Exotic hadrons from fragmentation functions

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Exotics from Heavy Ion Collisions

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(1) Introduction to fragmentation functions

(2) Determination of fragmentation functions

- For π, K, p

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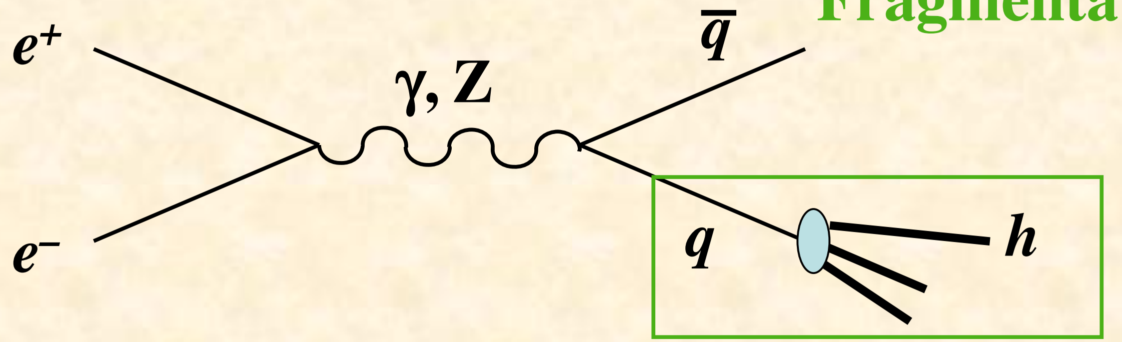
- For example, $f_0(980)$

Refs. (1) M. Hirai, S. Kumano, T.-H. Nagai, K. Sudoh, PRD 75 (2007) 094009.
(2) M. Hirai, S. Kumano, M. Oka, K. Sudoh, PRD 77 (2008) 017504.
S. Kumano, V. R. Pandharipande, PRD 38 (1988) 146.
F. E. Close, N. Isgur, S. Kumano, NPB 389 (1993) 513.

Major contribution has been made especially on (2) by Masanori Hirai (Tokyo University of Science).

Introduction to Fragmentation Functions

Fragmentation Functions



Fragmentation: hadron production from a quark, antiquark, or gluon

$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \quad s = Q^2$$

Fragmentation function is defined by

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz}$$

σ_{tot} = total hadronic cross section

Variable z

- **Hadron energy / Beam energy**
- **Hadron energy / Primary quark energy**

A fragmentation process occurs from quarks, antiquarks, and gluons, so that F^h is expressed by their individual contributions:

$$F^h(z,Q^2) = \sum_i \int_z^1 \frac{dy}{y} C_i\left(\frac{z}{y},Q^2\right) D_i^h(y,Q^2)$$

Calculated in perturbative QCD

Non-perturbative (determined from experiments)

$C_i(z,Q^2)$ = coefficient function

$D_i^h(z,Q^2)$ = fragmentation function of hadron h from a parton i

Momentum (energy) sum rule

$D_i^h(z,Q^2) =$ probability to find the hadron h from a parton i with the energy fraction z

Energy conservation: $\sum_h \int_0^1 dz\, z\, D_i^h(z,Q^2) = 1$

$h = \pi^+,\ \pi^0,\ \pi^-,\ K^+,\ K^0,\ \bar{K}^0,\ K^-,\ p,\ \bar{p},\ n,\ \bar{n},\ \cdots$

Favored and disfavored fragmentation functions

Simple quark model: $\pi^+(u\bar{d}),\ K^+(u\bar{s}),\ p(uud),\ \cdots$

Favored fragmentation: $D_u^{\pi^+},\ D_{\bar{d}}^{\pi^+}$, ...

(from a quark which exists in a naive quark model)

Disfavored fragmentation: $D_d^{\pi^+},\ D_{\bar{u}}^{\pi^+},\ D_s^{\pi^+}$, ...

(from a quark which does not exist in a naive quark model)

Parton Distribution Functions (PDFs)

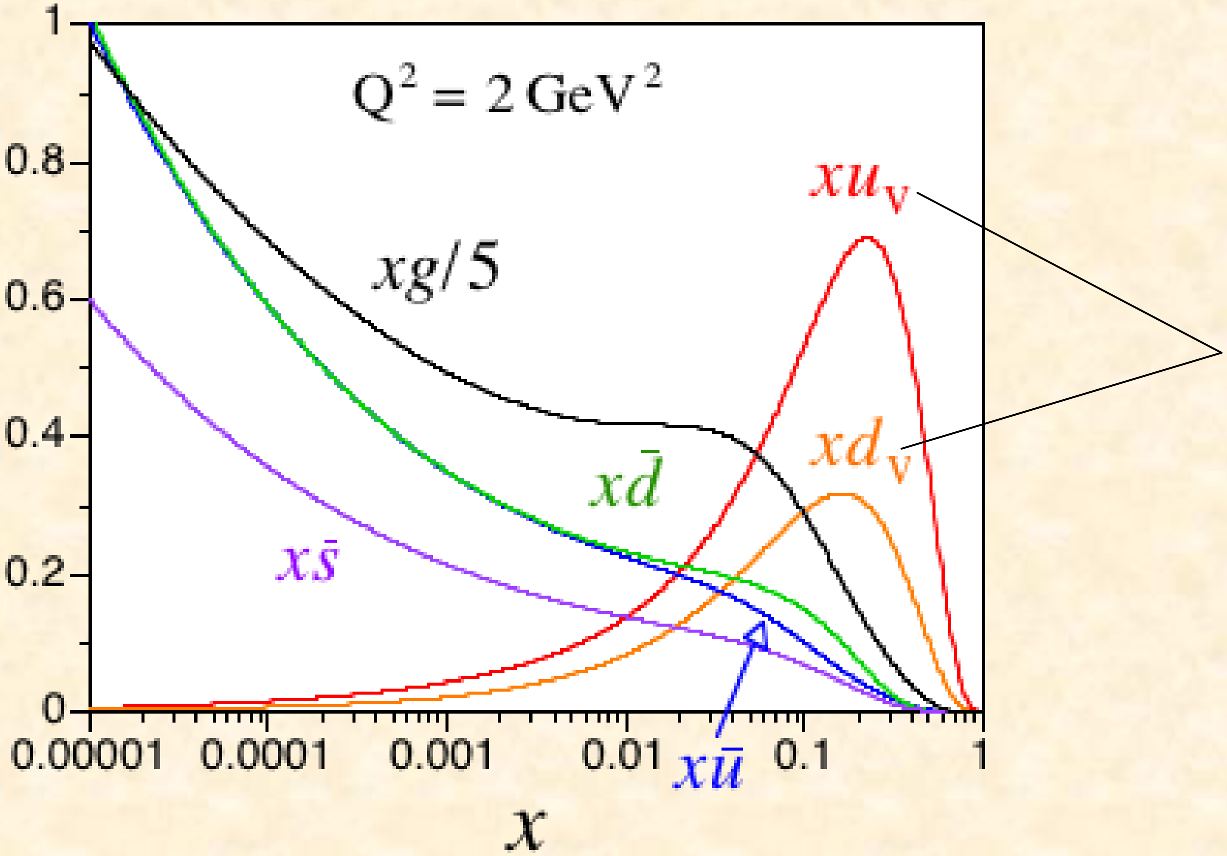
Favored **fragmentation functions**

$\leftrightarrow$ **Valence-quark** **distribution functions**

Disfavored **fragmentation functions**

$\leftrightarrow$ **Sea-quark and gluon** **distribution functions**

However, the PDFs would not be used for unstable exotic hadrons in studying internal configuration.



