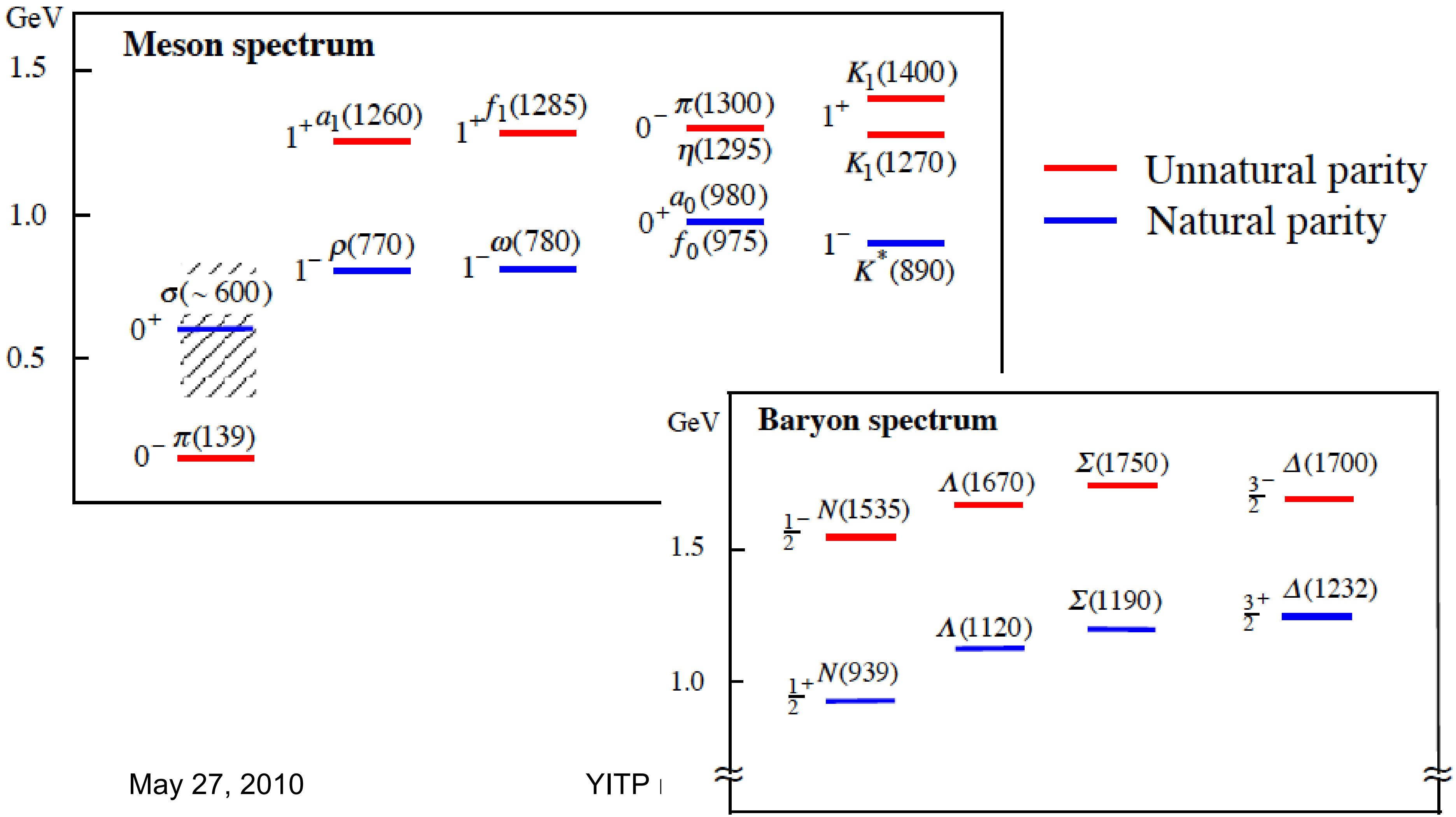
BUT

$$|B\rangle = g|A\rangle = ga_A{}^\dagger|0\rangle = ga_A{}^\dagger g^\dagger g|0\rangle = a_B{}^\dagger g|0\rangle = a_B{}^\dagger|0\rangle$$

Invariance of the vacuum $g|0\rangle = |0\rangle$ is assumed

Chiral symmetry is spontaneously broken

There is no mass degeneracy



Nambu’s contribution

- **Nambu** had communications with **Bardeen**
- He realized that the Goldber&Treiman’s **dispersion relation** is related to the **conservation of the axial current**
- He noticed the analogy of the **Dirac** theory and the **BCS** theory (Nambu-Gorkov equation)
- Discovery of the **SSB** of **chiral symmetry, NJL model**

2. Non-linear (dynamic) vs linear (algebraic)

Questions from Weinberg

• If chiral symmetry is spontaneously broken, no chiral partner is needed to write down chiral invariant Lagrangian in the ***non-linear*** realization.

• In that case, chiral symmetry is no longer ***algebraic***, but rather ***dynamic*** as it leads to low energy theorems.

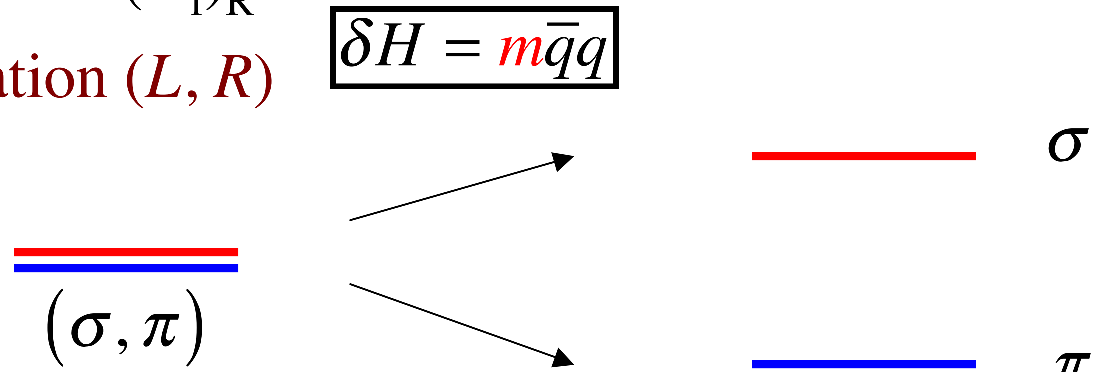
Weinberg:

"*how it is possible for a dynamical symmetry like chirality to have any algebraic consequences*?"

