

3-BODY STRUCTURE OF EXOTICS HADRONS

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EXHIC 2010, KYOTO, 20 MAY 2010

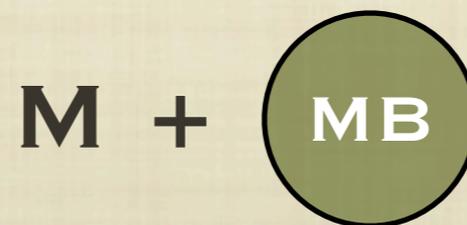
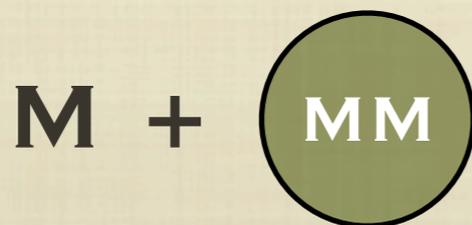
INTRODUCTION

- UNTIL RECENTLY, MESON-MESON AND MESON-BARYON STRUCTURE OF MESON AND BARYON RESONANCES HAS BEEN EXPLORED EXTENSIVELY USING CHIRAL DYNAMICS, E.G. :

$\sigma(600), f_0(980), a_0(980), \kappa, \Lambda(1405), \Lambda(1520), etc.$ ^{1, 2, 3, 4,5}



- SOME STATES IN THE MESON & BARYON SPECTRUM, HOWEVER, COULD VERY WELL POSSESS A MORE COMPLICATED MOLECULAR STRUCTURE



¹ J. A. OLLER, E. OSET, NUCL. PHYS. A 620 (1997) 438.

² J. A. OLLER, ULF-G. MEISSNER, PHYS. LETT. B 500 (2001) 263-272.

³ J. A. OLLER, E. OSET, J. R. PELÁEZ, PHYS. REV. D 59 074001 (199).

⁴ D. JIDO, J. A. OLLER, E. OSET, A. RAMOS, U. G. MEISSNER, NUCL. PHYS. A 725 (2003) 181-200.

⁵ L. ROCA, SOURAV SARKAR, V.K. MAGAS, E. OSET PHYS. REV. C73, 045208 (2006).

INDEED, THERE ARE EVIDENCES FOR SIGNIFICANT BRANCHING RATIOS TO THREE-HADRONS FOR SOME RESONANCES, E.G.,

$N(1710)$ DECAY MODES

The following branching fractions are our estimates, not fits or averages.

Mode	Fraction (Γ_i/Γ)
Γ_1 $N\pi$	10–20 %
Γ_2 $N\eta$	(6.2 ± 1.0) %
Γ_3 $N\omega$	(13.0 ± 2.0) %
Γ_4 ΛK	5–25 %
Γ_5 ΣK	
Γ_6 $N\pi\pi$	40–90 %
Γ_7 $\Delta\pi$	15–40 %
Γ_8 $\Delta(1232)\pi$, <i>P</i> -wave	
Γ_9 $N\rho$	5–25 %
Γ_{10} $N\rho$, $S=1/2$, <i>P</i> -wave	
Γ_{11} $N\rho$, $S=3/2$, <i>P</i> -wave	
Γ_{12} $N(\pi\pi)_{S\text{-wave}}^{I=0}$	10–40 %
Γ_{13} $p\gamma$	0.002–0.05%
Γ_{14} $p\gamma$, helicity=1/2	0.002–0.05%
Γ_{15} $n\gamma$	0.0–0.02%
Γ_{16} $n\gamma$, helicity=1/2	0.0–0.02%

$\Sigma(1620)$ DECAY MODES (PRODUCTION EXPERIMENTS)

Mode
Γ_1 $N\bar{K}$
Γ_2 $\Lambda\pi$
Γ_3 $\Sigma\pi$
Γ_4 $\Lambda\pi\pi$
Γ_5 $\Sigma(1385)\pi$
Γ_6 $\Lambda(1405)\pi$

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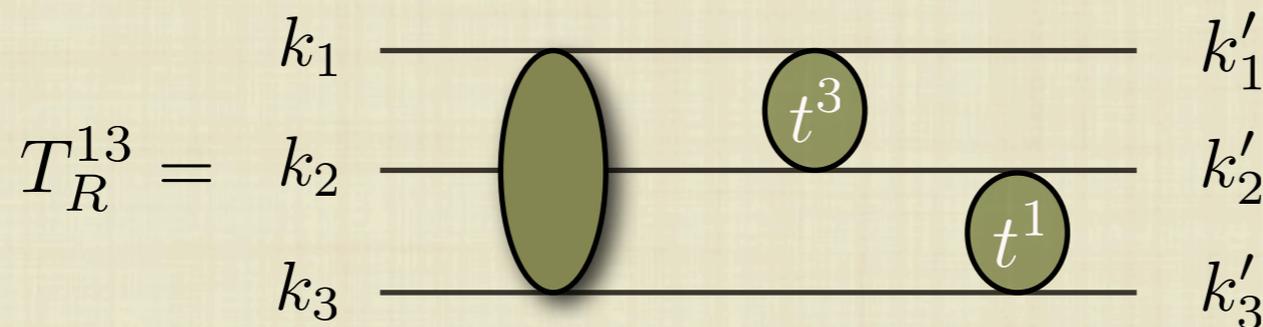
THE MODEL

- WE SOLVE THE FADDEEV EQUATIONS

$$T = T^1 + T^2 + T^3$$

$$T^i = t^i \delta^3(\vec{k}'_i - \vec{k}_i) + T_R^{ij} + T_R^{ik}$$

- THE T_R^{ij} MATRICES CONTAIN ALL THE POSSIBLE DIAGRAMS WHERE THE LAST TWO SUCCESSIVE INTERACTIONS ARE T^i AND T^j



- AND THEY SATISFY THE EQUATIONS:

$$T_R^{12} = t^1 g^{12} t^2 + t^1 \left[G^{121} T_R^{21} + G^{123} T_R^{23} \right]$$

$$T_R^{13} = t^1 g^{13} t^3 + t^1 \left[G^{131} T_R^{31} + G^{132} T_R^{32} \right]$$

$$T_R^{21} = t^2 g^{21} t^1 + t^2 \left[G^{212} T_R^{12} + G^{213} T_R^{13} \right]$$

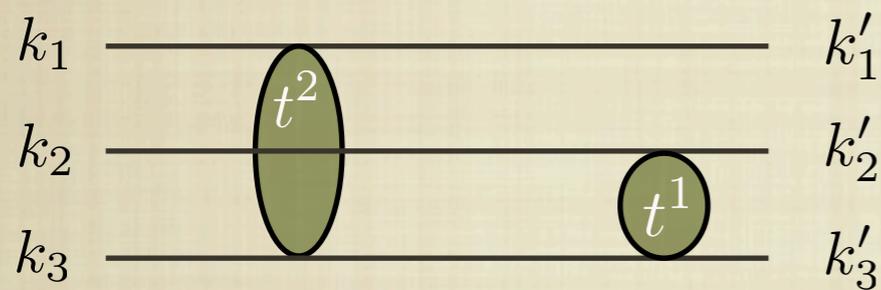
$$T_R^{23} = t^2 g^{23} t^3 + t^2 \left[G^{231} T_R^{31} + G^{232} T_R^{32} \right]$$

$$T_R^{31} = t^3 g^{31} t^1 + t^3 \left[G^{312} T_R^{12} + G^{313} T_R^{13} \right]$$

$$T_R^{32} = t^3 g^{32} t^2 + t^3 \left[G^{321} T_R^{21} + G^{323} T_R^{23} \right]$$

■ t^l IS THE TWO BODY t -MATRIX $\longrightarrow t = V + V\tilde{g}t$

■ g^{ij} IS THE THREE-BODY GREEN FUNCTION.

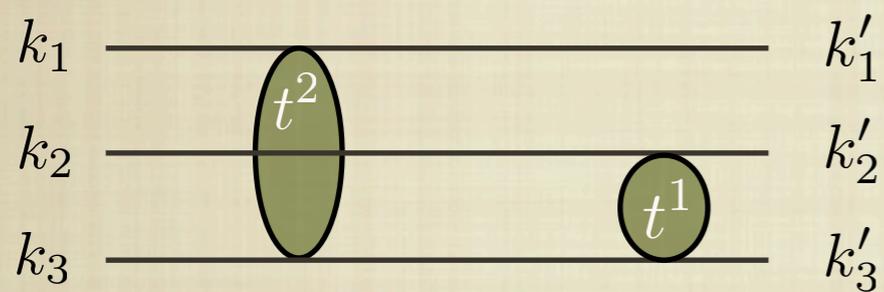


$$g^{ij}(\vec{k}_i', \vec{k}_j) = \left(\prod_{r=1}^D \frac{N_r}{2E_r} \right) \frac{1}{\sqrt{s} - E_i(\vec{k}_i') - E_l(\vec{k}_i' + \vec{k}_j) - E_j(\vec{k}_j)}$$

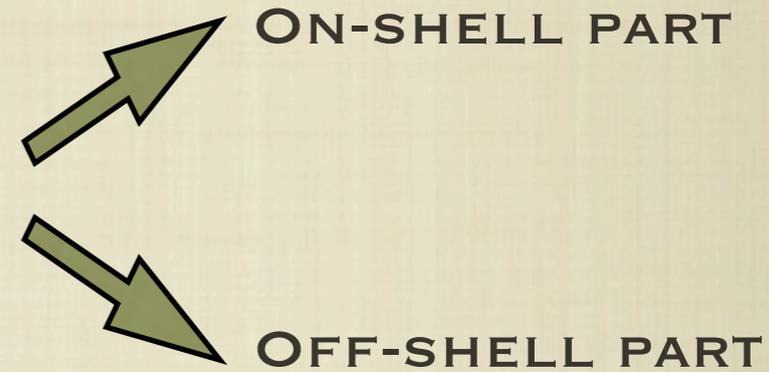
$$N_r = \begin{cases} 1 & \text{meson-meson interaction} \\ 2M_r & \text{meson-baryon interaction} \end{cases}$$

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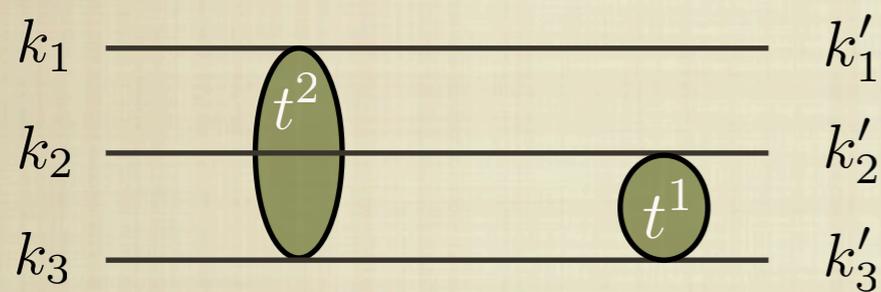


CHIRAL AMPLITUDES

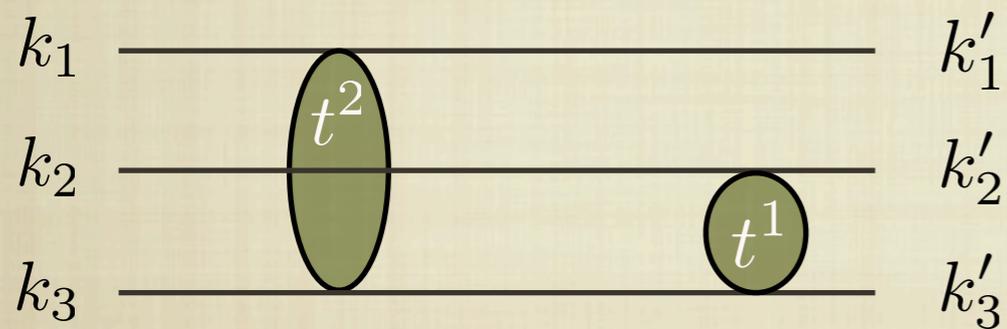


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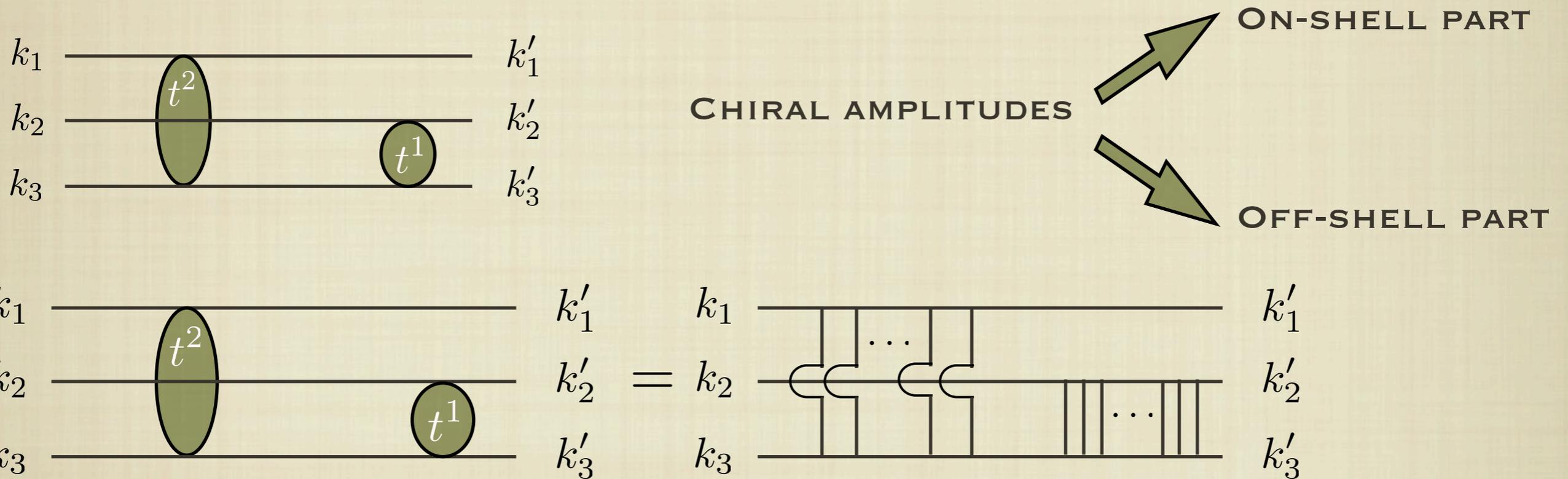


