Slow-roll dark energy

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  thawing model
  - minimally coupled scalar
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  freezing model

- Summary

Introduction

Q: Why dynamical dark energy?

Λ is good enough

$w=p/\rho=-1\pm0.07$ (WMAP+BAO+SN)

Ans: 1. We need to understand to what extent

Λ is good enough

2. If non Λ, w is time-dependent in general.

We need a form of w(a) to parametrize

the deviations from Λ

Q: Then, a linear (Chevallier-Polarski-Linder) parametrization,

$w(a)=w_0+w_a(1-a)$,

is widely used . Isn't it enough?

Ans: No not enough. Even if dark energy moves slowly (so that $w \cong -1$), its w(a) is never linear in a.

We demonstrate this for certain quintessence models.

- CPL parametrization

  $w(a)=w_0+w_a(1-a)$

  is not derived for any dark energy models

- Rather, we derive w(a) for slowly-rolling quintessence

We show that for quintessence models (called thawing models: $w \cong -1 \rightarrow w \neq -1$)

$$1 + w(a) =$$

$$(1 + w_0)a^{3(K-1)}\left(\frac{(K - F(a))(F(a)+1)^K + (K + F(a))(F(a)-1)^K}{(K - \Omega^{-1/2})(\Omega^{-1/2}+1)^K + (K + \Omega^{-1/2})(\Omega^{-1/2}-1)^K}\right)^2$$

$$F(a) = \sqrt{1 + (\Omega^{-1/2} - 1)a^{-3}}$$

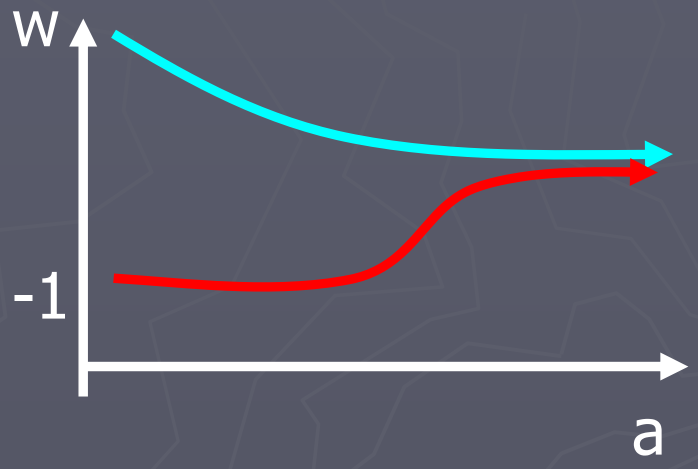
$w(a)$ is never $w(a) = w_0 + w_a(1-a)$

slow-roll conditions for scalar field dark energy

Slow-Roll Thawing Quintessence

- Freezing models: w ↘ -1
  (runaway potential)
- Thawing models: w≅-1 ↗
  (axion, ϕ^2)