Valence-quark distributions

Purposes of investigating fragmentation functions

Semi-inclusive reactions have been used for investigating

- **origin of proton spin**

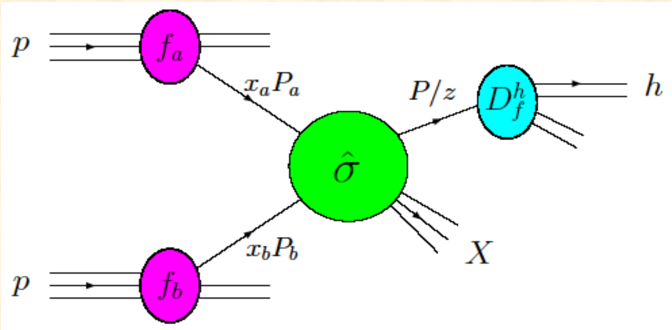
  $\vec{e} + \vec{p} \to e' + h + X, \quad \vec{p} + \vec{p} \to h + X$ **(RHIC-Spin)**

  Quark, antiquark, and gluon contributions to proton spin

  (flavor separation, gluon polarization)

- **properties of quark-hadron matters** $A + A' \to h + X$ **(RHIC, LHC)**

  Nuclear modification (recombination, energy loss, …)



$$\sigma = \sum_{a,b,c} f_a(x_a, Q^2) \otimes f_b(x_b, Q^2) \otimes \hat{\sigma}(ab \to cX) \otimes D_c^{\pi}(z, Q^2)$$

- **Exotic-hadron search**

Exotic-hadron search by fragmentation functions

"Favored" and "disfavored" (unfavored) fragmentation functions

- **Possibility of finding exotic hadrons in high-energy processes**

$$e.g.\ \text{if}\ f_0(980) = s\bar{s}:\quad \text{favored}\ \ s \to f_0,\ \bar{s} \to f_0$$

$$\text{disfavored}\ \ u \to f_0,\ \ d \to f_0,\ \ \bar{u} \to f_0,\ \ \bar{d} \to f_0, \cdots$$

$$f_0(980) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}),\quad s\bar{s},\quad \frac{1}{\sqrt{2}}(u\bar{u}s\bar{s} + d\bar{d}s\bar{s}),\quad K\bar{K},\quad \text{or}\ \ gg$$

f_0(980) in heavy-ion reactions: *e.g.* L. Maiani *et al.*, PLB 645 (2007) 138;
C. Nonaka *et al.*, PRC69 (2004) 031902. (I am sorry if I miss your works.)

→ tomorrow's (May 21) program at this workshop

Situation of fragmentation functions (before 2007)

There are two widely used fragmentation functions by Kretzer and KKP.

An updated version of KKP is AKK.

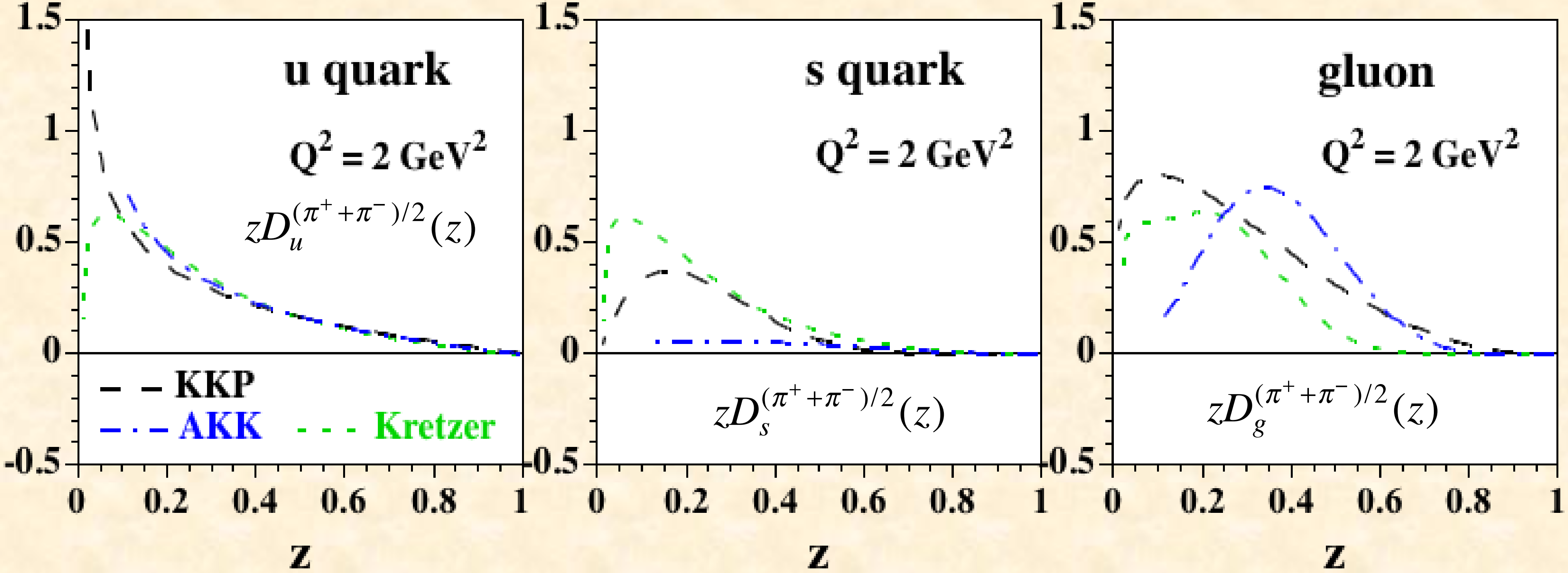
See also Bourhis-Fontannaz-Guillet-Werlen (2001) for FFs without hadron separation.

(Kretzer) S. Kretzer, PRD 62 (2000) 054001

(KKP) B. A. Kniehl, G. Kramer, B. Pötter, NPB 582 (2000) 514

(AKK) S. Albino, B.A. Kniehl, G. Kramer, NPB 725 (2005) 181

The functions of Kretzer and KKP (AKK) are very different.



Determination of Fragmentation Functions (π, K, p / $\bar{p}$)

Ref. M. Hirai, SK, T.-H. Nagai, K. Sudoh
Phys. Rev. D75 (2007) 094009.

Code for calculating the fragmentation functions is available at
http://research.kek.jp/people/kumanos/ffs.html .

New aspects in our analysis (compared with Kretzer, KKP, AKK)

- Determination of fragmentation functions (FFs) and their uncertainties (our work is the first uncertainty estimate in FFs) in LO and NLO.
- Discuss NLO improvement in comparison with LO by considering the uncertainties.
  (Namely, roles of NLO terms in the determination of FFs)
- Comparison with other parametrizations
- Avoid assumptions on parameters as much as we can,
  Avoid contradiction to the momentum sum rule
- SLD (2004) data are included.

Initial functions for pion

Note: constituent-quark composition $\pi^+ = u\bar{d}, \quad \pi^- = \bar{u}d$

$$D_q^{\pi^-} = D_{\bar{q}}^{\pi^+}$$

$$D_u^{\pi^+}(z,Q_0^2) = N_u^{\pi^+} z^{\alpha_u^{\pi^+}} (1-z)^{\beta_u^{\pi^+}} = D_{\bar{d}}^{\pi^+}(z,Q_0^2)$$

$$D_{\bar{u}}^{\pi^+}(z,Q_0^2) = N_{\bar{u}}^{\pi^+} z^{\alpha_{\bar{u}}^{\pi^+}} (1-z)^{\beta_{\bar{u}}^{\pi^+}} = D_d^{\pi^+}(z,Q_0^2) = D_s^{\pi^+}(z,Q_0^2) = D_{\bar{s}}^{\pi^+}(z,Q_0^2)$$

$$D_c^{\pi^+}(z,m_c^2) = N_c^{\pi^+} z^{\alpha_c^{\pi^+}} (1-z)^{\beta_c^{\pi^+}} = D_{\bar{c}}^{\pi^+}(z,m_c^2)$$

$$D_b^{\pi^+}(z,m_b^2) = N_b^{\pi^+} z^{\alpha_b^{\pi^+}} (1-z)^{\beta_b^{\pi^+}} = D_{\bar{b}}^{\pi^+}(z,m_b^2)$$

$$D_g^{\pi^+}(z,Q_0^2) = N_g^{\pi^+} z^{\alpha_g^{\pi^+}} (1-z)^{\beta_g^{\pi^+}}$$