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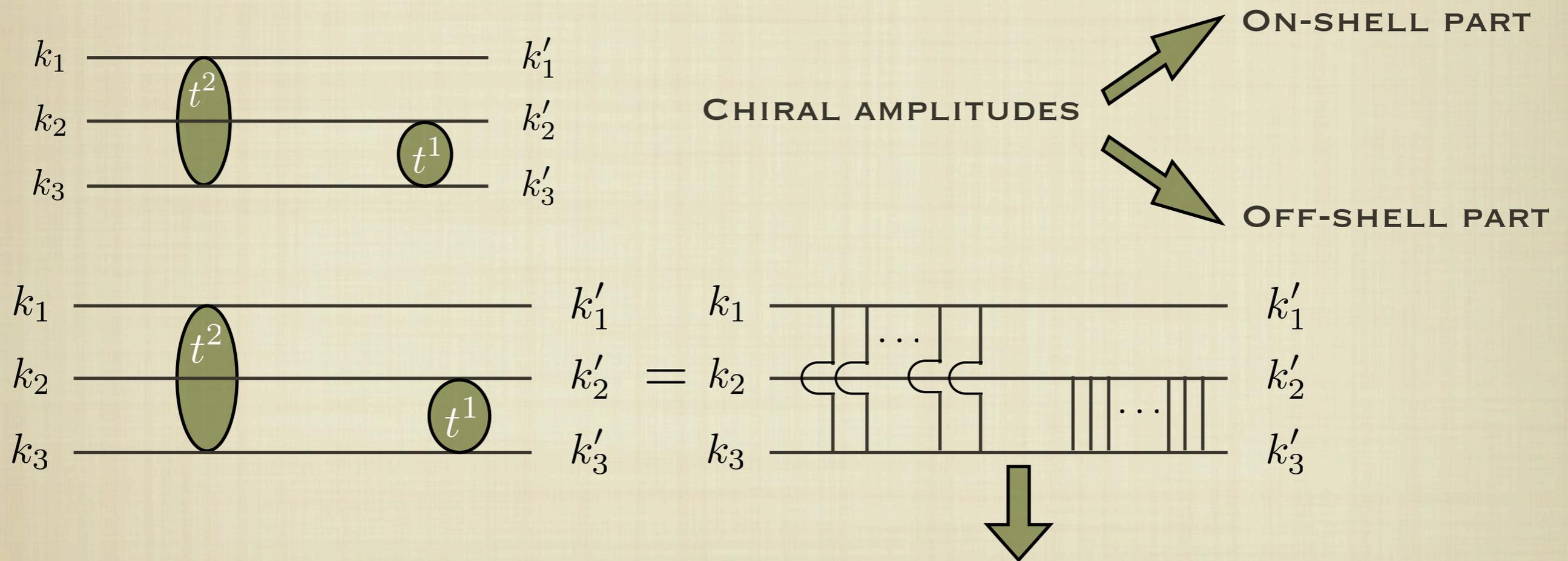
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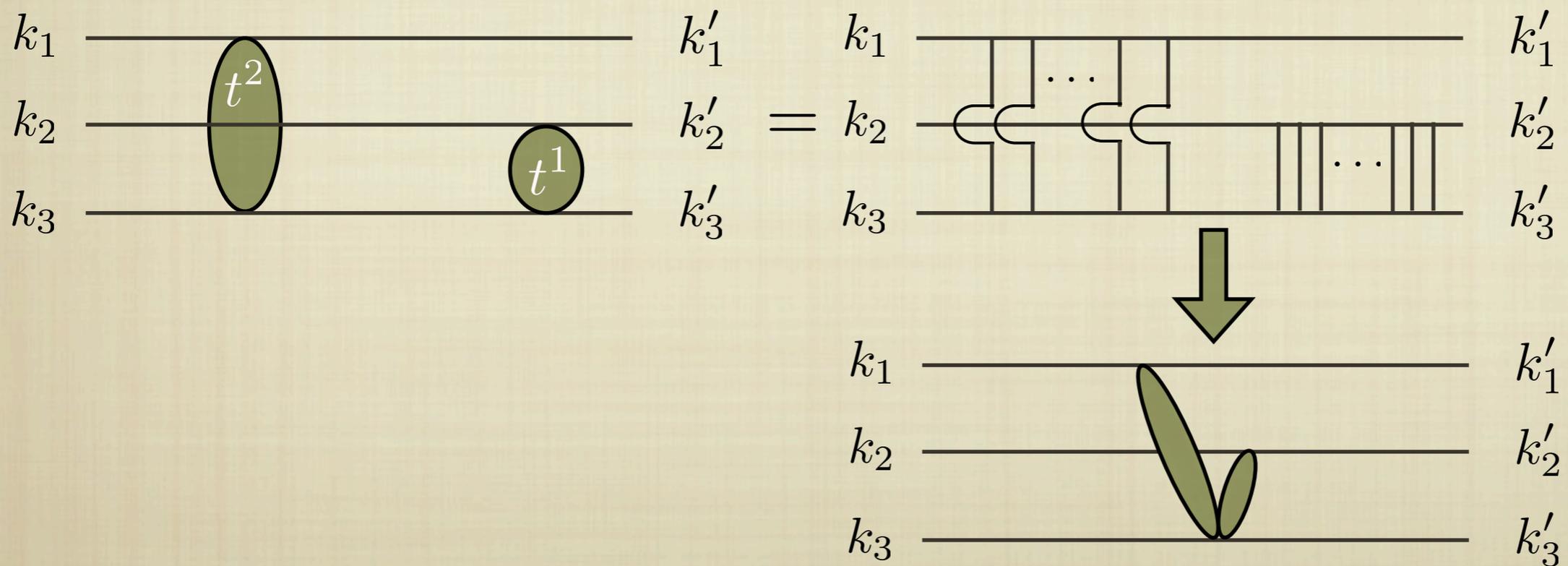
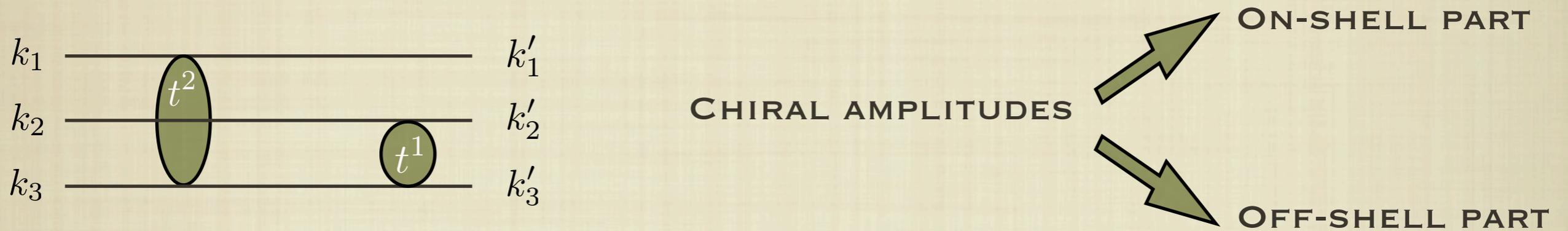
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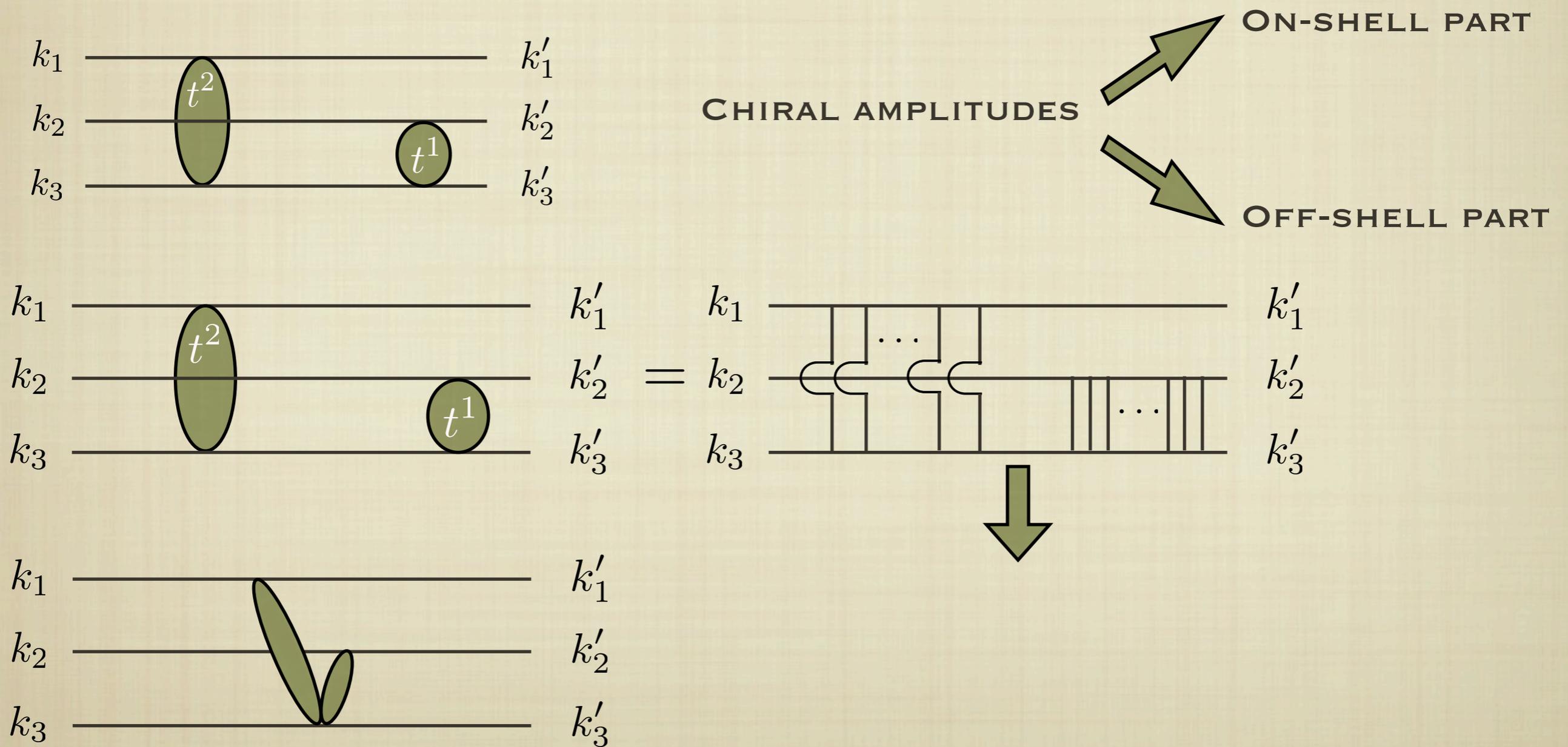
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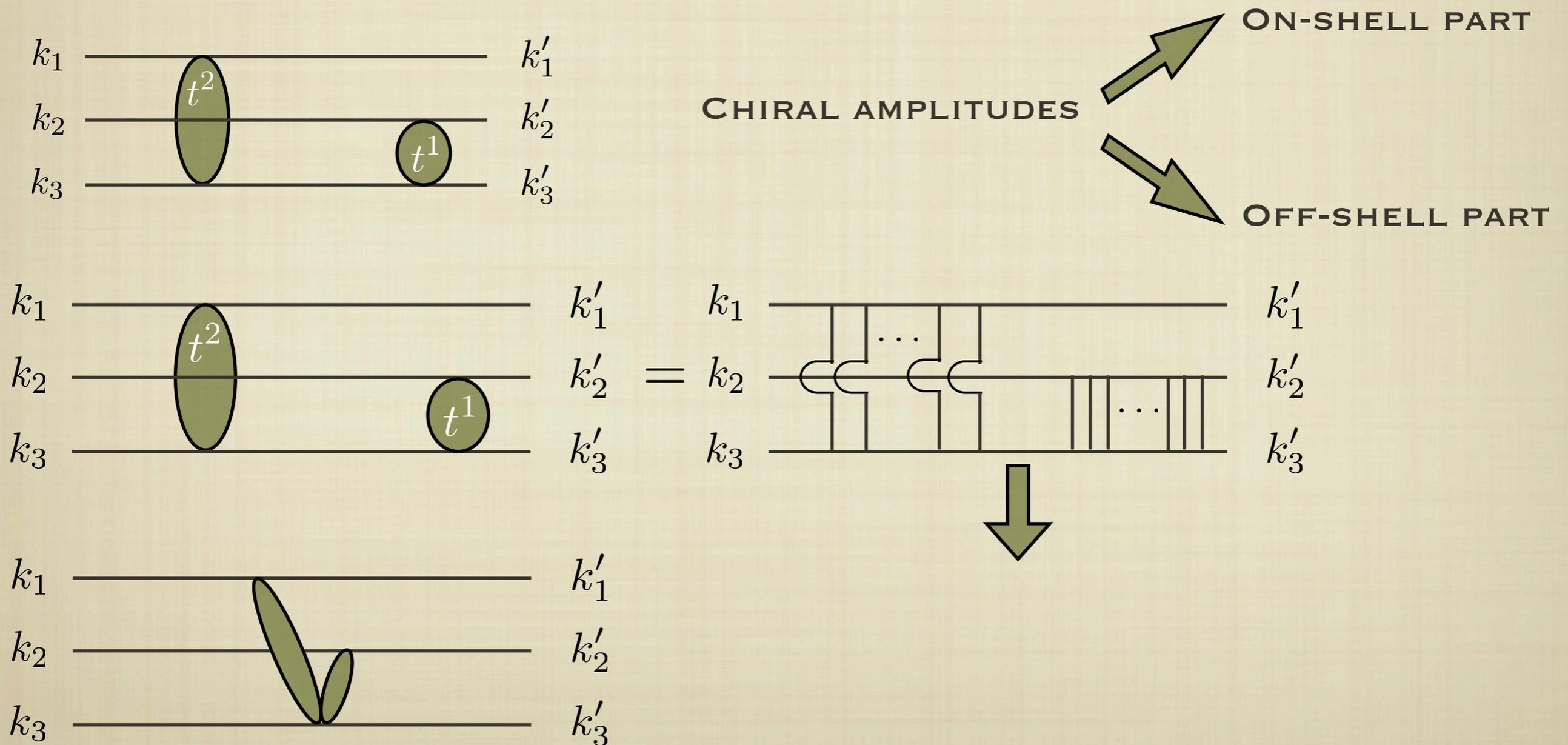
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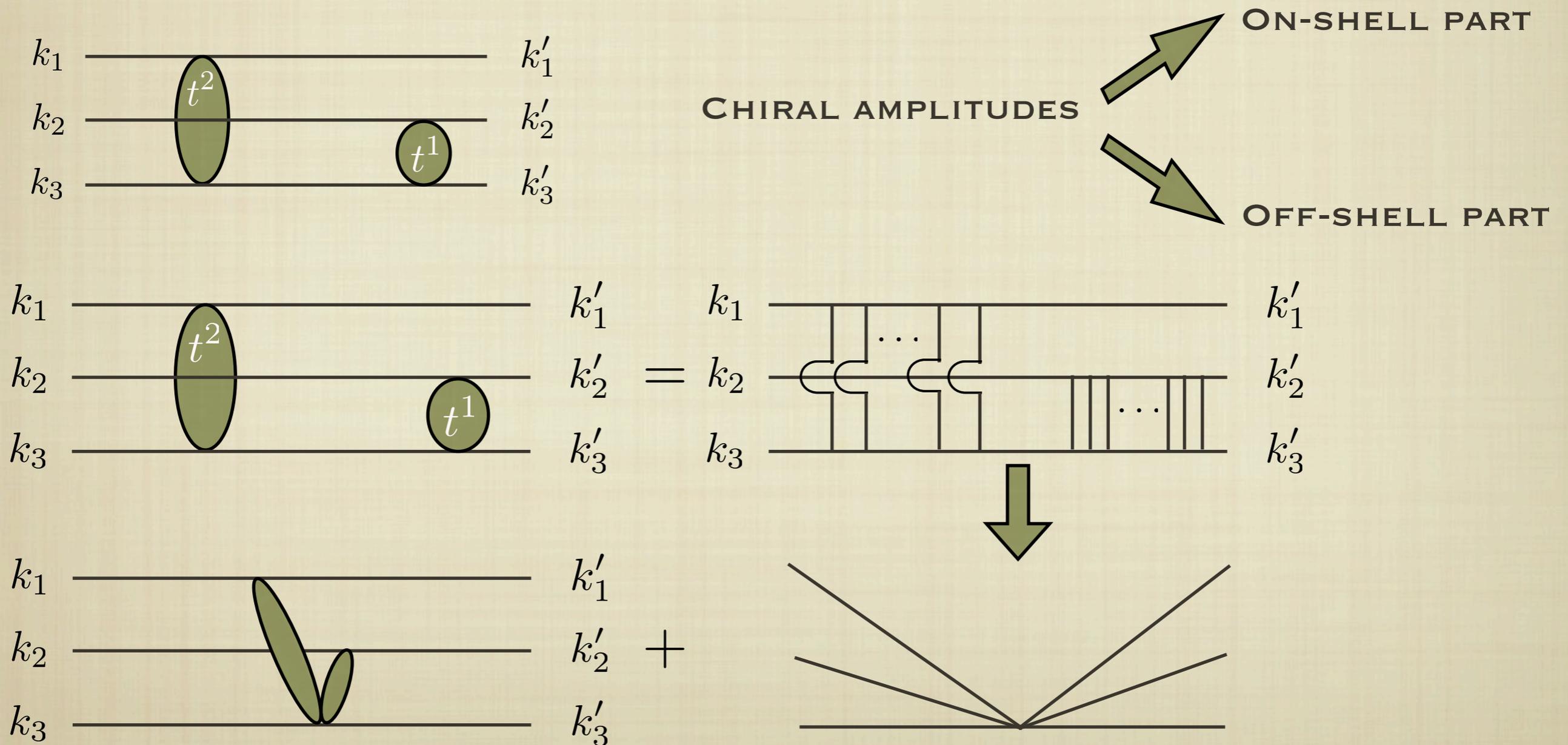
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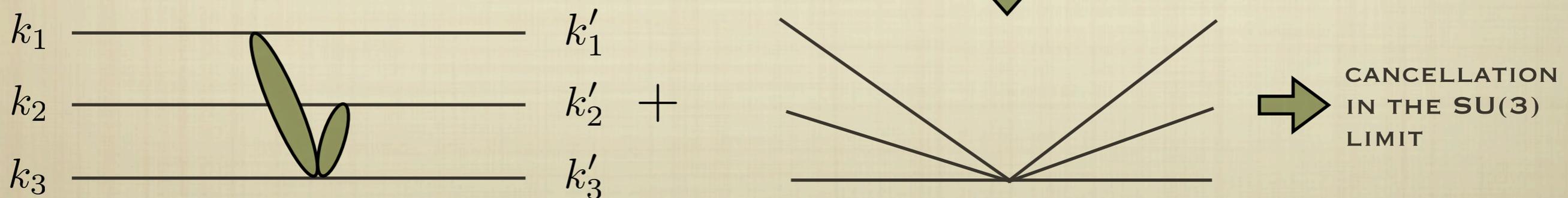
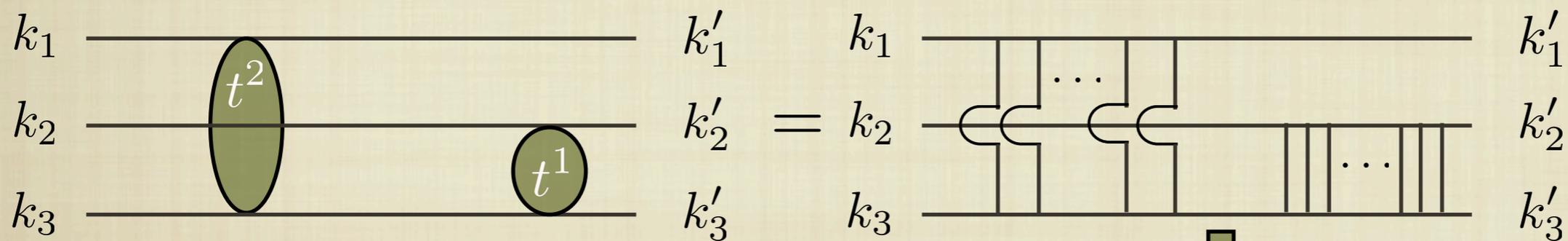
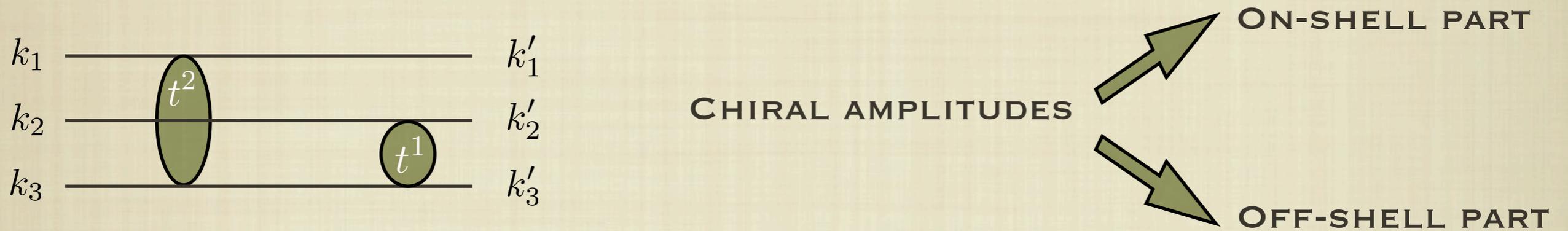
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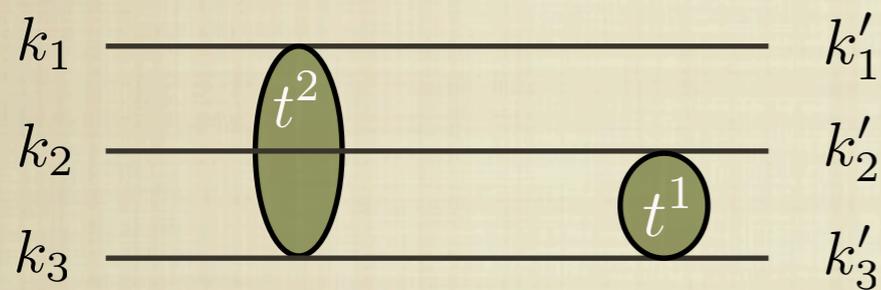
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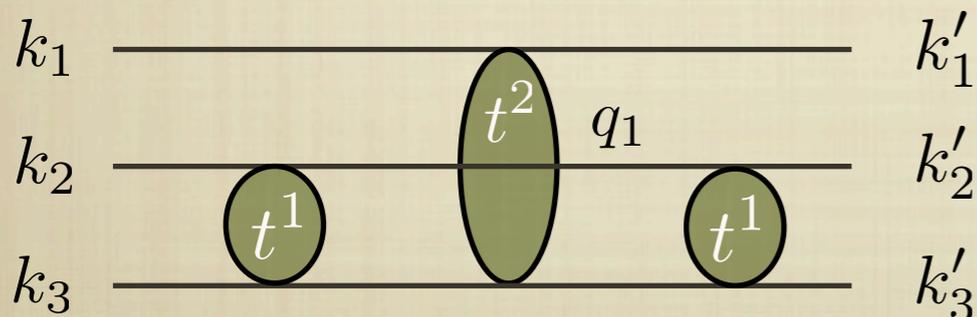
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$$N_r = \begin{cases} 1 & \text{meson-meson interaction} \\ 2M_r & \text{meson-baryon interaction} \end{cases}$$

■ G^{ijk} IS THE LOOP FUNCTION FOR DIAGRAMS INVOLVING THREE t MATRICES.



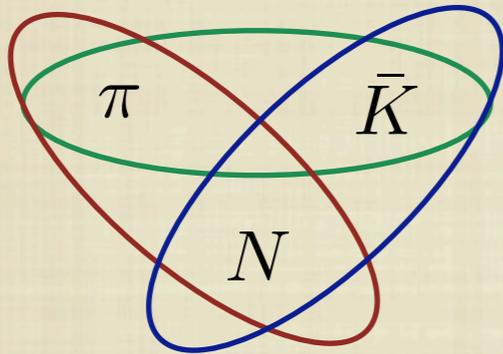
$$\int \frac{d^3 q_1}{(2\pi)^3} t^1(\sqrt{s_{23}}) g^{12} t^2(q_1^2) g^{21} t^1(\sqrt{s_{23}})$$

$$G^{121} = \int \frac{d^3 q_1}{(2\pi)^3} g^{12} t^2(q_1^2) g^{21} [g^{21}(\vec{k}'_2, \vec{k}_1)]^{-1} [t^2(\sqrt{s_{23}})]^{-1}$$

$$t^1(\sqrt{s_{23}}) G^{121} t^2(\sqrt{s_{13}}) g^{21}(\vec{k}'_2, \vec{k}_1) t^1(\sqrt{s_{23}})$$

THE $\pi\bar{K}N$ SYSTEM

- WE STARTED STUDYING THE $\pi\bar{K}N$ SYSTEM:



$$\Lambda(1405) \Rightarrow \bar{K}N, \pi\Sigma, \pi\Lambda, \eta\Sigma, \eta\Lambda, K\Xi^7$$

$$N^*(1535) \Rightarrow \pi N, K\Sigma, K\Lambda, \eta N^8$$

$$\kappa(700) \Rightarrow \pi\bar{K}, \eta\bar{K}^9$$

- ALL THE INTERACTIONS ARE IN S-WAVE $\Rightarrow J^\pi = 1/2^+$.
- THERE ARE SOME $S=-1, 1/2^+$ BARYONIC STATES IN THE ENERGY REGION 1500-1800 MEV WHOSE PROPERTIES, AS SPIN-PARITY, ARE NOT WELL UNDERSTOOD.
- SIGNATURES OF $\Sigma(1660), \Lambda(1600)$ HAVE BEEN FOUND IN SOME RECENT EXPERIMENTS, E.G.,

$$K^-p \rightarrow \pi\pi\Sigma^{10} \quad K^-p \rightarrow \pi\pi\Lambda^{11}$$

⁷ D. JIDO, J. A. OLLER, E. OSET, A. RAMOS, U. G. MEISSNER, NUCL. PHYS. A 725 (2003) 181-200.

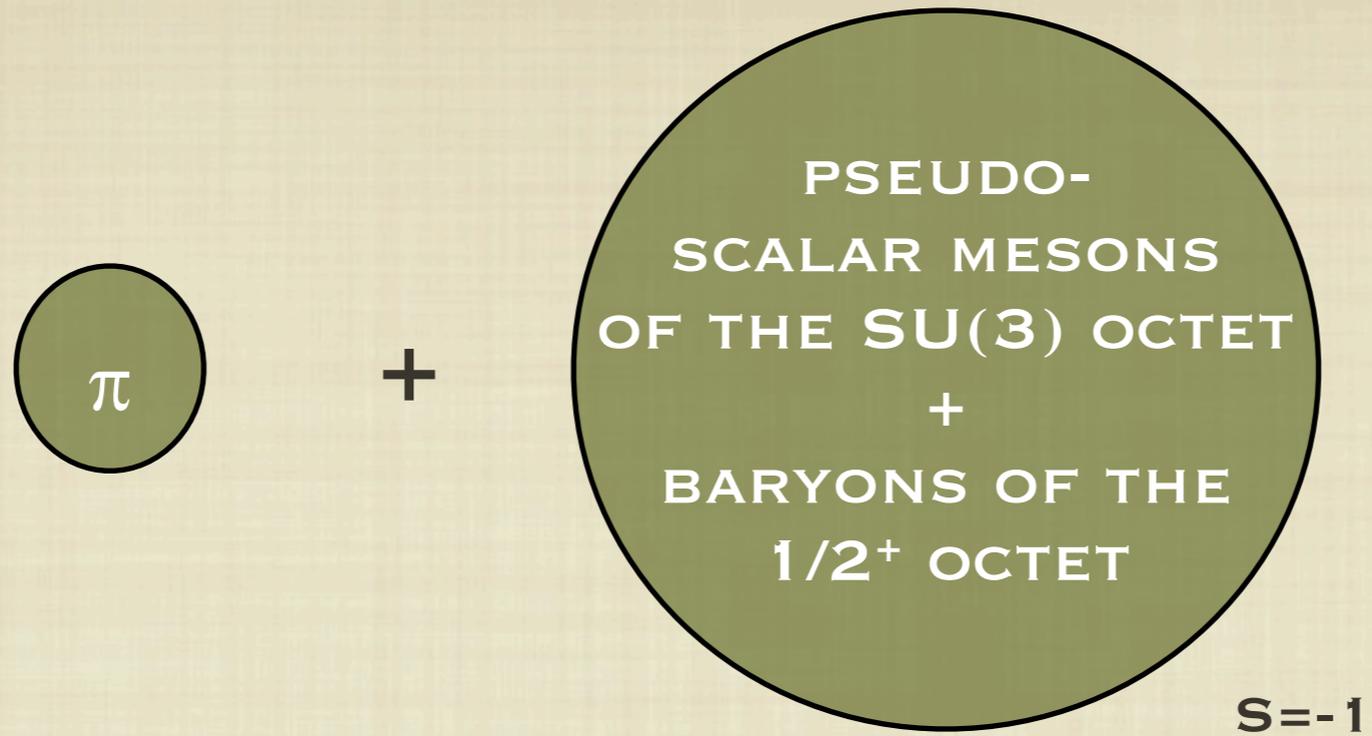
⁸ T. INOUE, E. OSET, M. J. VICENTE VACAS, PHYS. REV. C 65 035204 .

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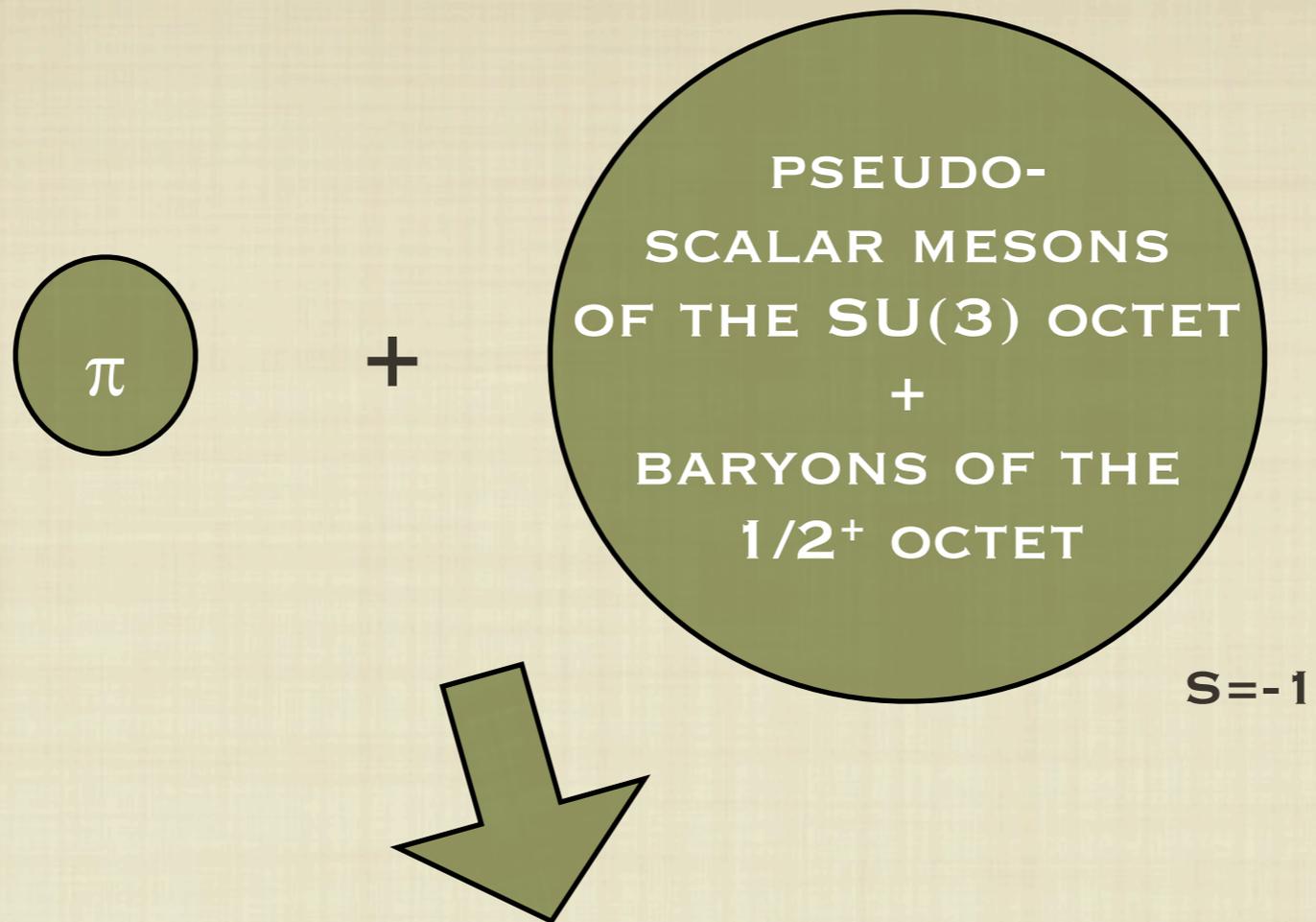
¹⁰ S. PRAKHOV ET AL. PHYS. REV. C 69, 042202 (2004).

¹¹ S. PRAKHOV ET AL. PHYS. REV. C 70, 034605 (2004).