$$\frac{1}{2}\dot{\phi}^2 << V \Leftrightarrow w \cong -1$$

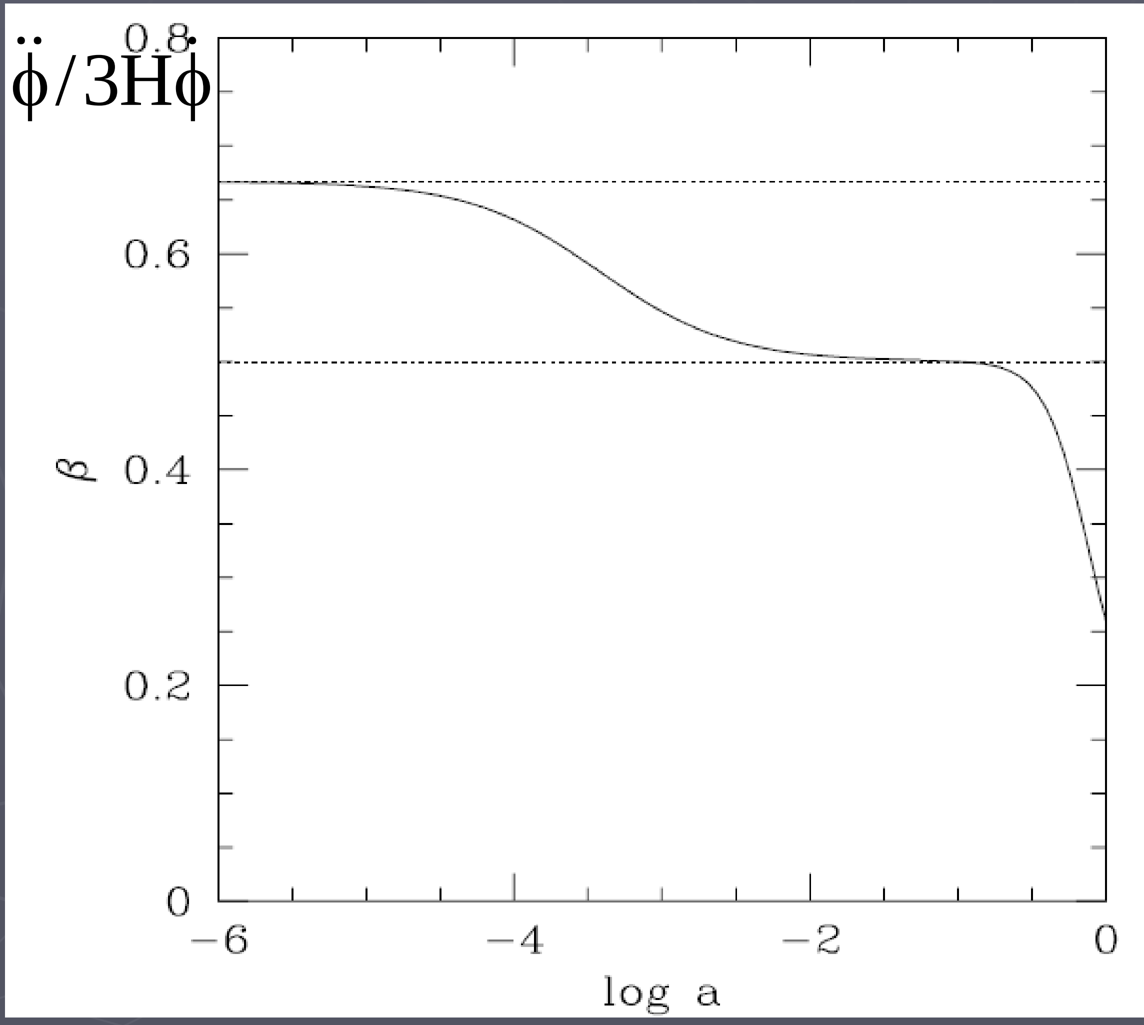


  - observations favors thawing models (w≅-1)
  - Inflation can provide natural initial conditions
    (Ringeval et al. 2010)

Slow-Roll Conditions

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

- ☹ acceleration is not negligible!
- ☺ But the ratio $\beta = \ddot{\phi}/3H\dot{\phi}$ (introduced by Crittenden et al 07) is approximately constant



slow-roll conditions TC (2009)

equation of motion: $\ddot{\phi} + 3H\dot{\phi} + V' = 0$

$\beta = \frac{\ddot{\phi}}{3H\dot{\phi}}$ is not small but is approximately constant

so $\dot{\phi} = -\frac{V'}{3(1+\beta)H}$

slow-roll $\frac{1}{2}\dot{\phi}^2 \ll V$ is consistent if

$\epsilon := \frac{V'^2}{6H^2V} \ll 1$: the first condition

slow-roll conditions(cont.)

the second condition:
from the time derivative of $\dot{\phi} = -\frac{V'}{3(1+\beta)H}$

$$\beta = \frac{\ddot{\phi}}{3H\dot{\phi}} \simeq -\frac{V''}{9(1+\beta)H^2} + \frac{(1+w_B)}{2}$$