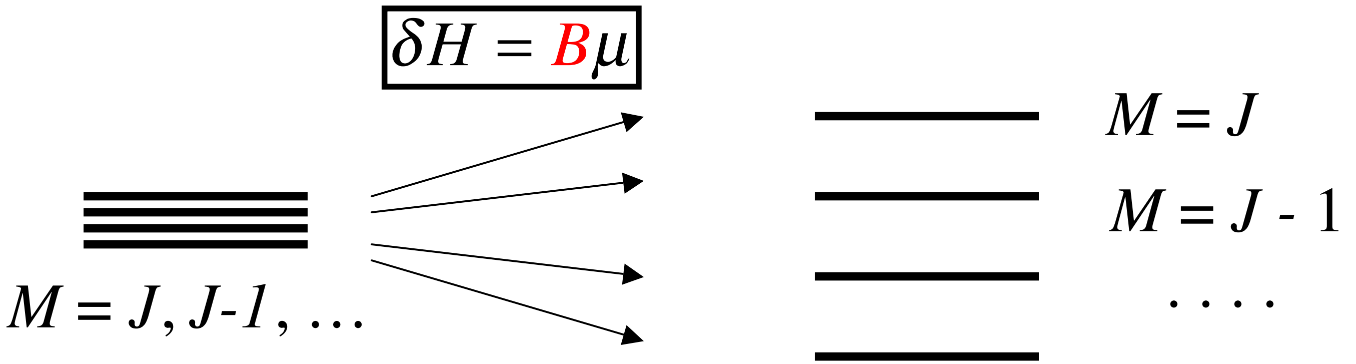
Resolving degeneracy

$SU(N_f)_L \times SU(N_f)_R$ $\qquad\qquad\qquad\qquad$ $SU(N_f)_V$

Representation (L, R)



Analogy to rotation



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Representation (L, R)

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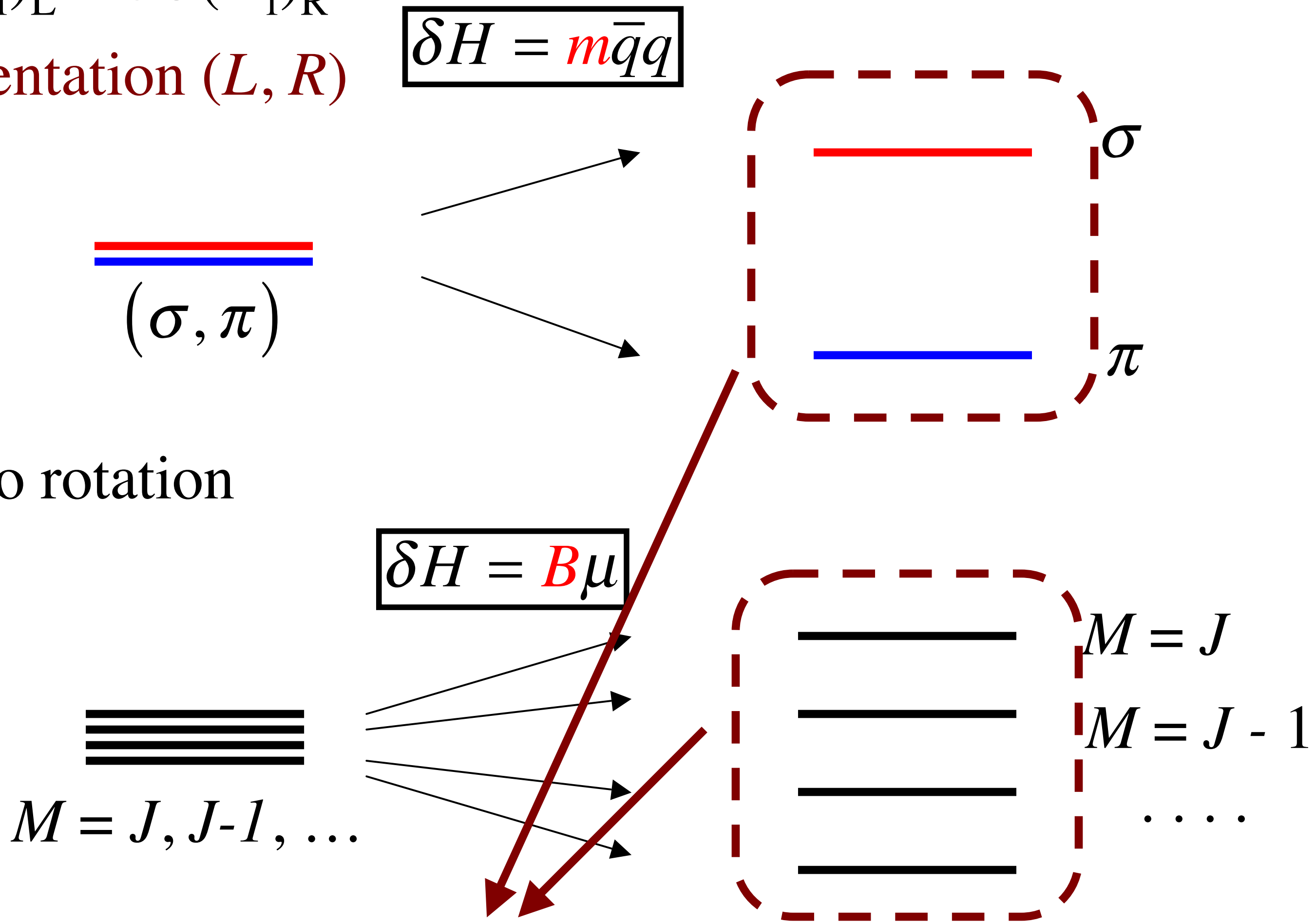


Broken world may have some trace of symmetry

QCD ~ linear rep.

$$L_{QCD} = \sum_f \bar{q}_f (i\not{D} - m_f) q_f + \frac{\mathrm{tr}}{4} F_{\mu\nu}^2$$

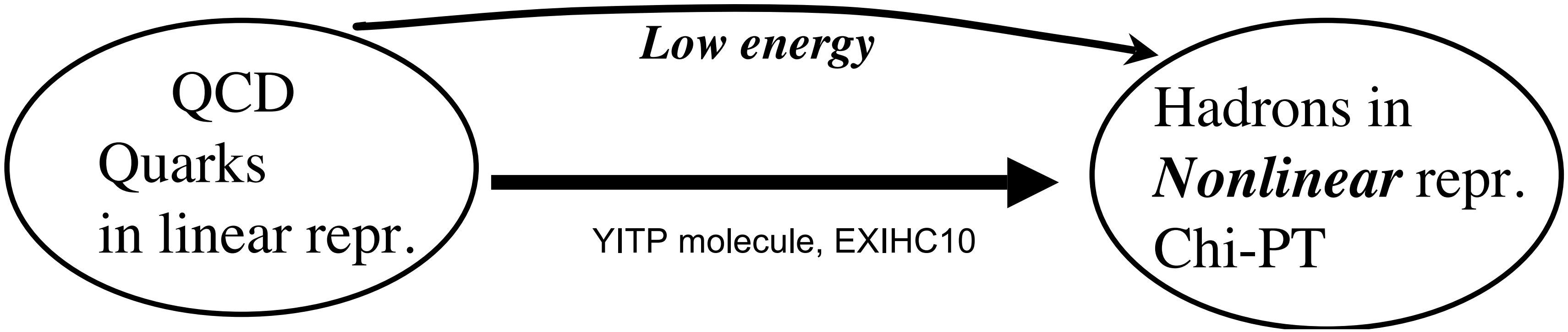
For massless quarks, $f = u, d$

$$L_{QCD} = \bar{q} i\not{D} q + \ldots = \bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R + \ldots$$

$$q_{L,R} = \frac{1 \mp \gamma_5}{2} q, \quad q = q_L + q_R$$

This is invariant under $SU(2)_L$ x $SU(2)_R$, and furthermore, quarks are in the ***linear fundamental representation***.