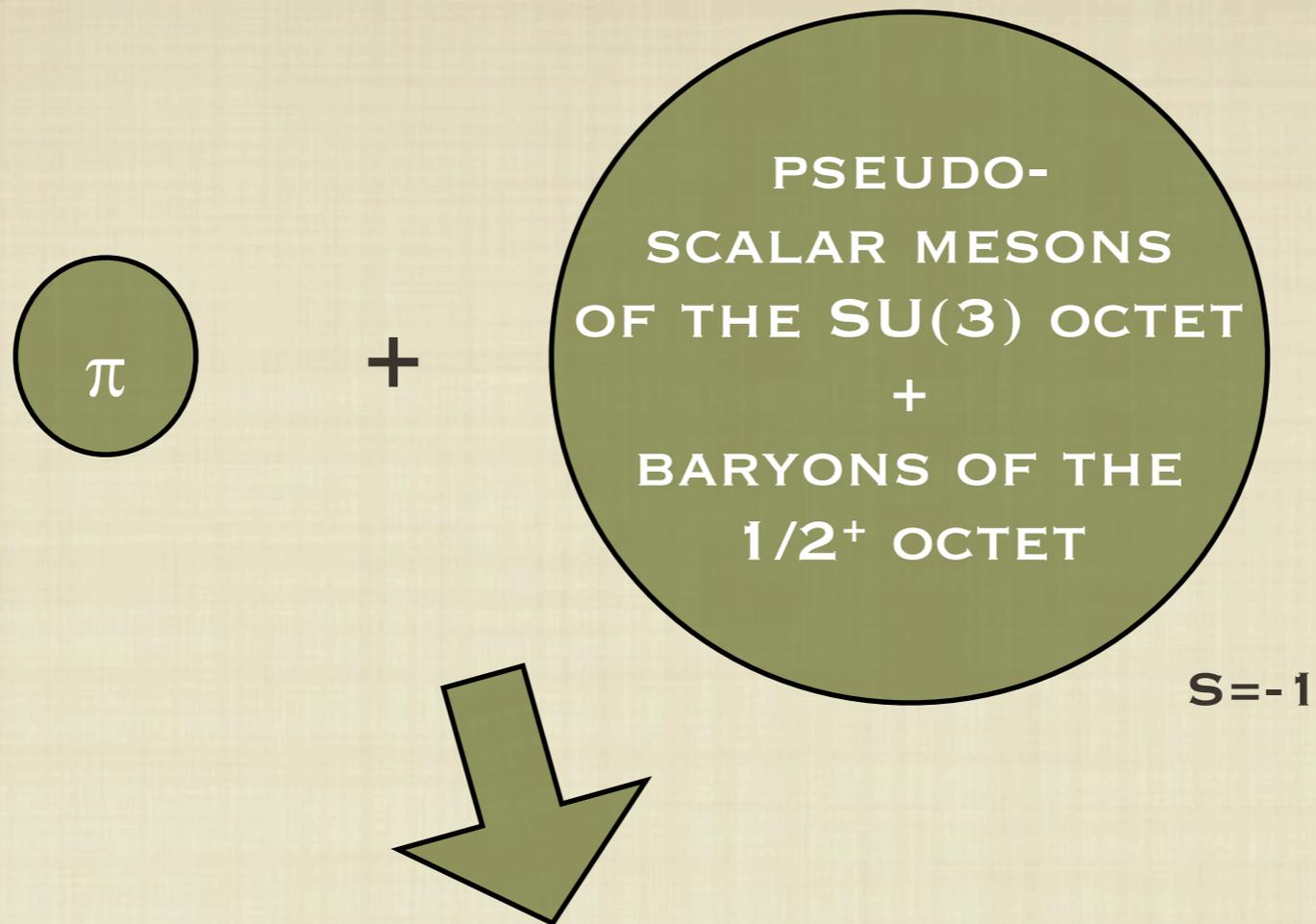
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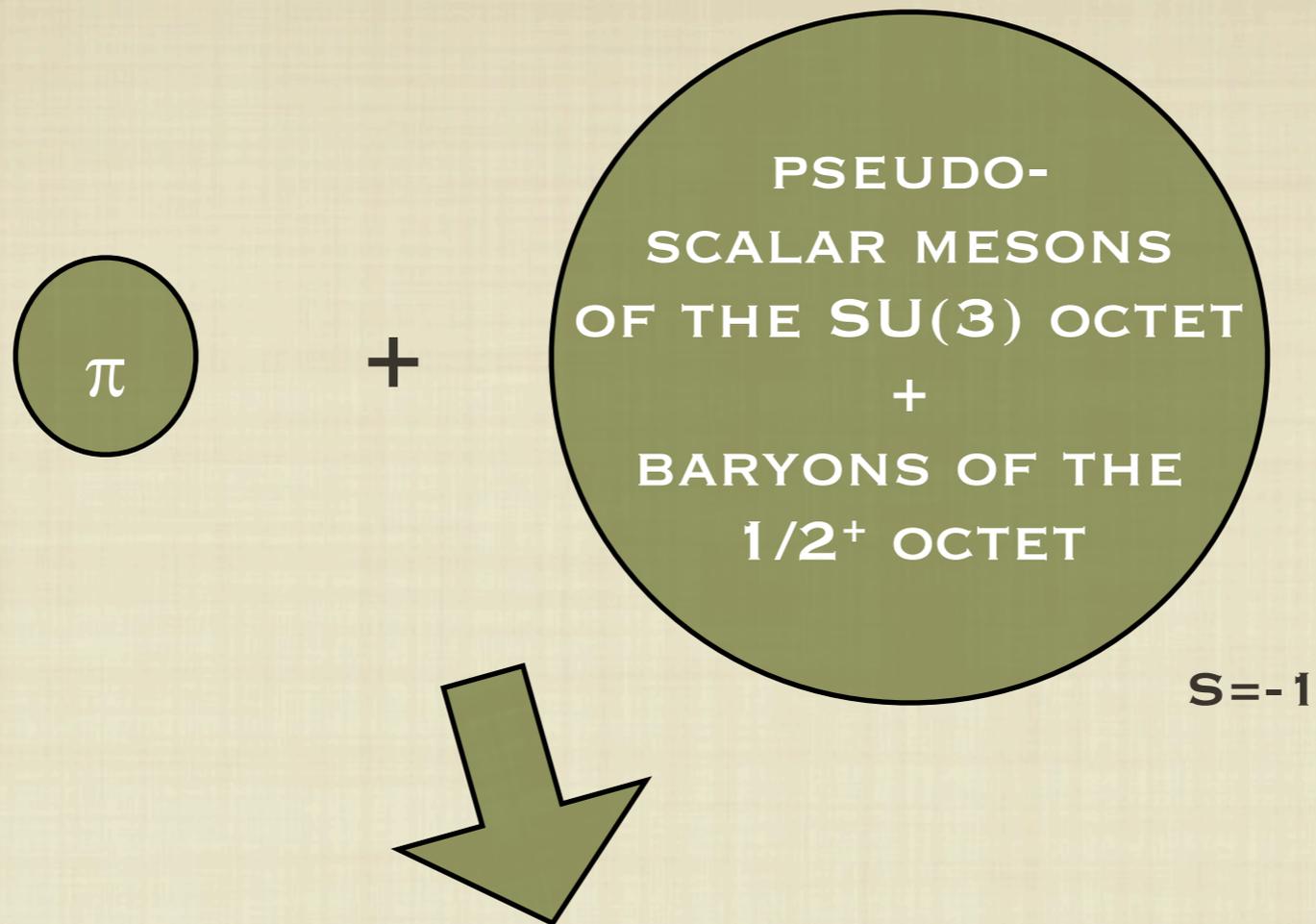


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$\pi^0 \eta \Lambda, \pi^0 K^+ \Xi^-, \pi^0 K^0 \Xi^0,$

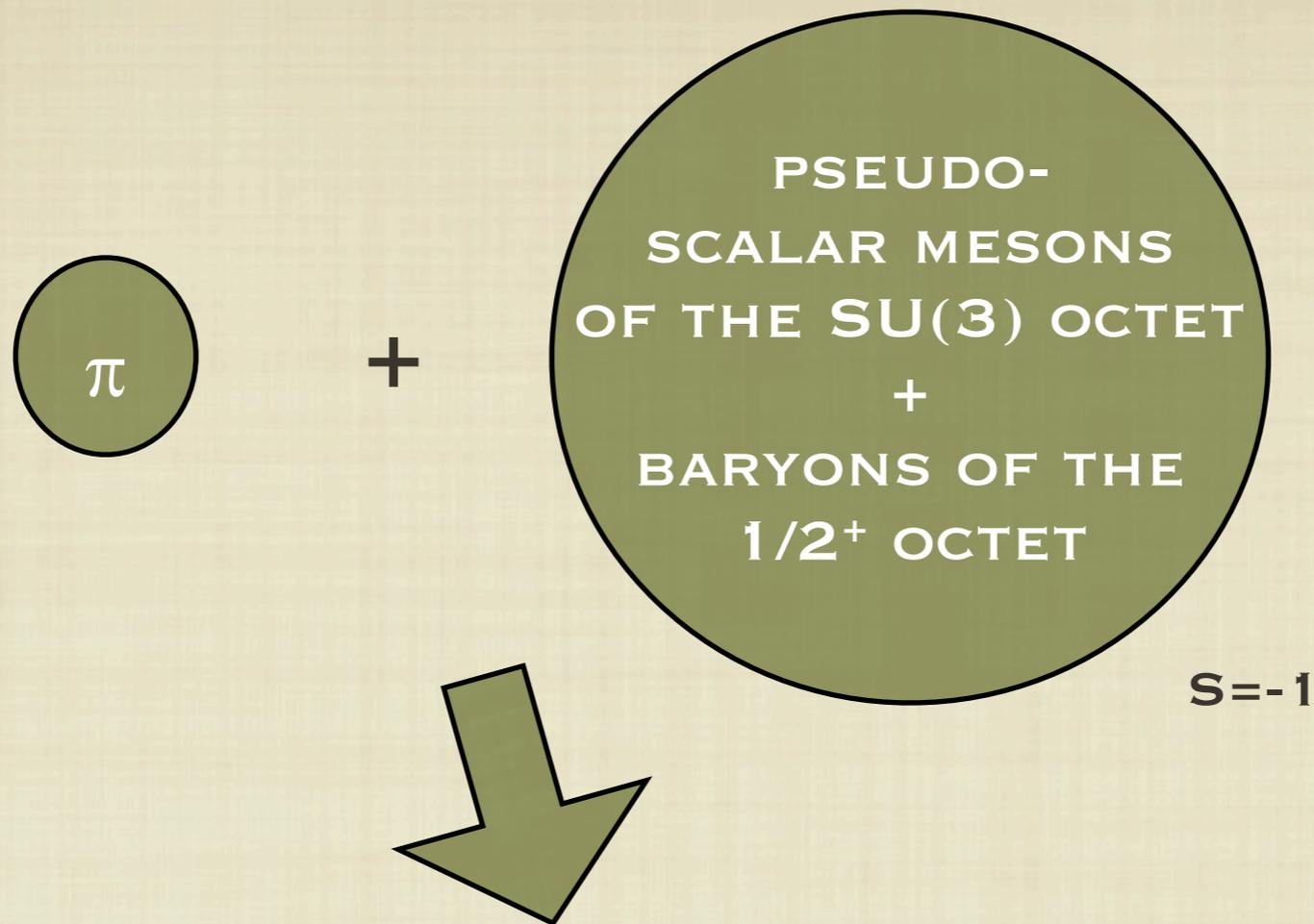
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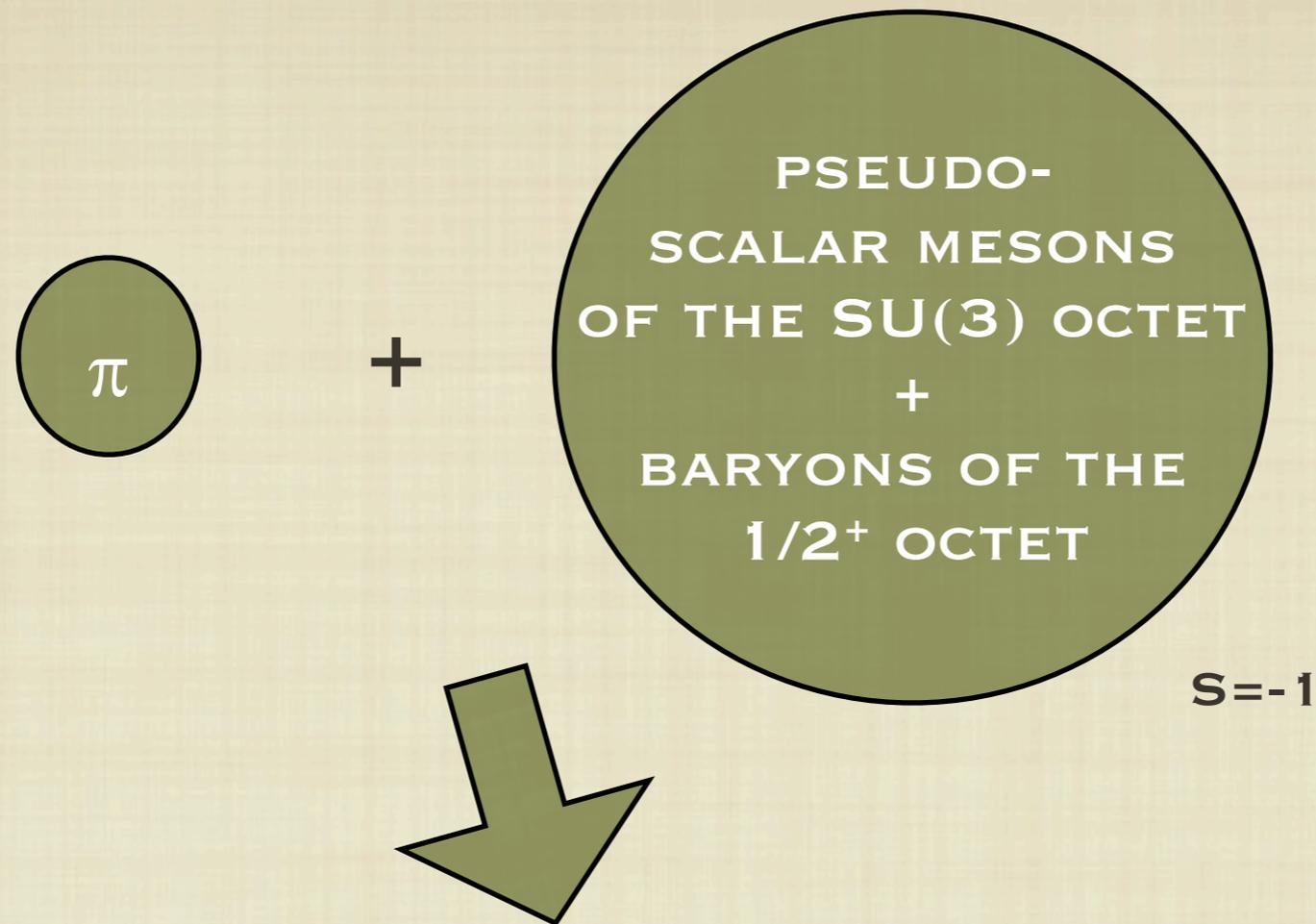


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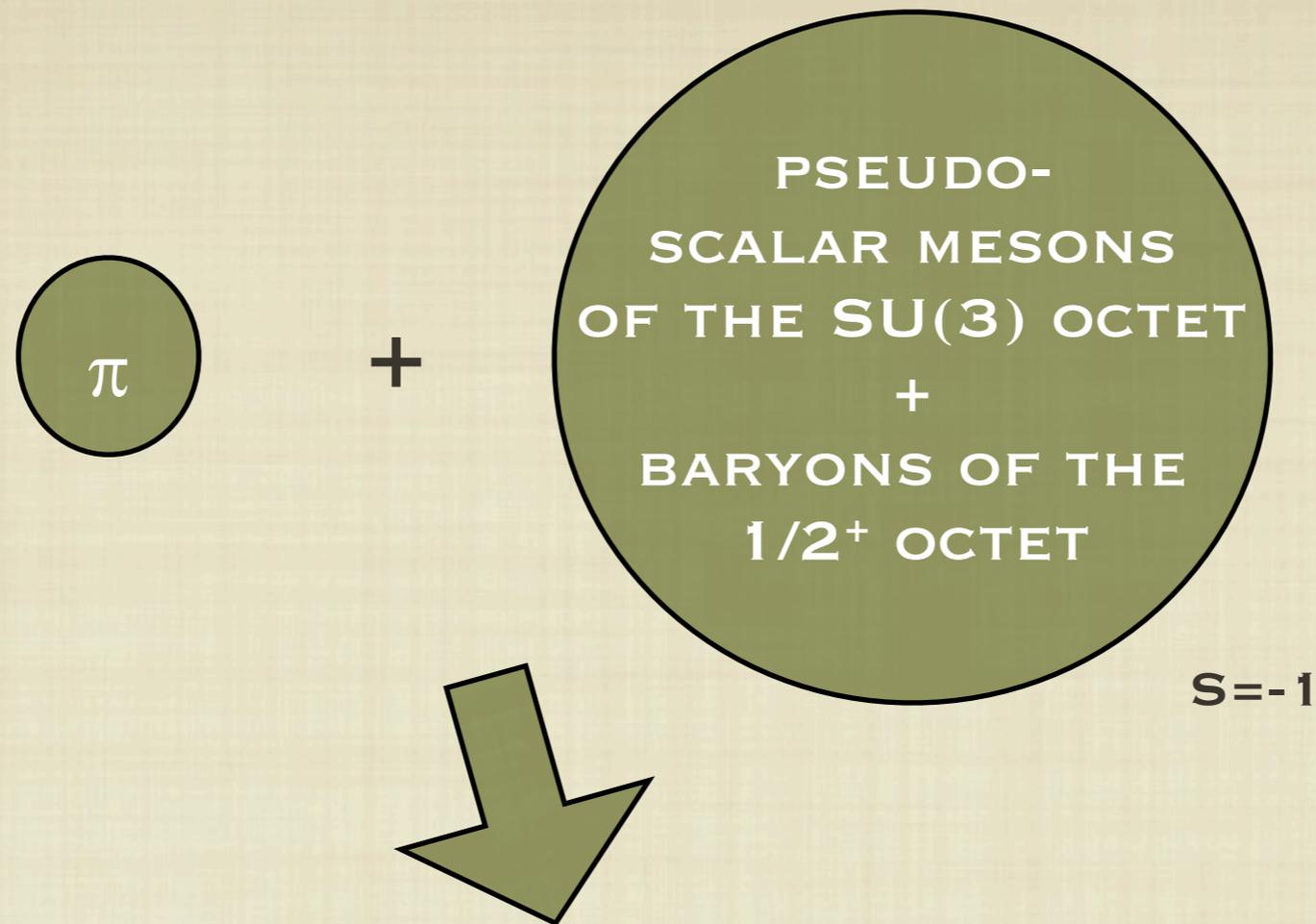


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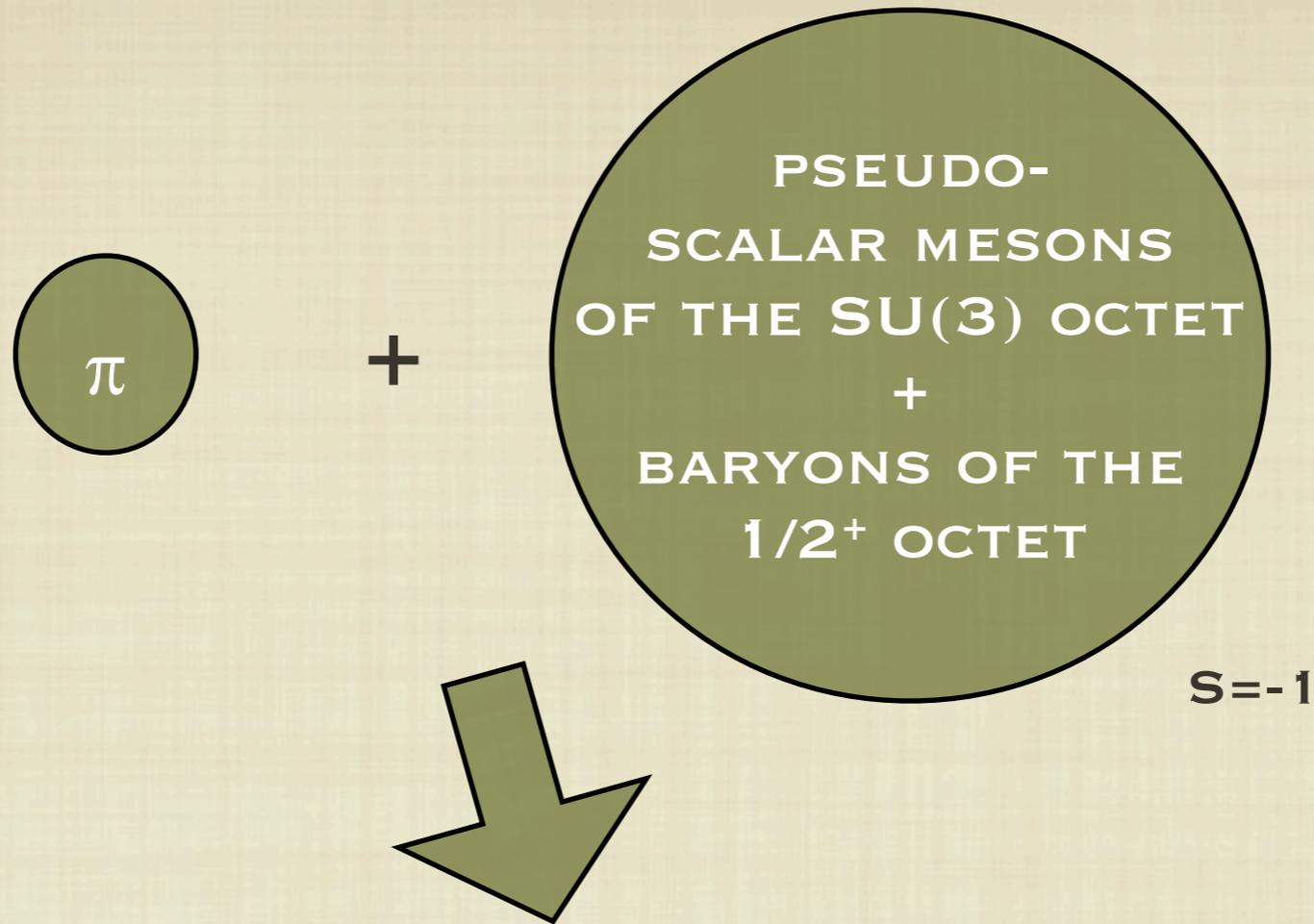
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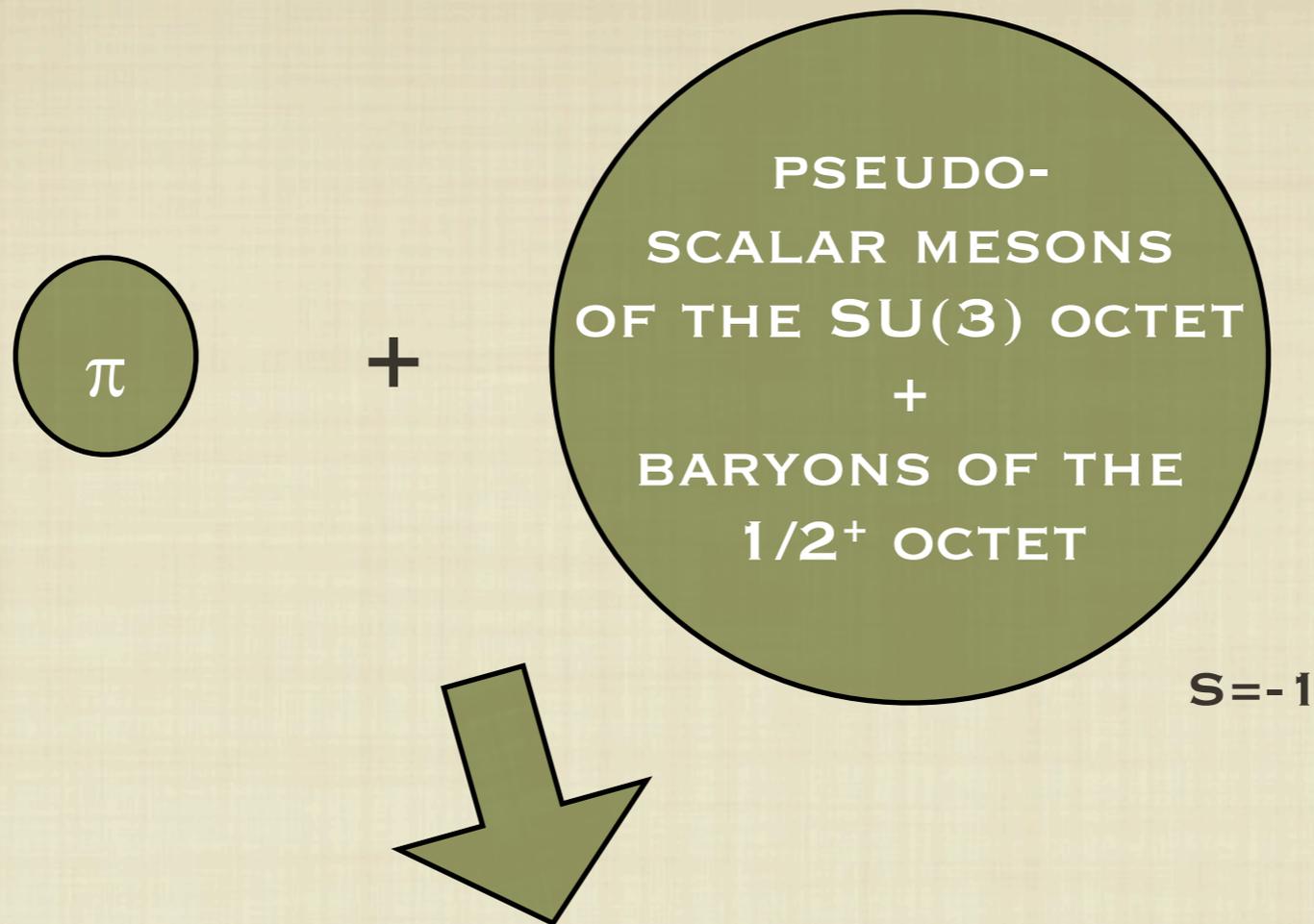
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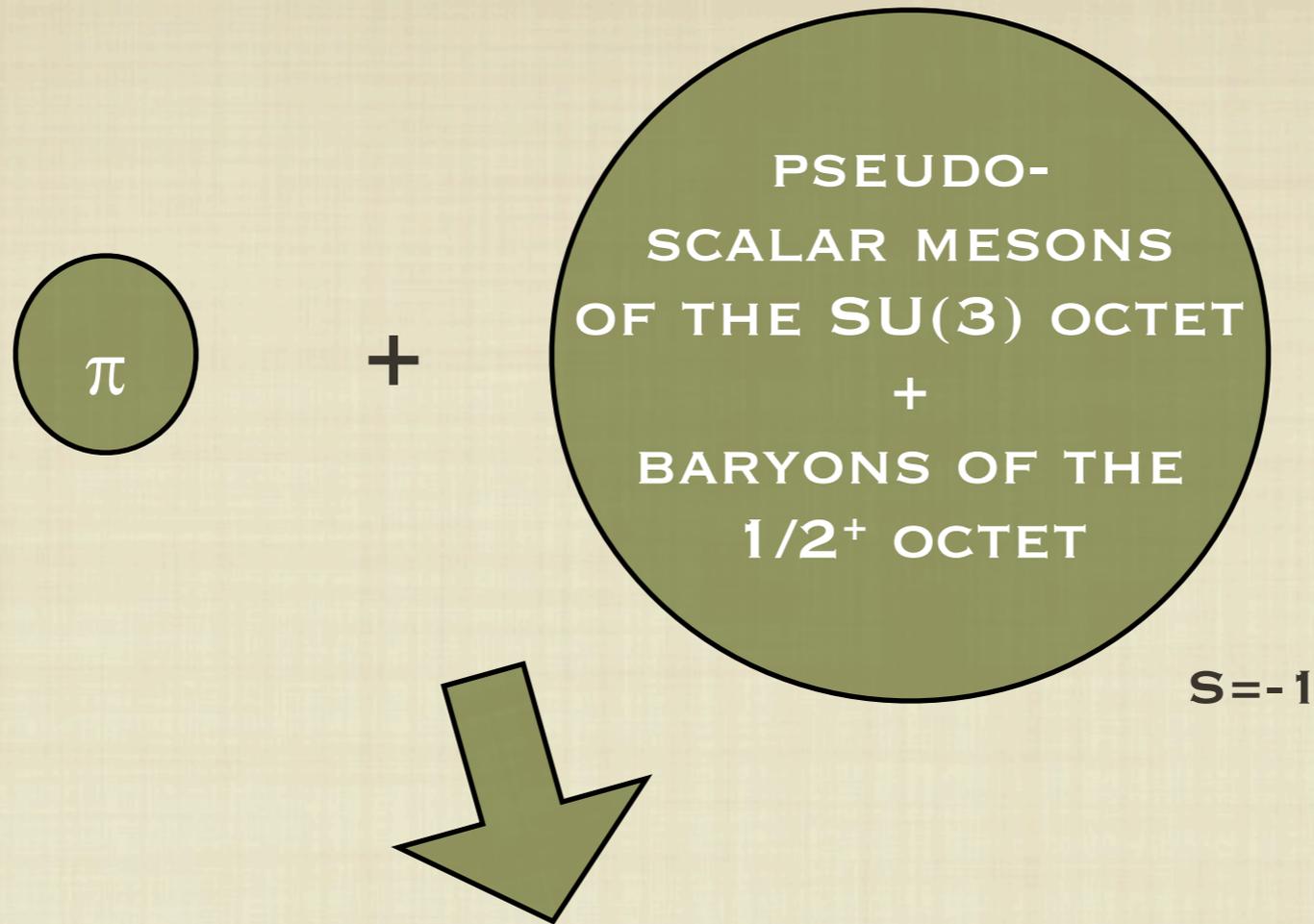
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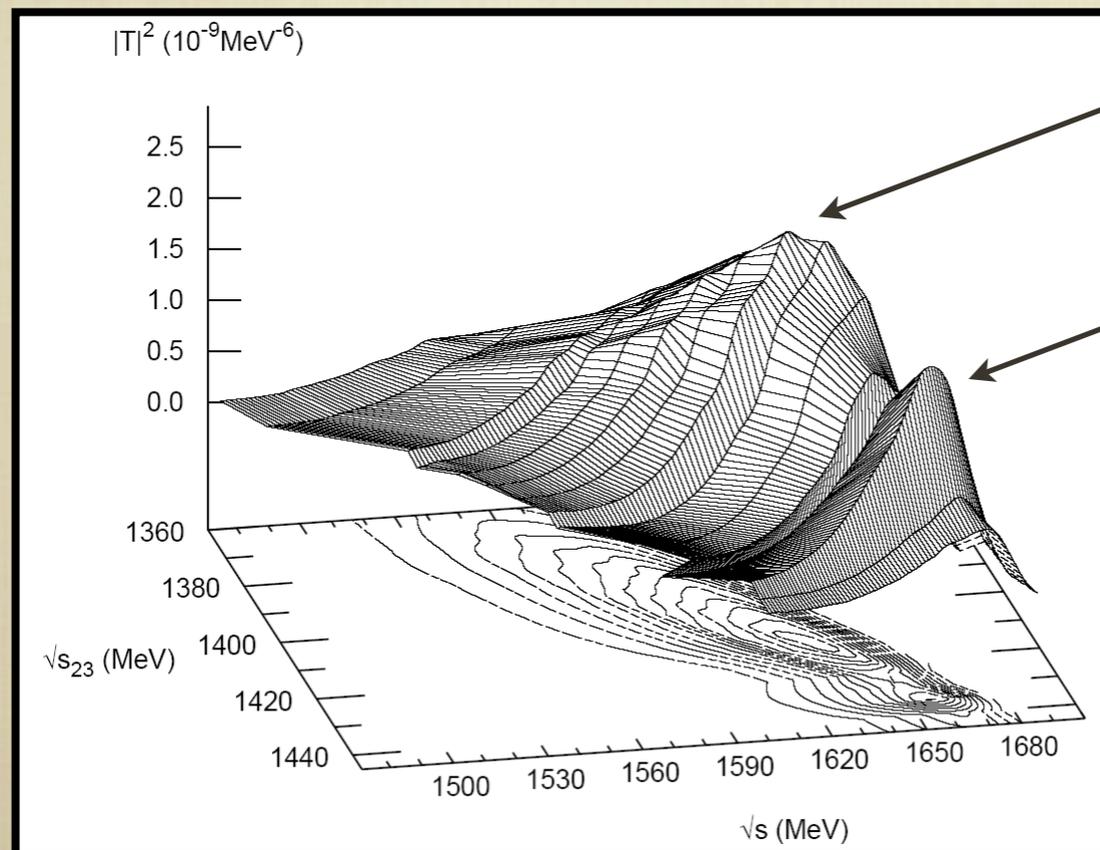
$\pi^0 \eta \Lambda, \pi^0 K^+ \Xi^-, \pi^0 K^0 \Xi^0, \pi^+ K^- n, \pi^+ \pi^0 \Sigma^-, \pi^+ K^0 \Xi^-, \pi^- \bar{K}^0 p,$
 $\pi^+ \pi^- \Sigma^0, \pi^+ \pi^- \Lambda, \pi^+ \eta \Sigma^-, \pi^- \pi^0 \Sigma^+, \pi^- \pi^+ \Sigma^0, \pi^- \pi^+ \Lambda,$
 $\pi^0 K^- p, \pi^0 \bar{K}^0 n, \pi^0 \pi^0 \Sigma^0, \pi^0 \pi^+ \Sigma^-, \pi^0 \pi^- \Sigma^+, \pi^0 \pi^0 \Lambda, \pi^0 \eta \Sigma^0, \pi^- \eta \Sigma^+, \pi^- K^+ \Xi^0$