time indep time dep

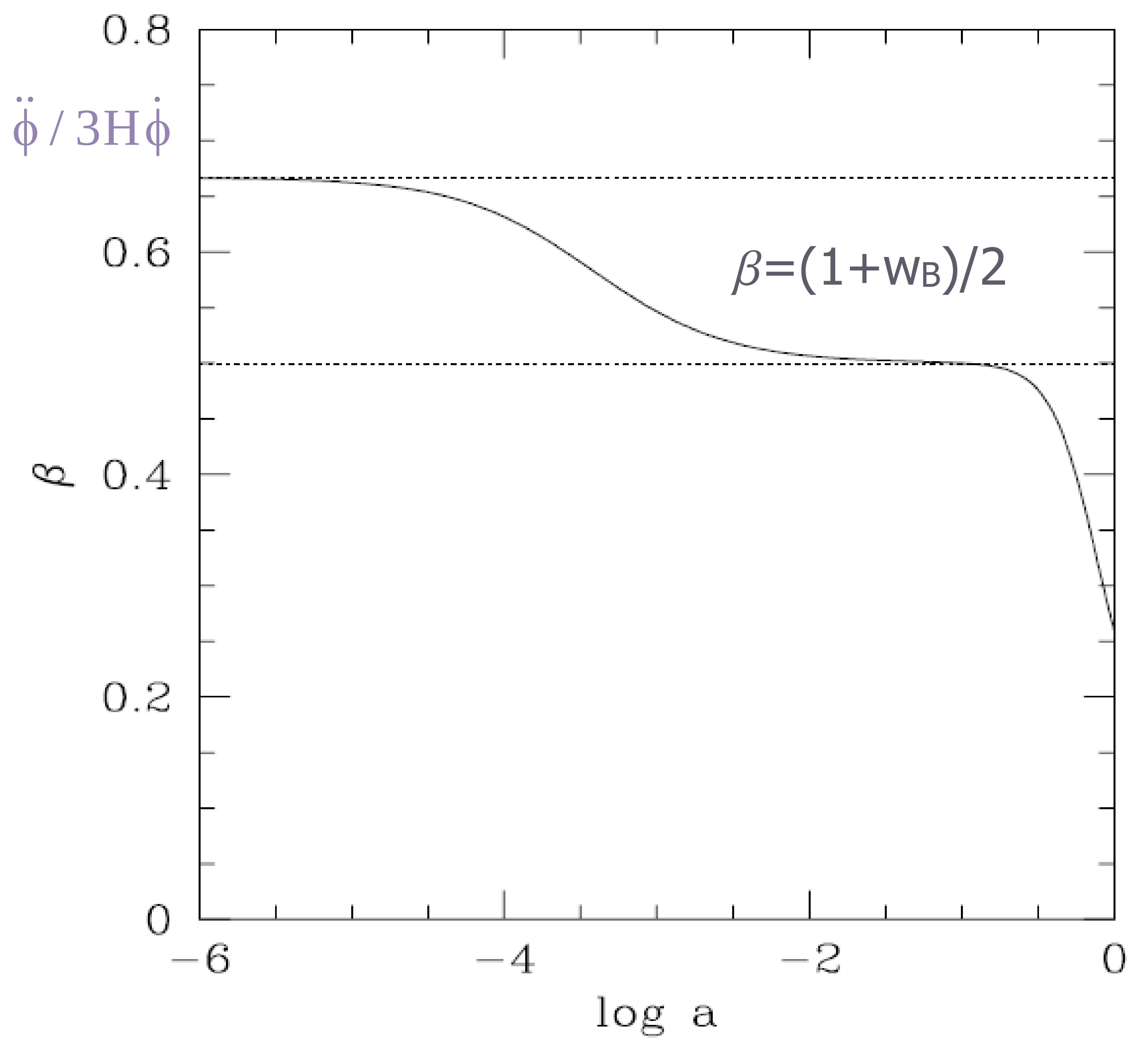
the equality holds if

$$\eta := \frac{V''}{3H^2} \ll 1$$

:the second condition

so that

$$\beta = \frac{\ddot{\phi}}{3H\dot{\phi}} = \frac{1+w_B}{2}$$



$\ddot{\phi}/3H\dot{\phi}$
$\beta=(1+w_B)/2$
0.8
0.6
0.4
0.2
0
β
−6
−4
−2
0
log a

slow-roll conditions: summary

dark energy (subdominant scalar):

matter/radiation is a clock

$$\epsilon := \frac{V'^2}{6H^2V} \ll 1$$

$$\eta := \frac{V''}{3H^2} \ll 1,$$

inflation:

inflaton is a clock

$$(V'/V)^2 \ll 1$$

$$|V''|/V \ll 1$$

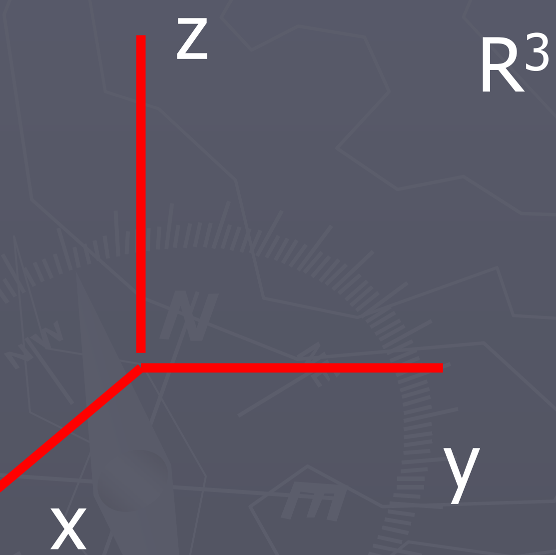
$$\dot{\phi} = -\frac{V'}{3(1+\beta)H}$$

$$\beta = \frac{\ddot{\phi}}{3H\dot{\phi}} = \frac{1+w_B}{2}$$

$$\beta \ll 1$$

equation of state of slow-roll scalar field

introduce “coordinate” (parameters) in dark energy “theory space”



w(a)

?