$$q \sim \left(\tfrac{1}{2}, 0\right) + \left(0, \tfrac{1}{2}\right)$$



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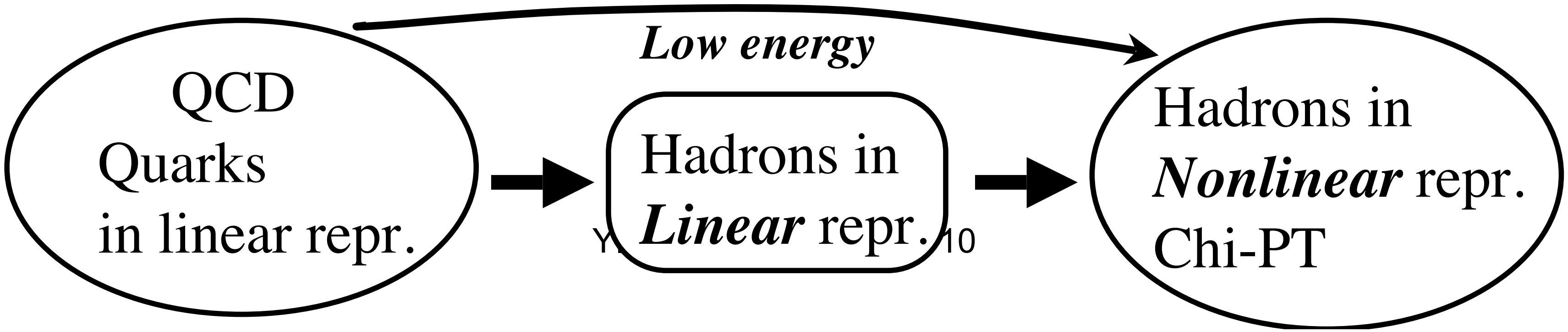
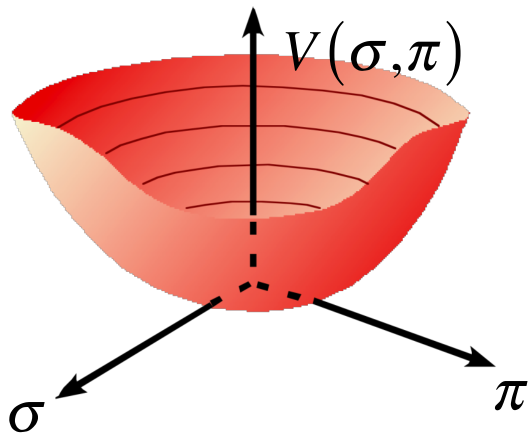


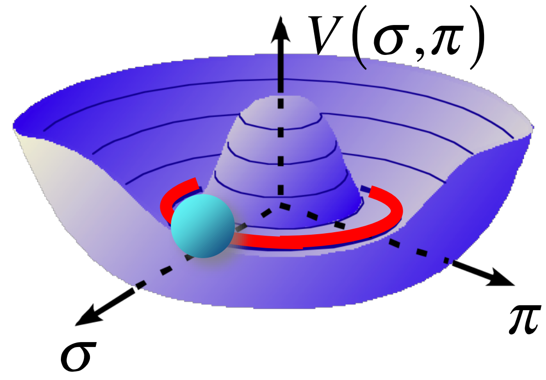
From linear to non-linear

Linear sigma model

$$L = \bar{q} i \partial_\mu \gamma^\mu q - g \bar{q} \left(\sigma + i \vec{\tau} \cdot \vec{\pi} \gamma_5 \right) q + \frac{1}{2} \left((\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2 \right) - V(\sigma, \vec{\pi})$$



$\langle \sigma \rangle = f_\pi$



Freeze out the massive σ mode,
Consider only the massless π

Angle variables

$$\frac{\sigma + i \vec{\tau} \cdot \vec{\pi} \gamma_5}{f_\pi} = U_5 = e^{i \vec{\tau} \cdot \vec{\phi} \gamma_5 / f_\pi}, \quad \frac{\sigma + i \vec{\tau} \cdot \vec{\pi}}{f_\pi} = U$$

Assumptions are

$q \sim \left(\frac{1}{2},0\right)+\left(0,\frac{1}{2}\right)$ Fundamental representation

$\left(\sigma,\vec{\pi}\right) \sim \left(\frac{1}{2},\frac{1}{2}\right)$ Chiral four-vector representation

As a consequence $m_q = g f_\pi$ (Golberger-Treiman relation)

with $g_A = 1$

The value of g_A is determined by the representation.
In a non-linear theory, g_A can take an ***arbitrary value.***
In other words, in a broken phase, representation mixing occurs.

Isospin analogue: Isospin charge of $N = 1/2$
$\Delta = 3/2$

Linear representations

SU(2)xSU(2) -> SU(2)

Linear chiral Non-linear chiral

↓ Larger symmetry

More constraints on dynamics
Various couplings, scattering amplitudes, etc

Question:
What chiral multiplets do hadrons belong to?

Further needs of linear reps.

At high temperature and/or density,
chiral symmetry is restored

=>

• Hadrons (if survive) ***must*** belong to linear chiral representations, and a hadron theory may be constructed by a linear chiral model

• Character change in a massive (Higgs) mode (σ) may be observed in experiments

3. Representations and constraints

Chiral representations

$$\mathrm{SU(2)_L \times SU(2)_R} \;\rightarrow\; (\mathrm{D_L}, \mathrm{D_R})$$

$$\mathrm{D_L + D_R} \sim \text{Isospin}$$

Quark: $\psi = \psi_L + \psi_R \sim \left(\tfrac{1}{2}, 0\right) + \left(0, \tfrac{1}{2}\right)$ Fundamental

(Nucleon in the linear sigma model)

Scalar mesons $(\sigma, \vec{\pi}), (\eta, \vec{a}_0) \sim \left(\tfrac{1}{2}, \tfrac{1}{2}\right)$ Chiral four-vector

$$\tau_\mu = (1, \tau_1, \tau_2, \tau_3) \rightarrow \left(\bar{\psi}\tau_\mu\psi, \bar{\psi}\tau_\mu\gamma_5\psi\right)$$

$$\rightarrow \left(\bar{\psi}\tau_0\psi, \bar{\psi}\vec{\tau}\gamma_5\psi\right) + \left(\bar{\psi}\tau_0\gamma_5\psi, \bar{\psi}\vec{\tau}\psi\right) \sim (\sigma, \vec{\pi}) + (\eta, \vec{a}_0)$$