Constraint: 2nd moment should be finite and less than 1

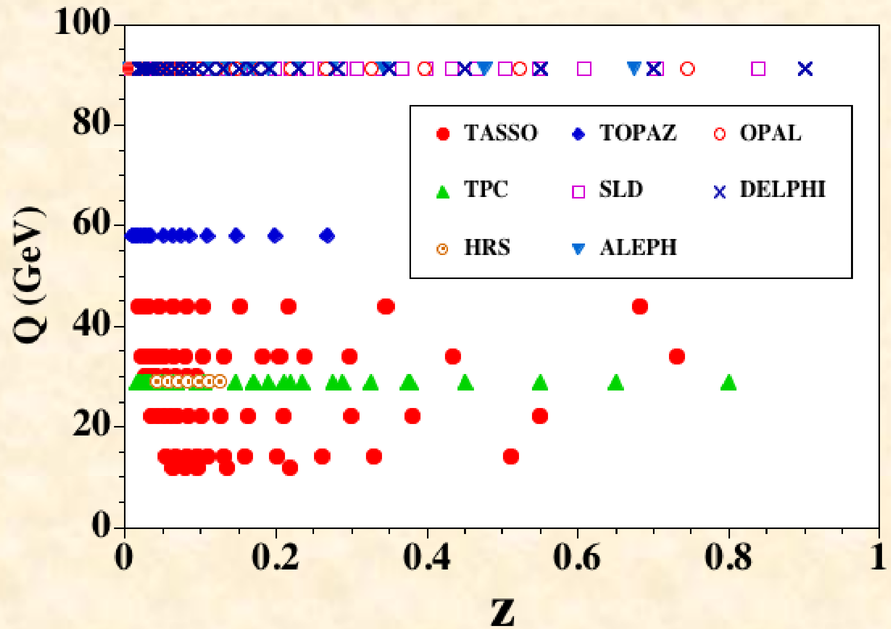
$$N = \frac{M}{B(\alpha+2,\beta+1)}, \qquad M \equiv \int_0^1 zD(z)\,dz \quad \text{(2nd moment)}, \quad B(\alpha+2,\beta+1) = \text{ beta function}$$

$$0 < M_i^h < 1 \quad \text{because of the sum rule } \sum_h M_i^h = 1$$

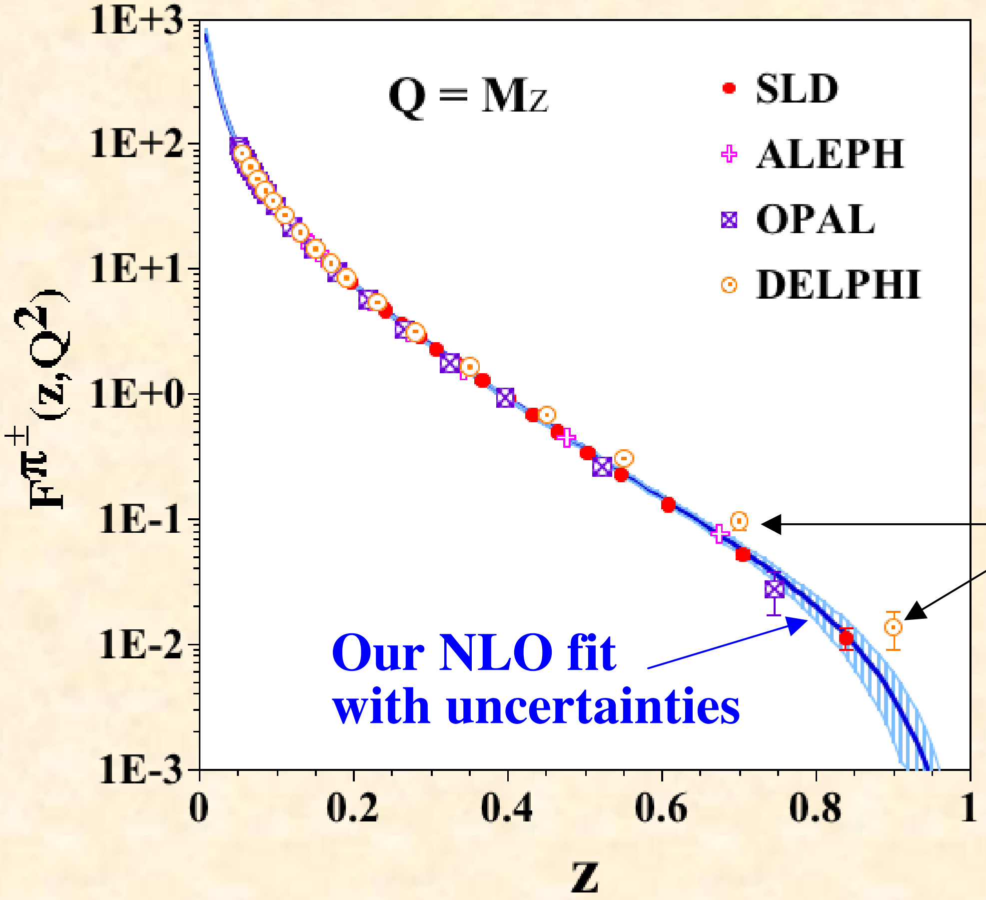
Experimental data for pion

Total number of data: 264

Collaboration	Lab	$\sqrt{s}$ (GeV)	# of data
TASSO	DESY	12,14,22,30,34,4	29
TPC	SLAC	4	18
HRS	SLAC	29	2
TOPAZ	KEK	29	4
SLD	SLAC	58	29
SLD [light quark]		91.2	29
SLD [c quark]			29
SLD [b quark]			29
ALEPH	CERN		22
OPAL	CERN	91.2	22
DELPHI	CERN	91.2	17
DELPHI [light quark]		91.2	17
DELPHI [b quark]			17



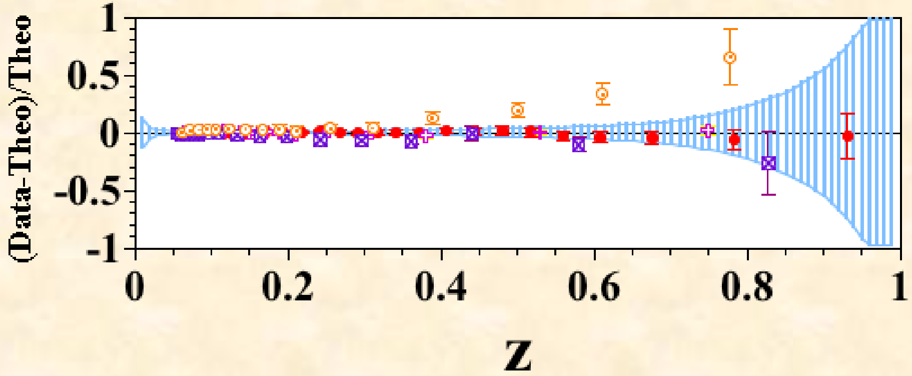
Comparison with pion data



$$F^{\pi^\pm}(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to \pi^\pm X)}{dz}$$

Our fit is successful to reproduce the pion data.

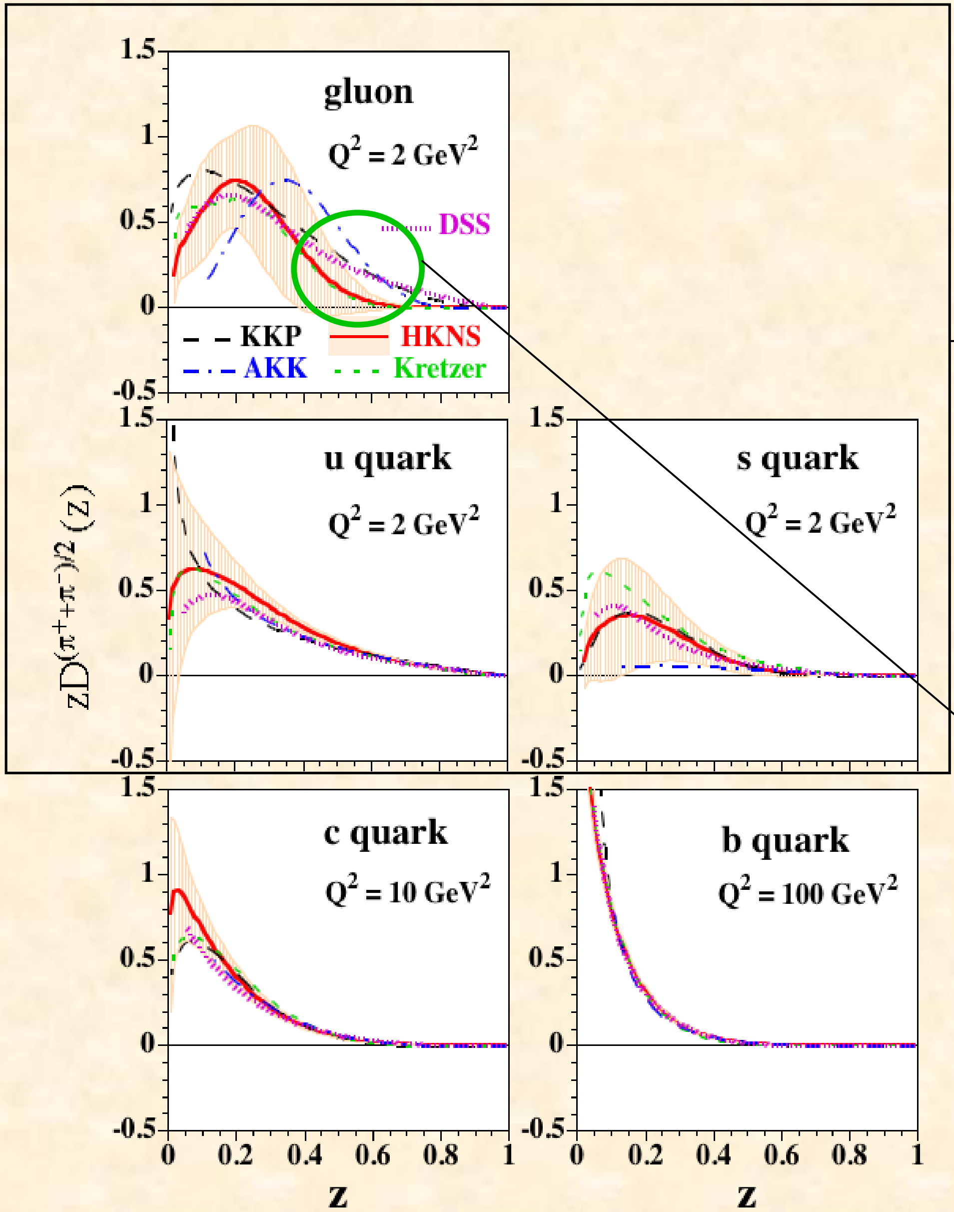
The DELPHI data deviate from our fit at large z.