$$p = w_0 \rho_0 + c_s^2(\rho - \rho_0)$$

(X-matter:TC,Sugiyama,Nakamura,1997)

w_0, w_a … ?

approaches Λ smoothly?

equation of state

equation of motion: $\ddot{\phi} + 3H\dot{\phi} + V' = 0$

can be solved assuming w≅-1 (ΛCDM) $a(t) = \left(\frac{1-\Omega_{\phi 0}}{\Omega_{\phi 0}}\right)^{1/3} \sinh^{2/3}(t/t_\Lambda)$

for a Taylor expanded potential

$$V(\phi) = V(\phi_i) + V'(\phi_i)(\phi - \phi_i) + \frac{1}{2}V''(\phi_i)(\phi - \phi_i)^2$$

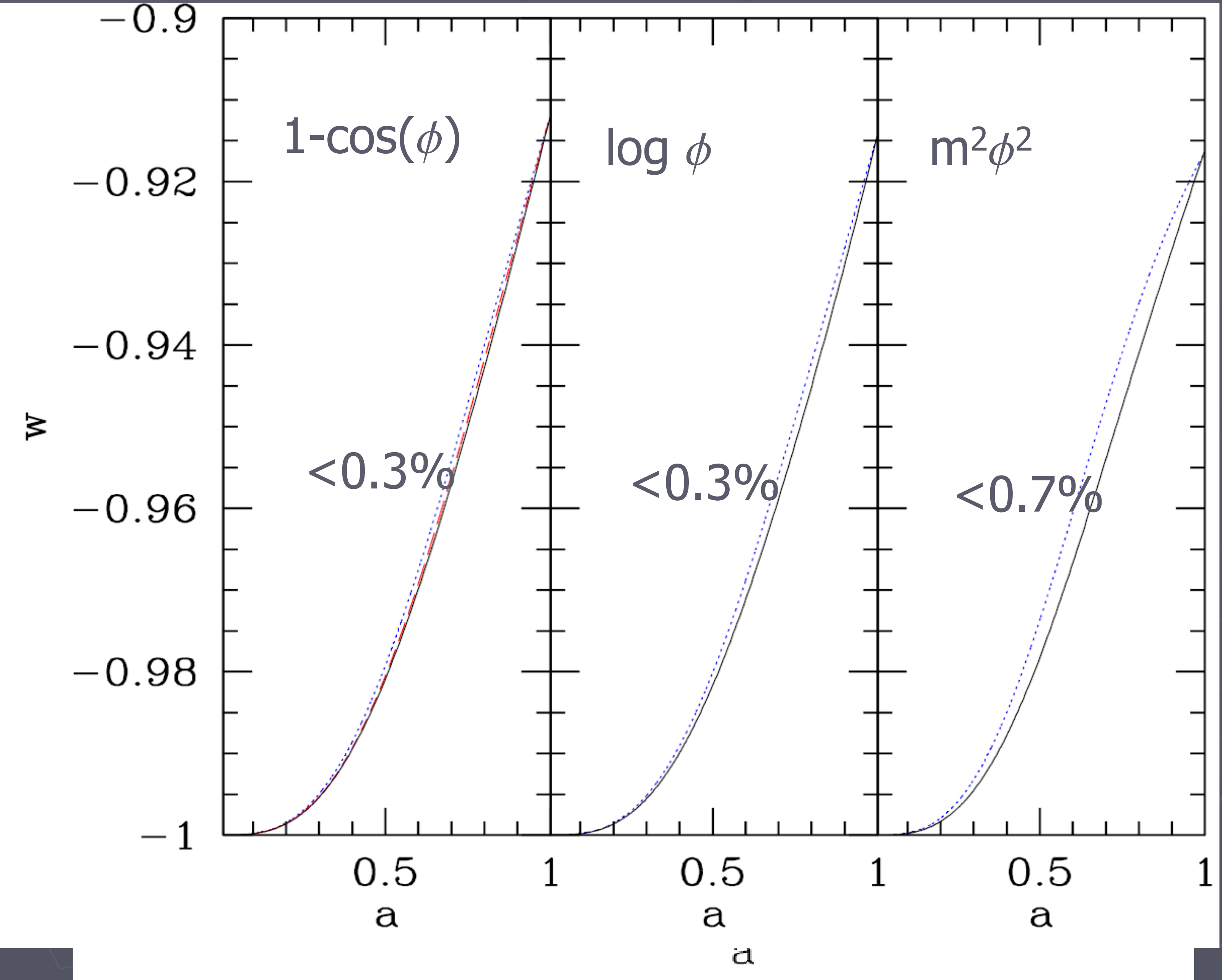
$$1 + w(a) =$$

$$(1+w_0)a^{3(K-1)}\left(\frac{(K-F(a))(F(a)+1)^K + (K+F(a))(F(a)-1)^K}{(K-\Omega_\phi^{-1/2})(\Omega_\phi^{-1/2}+1)^K + (K+\Omega_\phi^{-1/2})(\Omega_\phi^{-1/2}-1)^K}\right)^2$$

$$F(a) = \sqrt{1+(\Omega_\phi^{-1/2}-1)a^{-3}},\ K = \sqrt{1-\frac{4V''(\phi_i)}{3V(\phi_i)}}$$

two parameters

good agreement



black: numerical

blue: w(a)