In general various higher representations are possible

Chiral structure of quark composites

Mesons $\bar{q}q \sim \left[\left(\tfrac{1}{2},0\right)+\left(0,\tfrac{1}{2}\right)\right]\times\left[\left(\tfrac{1}{2},0\right)+\left(0,\tfrac{1}{2}\right)\right]$

$$= \underbrace{(0,0)}_{\text{Scalar}} + \underbrace{(1,0)+(0,1)}_{\text{Anti-symm tensor}} + \underbrace{\left(\tfrac{1}{2},\tfrac{1}{2}\right)}_{\text{Four-vector}}$$

Baryons $qqq \sim \left[\left(\tfrac{1}{2},0\right)+\left(0,\tfrac{1}{2}\right)\right]^3$

$$= \underbrace{\left(\tfrac{1}{2},0\right)+\left(0,\tfrac{1}{2}\right)}_{\text{Fundamental}} + \underbrace{\left(1,\tfrac{1}{2}\right)+\left(\tfrac{1}{2},1\right) + \left(\tfrac{3}{2},0\right)+\left(0,\tfrac{3}{2}\right)}_{\text{Higher repr.}}$$

$qqqq\bar{q} \sim \left[\left(\tfrac{1}{2},0\right)+\left(0,\tfrac{1}{2}\right)\right]^5$

$$= \underbrace{\left(0,\tfrac{1}{2}\right)+\left(\tfrac{1}{2},0\right)}_{\text{Mirror}} + \left(1,\tfrac{3}{2}\right)+\left(\tfrac{3}{2},1\right)+\ldots$$

For multiquarks, more complicated structures are possible

A technical question

Operators vs States

$$N_1 = (\tilde{q}q)q, \quad N_2 = (\tilde{q}\gamma_5 q)\gamma_5 q \qquad \tilde{q} = q^T C\gamma_5 (i\tau_2)$$

These are both $I = 1/2$, and (1/2,0)+(0,1/2)

However, isospin is conserved but chiral is not in general

$$\langle 0|\bar{N}N|0\rangle \sim \sum_B \langle 0|\bar{N}|B\rangle\langle B|N|0\rangle$$

B has the definite $I = 1/2$, but with any (L, R)

g_A values

Fundamental	$\left(\frac{1}{2},0\right)+\left(0,\frac{1}{2}\right)$	$g_A = 1$
Mirror	$\left(0,\frac{1}{2}\right)+\left(\frac{1}{2},0\right)$	$g_A = -1$
Higher repr.	$\left(1,\frac{1}{2}\right)+\left(\frac{1}{2},1\right)$	$g_A = \frac{5}{3}$
Arbitrary N_c	$\left(\frac{N_c+1}{4},\frac{N_c-1}{4}\right)+\left(\frac{N_c-1}{4},\frac{N_c+1}{4}\right)$	$g_A = \frac{N_c+2}{3}$

Determination g_A value of a baryon is useful to know its structure

4. Weinberg's consistency conditions

Algebraic constraints

Weinberg's bottom-up procedure (pre QCD)

Any Lagrangian built from such ingredients will be highly nonlinear and replete with derivative interactions.

"The algebraic aspects of chiral symmetry arise from the need for cancellations which insure reasonable asymptotic behavior at high energy."

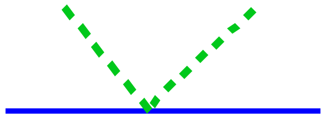
… to use them in the tree approximation, …

We shall require *that the rapidly growing terms contributed by the tree graphs shall cancel among themselves*

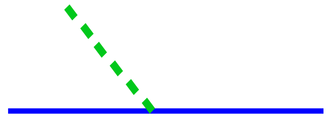
Lagrangian

Non-linear chiral Lagrangian

$$L = \frac{2i}{f_\pi^2} V_\mu^a \varepsilon_{abc} \pi^b \partial_\mu \pi^c \quad + \quad \frac{1}{f_\pi} \tilde{A}_\mu^a \partial_\mu \pi^a \quad \sim \quad O(q^1)$$



Tomozawa-Weinberg



Yukawa

$$\left\langle N^\beta s' \right| \int d^3x\, V_0^a \left| N^\alpha s \right\rangle = \left(T^a \right)_{\beta\alpha}$$

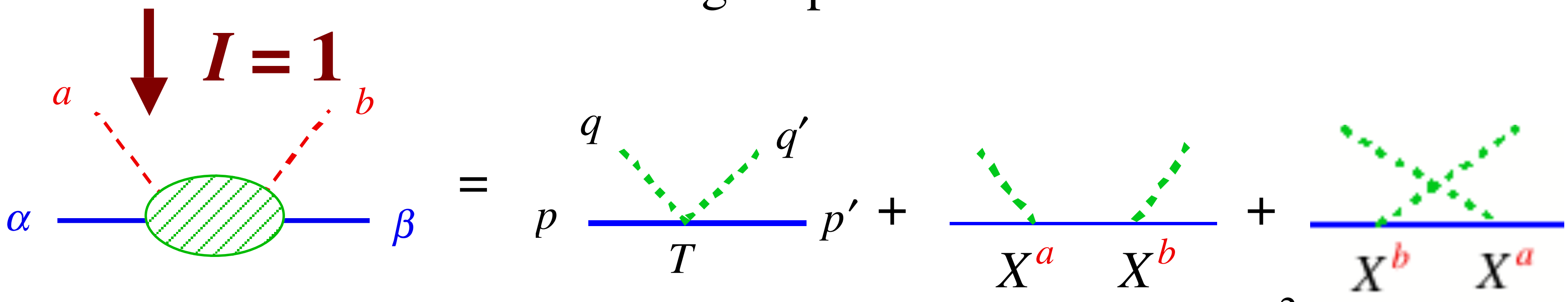
Isospin charge

$$\left\langle N^\beta s' \right| \int d^3x\, A_{\mu=3}^a \left| N^\alpha s \right\rangle = s\delta_{ss'} \left(X^a \right)_{\beta\alpha}$$

Axial charge

Consistency conditions 1

Scattering amplitude of πN -> πN



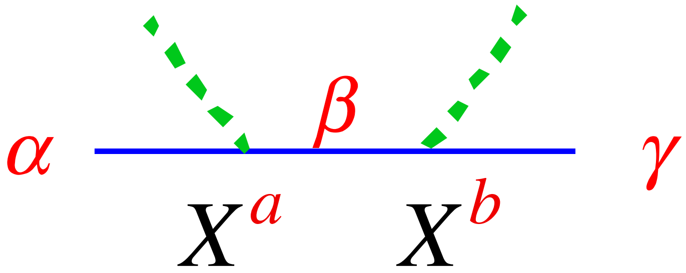
$O(q)$ $\qquad\qquad$ $O(q^2)$

$$\xrightarrow{q\to\infty} \frac{\omega}{f_\pi^{\,2}}\left\{-i\varepsilon_{abc}T^c+\left[X^a,X^b\right]\right\}\to 0$$