RESULTS $\pi\bar{K}N$ SYSTEM

$\Sigma(1620) S_{11} [I(J^P)=1(1/2^-)] **$

$\Sigma(1660) P_{11} [I(J^P)=1(1/2^+)] ***$



$1630 - i39/2 \text{ MeV}$

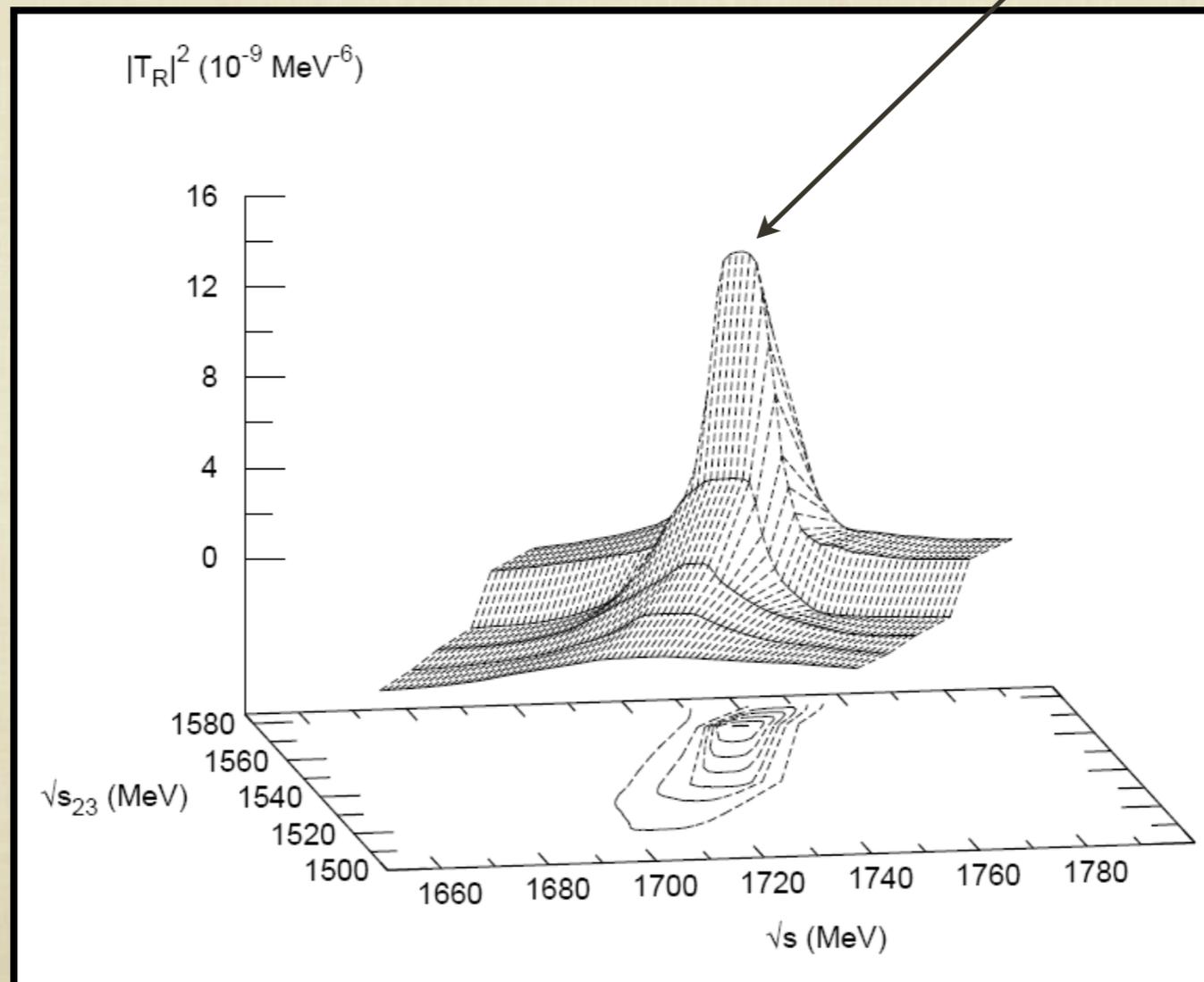
$1656 - i30/2 \text{ MeV}$

R. ARMENTEROS ET AL. NUCL. PHYS. B 8, 183 (1968).
B. R. MARTIN ET AL, NUCL. PHYS. B 127, 349 (1977).

RESULTS $\pi\bar{K}N$ SYSTEM

$\Lambda(1810)$ P_{01} [$I(J^P)=0(1/2^+)$] ***
1750 TO 1850 (~ 1810) OUR ESTIMATE

$1740 - i24/2 \text{ MeV}$



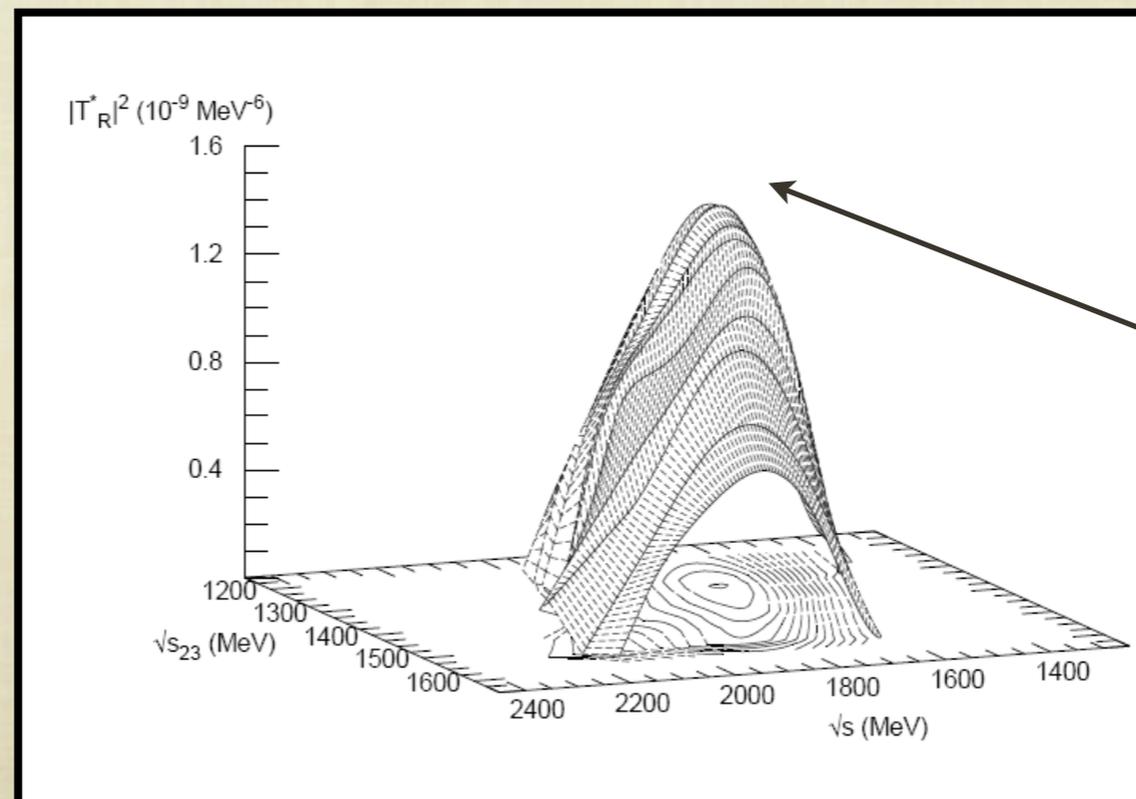
RESULTS $\pi\bar{K}N$ SYSTEM

	Γ (PDG) (MeV)	PEAK POSITION (THIS WORK) (MEV)	Γ (THIS WORK) (MEV)
ISOSPIN = 1			
$\Sigma(1560)$	10-100	1590	70
$\Sigma(1620)$	10-100	1630	39
$\Sigma(1660)$	40-200	1656	30
$\Sigma(1770)$	60-100	1790	24
ISOSPIN = 0			
$\Lambda(1600)$	50-250	1568, 1700	60, 136
$\Lambda(1810)$	50-250	1740	20

THE $\pi\pi N$ SYSTEM

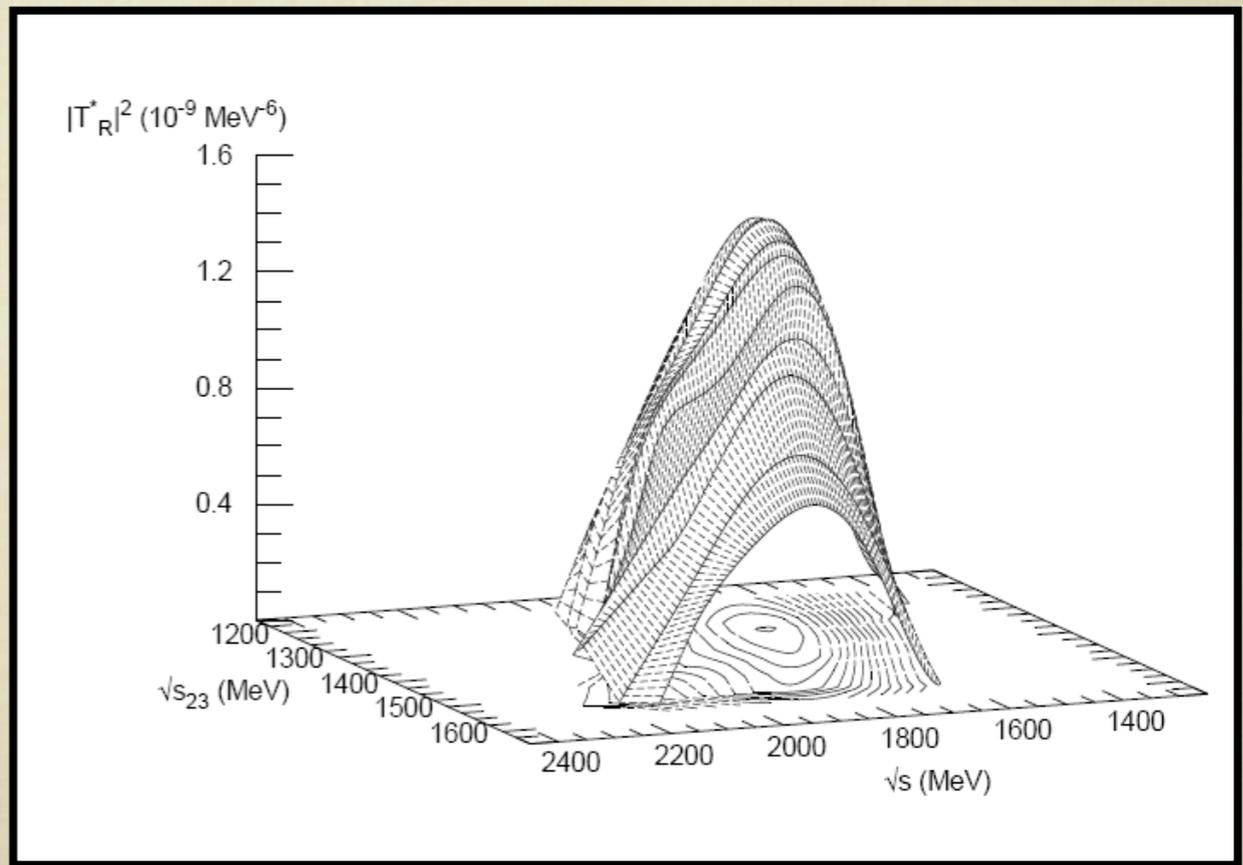
- WE CONSIDER THE CHANNELS $\pi^0\pi^0n, \pi^+\pi^-n, \pi^-\pi^+n, \pi^0\pi^-p, \pi^-\pi^0p$.
- NEGLIGIBLE EFFECT OF THE $\pi K\Sigma, \pi K\Lambda,$ AND $\pi\eta N$ CHANNELS IN THE ENERGY REGION EXPLORED.

$N^*(1710) P_{11} [I(J^P)=1/2(1/2^+)] *$**

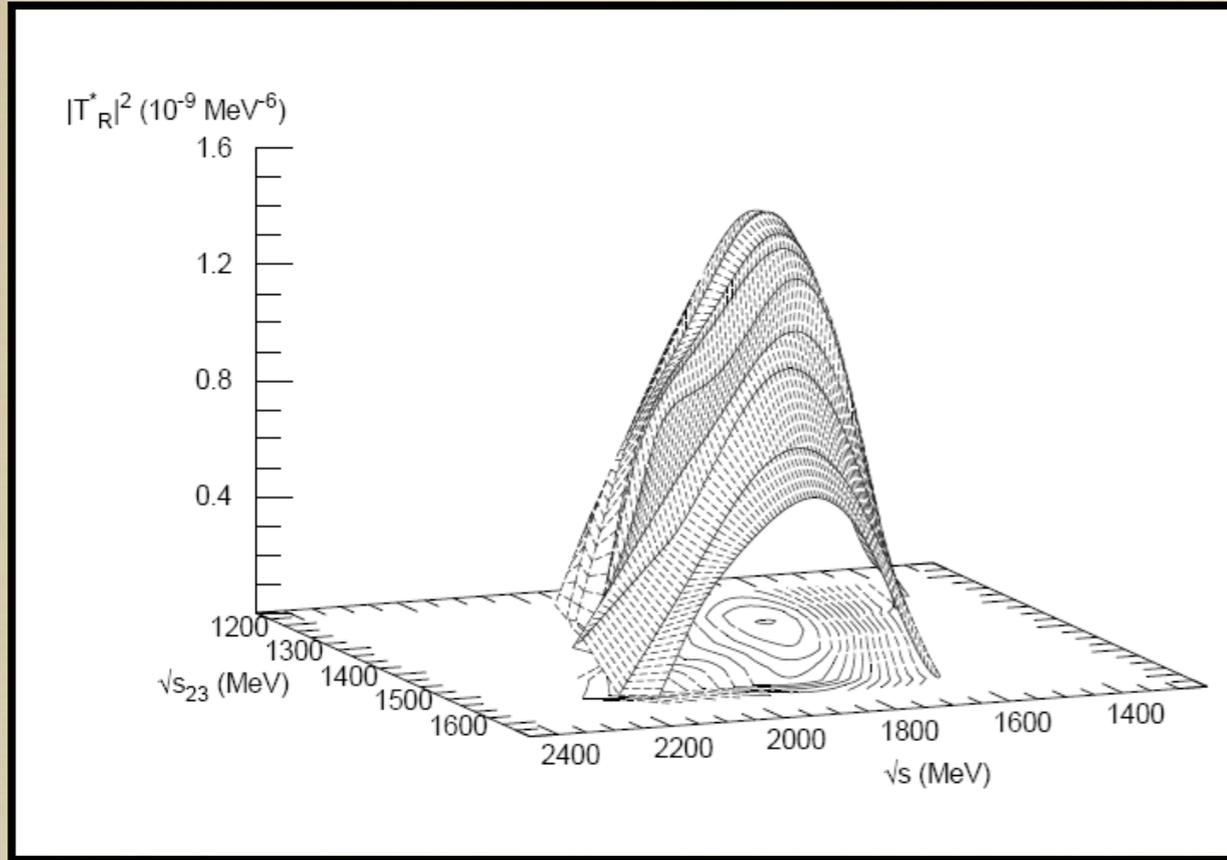


$1704 - i375/2 \text{ MeV}$

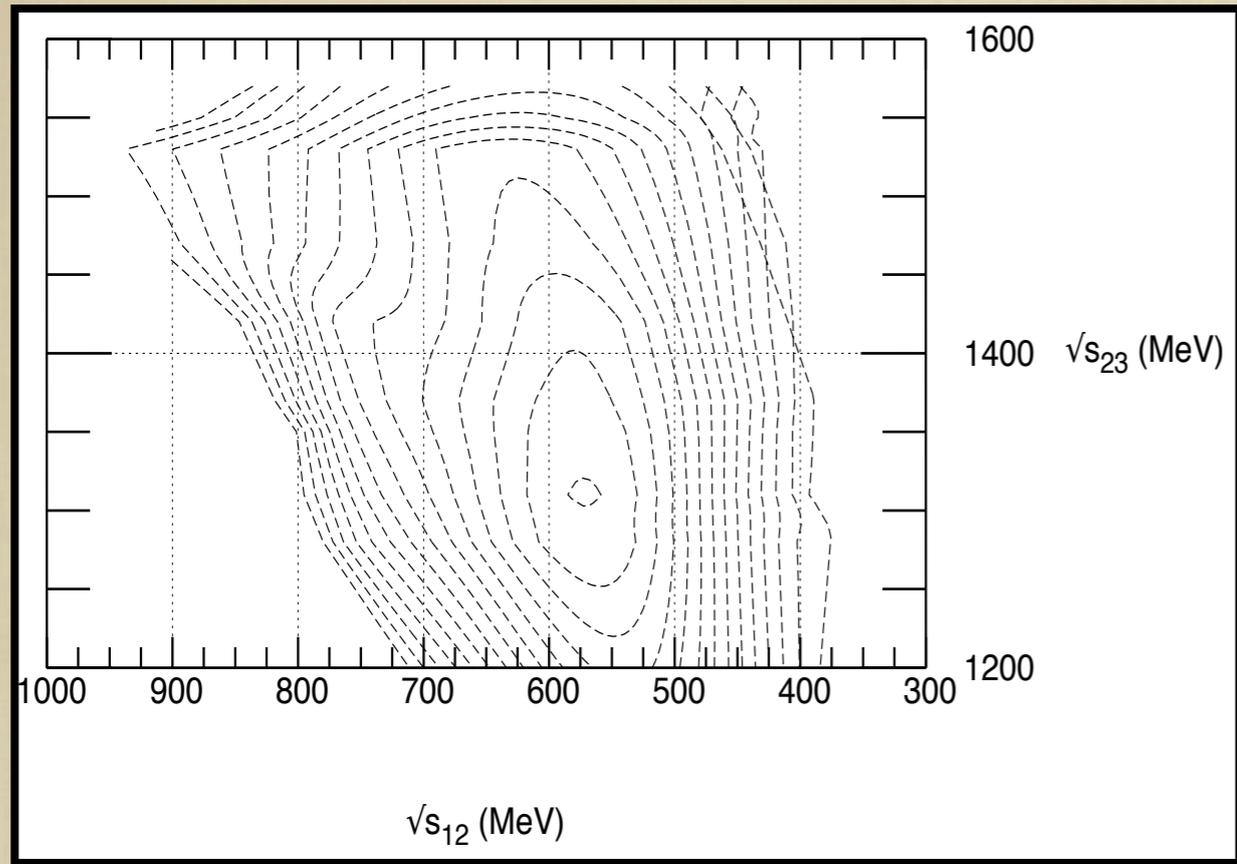
$N^*(1710) P_{11} [I(J^P)=1/2(1/2^+)] *$**