Rational difference between data and theory

$$\frac{F^{\pi^\pm}(z,Q^2)_{\text{data}} - F^{\pi^\pm}(z,Q^2)_{\text{theory}}}{F^{\pi^\pm}(z,Q^2)_{\text{theory}}}$$

Comparison with other parametrizations in pion



(KKP) Kniehl, Kramer, Pötter

(AKK) Albino, Kniehl, Kramer

(HKNS) Hirai, Kumano, Nagai, Sudoh

(DSS) De Florian, Sassot, Stratmann

- Gluon and light-quark disfavored fragmentation functions have large differences, but they are within the uncertainty bands.

  → The functions of KKP, Kretzer, AKK, DSS, and HKNS are consistent with each other.

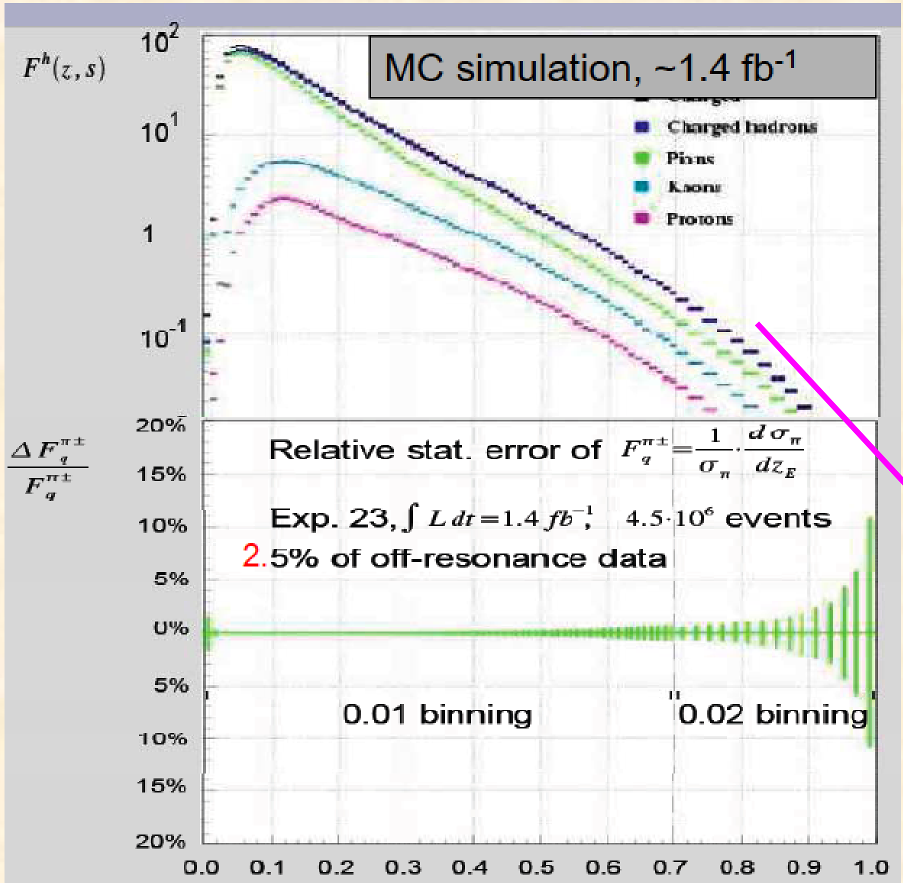
$$\hat{s} = x_a x_b s \sim (0.1)^2 (200 \text{ GeV})^2 \quad \text{for RHIC}$$

$$\sqrt{\hat{s}} = 0.1 \cdot 200 = 20 \text{ GeV}$$

$$z \sim \frac{p_T}{\sqrt{\hat{s}}/2} = \frac{p_T}{10} \sim 0.5 \quad \text{(relatively large } z\text{)}$$

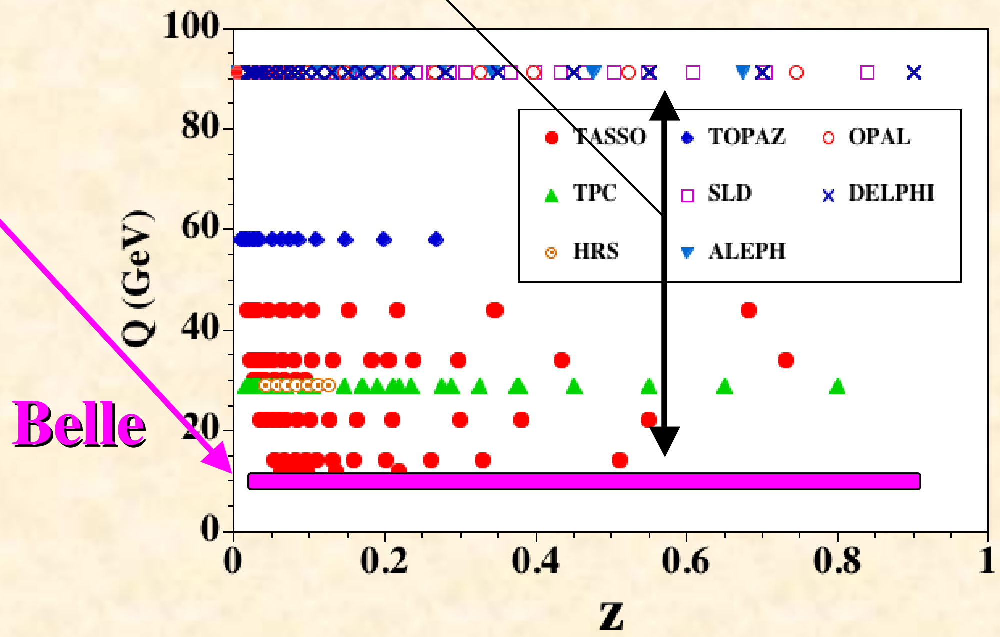
Hadron model: T. Ito, W. Bentz, I. C. Cloet, A. W. Thomas, K. Yazaki, PRD 80 (2009) 074008.

Expected fragmentation functions by Belle



Expected Belle data by R. Seidl

Scaling violation
= Determination of gluon
fragmentation function



Current data

Summary I

Determination of the optimum fragmentation functions for π, K, p in LO and NLO by a global analysis of $e^+ + e^- \rightarrow h + X$ data.

- **It was the first time that uncertainties of the fragmentation functions are estimated.**
- **Gluon and disfavored light-quark functions have large uncertainties.**
  - → **The uncertainties could be important for discussing physics in** $\vec{p} + \vec{p} \rightarrow \pi^0 + X$, $A + A' \rightarrow h + X$ **(RHIC, LHC), HERMES, COMPASS, JLab, ...**
  - → **Need accurate data at low energies (Belle and BaBar).**
- **For the pion and kaon, the uncertainties are reduced in NLO in comparison with LO.**
  For the proton, such improvement is not obvious.
- **Heavy-quark functions are well determined.**
- **Code for calculating the fragmentation functions is available at**
  http://research.kek.jp/people/kumanos/ffs.html .