Combined with
the isospin algebra

$$[T^a,T^b]=i\varepsilon_{abc}T^c$$
$$[T^a,X^b]=i\varepsilon_{abc}X^c$$
$$[X^a,X^b]=i\varepsilon_{abc}T^c$$

Representations



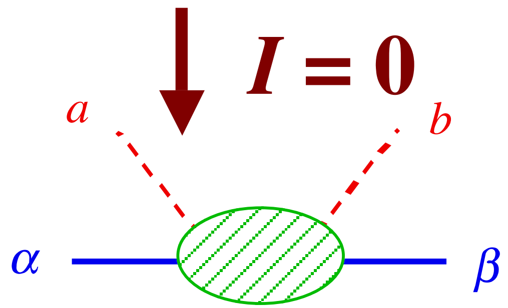
$$\sim \quad \left(X^b\right)_{\beta\gamma}\left(X^a\right)_{\gamma\alpha}$$

Hadron states α, β, γ can be components of any representation of chiral symmetry group that are connected by X^a

Pre-QCD, the choice of representations was phenomenological.

Now, can we hope that we determine it from QCD?

Consistency condition 2



Isospin I = 2 part does not contribute to this amplitude
<= No exotics

This lead to a constraint on the mass matrix

$$m^2 = m_0^2 + m_4^2$$

Chiral scalar — Chiral four vector

$$\left(m_a^2\ (a=1,2,3)=0\right)$$

$$\left[X^a, m_0^2\right]=0,\quad \left[X^a, m_4^2\right]=i m_a^2$$

Predictions

Mesons

$\pi, \sigma, \rho, a_1 \quad \left(\tfrac{1}{2}, \tfrac{1}{2}\right) + \left[(1,0) + (0,1)\right]$ with 45° mixing

σ-ρ and π-a_1 mixing

$$m_\sigma = m_\rho, \quad m_{a_1} = \sqrt{2} m_\rho$$

$$\Gamma_\sigma / \Gamma_\rho = \frac{9}{2}\left(1 - 4\frac{m_\pi^2}{m_\rho^2}\right)^{-1}$$

Baryons

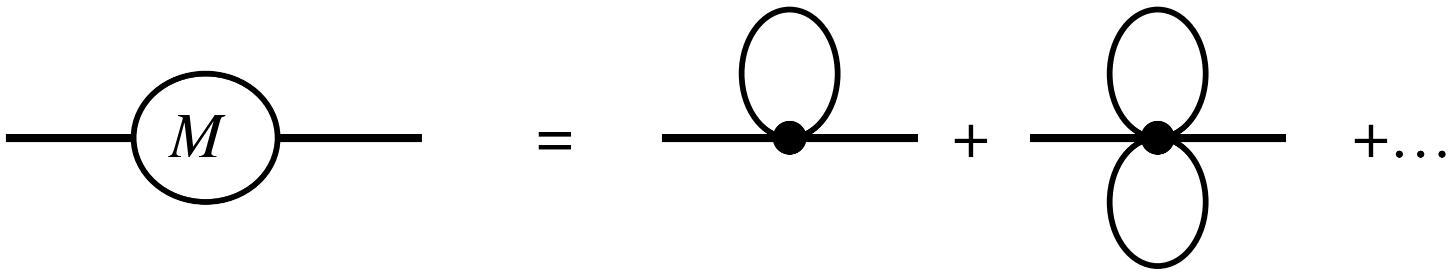
$$N, D, Roper \quad \left[\left(\tfrac{1}{2},0\right)+\left(0,\tfrac{1}{2}\right)\right], \left[\left(1,\tfrac{1}{2}\right)+\left(\tfrac{1}{2},1\right)\right]$$

$$\theta \sim 52^\circ \qquad g_A \sim 1.25$$

$$\theta \sim 45^\circ \qquad g_A \sim 4/3 \sim 1.333$$

$$m_N^2 + m_R^2 = 2m_\Delta^2$$

Mass matrix



$\langle \bar{q} O q \rangle$ $\qquad$ $\langle \bar{q} O q \bar{q} O q \rangle$

From $\langle \bar{q} O q \rangle$ we can construct only one Lorentz scalar $\langle \bar{q} 1 q \rangle$

But this is the fourth component of a chiral four-vector

$$\left(\tfrac{1}{2},\ \tfrac{1}{2}\right) \sim \left(\sigma, \pi_1, \pi_2, \pi_3\right)$$

Hence, if the mass has a qq^{bar} origin, it must be of chiral four-vector

Chiral scalar may be furnished by gluons or multiquarks

5. Linear sigma models with mirror baryons

Simple models

A chiral doublet N and N^* ~ can be any chiral repr.

Simple choices $\left(\frac{1}{2},0\right)+\left(0,\frac{1}{2}\right)$ and/or $\left(0,\frac{1}{2}\right)+\left(\frac{1}{2},0\right)$

Naive: $(1/2, 0)+(0, 1/2)$

$\psi_{\mathrm{L}} \quad + \quad \psi_{\mathrm{R}}$

qqq

$g_A = +1 \qquad g_{\pi BB} = +1$

Mirror: $(0, 1/2)+(1/2, 0)$

$\psi_{\mathrm{L}} \quad + \quad \psi_{\mathrm{R}}$

$qqqq\bar{q}$

$g_A = -1 \qquad g_{\pi BB} = -1$

Axial-charge carry information on internal structure

Naïve model

Lagrangian

$$L = \bar{N}_1 i \partial N_1 - g_1 \underline{\bar{N}_1(\sigma} + i\gamma_5 \pi_a \tau^a) N_1$$
$$+ \bar{N}_2 i \partial N_2 + g_2 \underline{\bar{N}_2(\sigma} + i\gamma_5 \pi_a \tau^a) N_2$$
$$+ g_{12} \left[\underline{\bar{N}_1(\sigma} + i\pi_a \tau^a \gamma_5) N_2 + \underline{\bar{N}_2(\sigma} + i\pi_a \tau^a \gamma_5) N_1 \right] + \cdots$$

Chiral scalar mass term is not possible

Lorentz invariance requires a structure $L^\dagger R + R^\dagger L$

Mass term after $\sigma \to \langle \sigma \rangle \equiv \sigma_0$

$$\left(\bar{N}_1, \bar{N}_2\right) \sigma_0 U_5 \begin{pmatrix} g_1 & g_{12}\gamma_5 \\ g_{12}\gamma_5 & -g_2 \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}$$