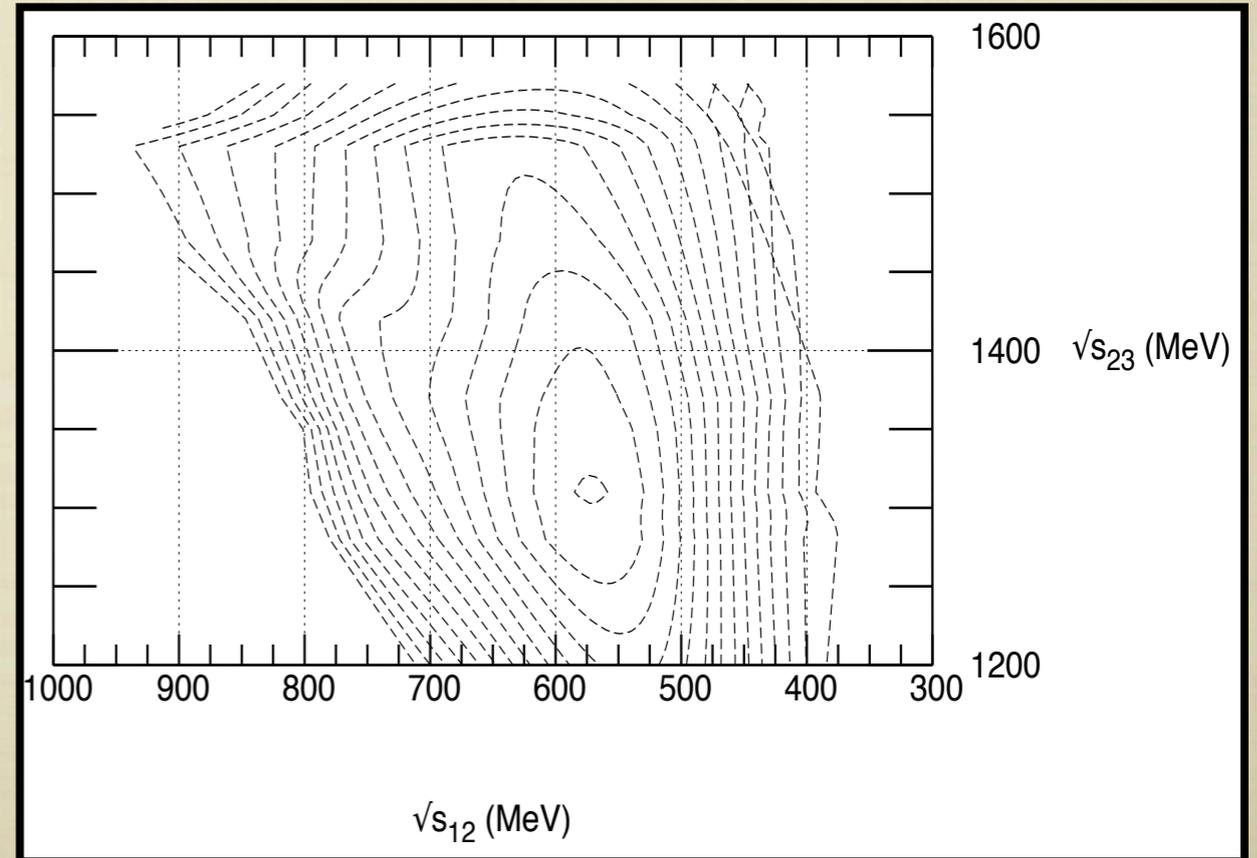
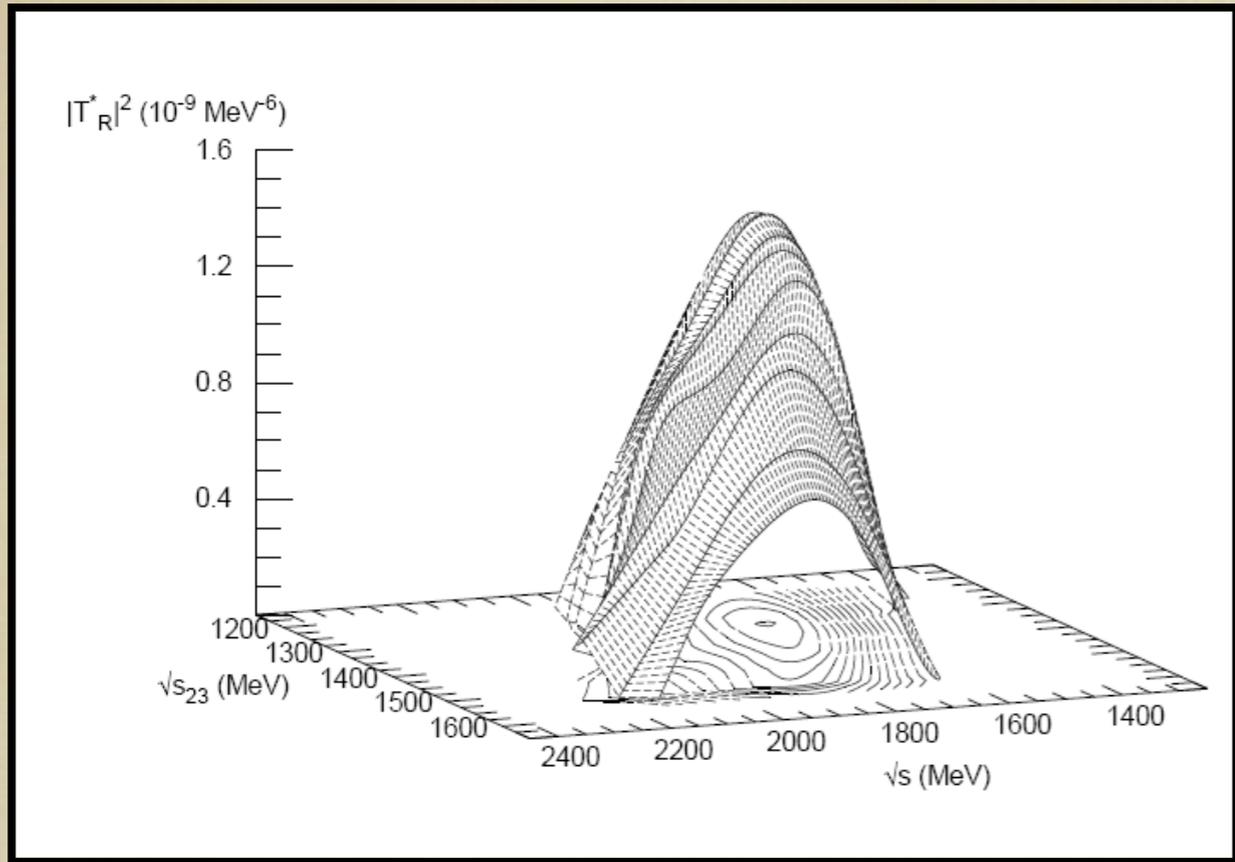
$N^*(1710) P_{11} [I(J^P)=1/2(1/2^+)] *$**



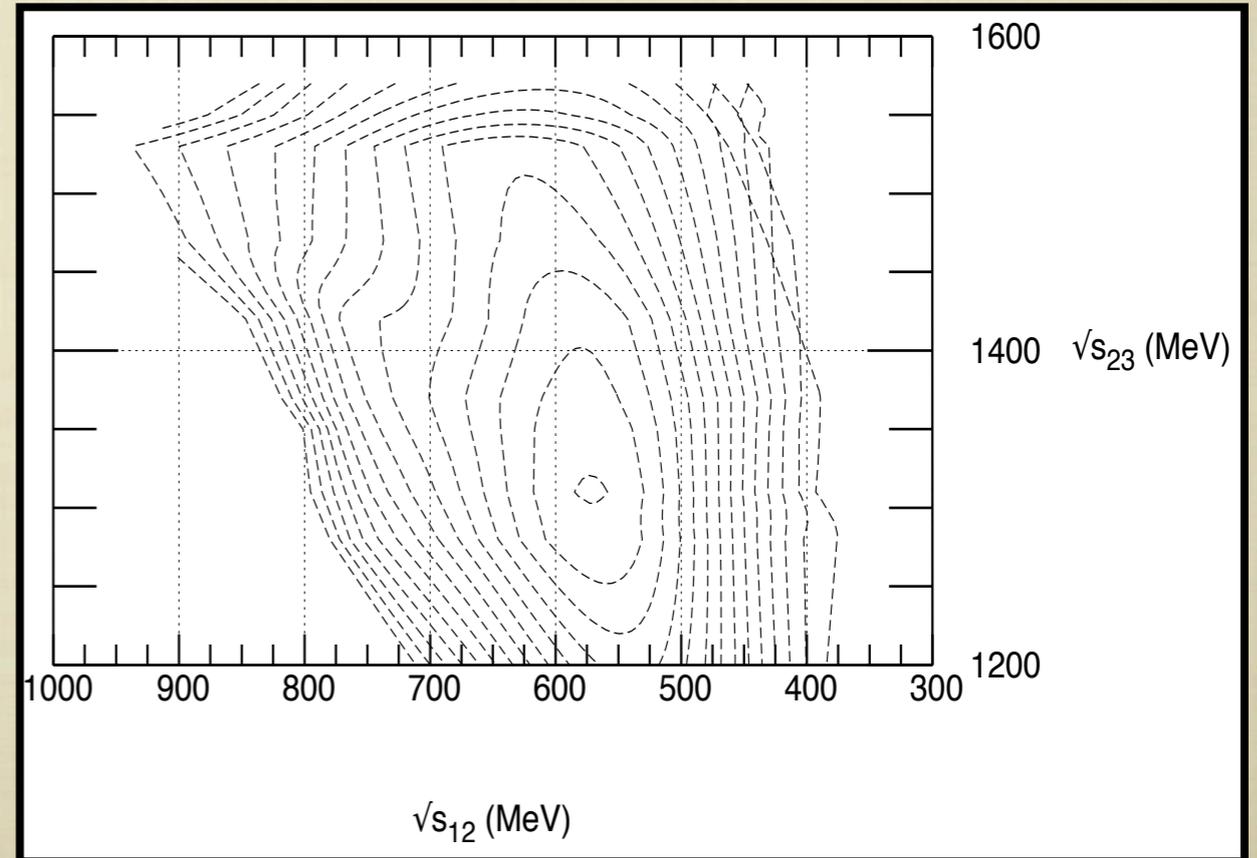
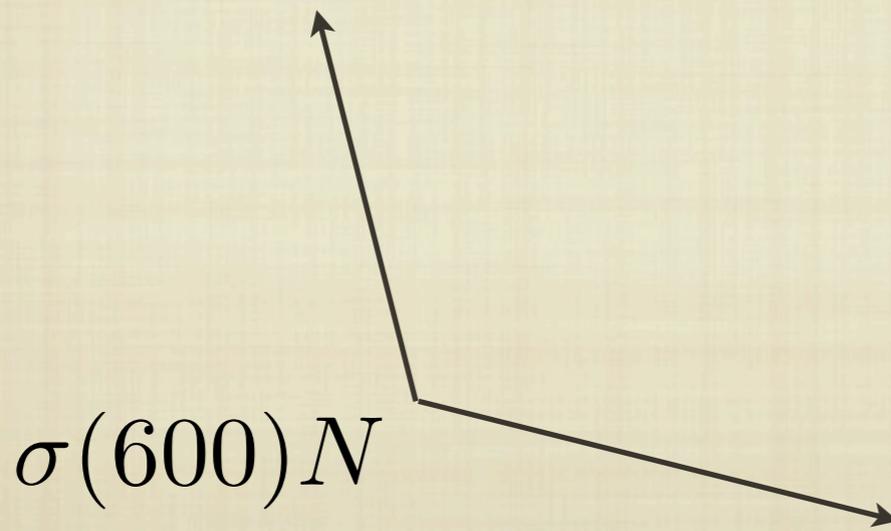
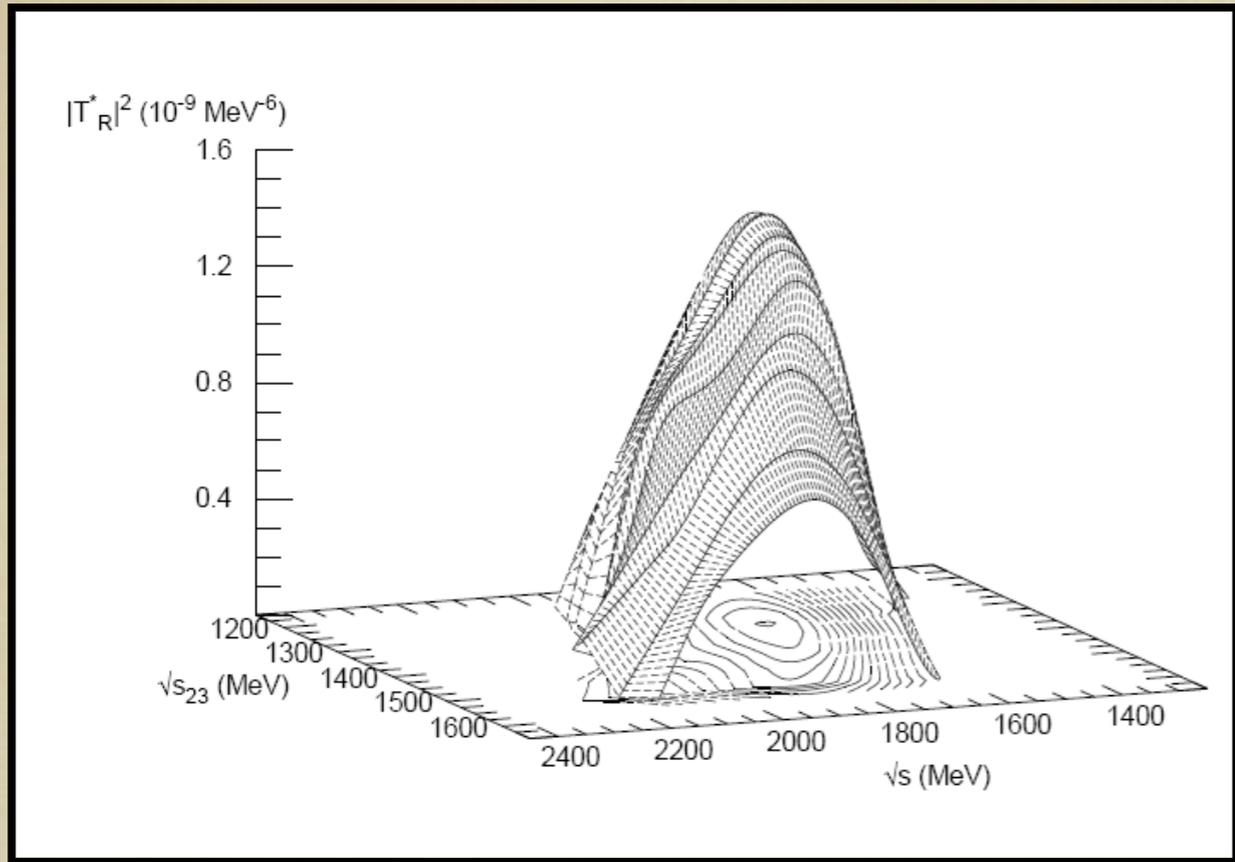
$N^*(1710) P_{11} [I(J^P)=1/2(1/2^+)] *$**



$N^*(1710) P_{11} [I(J^P)=1/2(1/2^+)] *$**

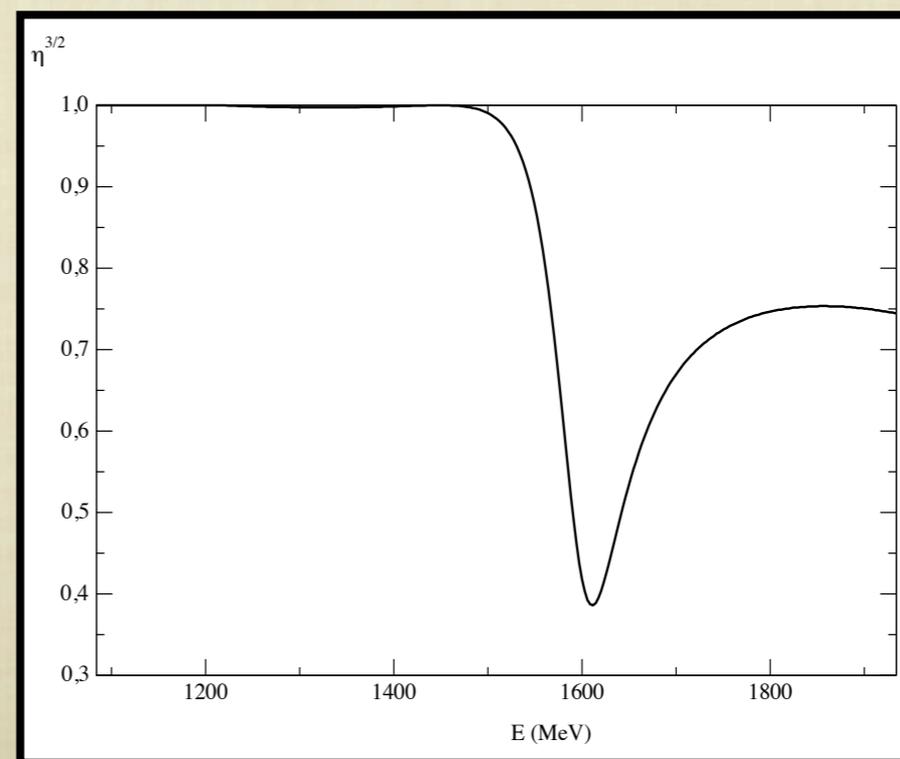
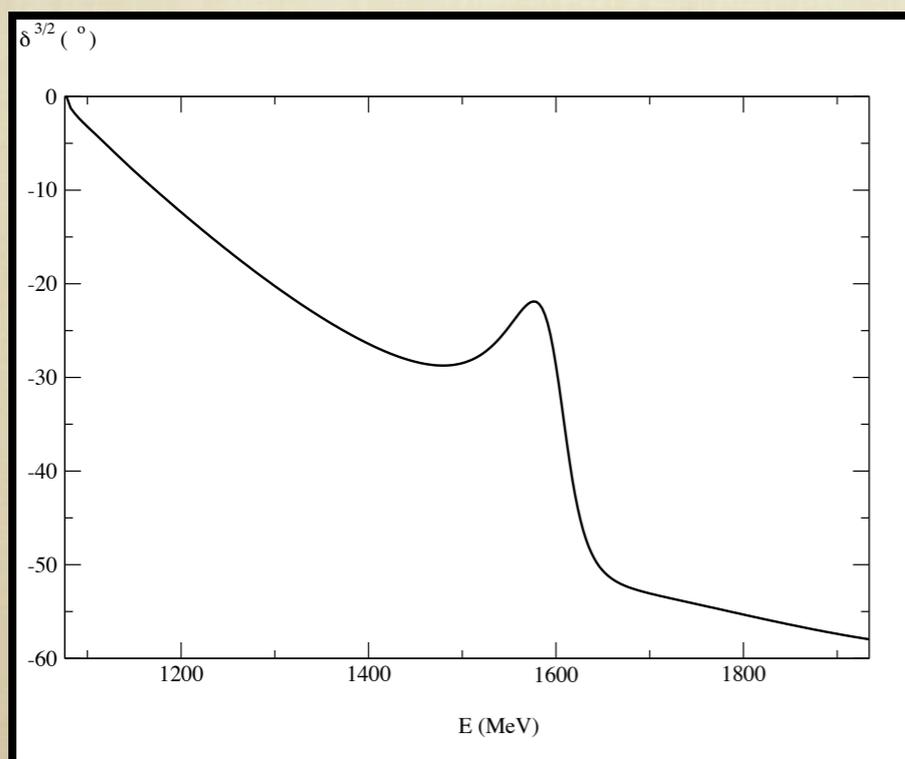
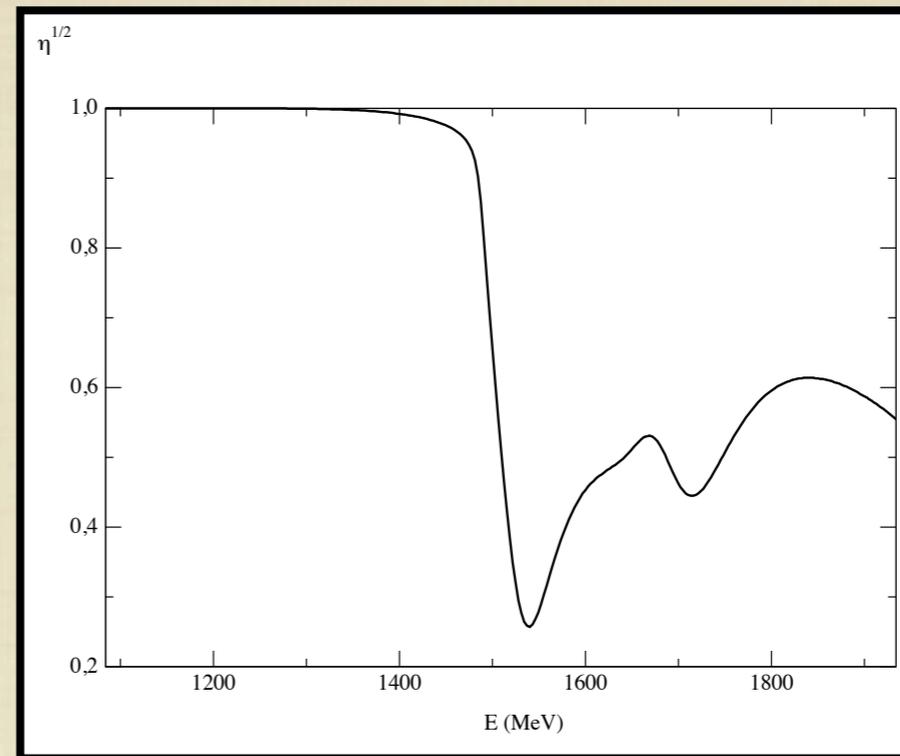
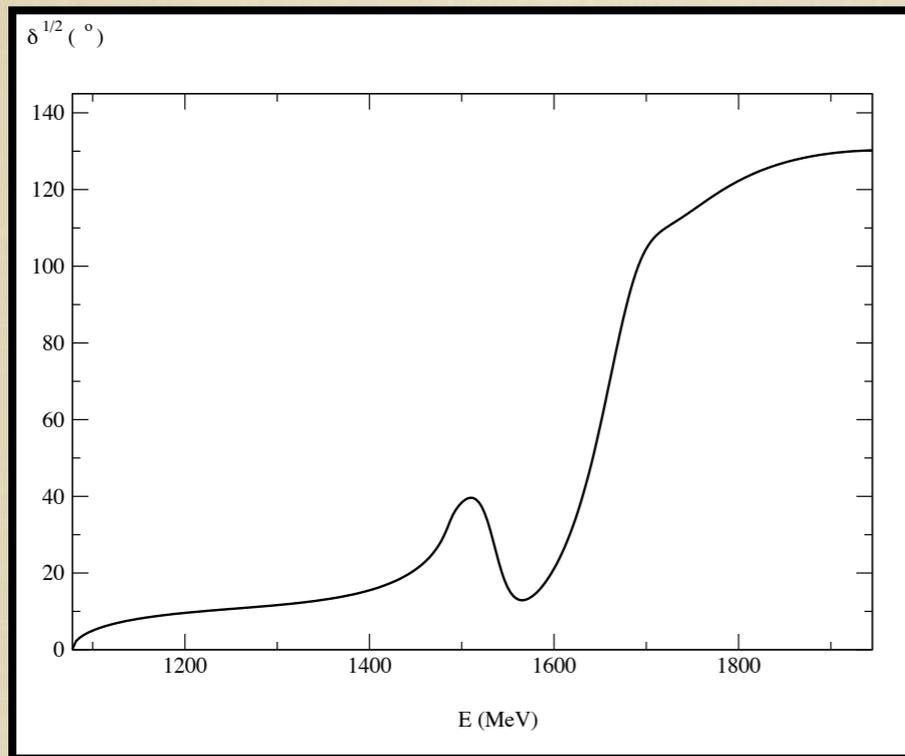