Fragmentation Functions for Exotic-Hadron Search

$f_0(980)$ as an example

Refs. S. Kumano, V. R. Pandharipande, PRD 38 (1988) 146.
F. E. Close, N. Isgur, S. Kumano, NPB 389 (1993) 513.
M. Hirai, S. Kumano, M. Oka, K. Sudoh, PRD 77 (2008) 017504.

- **Introduction to exotic hadrons**

  Exotic hadrons at $M \sim 1$ GeV, especially $f_0(980)$

- **Criteria for determining internal configurations by fragmentation functions**

  Functional forms, Second moments

- **Analysis of $e^+ + e^- \to f_0 + X$ data for determining fragmentation functions for $f_0(980)$**

  Analysis method, Results, Discussions

- **Summary II**

Recent progress in exotic hadrons

$q\bar{q}$ Meson
q^3 Baryon

$q^2\bar{q}^2$ Tetraquark
$q^4\bar{q}$ Pentaquark
q^6 Dibaryon
…
$q^{10}\bar{q}$ e.g. Strange tribaryon
…
gg Glueball
…

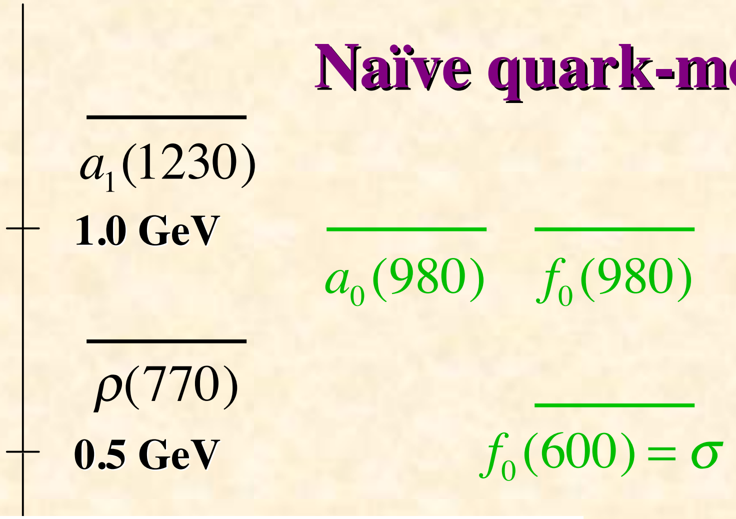
(Japanese ?) Exotics

- $\Theta^+(1540)$?: LEPS
  Pentaquark?
  $uudd\bar{s}$?
- $S^0(3115)$, $S^+(3140)$: KEK-PS
  Strange tribaryons?
  $K^- pnn$
  $K^- ppn$?
- X (3872), Y(3940): Belle
  Tetraquark, $D\bar{D}$ molecule
  $c\bar{c}$
  $D^0(c\bar{u})\bar{D}^0(\bar{c}u)$
  $D^+(c\bar{d})D^-(\bar{c}d)$?
- $D_{sJ}(2317)$, $D_{sJ}(2460)$: BaBar, CLEO, Belle
  Tetraquark, DK molecule
  $c\bar{s}$
  $D^0(c\bar{u})K^+(u\bar{s})$
  $D^+(c\bar{d})K^0(d\bar{s})$?
- Z (4430): Belle
  Tetraquark, …
  $c\bar{c}u\bar{d}$, D molecule?

Note: $Z(4430) \neq q\bar{q}$?

Scalar mesons $J^P=0^+$ at $M\sim 1$ GeV

Naïve quark-model



$$\sigma = f_0(600) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$f_0(980) = s\bar{s} \;\rightarrow \text{denote } f_0 \text{ in this talk}$$

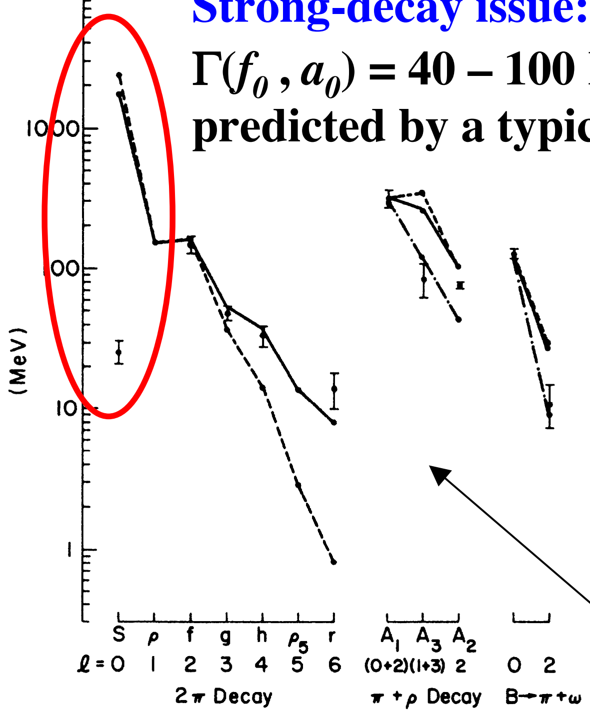
$$a_0(980) = u\bar{d},\ \ \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}),\ \ d\bar{u}$$

Naive model: $m(\sigma) \sim m(a_0) < m(f_0)$

⇕ **contradiction**

Experiment: $m(\sigma) < m(a_0) \sim m(f_0)$

Strong-decay issue: The experimental widths $\Gamma(f_0, a_0)$ = 40 – 100 MeV are too small to be predicted by a typical quark model.



These issues could be resolved if f_0 is a tetraquark ($qq\bar{q}\bar{q}$) or a $K\bar{K}$ molecule, namely an "exotic" hadron.

R. Kokoski and N. Isgur, Phys. Rev. D35 (1987) 907;
SK and V. R. Pandharipande, Phys. Rev. D38 (1988) 146.

Determination of $f_0(980)$ structure by electromagnetic decays

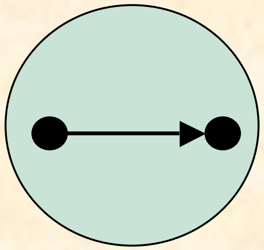
F. E. Close, N. Isgur, and SK, Nucl. Phys. B389 (1993) 513.

Radiative decay: $\phi \to S\gamma$

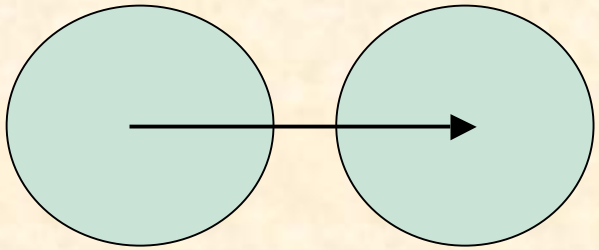
$S = f_0(980)$, $a_0(980)$

$J^P = 1^- \to 0^+$ **E1 transition**

Electric dipole:
$e\vec{r}$ (distance!)



$q\bar{q}$ model:
Γ = small



$K\bar{K}$ molecule
or $qq\bar{q}\bar{q}$: Γ = large

Experimental results of VEPP-2M and DAΦNE suggest that f_0 is a tetraquark state (or a $K\bar{K}$ molecule?).

CMD-2 (1999): $B(\phi \to f_0\gamma) = (1.93 \pm 0.46 \pm 0.50) \times 10^{-4}$
SND (2000): $(3.5 \pm 0.3^{+1.3}_{-0.5}) \times 10^{-4}$
KLOE (2002): $(4.47 \pm 0.21_{\text{stat+syst}}) \times 10^{-4}$

For recent discussions,
N. N. Achasov and A. V. Kiselev, PRD 73 (2006) 054029; D74 (2006) 059902(E); D76 (2007) 077501;
Y. S. Kalashnikova *et al.*, Eur. Phys. J. A24 (2005) 437.