Diagonalization

$$\begin{pmatrix} N_+ \\ N_- \end{pmatrix} = \begin{pmatrix} \cos\theta & \gamma_5 \sin\theta \\ -\gamma_5 \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} \qquad \tan 2\theta = \frac{2g_{12}}{g_1 + g_2}$$

$$m_\pm = \frac{\sigma_0}{2}\left(\sqrt{(g_1 + g_2)^2 + 4g_{12}^2} \pm (g_1 - g_2) \right)$$

- N_1 and N_2 decouple
- Masses are solely generated by σ_0

Chiral four-vector

Mirror

$$L_{free} = \bar{\psi}_1 i\partial\psi_1 + \bar{\psi}_2 i\partial\psi_2 - m_0\left(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2\right)$$

Axial charge $g_A = \begin{array}{cc} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} & \begin{array}{c} \psi_1 \\ \psi_2 \end{array} \end{array}$ diagonal

But mass matrix $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Non-diagonal

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

45^{o} rotation

Mirror

$$
\begin{aligned}
L &= \bar{\psi}_1 i\partial\psi_1 - g_1\,\bar{\psi}_1 \underline{(\sigma} + i\gamma_5\pi_a\tau^a)\psi_1 \\
&+ \bar{\psi}_2 i\partial\psi_2 - g_2\,\bar{\psi}_2 \underline{(\sigma} + i\gamma_5\pi_a\tau^a)\psi_2 \\
&- \underline{m_0}\,(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2) + \cdots
\end{aligned}
$$

Chiral four-vector

Chiral scalar

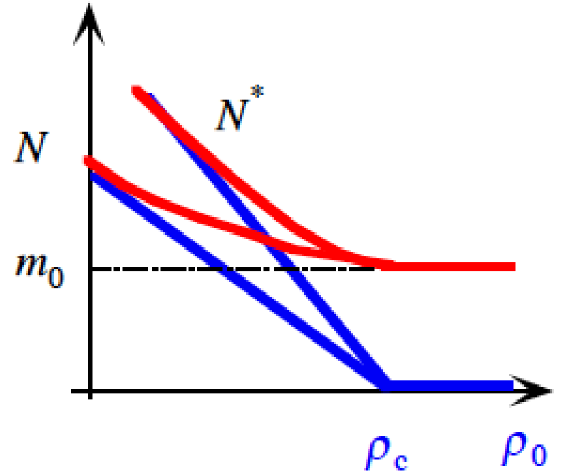
is now available when ψ_1 and ψ_2 transform inversely under chiral transformations.

Diagonalization

$$
\begin{pmatrix} N_+ \\ N_- \end{pmatrix} = \begin{pmatrix} \cos\theta & \gamma_5 \sin\theta \\ -\gamma_5 \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \qquad \tan 2\theta = \frac{m_0}{\sigma_0}\frac{2}{g_1+g_2}
$$

$$
m_\pm = \frac{1}{2}\left(\sqrt{(g_1+g_2)^2\sigma_0^2 + 4m_0^2} \pm (g_1 - g_2)\sigma_0\right)
$$

Masses

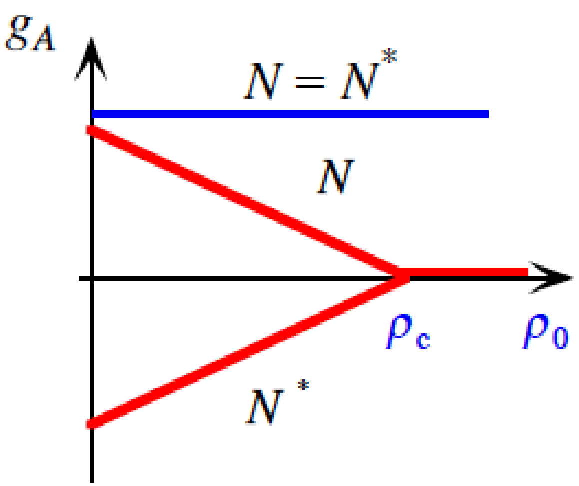


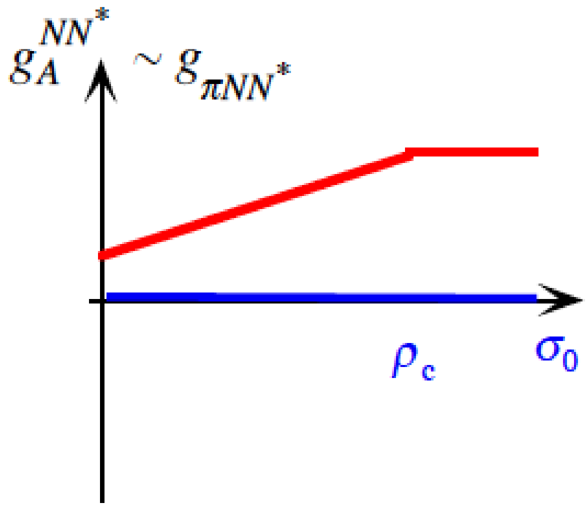
Naive

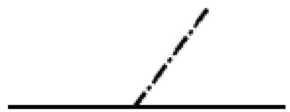
Mirror

$m_0 = \text{const}$

Axial couplings





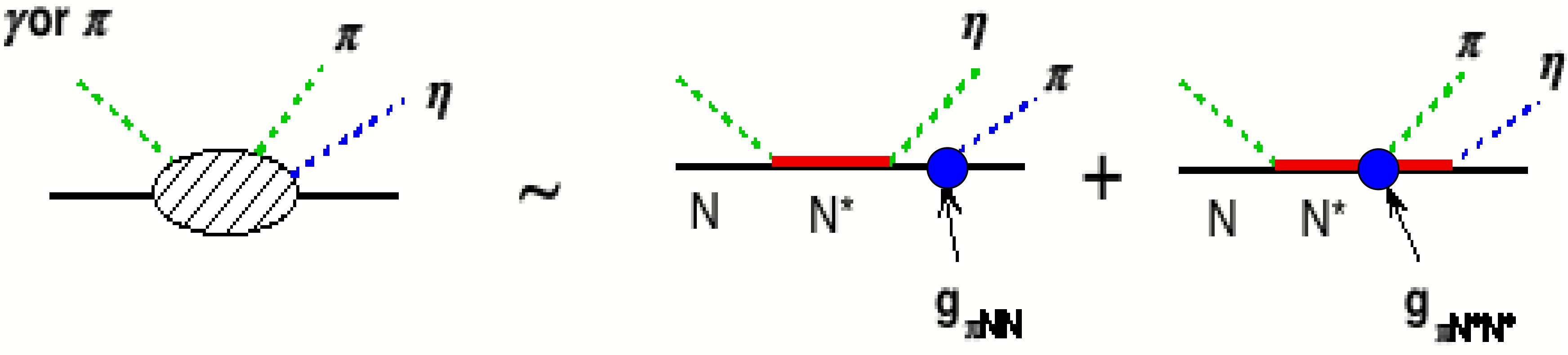


diagonal

off diagonal

Observe the sign of g_A

Assume minimal processes with N and $N(1535)$



+ ….