(red: K at the maximum)

w(a) is universal: even for k-essence/phantom

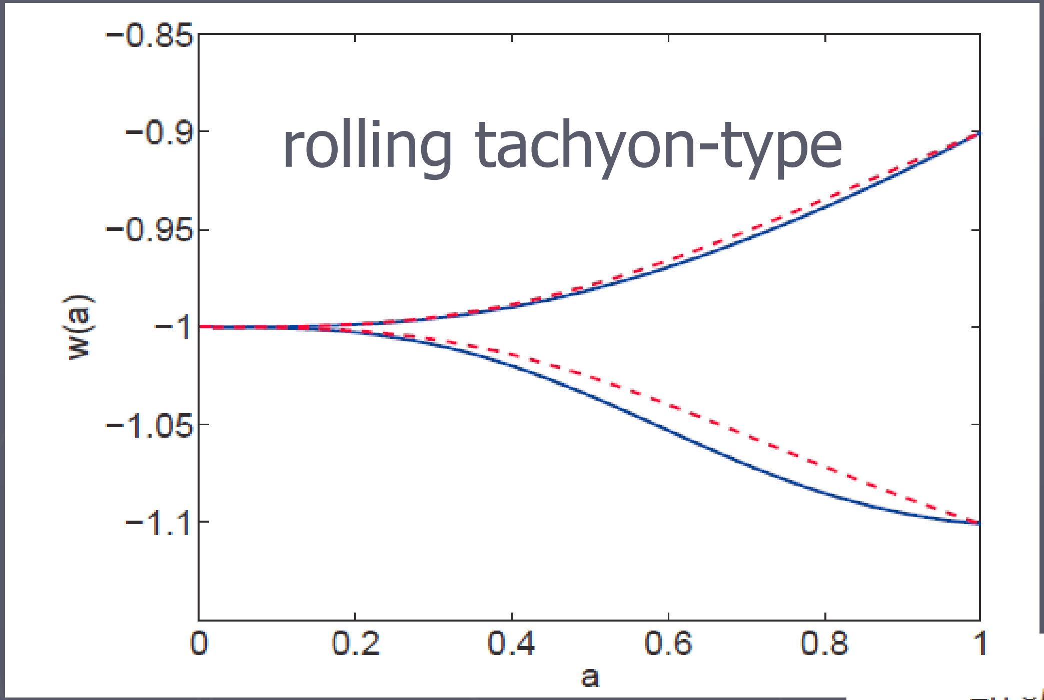
(TC,Dutta,Scherrer)

because

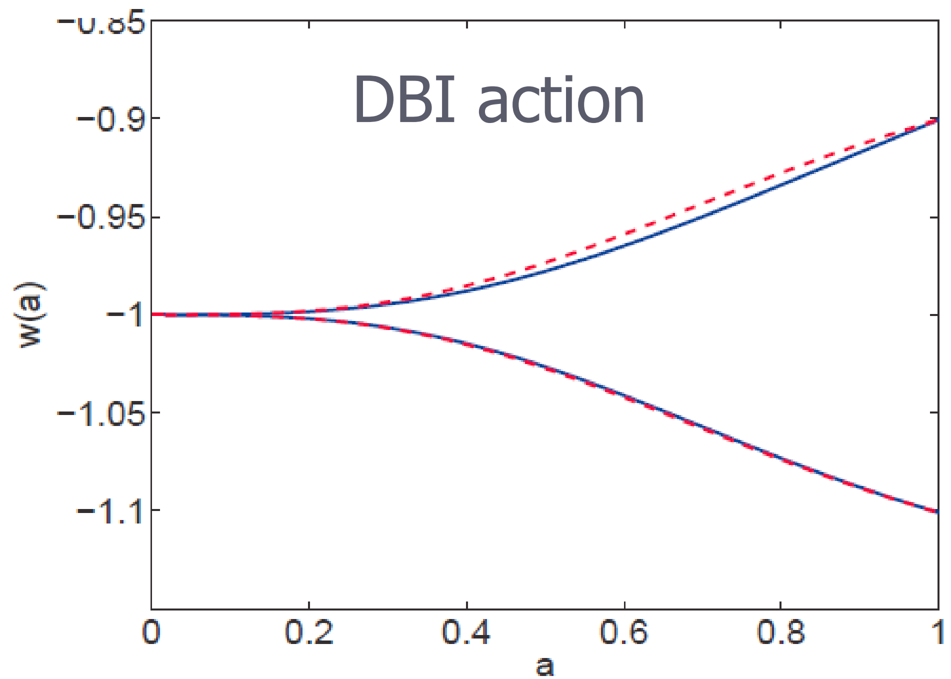
$p(\phi,X) \cong p(\phi,0) + (\partial p(\phi,0)/\partial X)X + \ldots$

for small $X=(d\phi/dt)^2/2$,
which reduces to slow-roll quintessence by field redefinition