See also Belle (2007)
$\Gamma(f_0 \to \gamma\gamma) = 0.205^{+0.095}_{-0.083}(\text{stat})^{+0.147}_{-0.117}(\text{syst})$ keV

Criteria for determining internal structure by fragmentation functions

(Naïve estimates)

M. Hirai, S. Kumano, M. Oka, K. Sudoh, PRD 77 (2008) 017504.

Criteria for determining f_0 structure by its fragmentation functions

Possible configurations of $f_0(980)$

(1) **ordinary u,d - meson** $\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$

Contradicts with experimental widths

$\Gamma_{\text{theo}}(f_0 \to \pi\pi) = 500 - 1000 \text{ MeV} \gg \Gamma_{\text{exp}} = 40 - 100 \text{ MeV}$

$\Gamma_{\text{theo}}(f_0 \to \gamma\gamma) = 1.3 - 1.8 \text{ keV} \gg \Gamma_{\text{exp}} = 0.205 \text{ keV}$

(2) **strange meson,** $s\bar{s}$

(3) **tetraquark ($K\bar{K}$),** $\frac{1}{\sqrt{2}}(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})$

(4) **glueball** gg

Contradicts with lattice-QCD estimate

$m_{\text{lattice}}(f_0) = 1600 \text{ MeV} \gg m_{\text{exp}} = 980 \text{ MeV}$

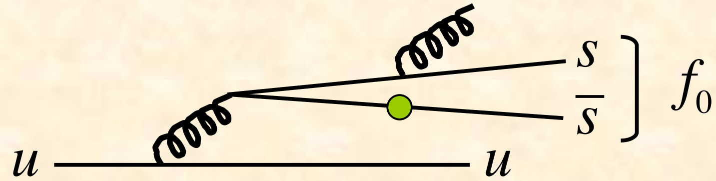
Discuss 2nd moments and functional forms (peak positions) of the fragmentation functions for f_0 by assuming the above configurations, (1), (2), (3), and (4).

$s\bar{s}$ picture for $f_0(980)$

$$M \equiv \int_0^1 zD(z)\,dz \quad \text{(2nd moment)}$$

2nd moment:	$M(u) < M(s) \lesssim M(g)$
Peak of function:	$z_{\max}(u) < z_{\max}(s) \simeq z_{\max}(g)$

u (disfavored)

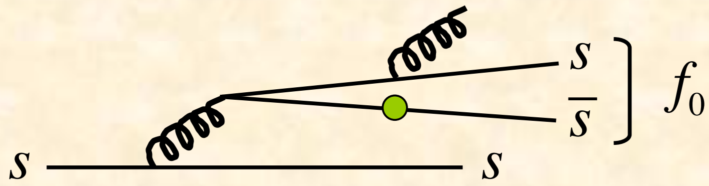


O (g^3)

+ one O (g^3) term of gluon radiation from the antiquark

s (favored)

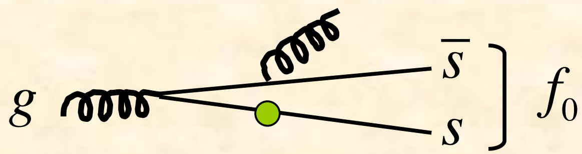
O (g^2)



O (g^3)

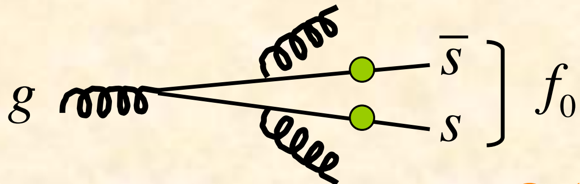
+ one O (g^3) term of gluon radiation from the antiquark

g



O (g^2)

+ one O (g^2) term of gluon radiation from the quark



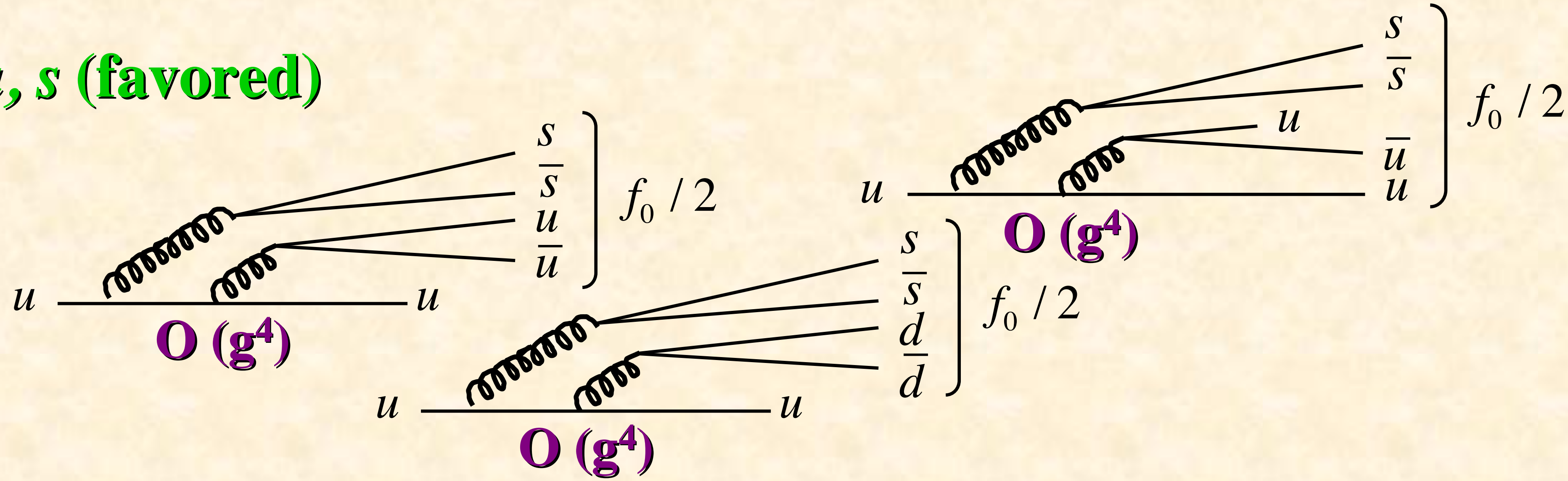
O (g^3)

+ two O (g^3) terms of gluon radiation from the quark or antiquark

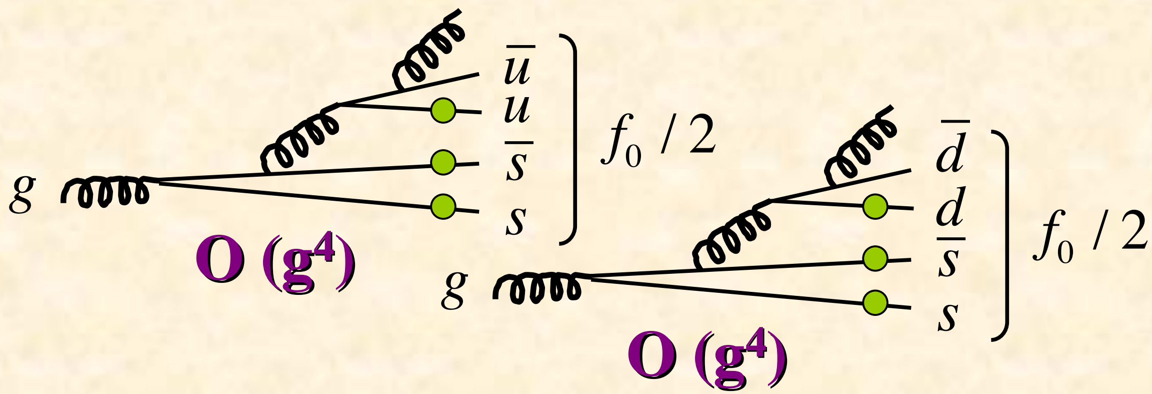
$n\bar{n}s\bar{s}$ picture for $f_0(980)$ $\quad f_0 = (u\bar{u}s\bar{s} + d\bar{d}s\bar{s})/\sqrt{2}$

$K\bar{K}$ picture for $f_0(980)$ $\quad f_0 = \left[K^+(u\bar{s})K^-(\bar{u}s) + K^0(d\bar{s})\bar{K}^0(\bar{d}s)\right]/\sqrt{2}$

u, s (favored)



g



+ six O (g^4) terms of gluon radiation from other quarks ●

2nd moment:	$M(u) = M(s) \lesssim M(g)$
Peak of function:	$z_{max}(u) = z_{max}(s) \simeq z_{max}(g)$