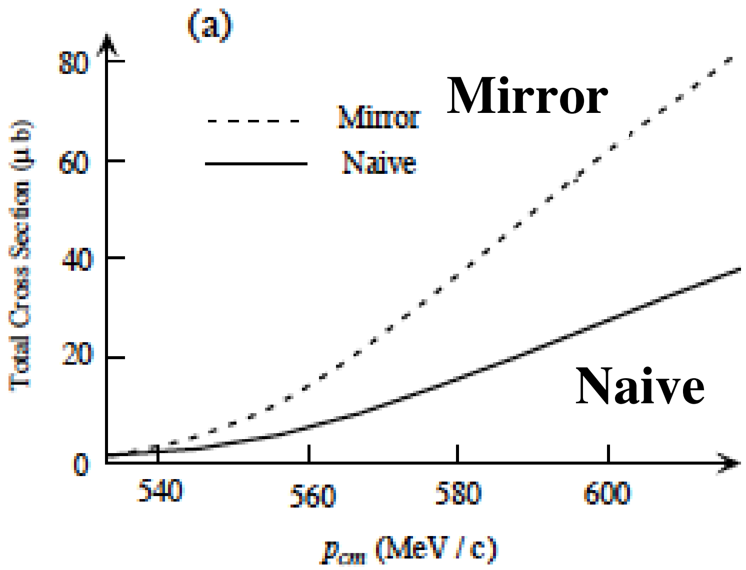
Naive: $g_{\pi N^* N^*} = g_{\pi NN}$ -> constructive

Mirror: $g_{\pi N^* N^*} = -g_{\pi NN}$ -> destructive

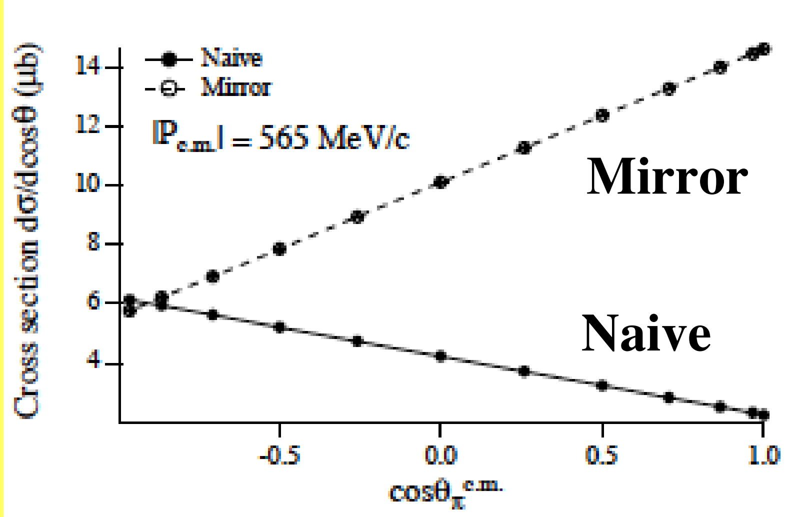
Pion-induced production

Jido, Oka, Hosaka
Prog. Theor. Phys. 106, 873, 2001

Total cross section



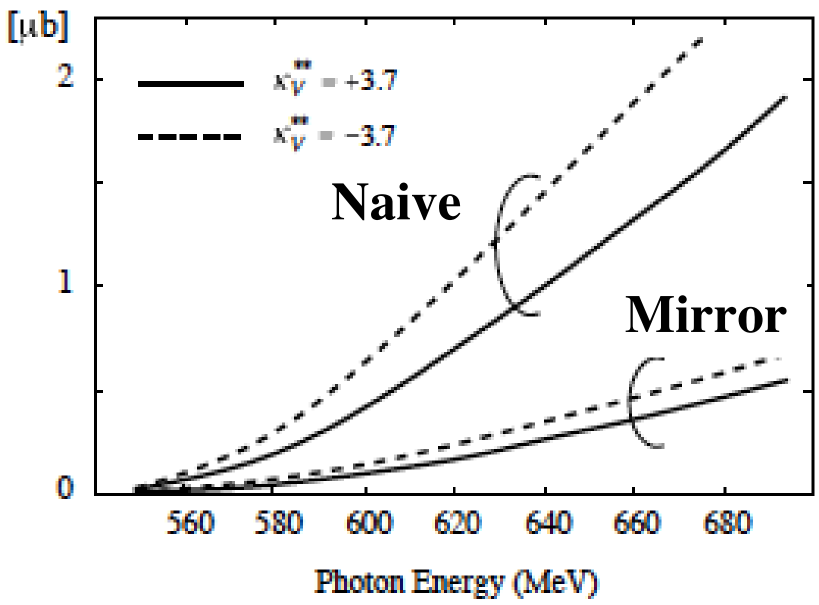
Angular distributions



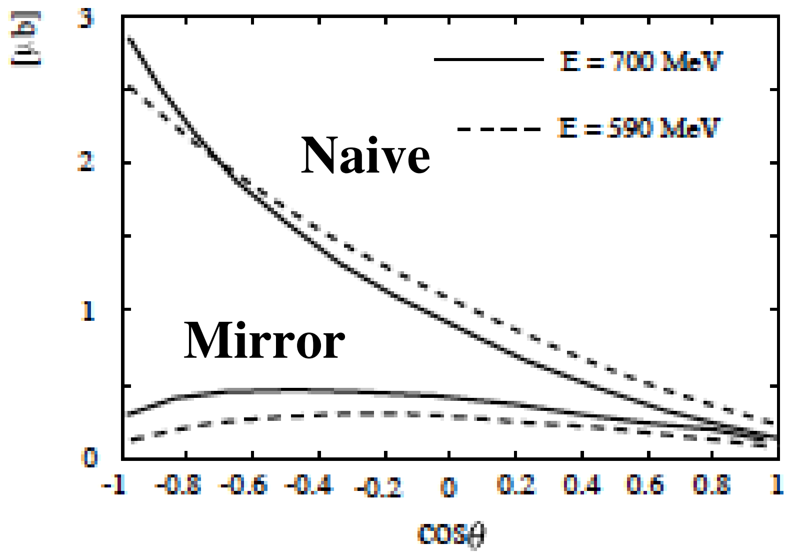
Photoproduction

Jido, Oka, Hosaka
Prog. Theor. Phys. 106, 873, 2001

Total cross section



Angular distributions



6. Extension to SU(3)

SU(2) $qqq \sim \left[\left(\tfrac{1}{2},0\right)+\left(0,\tfrac{1}{2}\right)\right]^3$

$$= \underbrace{\left(\tfrac{1}{2},0\right)+\left(0,\tfrac{1}{2}\right)}_{\text{Fundamental}} \quad + \quad \underbrace{\left(1,\tfrac{1}{2}\right)+\left(\tfrac{1}{2},1\right) \quad + \quad \left(\tfrac{3}{2},0\right)+\left(0,\tfrac{3}{2}\right)}_{\text{Higher repr.}}$$

SU(3) $qqq \sim \left[(3,0)+(0,3)\right]^3$

$$= (3,\bar{3})+(\bar{3},3)+(8,1)+(1,8) \quad + \quad (3,6)+(6,3)$$

$$+ \quad (10,1)+(1,10)$$

γ_μ, derivatives d_μ, multi q components form various reprs.