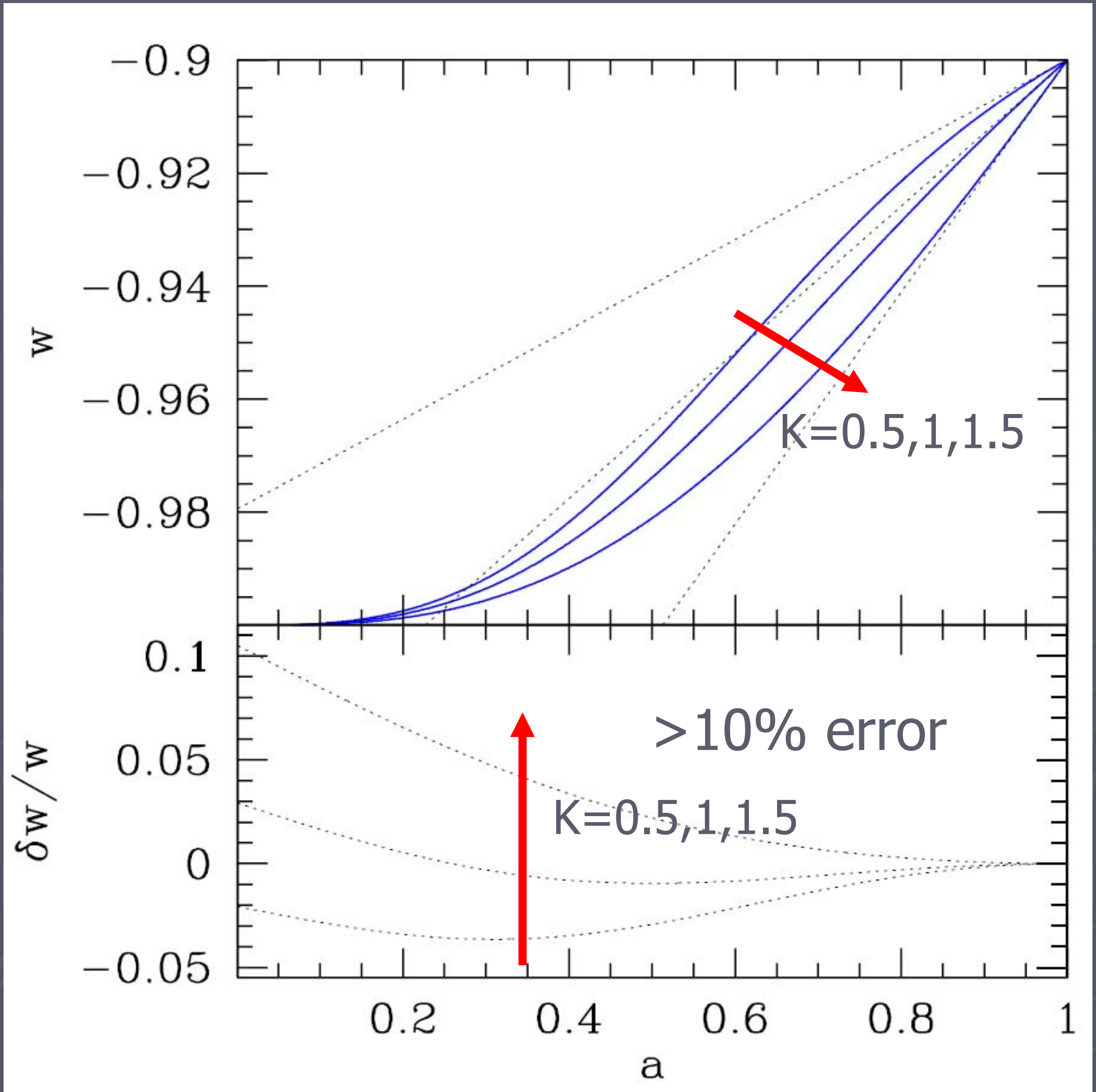
rolling tachyon, DBI, ...