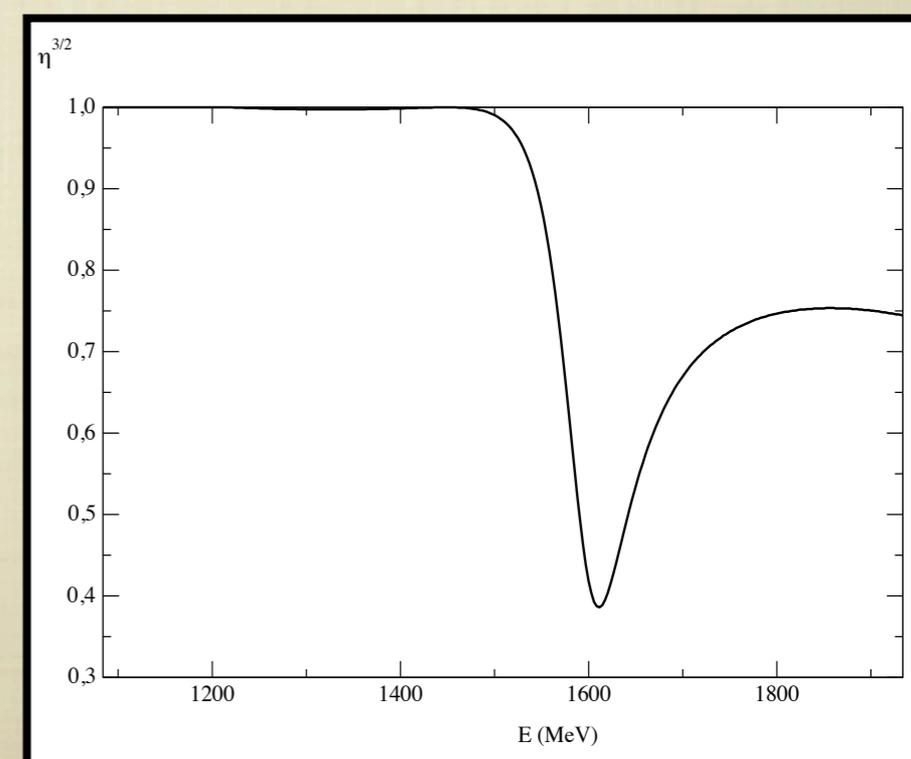
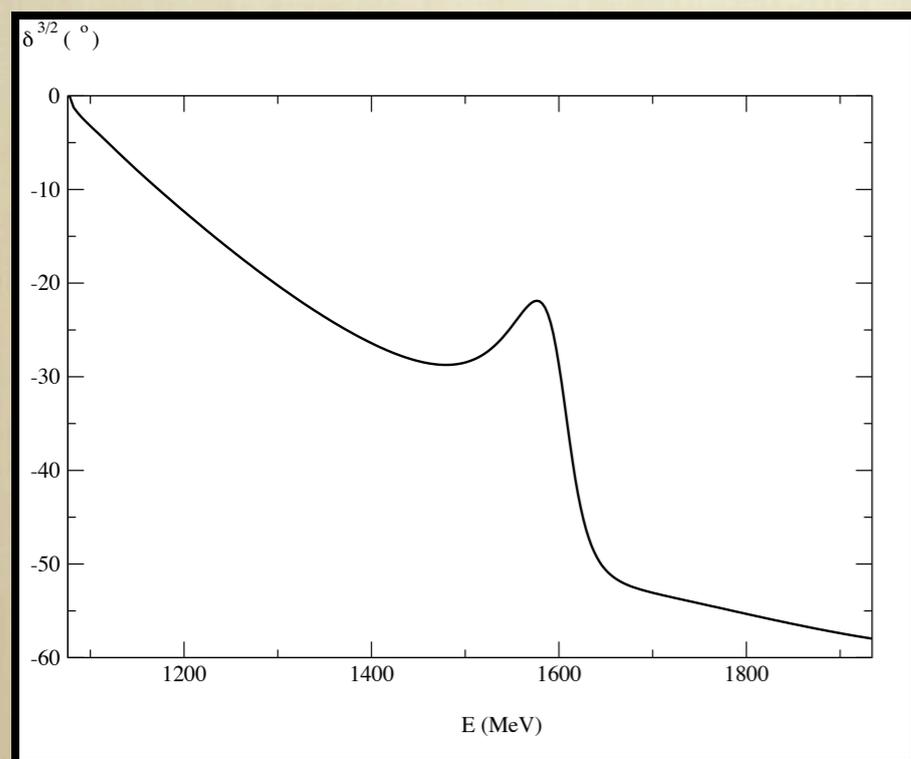
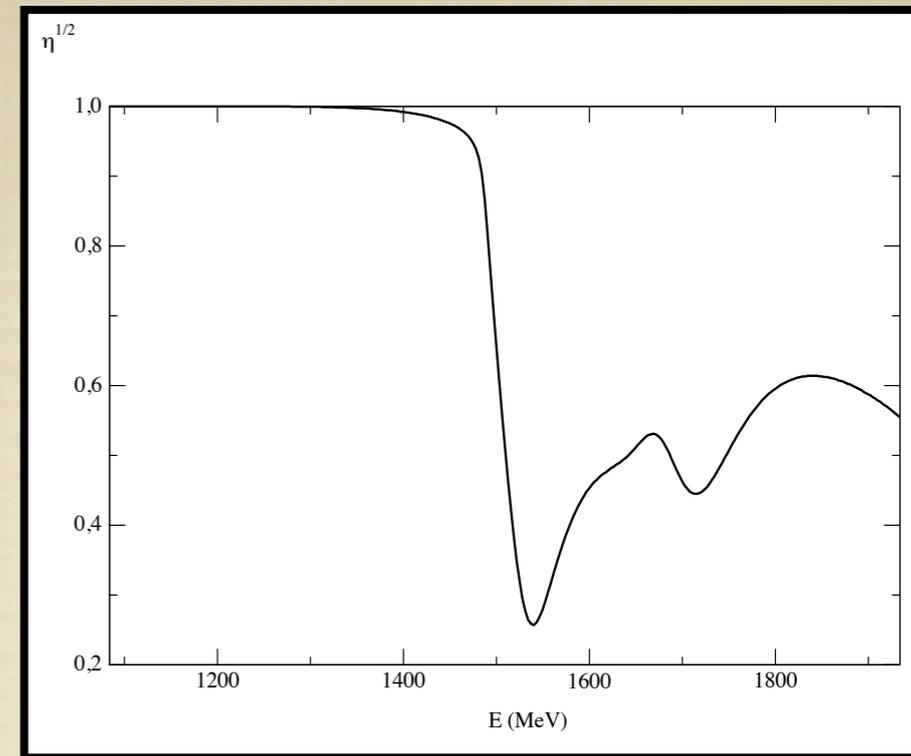
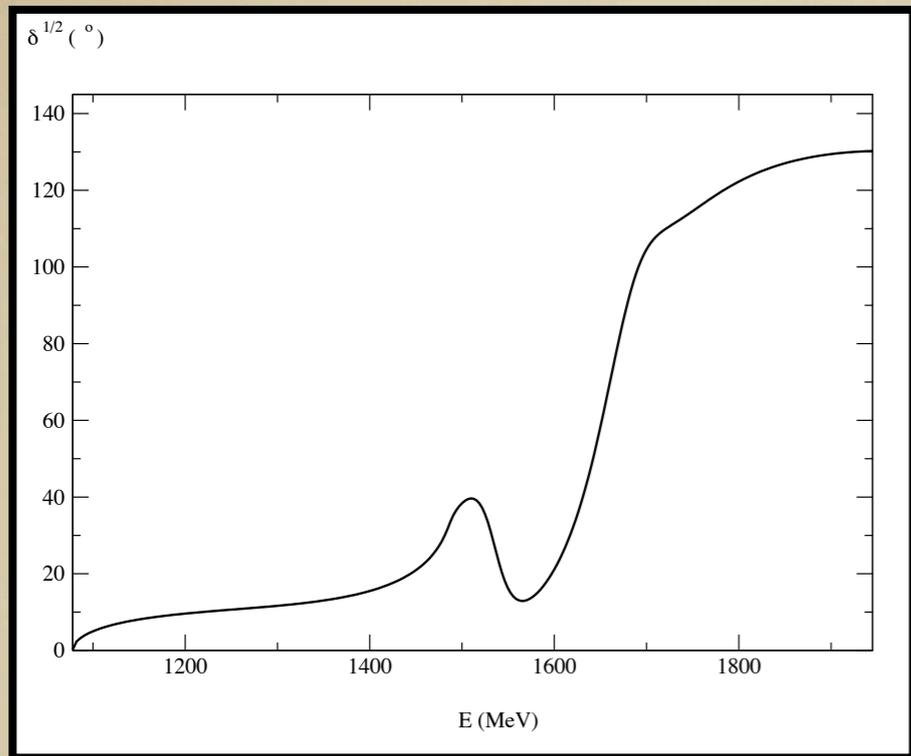
$N^*(1710) P_{11} [I(J^P)=1/2(1/2^+)] *$**

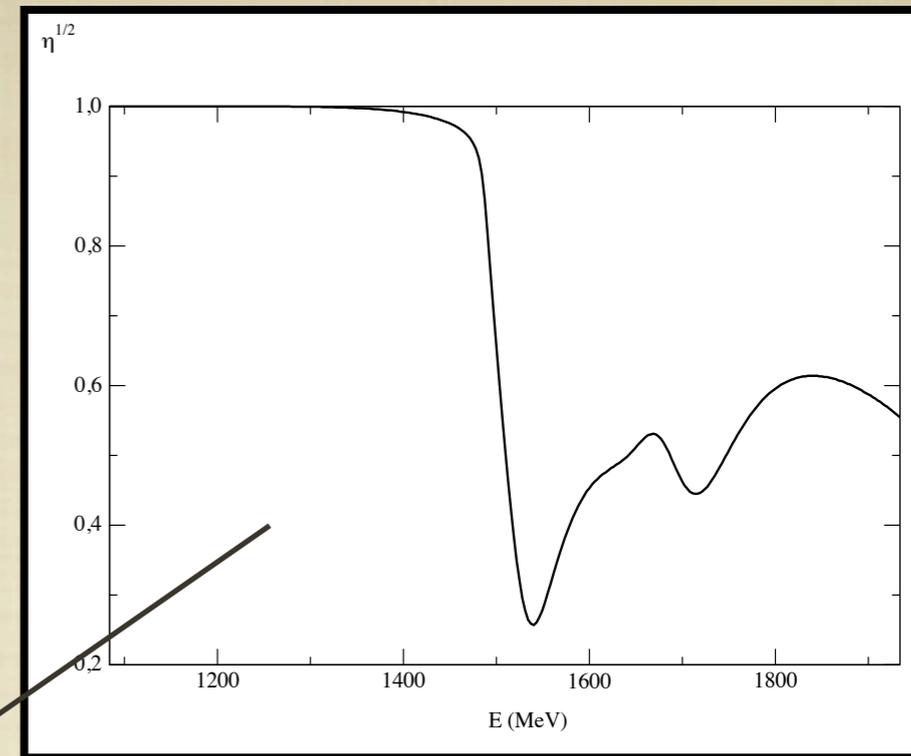
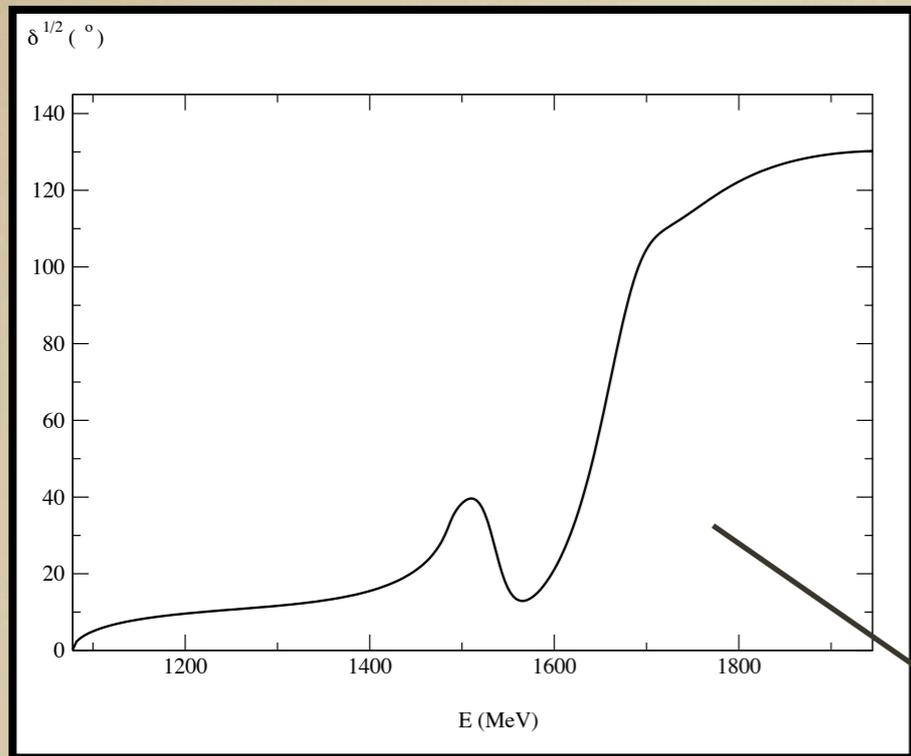


■ THERE ARE OTHER $1/2^+$ STATES: $N^*(2100)$, $\Delta(1750)$, $\Delta(1910)$.

■ THE πN T-MATRIX USED GENERATES THE $N^*(1535)$, BUT NOT THE $N^*(1650)$.

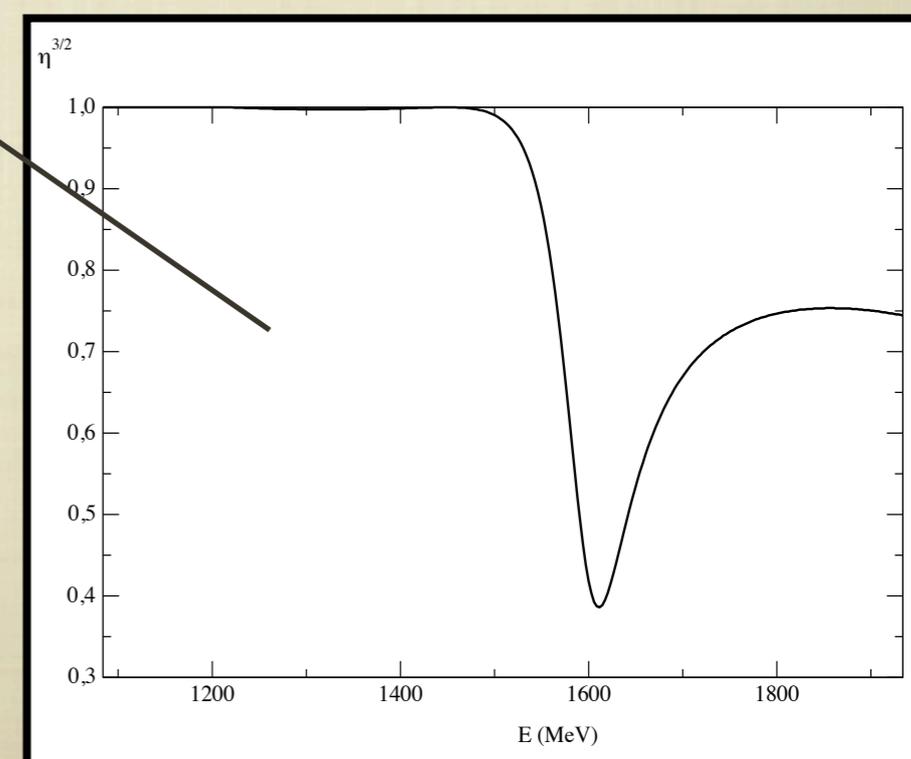
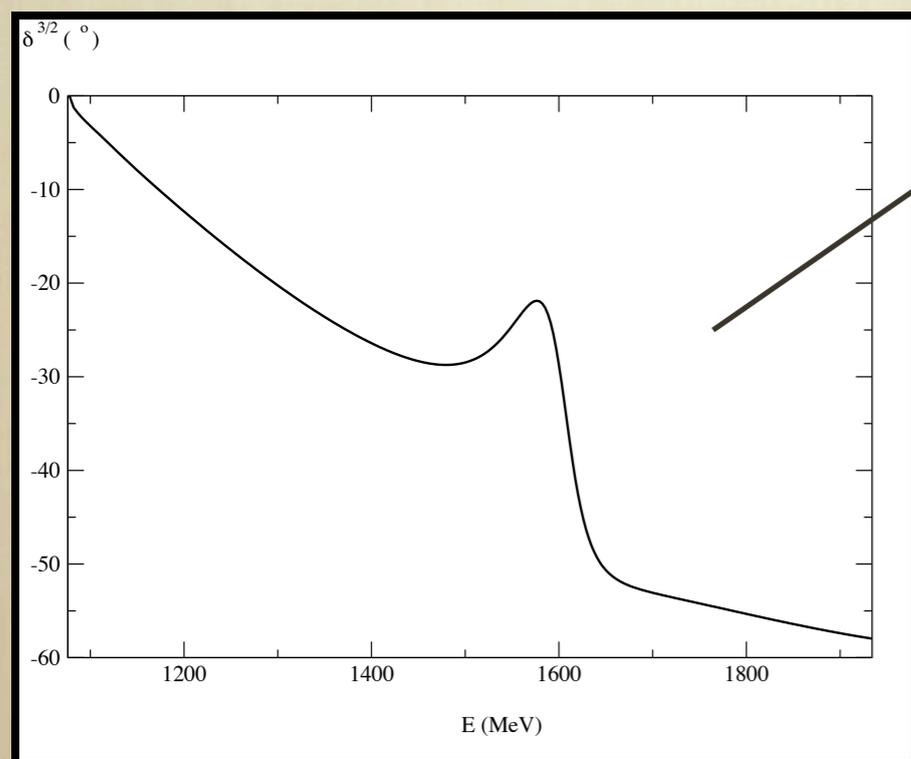




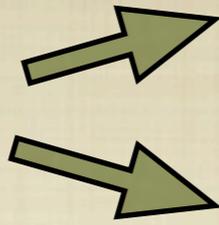


$$f^I = \frac{\eta^I e^{2i\delta^I} - 1}{2iq}$$

$$t^I = -\frac{4\pi E}{M} f^I, \quad I=1/2, 3/2$$



■ πN INTERACTION



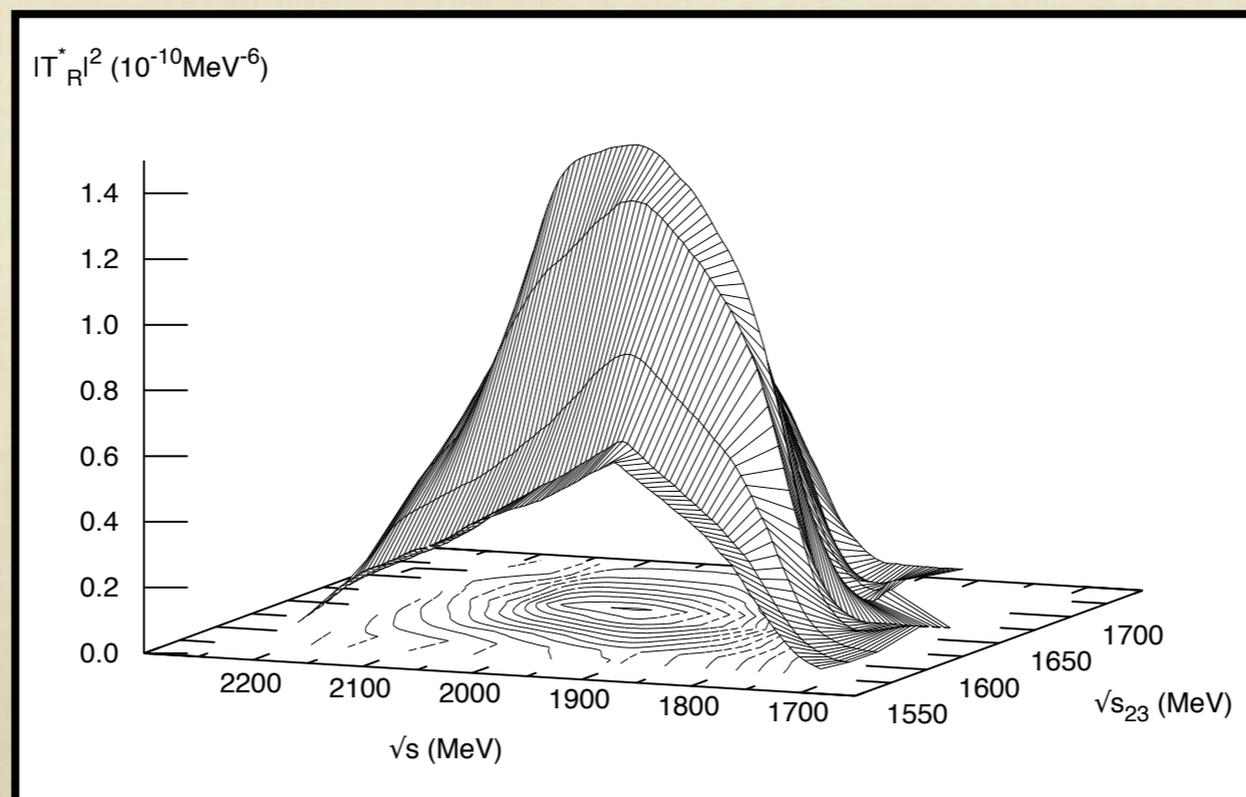
MODEL FOR THE $N^*(1535)$ BELOW THRESHOLD.

EXPERIMENTAL RESULTS ABOVE THRESHOLD.

■ $\pi\pi$ INTERACTION CALCULATED WITH THE MODEL OF NUCL. PHYS. A 620, 438-456, 1997.