Judgment

Type	Configuration	2nd Moment	Peak z
Nonstrange $q\bar{q}$	$(u\bar{u}+d\bar{d})/\sqrt{2}$	$M(s)<M(u)<M(g)$	$z_{\max}(s)<z_{\max}(u)\simeq z_{\max}(g)$
Strange $q\bar{q}$	$s\bar{s}$	$M(u)<M(s)\lesssim M(g)$	$z_{\max}(u)<z_{\max}(s)\simeq z_{\max}(g)$
Tetraquark	$(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})/\sqrt{2}$	$M(u)=M(s)\lesssim M(g)$	$z_{\max}(u)=z_{\max}(s)\simeq z_{\max}(g)$
$K\bar{K}$ Molecule	$(K^+K^-+K^0\bar{K}^0)/\sqrt{2}$	$M(u)=M(s)\lesssim M(g)$	$z_{\max}(u)=z_{\max}(s)\simeq z_{\max}(g)$
Glueball	gg	$M(u)=M(s)<M(g)$	$z_{\max}(u)=z_{\max}(s)<z_{\max}(g)$

Since there is no difference between $D_u^{f_0}$ and $D_d^{f_0}$ in the models, they are assumed to be equal. On the other hand, $D_s^{f_0}$ and $D_g^{f_0}$ are generally different from them, so that they should be used for finding the internal structure. Therefore, simple and "model-independent" initial functions are

$$D_u^{f_0}(z,Q_0^2)=D_{\bar{u}}^{f_0}(z,Q_0^2)=D_d^{f_0}(z,Q_0^2)=D_{\bar{d}}^{f_0}(z,Q_0^2),\quad D_s^{f_0}(z,Q_0^2)=D_{\bar{s}}^{f_0}(z,Q_0^2),$$

$$D_g^{f_0}(z,Q_0^2),\qquad D_c^{f_0}(z,m_c^2)=D_{\bar{c}}^{f_0}(z,m_c^2),\qquad D_b^{f_0}(z,m_b^2)=D_{\bar{b}}^{f_0}(z,m_b^2).$$

2nd moments of favored • and disfavored • fragmentation functions

Actual HKNS07 analysis results (M. Hirai *et al*., PRD75 (2007) 094009) for the 2nd moments: $M \equiv \int_0^1 zD(z)dz$

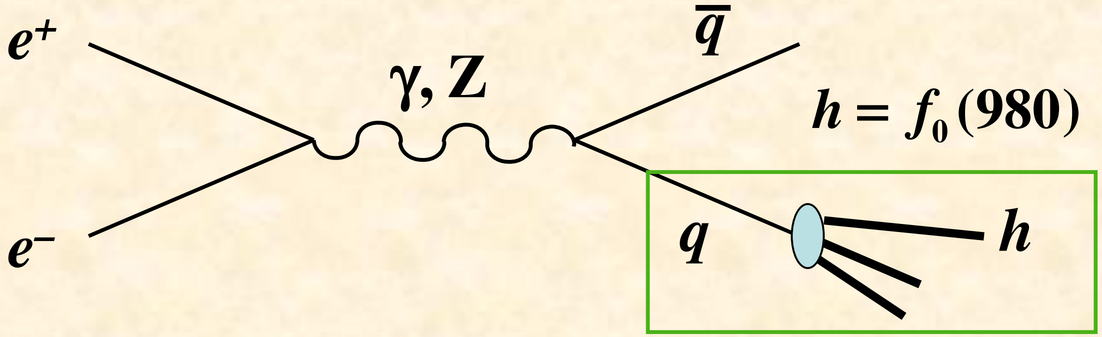
	2nd moment
• $D_u^{\pi^+}$	0.401 ± 0.052
• $D_{\bar{u}}^{\pi^+}$	0.094 ± 0.029
• $D_c^{\pi^+}$	0.178 ± 0.018
• $D_b^{\pi^+}$	0.236 ± 0.009
• $D_g^{\pi^+}$	0.238 ± 0.029
• $D_u^{K^+}$	0.0740 ± 0.0268
• $D_{\bar{s}}^{K^+}$	0.0878 ± 0.0506
• $D_{\bar{u}}^{K^+}$	0.0255 ± 0.0173
• $D_c^{K^+}$	0.0583 ± 0.0052
• $D_b^{K^+}$	0.0522 ± 0.0024
• $D_g^{K^+}$	0.0705 ± 0.0099

There is a tendency that 2nd moments are larger for the favored functions.

→ It suggests that the 2nd moments could be used for exotic hadron determination (quark / gluon configuration in hadrons).

Global analysis for fragmentation functions of $f_0(980)$

Fragmentation functions for $f_0(980)$



$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \quad s = Q^2$$

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz}$$

σ_{tot} = total hadronic cross section

$$F^h(z,Q^2) = \sum_i \int_z^1 \frac{dy}{y} C_i\left(\frac{z}{y}, Q^2\right) D_i^h(y,Q^2)$$

Initial functions

$$D_u^{f_0}(z,Q_0^2) = D_d^{f_0}(z,Q_0^2) = N_u^{f_0} z^{\alpha_u^{f_0}} (1-z)^{\beta_u^{f_0}}$$

$$D_s^{f_0}(z,Q_0^2) = N_s^{f_0} z^{\alpha_s^{f_0}} (1-z)^{\beta_s^{f_0}}$$

$$D_g^{f_0}(z,Q_0^2) = N_g^{f_0} z^{\alpha_g^{f_0}} (1-z)^{\beta_g^{f_0}}$$

$$D_c^{f_0}(z,m_c^2) = N_c^{f_0} z^{\alpha_c^{f_0}} (1-z)^{\beta_c^{f_0}}$$

$$D_b^{f_0}(z,m_b^2) = N_b^{f_0} z^{\alpha_b^{f_0}} (1-z)^{\beta_b^{f_0}}$$

- $D_q^{f_0}(z,Q_0^2) = D_{\bar{q}}^{f_0}(z,Q_0^2)$
- $Q_0 = 1$ GeV
  $m_c = 1.43$ GeV
  $m_b = 4.3$ GeV

$$N = M \frac{\Gamma(\alpha+\beta+3)}{\Gamma(\alpha+2)\Gamma(\beta+1)}, \quad M \equiv \int_0^1 zD(z)dz$$

Experimental data for f_0

Total number of data: only 23

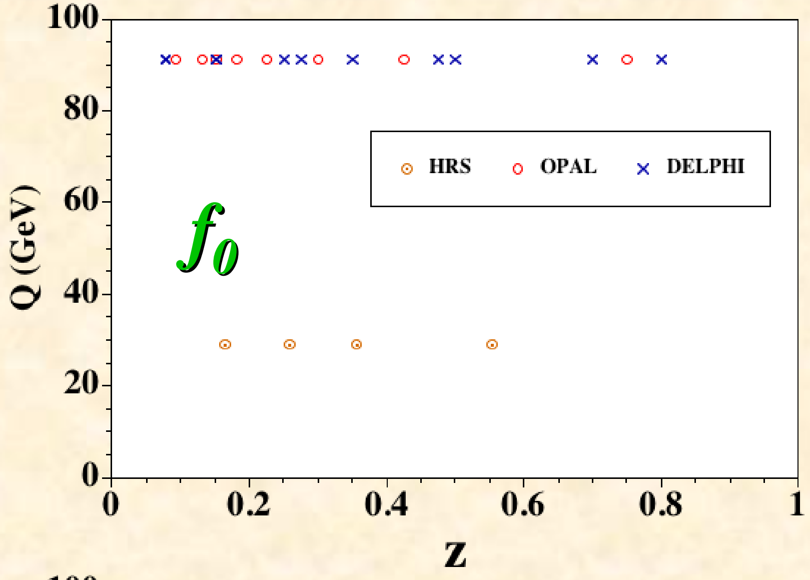
Exp. collaboration	$\sqrt{s}$ (GeV)	# of data
HRS	29	4
OPAL	91.2	8
DELPHI	91.2	11