Actual calculation

Local fields

Singlet $$\Lambda \equiv \epsilon^{ABC}\epsilon_{abc}(q_A^{aT}C\Gamma_1 q_B^b)\Gamma_2 q_C^c.$$

Octet $$N^N \equiv \epsilon^{ABD}(\boldsymbol{\lambda}^N)_{DC}\epsilon_{abc}(q_A^{aT}C\Gamma_1 q_B^b)\Gamma_2 q_C^c.$$

Decuplet $$\Delta^P \equiv S_P^{ABC}\epsilon_{abc}(q_A^{aT}C\Gamma_1 q_B^b)\Gamma_2 q_C^c.$$

$$\Lambda_1 = \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}Cq_B^b)\gamma_5 q_C^c,$$
$$\Lambda_2 = \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}C\gamma_5 q_B^b)q_C^c,$$
$$\Lambda_3 = \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}C\gamma_\mu\gamma_5 q_B^b)\gamma^\mu q_C^c,$$
$$\Lambda_4 = \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}C\gamma_\mu q_B^b)\gamma^\mu\gamma_5 q_C^c,$$
$$\Lambda_5 = \epsilon_{abc}\epsilon^{ABC}(q_A^{aT}C\sigma_{\mu\nu} q_B^b)\sigma_{\mu\nu}\gamma_5 q_C^c.$$

Not all of them
are independent

Local fields

Flavor			Chiral
Octet	2	B_+, B-	$(3,\bar{3})+(\bar{3},3)$
			$(8,1)+(1,8)$
Singlet	1	Λ	$(3,\bar{3})+(\bar{3},3)$

γ_μ ∂^μ

$$N^N_{1\mu} = \partial^\mu \epsilon^{ABD} \lambda^N_{DC} (\tilde{q}_A \gamma_5 q_B) \gamma_\mu q_C$$

$$N^N_{3\mu\nu} = -\epsilon^{ABD} \lambda^N_{DC} (\tilde{q}_A \gamma_\mu q_B) \gamma_\nu q_C + (\mu \leftrightarrow \nu)$$

Octet	$B'_{(\mu)}$	$(6,3)+(3,6)$
Decuplet	$\Delta'_{(\mu)}$	
	$\Delta'_{(\mu\nu)}$	$(10,1)+(1,10)$

Chiral transformations

$$\mathbf{U(1)_A}: q \to \exp\left(i\gamma_5 \frac{\lambda^0}{2} b_0\right) q = q + \delta_5 q,$$

$$\mathbf{SU(3)_A}: q \to \exp\left(i\gamma_5 \frac{\vec{\lambda}}{2} \cdot \vec{b}\right) q = q + \delta_5^{\vec{b}} q,$$

$SU(3)_L \otimes SU(3)_R$	$SU(3)_F$		g_A^0	g_A^3	g_A^8	α
	1	Λ	-1	—	—	—
$(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	**8**	N_-	-1	1	-1	1
		Σ_-	-1	0	2	
		Ξ_-	-1	-1	-1	
		Λ_-	-1	—	-2	
$(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$	**8**	N_+	3	1	3	0
		Σ_+	3	1	0	
		Ξ_+	3	1	-3	
		Λ_+	3	—	0	
$(\mathbf{3}, \mathbf{6}) \oplus (\mathbf{6}, \mathbf{3})$	**8**	N_μ	1	5/3	1	3/5
		Σ_μ	1	2/3	2	
		Ξ_μ	1	$-1/3$	-3	
		Λ_μ	1	—	-2	
	10	Δ_μ	1	1/3	1	—
		Σ^*_μ	1	1/3	0	
		Ξ^*_μ	1	1/3	-1	
		Ω_μ	1	—	-2	
$(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$	**10**	$\Delta_{\mu\nu}$	3	1	3	—
		$\Sigma^*_{\mu\nu}$	3	1	0	
		$\Xi^*_{\mu\nu}$	3	1	-3	
		$\Omega_{\mu\nu}$	3	—	-6	

Mixing and g_A

case	field	$g_A^{(0)}$	$g_A^{(1)}$	F	D	$SU_L(3)\times SU_R(3)$
I	$N_1 - N_2$	-1	$+1$	0	$+1$	$(3,\bar{3})\oplus(\bar{3},3)$
II	$N_1 + N_2$	$+3$	$+1$	$+1$	0	$(8,1)\oplus(1,8)$
III	$N_1' - N_2'$	$+1$	-1	0	-1	$(\bar{3},3)\oplus(3,\bar{3})$
IV	$N_1' + N_2'$	-3	-1	-1	0	$(1,8)\oplus(8,1)$
0	$\partial_\mu(N_3^\mu + \frac{1}{3}N_4^\mu)$	$+1$	$+\frac{5}{3}$	$+\frac{2}{3}$	$+1$	$(6,3)\oplus(3,6)$

(0,I,II), (0, I, III) Mixing of ***three*** components with ***two*** angles

case	$g_{A\,\mathrm{expt.}}^{(3)}$	$g_{A\,\mathrm{expt.}}^{(0)}$	θ	φ	F	D	F/D
I-II	1.267	0.33	39.3^o	$28.0^o \pm 2.3^o$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04

Input

Exp $F/D = 0.571 \pm 0.005$

$$g_A^{(0)} \sim \Delta u + \Delta d + \Delta s$$

$$g_A^{(3)} \sim \Delta u - \Delta d$$

$$g_A^{(8)} \sim \Delta u + \Delta d - 2\Delta s$$

3 valence quarks => Δs = 0
F/D is not reproduced

Need of multi-quark components

Furthermore

Construct chiral invariant Lagrangians of SU(3)xSU(3)

Study of various properties: masses, couplings, decays

Property changes toward symmetry restoration

Problems/Questions

- What is the origin of the mass, especially of chiral scalar?
- Another source of g_A?
- How we can look into chiral structure of physical hadrons?
- What experiments are available to look into chiral structure?
- Do particles belonging to a chiral representation has the common spin?

Derivative terms

$$\left(\frac{g_A-1}{v^2}\right)\left[\left(\bar{\psi}\gamma_\mu\frac{\tau}{2}\psi\right)\cdot(\pi\times\partial^\mu\pi)+\left(\bar{\psi}\gamma_\mu\gamma_5\frac{\tau}{2}\psi\right)\cdot(\sigma\partial^\mu\pi-\pi\partial^\mu\sigma)\right]$$

Arbitrary g_A

Is this related to chiral representations?