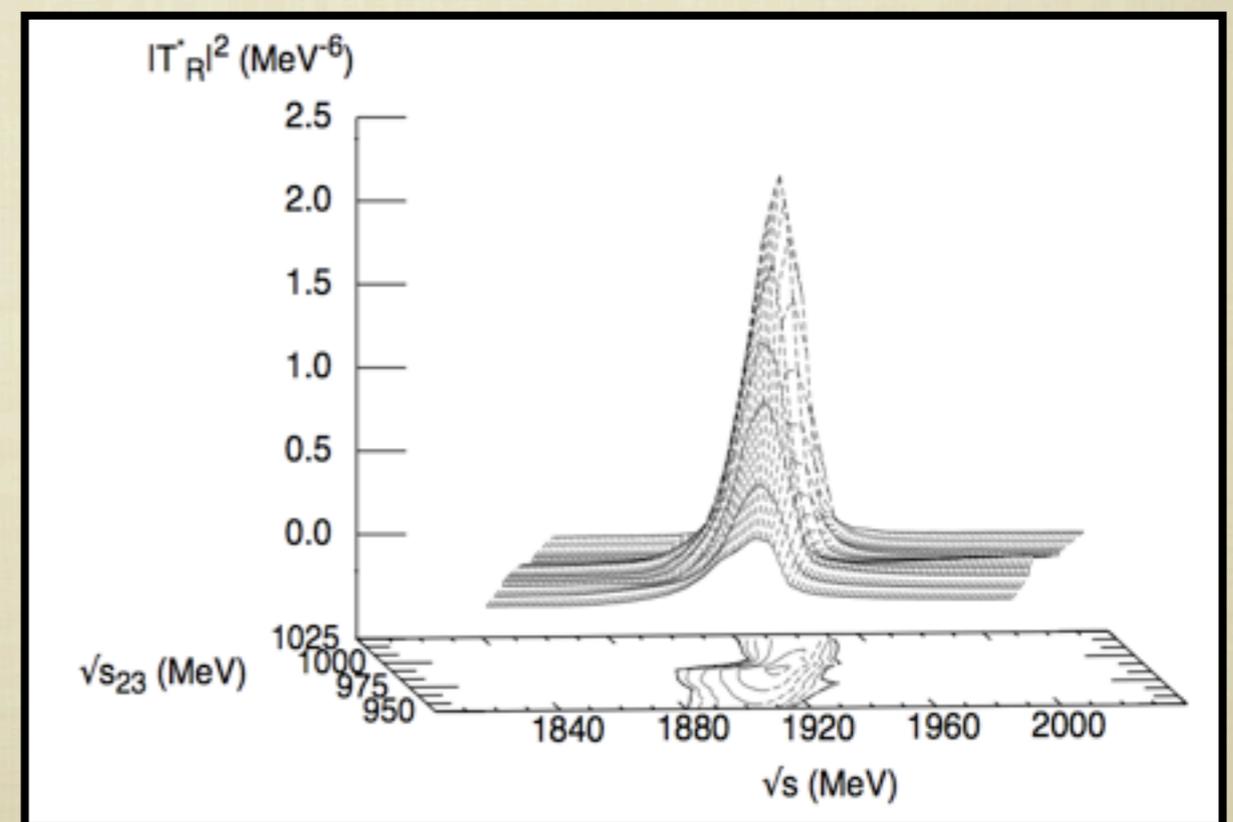
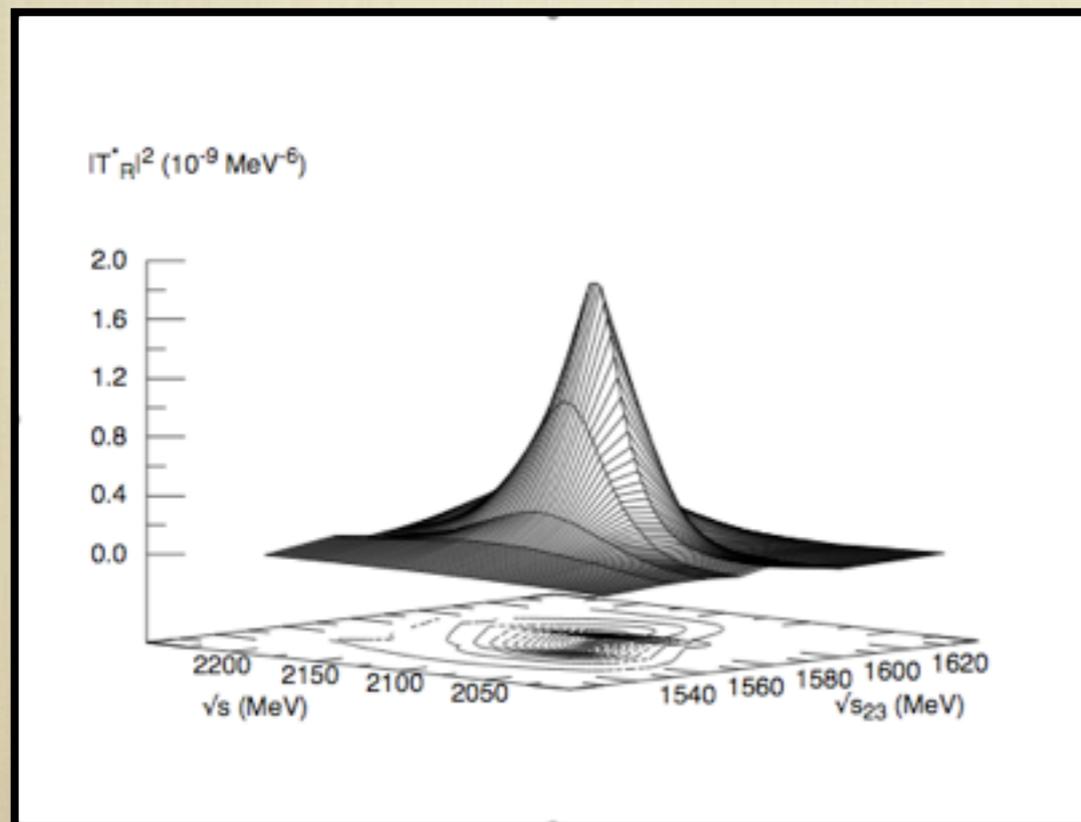
■ CONSIDERING ONLY THE $\pi\pi N$ CHANNELS $\pi^0\pi^0n, \pi^+\pi^-n, \pi^-\pi^+n, \pi^0\pi^-p, \pi^-\pi^0p$

$N^*(2100) P_{11} [I(J^P)=1/2(1/2^+)] *$



- WE ADD THE $\pi K \Sigma$, $\pi K \Lambda$ AND $\pi \eta N$ CHANNELS \longrightarrow $\Delta(1910)$.
- WE DON'T SEE ANY SIGNAL FOR $\Delta(1750)$.
- WE HAVE STUDY THE $a_0(980)N$, $f_0(980)N$ SYSTEMS BY INCLUDING THE COUPLED CHANNEL $K \bar{K} N$ (D. JIDO AND Y.KANADA-EN'YO, PHYS.REV. C78,035203 (2008)).

$\Delta(1910)$ P_{31} [$I(J^P)=3/2(1/2^+)$] ****



WHY STUDY THE $N\pi K$ SYSTEM?

- A PEAK IN THE $K^+\eta$ INVARIANT MASS IN THE $\gamma \eta \rightarrow K^+K^-\eta$ REACTION AT THE SPRING8/OSAKA PENTAQUARK

- CHIRAL LAGRANGIANS: K^+N INTERACTION IS REPULSIVE.



$\pi K N$

- SOME INVESTIGATIONS HAVE ALREADY BEEN DONE AND THE RESULTS DO NOT LOOK PROMISING^{15,16}

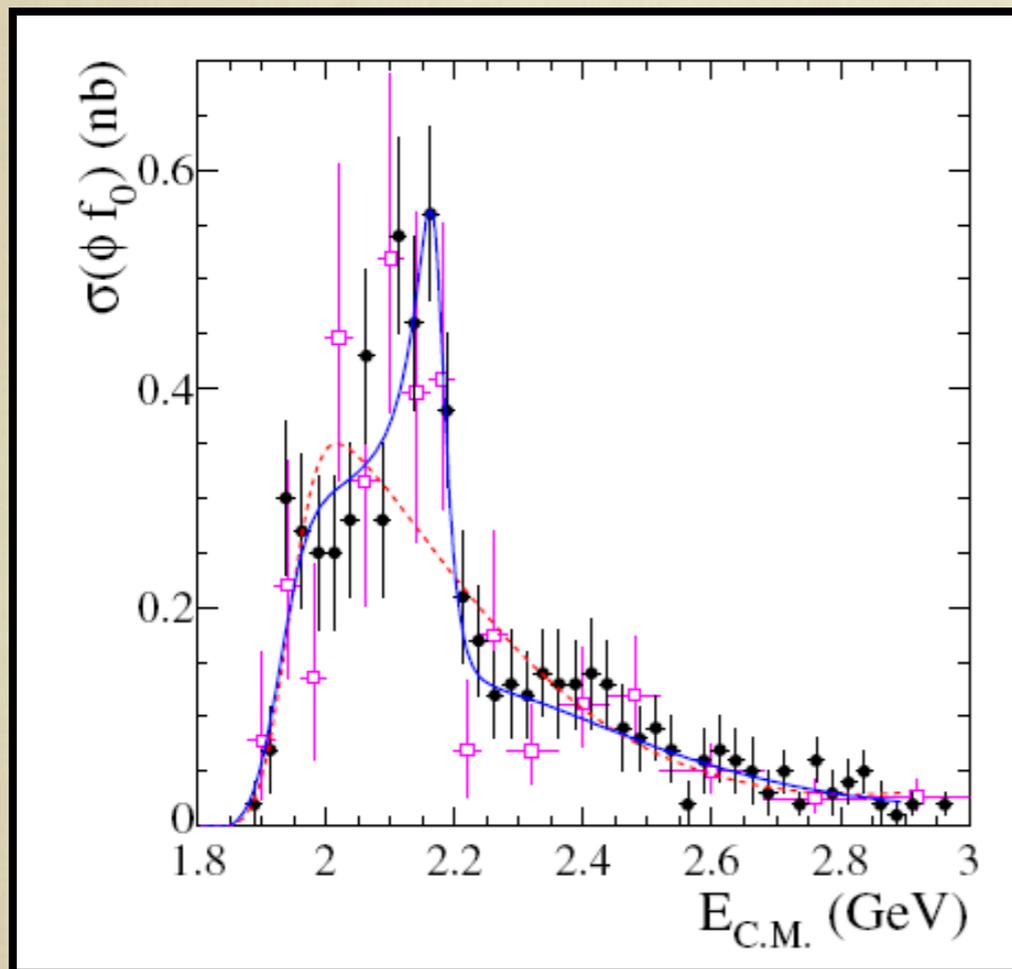
¹⁵ P. BICUDO, G. M. M. MARQUES, PHYS. REV. D 69 011503 (2004).

¹⁶ FELIPE J. LLANES-ESTRADA, E. OSET, V. MATEU, PHYS. REV. C 69 055203 (2004).

THREE-MESON SYSTEMS

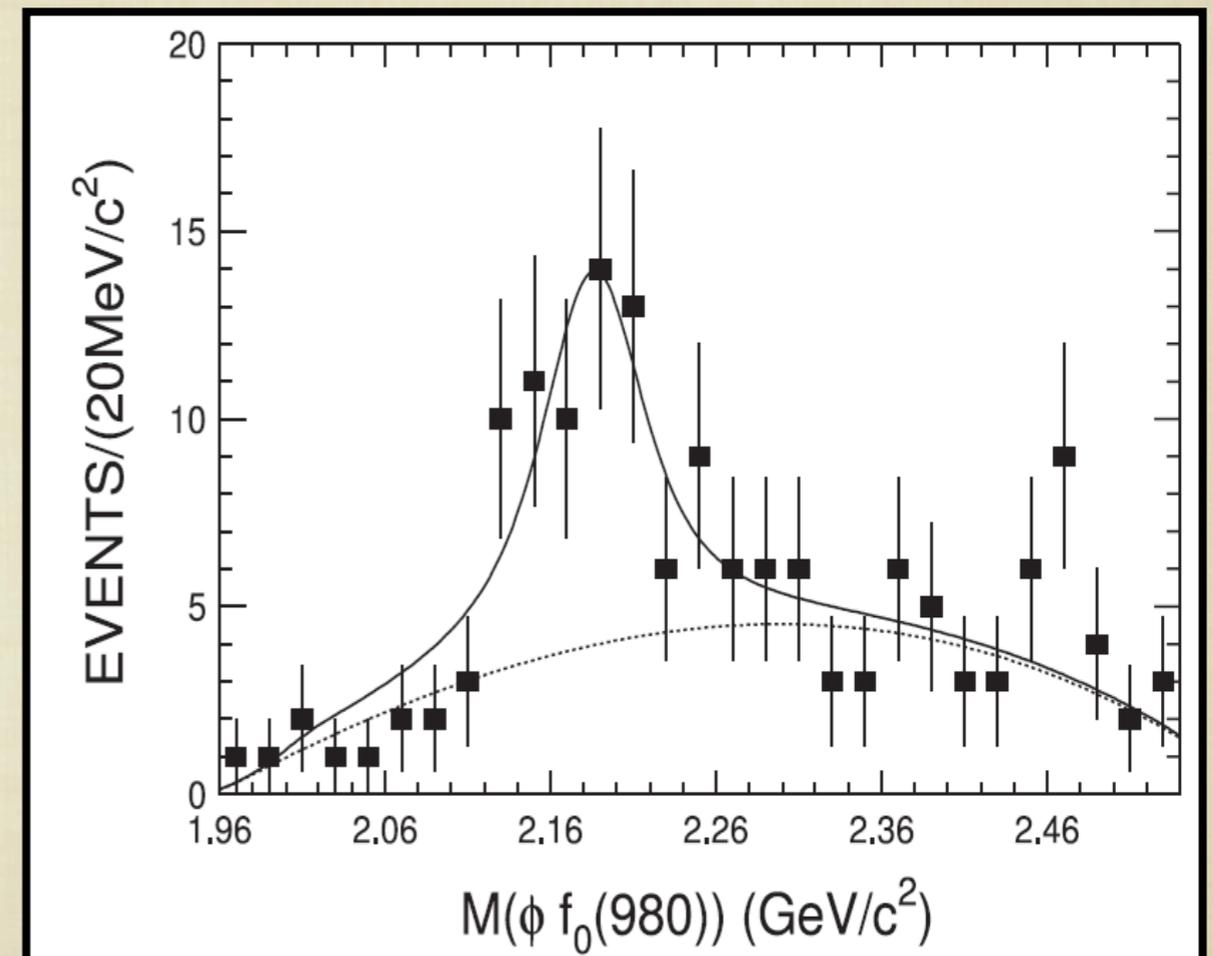
BABAR

$$e^+e^- \rightarrow \phi f_0(980)$$



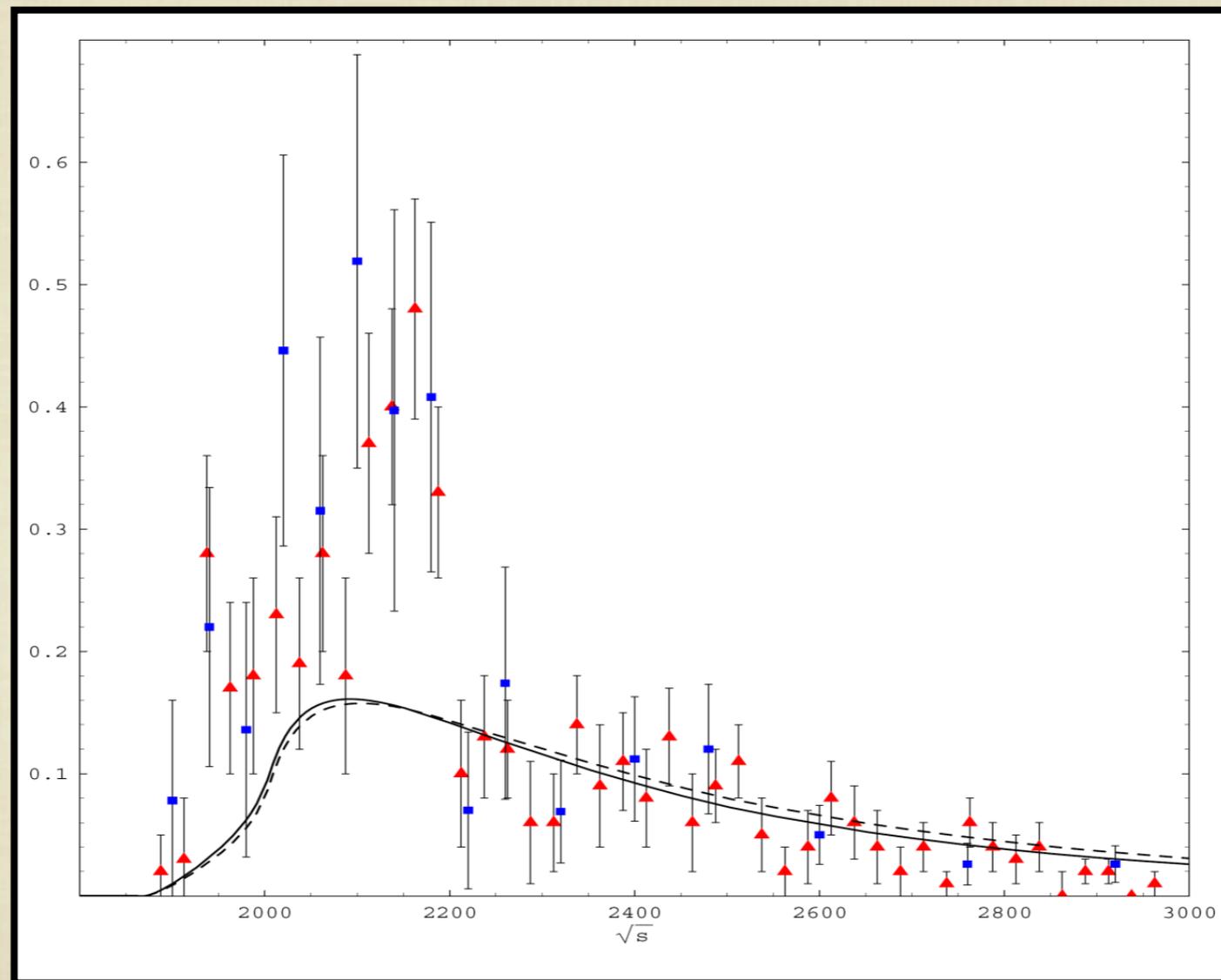
BES

$$J/\Psi \rightarrow \eta \phi f_0(980)$$



$X(2175)$

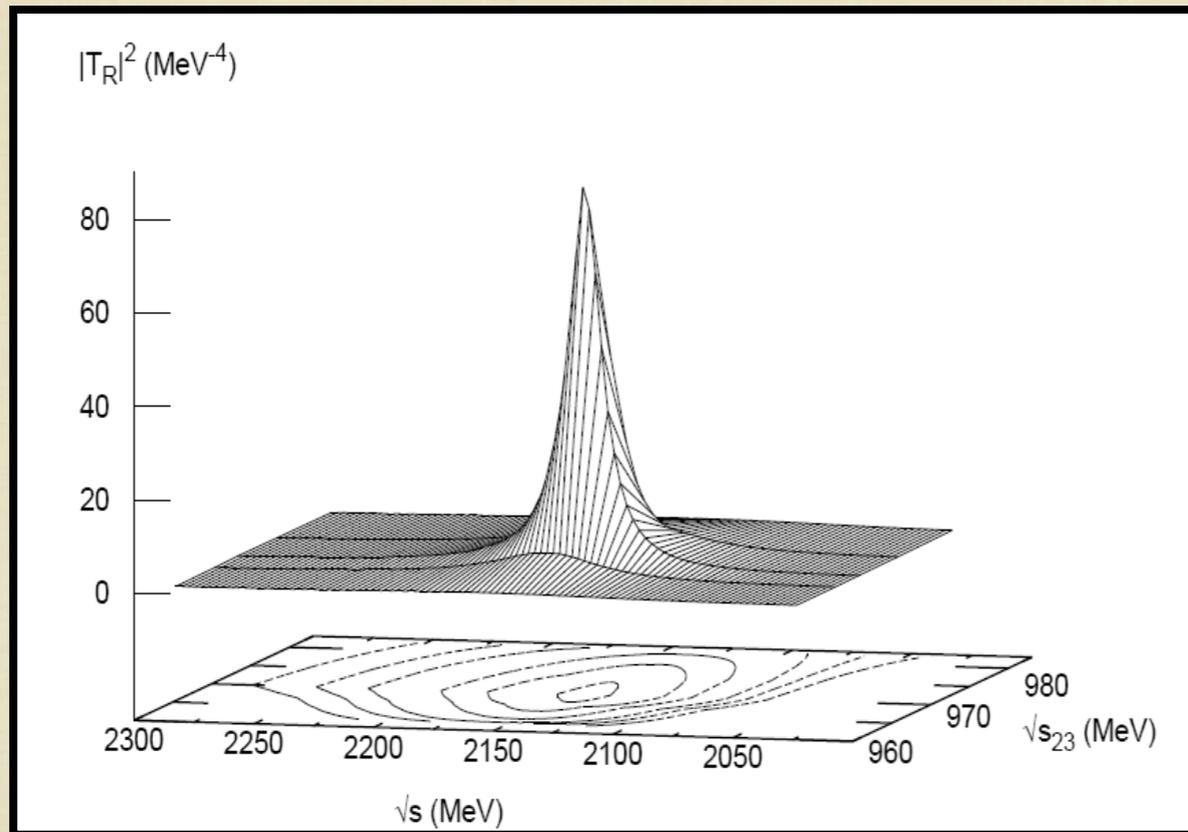
THEORETICAL INVESTIGATIONS*



*see a review on different studies (for example) : “New hadron states”, Shi-Lin Zhu , *Int.J.Mod.Phys.E17:283-322,2008*; e-Print: [hep-ph/0703225](https://arxiv.org/abs/hep-ph/0703225)

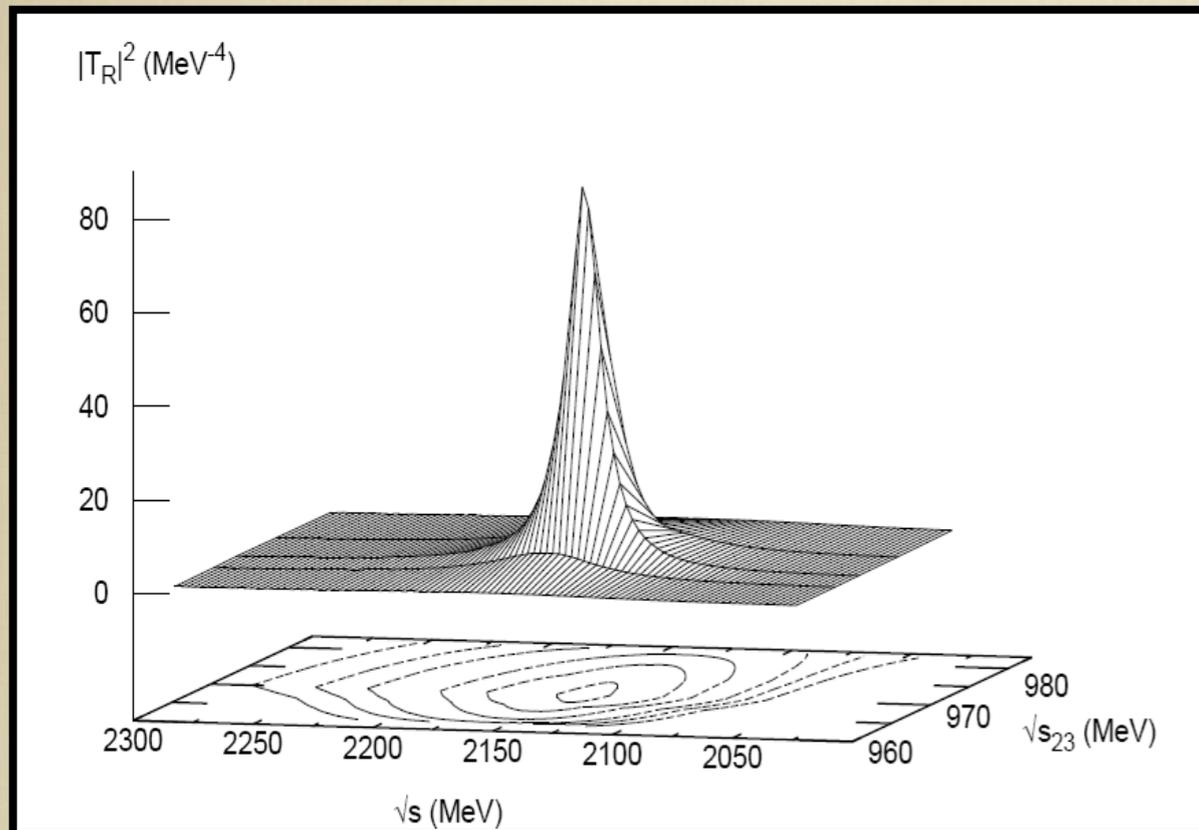
- WE HAVE STUDIED THE SYSTEM $\phi K \bar{K}, \phi \pi \pi$.
- WE GET A PEAK AROUND 2150 MEV WITH A WIDTH OF 27 MEV.

- WE HAVE STUDIED THE SYSTEM $\phi K \bar{K}, \phi \pi \pi$.
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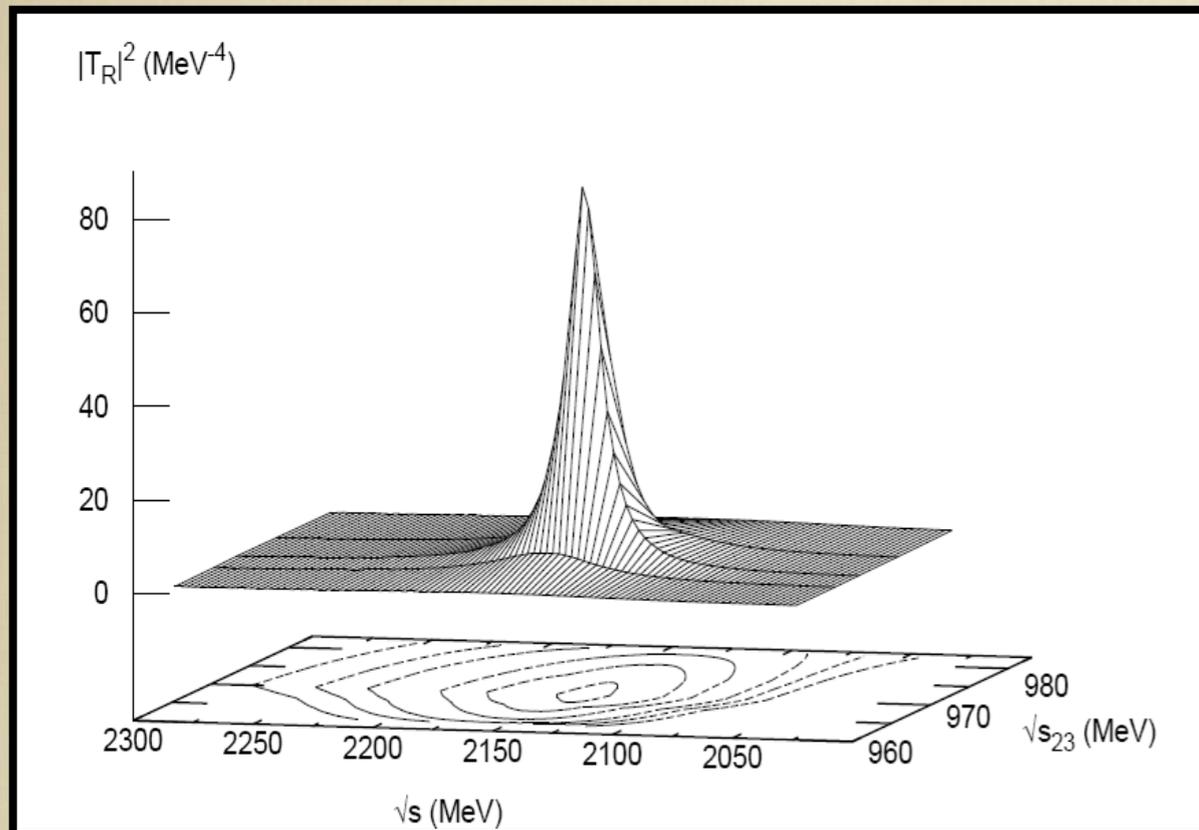
■ WE HAVE STUDIED THE SYSTEM $\phi K \bar{K}$, $\phi \pi \pi$.