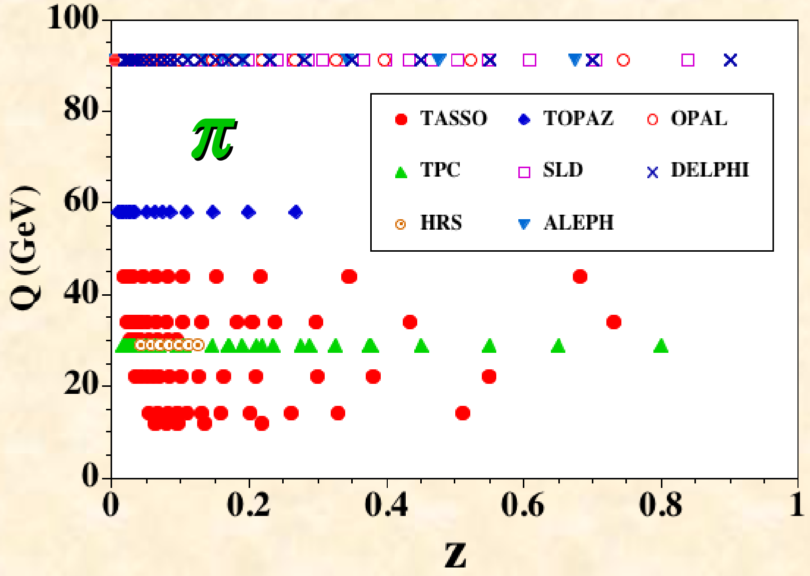
pion

Total number of data: 264

Exp. collaboration	$\sqrt{s}$ (GeV)	# of data
TASSO	12,14,22,30,34,44	29
TCP	29	18
HRS	29	2
TOPAZ	58	4
SLD	91.2	29
SLD [light quark]		29
SLD [c quark]		29
SLD [b quark]		29
ALEPH	91.2	22
OPAL	91.2	22
DELPHI	91.2	17
DELPHI [light quark]		17
DELPHI [b quark]		17

One could foresee the difficulty in getting reliable FFs for f_0 *at this stage*.





Results on the fragmentation functions

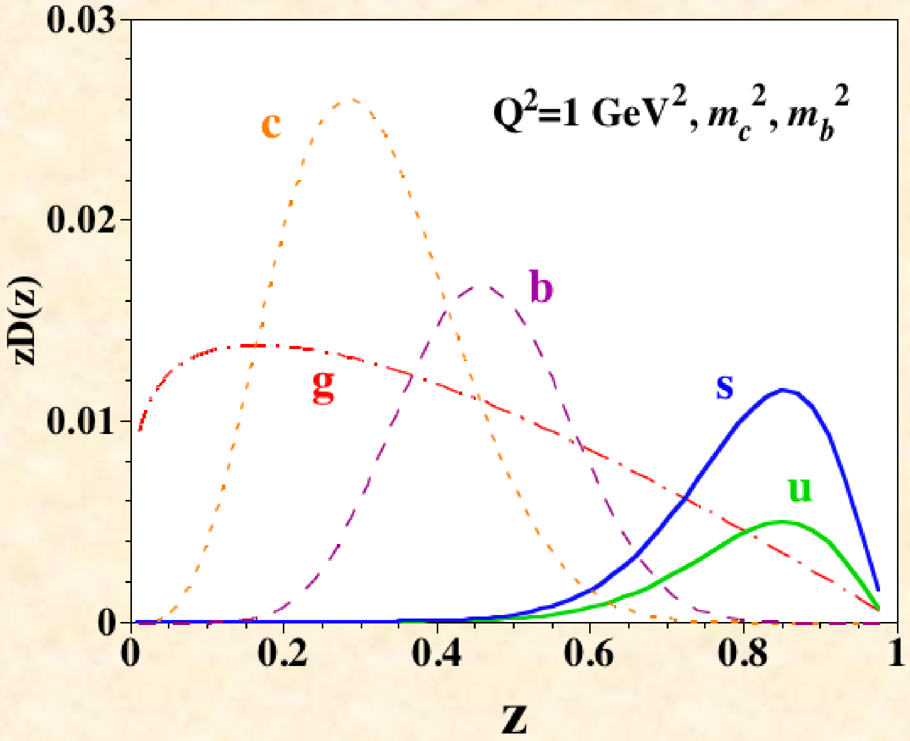
- **Functional forms**

  (1) $D_u^{f_0}(z)$, $D_s^{f_0}(z)$ have peaks at large z

  (2) $z_u^{\max} \sim z_s^{\max}$

(1) and (2) indicate tetraquark structure

$$f_0 \sim \frac{1}{\sqrt{2}}(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})$$



- **2nd moments:** $\frac{M_u}{M_s} = 0.43$

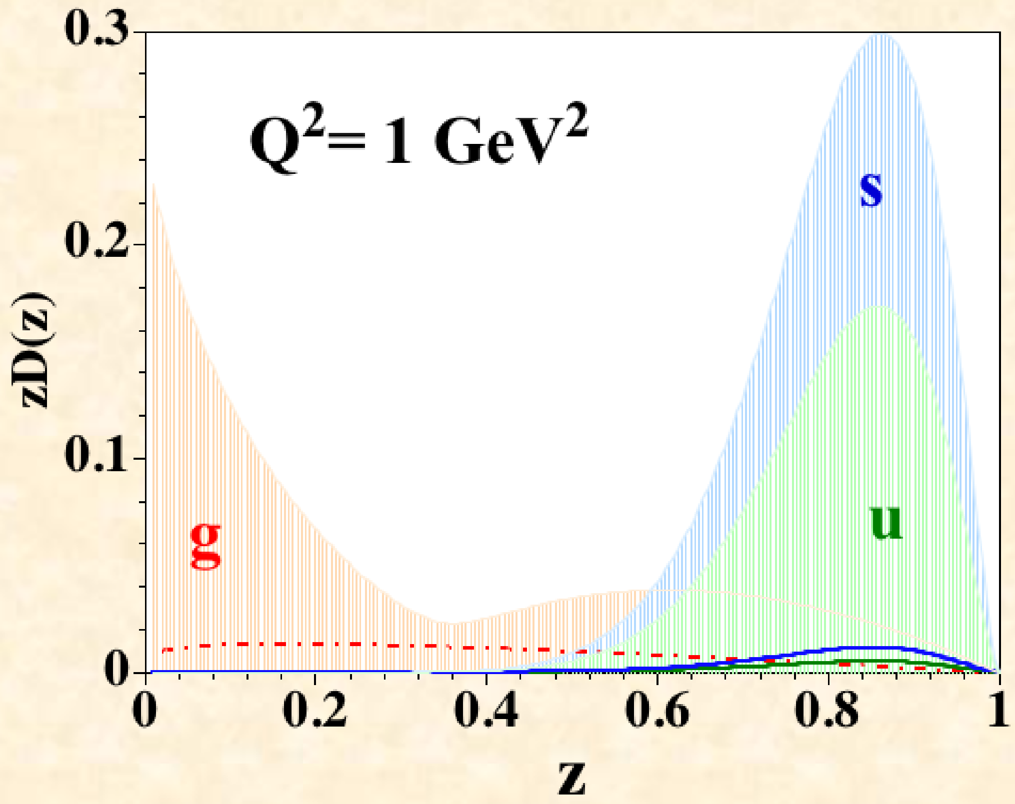
This relation indicates $s\bar{s}$-like structure (or admixture)

$$f_0 \sim s\bar{s}$$

$\Rightarrow$ **Why do we get the conflicting results?**

$\rightarrow$ **Uncertainties of the FFs should be taken into account (next page).**

Large uncertainties



2nd moments

$$M_u = 0.0012 \pm 0.0107$$
$$M_s = 0.0027 \pm 0.0183$$
$$M_g = 0.0090 \pm 0.0046$$

$$\rightarrow M_u / M_s = 0.43 \pm 6.73$$

The uncertainties are order-of-magnitude larger than the distributions and their moments themselves.

At this stage, the determined FFs are not accurate enough to discuss internal structure of $f_0(980)$.

→ Accurate data are awaited not only for $f_0(980)$ but also for other exotic and “ordinary” hadrons.

Summary II

Exotic hadrons could be found by studying fragmentation functions. As an example, the $f_0(980)$ meson was investigated.

(1) We proposed to use 2nd moments and functional forms as criteria for finding quark configuration.

(2) Global analysis of $e^+ + e^- \to f_0 + X$ data

The results *may* indicate $s\bar{s}$ or $qq\bar{q}\bar{q}$ structure. However, …

- **Large uncertainties in the determined FFs**

→ The obtained FFs are not accurate enough to discuss the quark configuration of $f_0(980)$.

(3) Accurate experimental data are important

→ Small-Q^2 data as well as large-Q^2 (M_z^2) ones

→ *c*- and *b*-quark tagging

Requests for experimentalist

- **Accurate data on $f_0(980)$ and other exotic hadrons, as well as ordinary ones**

- **Accurate data especially at small Q^2**

  e.g. Belle, c.m. energy = 10.58 GeV

  → Determination of scaling violation
  (mainly, gluon fragmentation function)

Our theoretical effort …

The End

The End