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■ WE HAVE STUDIED THE SYSTEM $\phi K \bar{K}$, $\phi \pi \pi$.

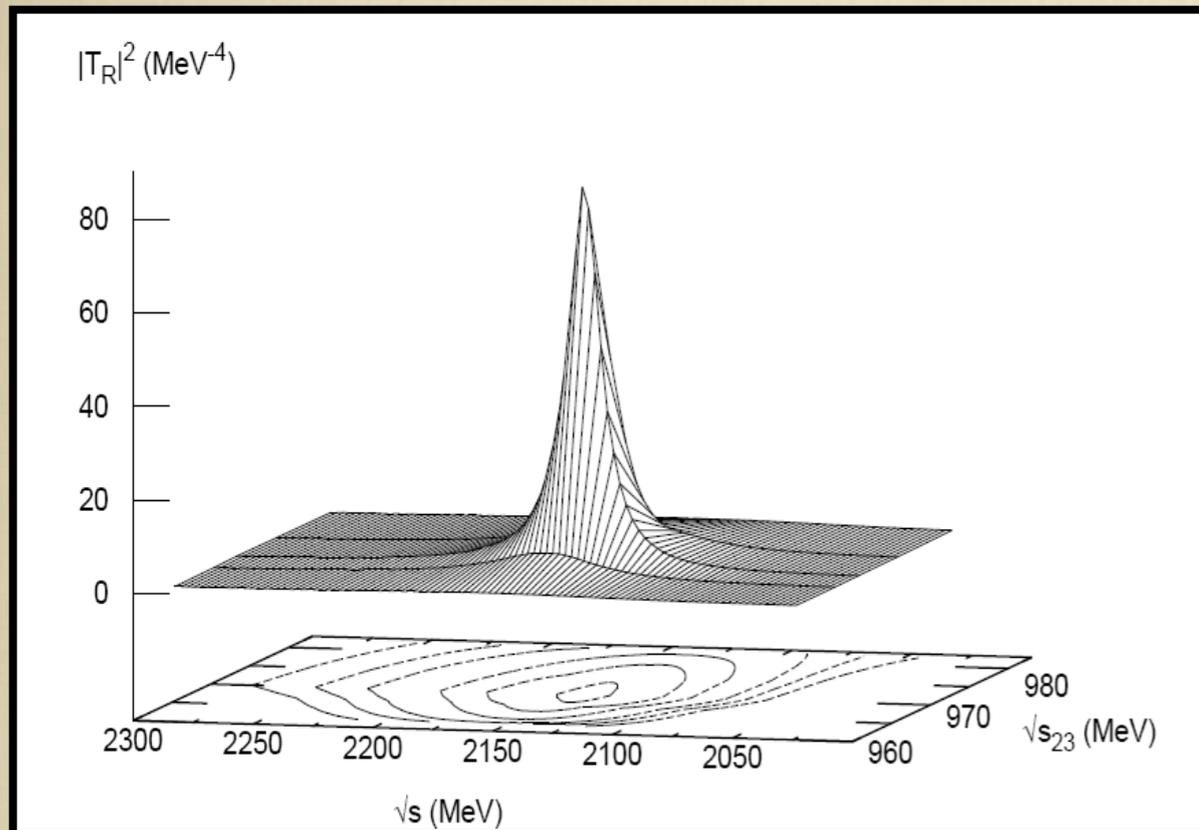
■ WE GET A PEAK AROUND 2150 MEV WITH A WIDTH OF 27 MEV.



$$T_{pw}^{\phi f_0} [1 + G_{\phi f_0} T_{\phi f_0}]$$

■ WE HAVE STUDIED THE SYSTEM $\phi K \bar{K}$, $\phi \pi \pi$.

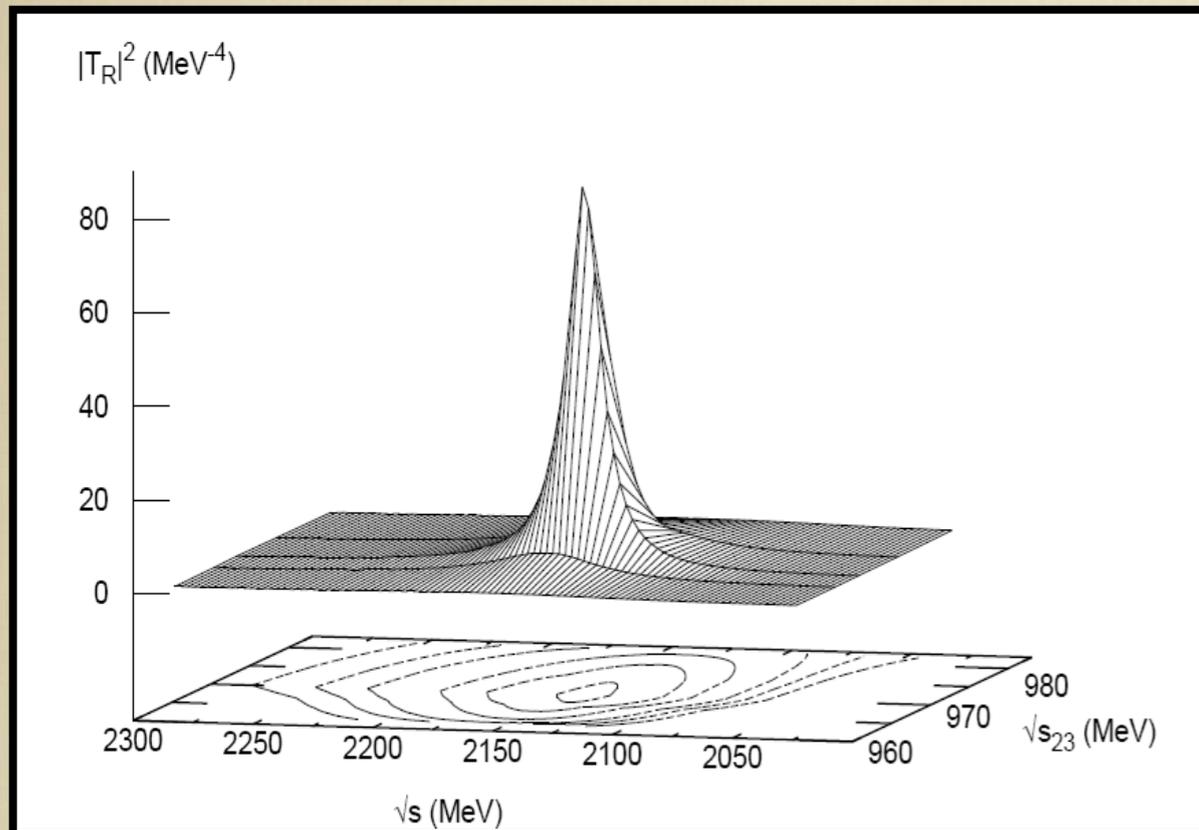
■ WE GET A PEAK AROUND 2150 MEV WITH A WIDTH OF 27 MEV.



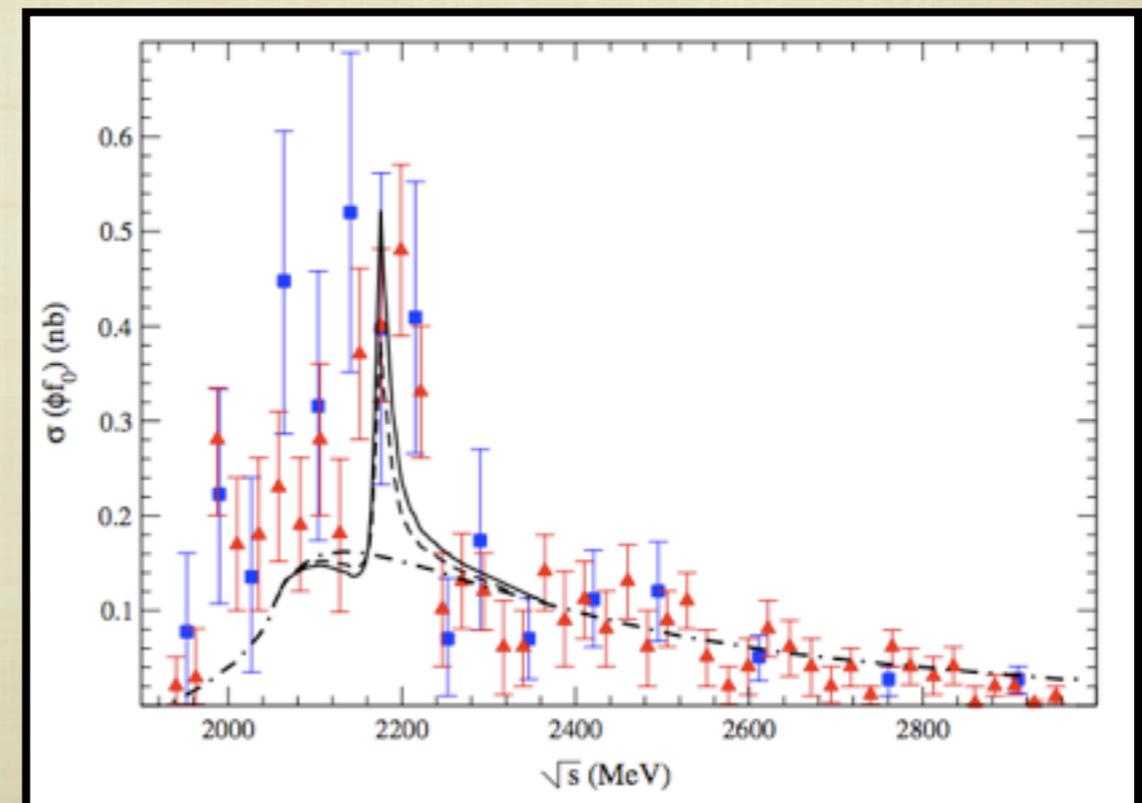
$$T_{pw}^{\phi f_0} [1 + G_{\phi f_0} T_{\phi f_0}]$$

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■ WE GET A PEAK AROUND 2150 MEV WITH A WIDTH OF 27 MEV.



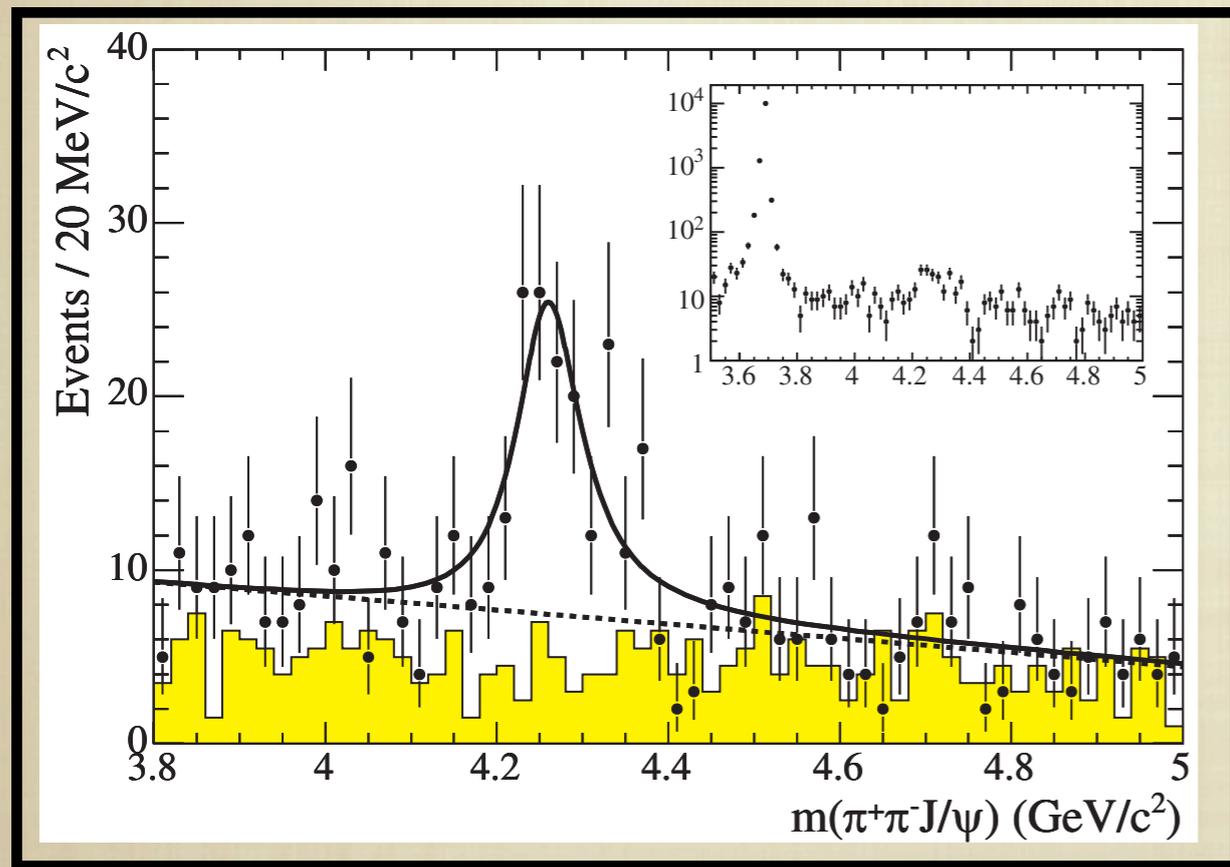
$$T_{pw}^{\phi f_0} [1 + G_{\phi f_0} T_{\phi f_0}]$$



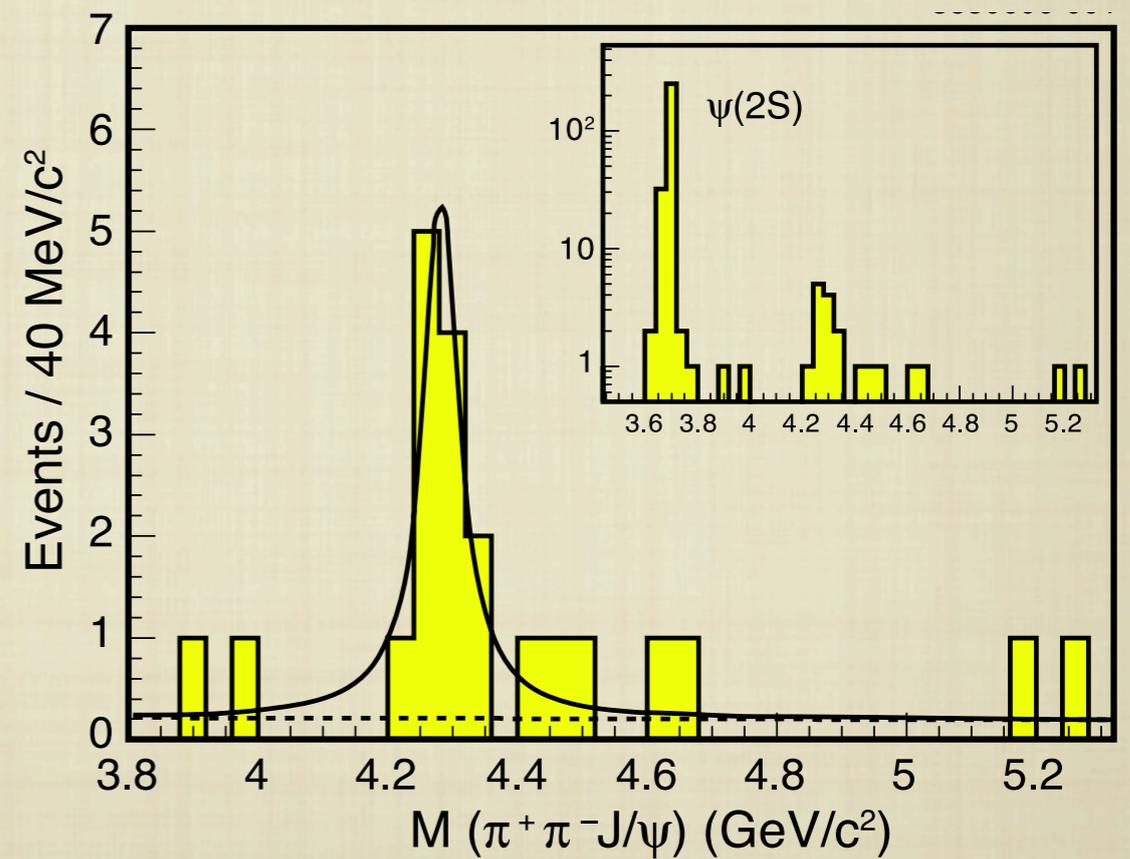
THE $Y(4260)$ RESONANCE

- Observed in the reaction $e^+e^- \rightarrow \pi^+\pi^- J/\psi$.

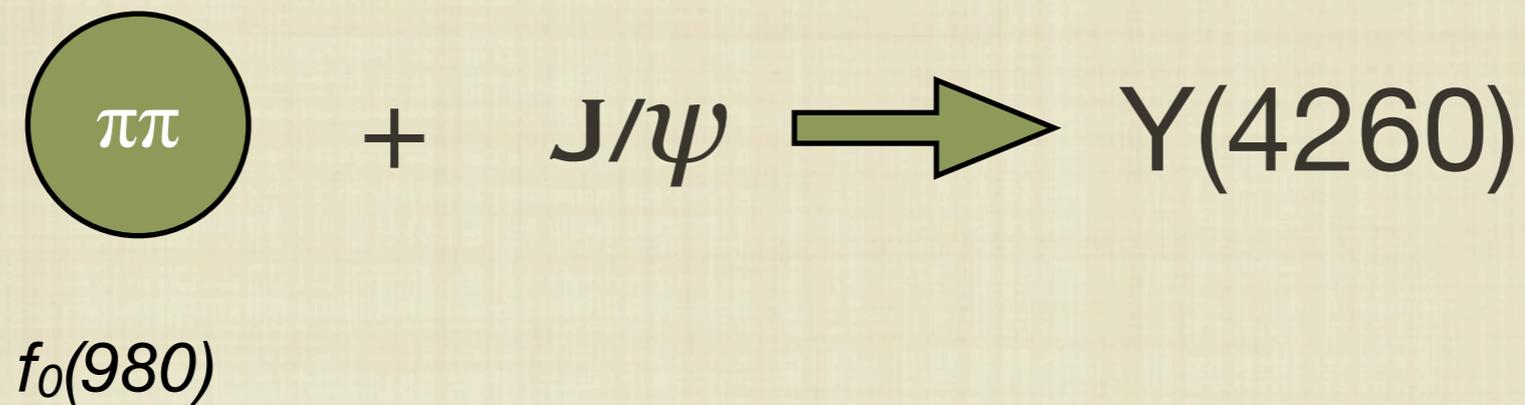
BaBar



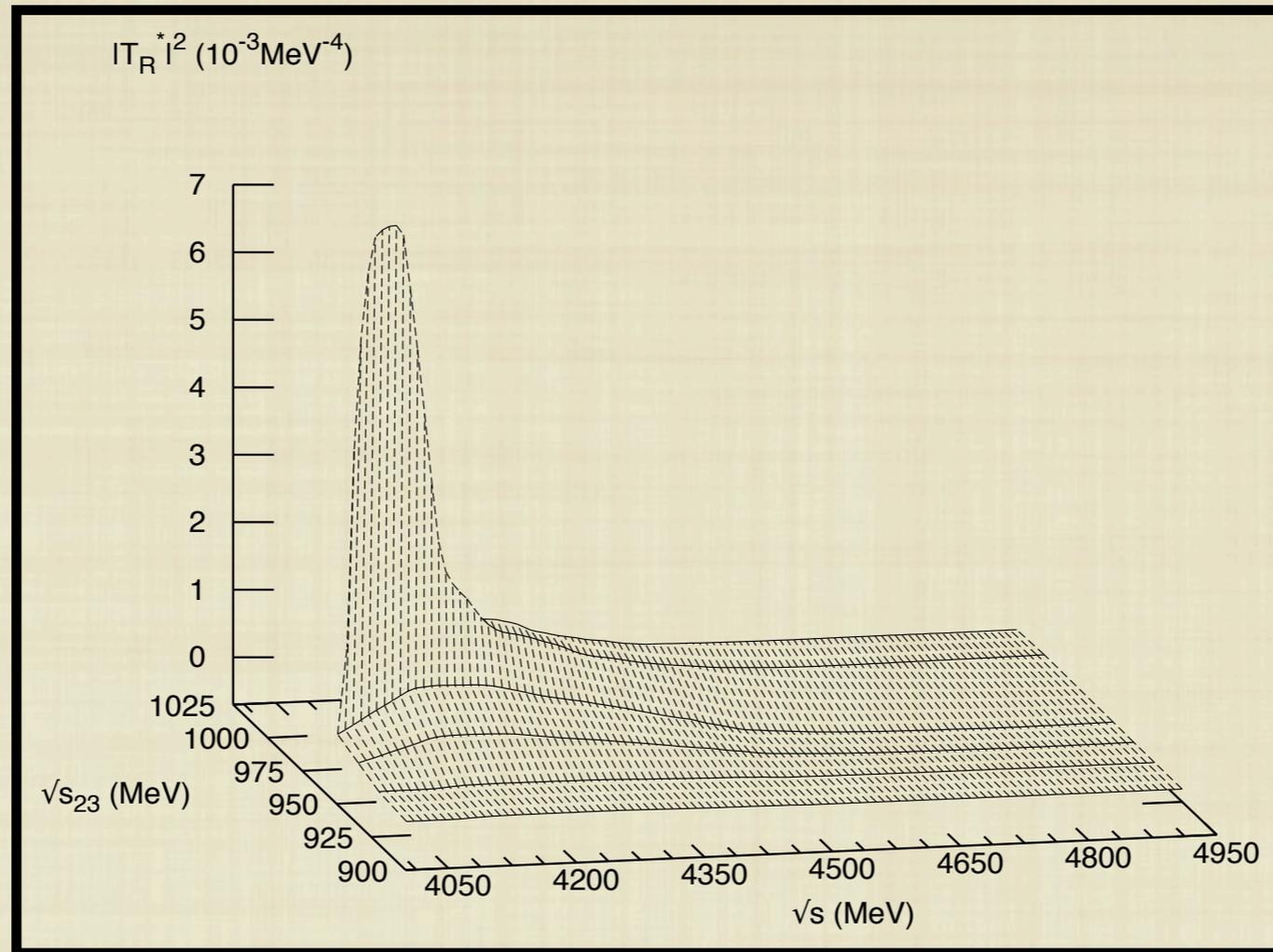
CLEO



- Enhancement near 1 GeV in the $\pi\pi$ invariant mass.
- Analogy with X(2175) :



- We consider $J/\psi\pi\pi$, $J/\psi K\bar{K}$ as coupled channels.



SUMMARY AND FUTURE PLANS

- WE HAVE OBTAINED FOUR Σ 'S AND TWO Λ 'S RESONANCES IN THE $\pi\bar{K}N$, WHICH CORRESPOND TO ALL THE $1/2^+$ Σ AND Λ STATES IN THE ENERGY REGION 1500-1870.
- WE OBSERVED THE $N^*(1710)$, $N^*(2100)$, $\Delta(1910)$ IN THE $\pi\pi N$ SYSTEM AND COUPLED CHANNELS AND A POSSIBLE $N^*(1910)$ WITH $J^P=1/2^+$ IN THE $K\bar{K}N$ SYSTEM .
- WE HAVE STUDIED THE THREE-MESON SYSTEMS, $\phi K\bar{K}$, $\phi\pi\pi$, WHERE WE GOT THE RESONANCE X(2175) .
- A BROAD BUMP IS OBTAINED IN THE STUDY OF THE πKN SYSTEM AROUND 1700 MEV.
- IN THE $J/\psi K\bar{K}$, $J/\psi\pi\pi$ SYSTEMS WE OBTAIN THE Y(4260).
- STUDY OF THE SYSTEMS $\omega\pi\pi$, $\rho\pi\pi$, $K^*\pi K$, *etc.* , TO GET THE LOW-LYING VECTOR RESONANCES AS W(1420), W(1650), ETC.

AND MANY MORE!!