TC,Dutta,Scherrer(2009)



w(a) is never linear in a



solid(blue): w(a)

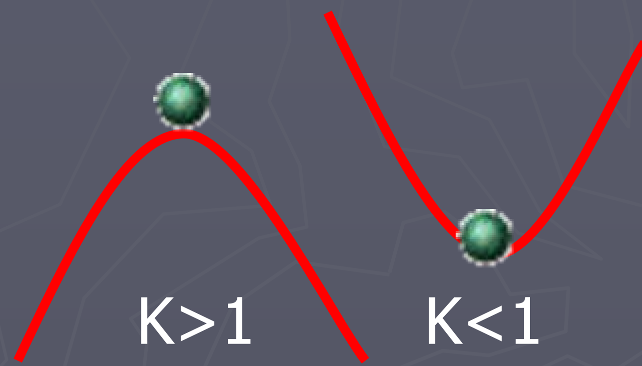
dotted: linear

Observational Constraints in (w_0,K) plane

Observational constraints in (w_0,K) plane

(TC,Dutta,Scherrer)

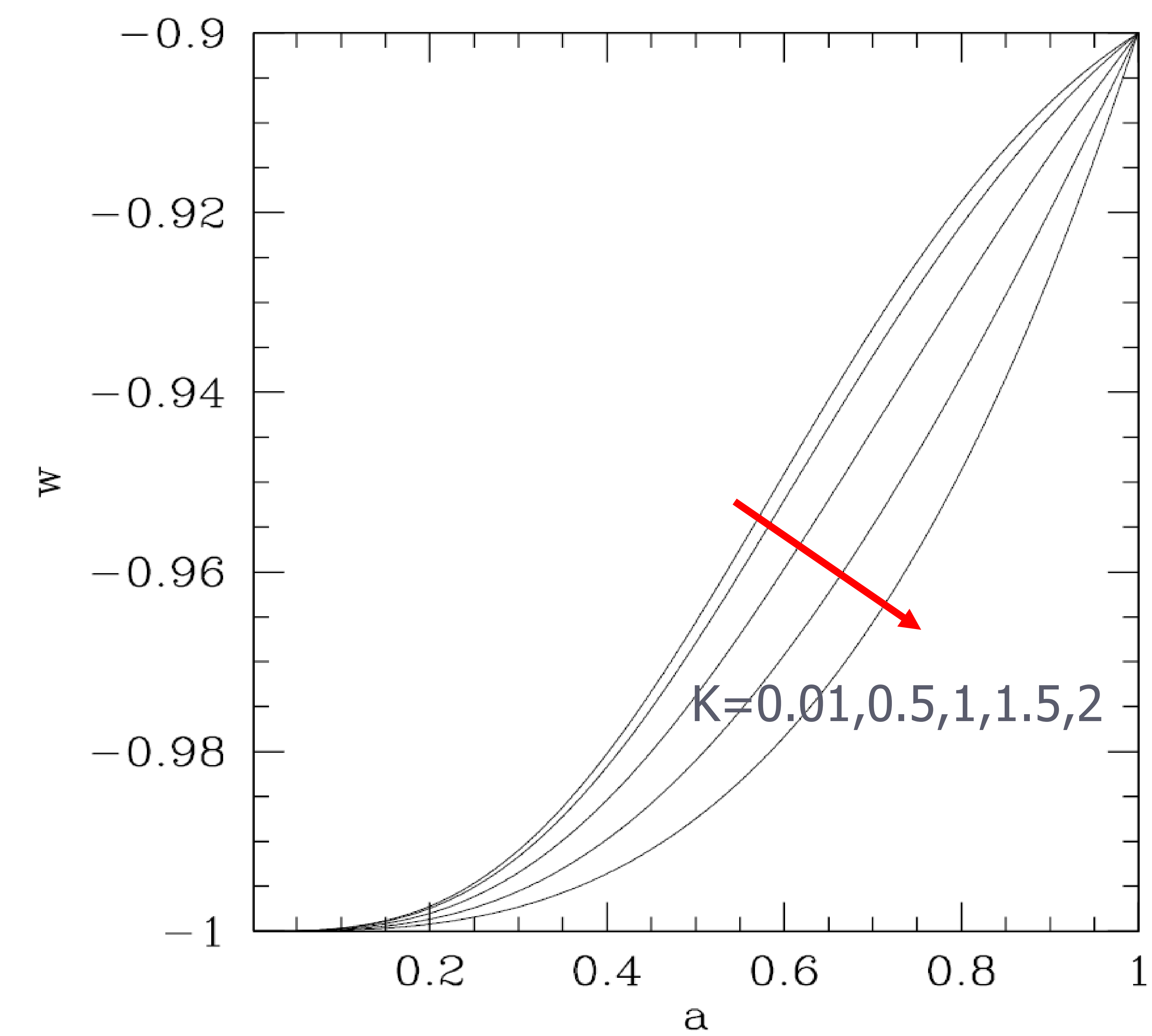
- w_0=-1: Lambda irrespective of K



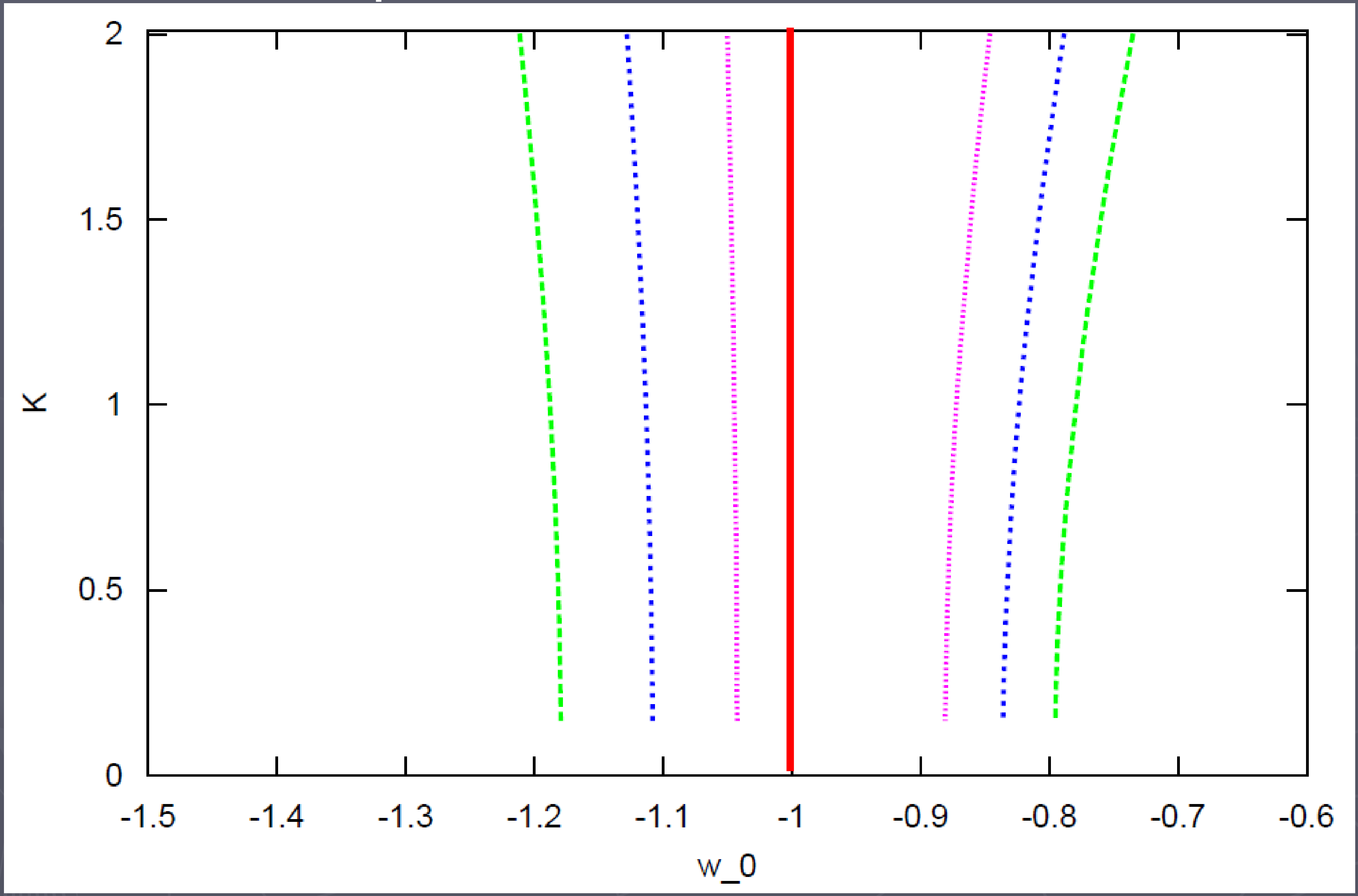
$$1 + w(a) = (1 + w_0) a^{3(K-1)} \left(\frac{(K - F(a))(F(a)+1)^K + (K + F(a))(F(a)-1)^K}{(K - \Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2} + 1)^K + (K + \Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2} - 1)^K} \right)^2$$

$$K = \sqrt{1 - \frac{4}{3} \frac{V''(\phi_i)}{V(\phi_i)}}$$

$$= \sqrt{1 - \frac{4}{3} \frac{V''(\phi_i)}{F_X(0) V(\phi_i)^2}}$$

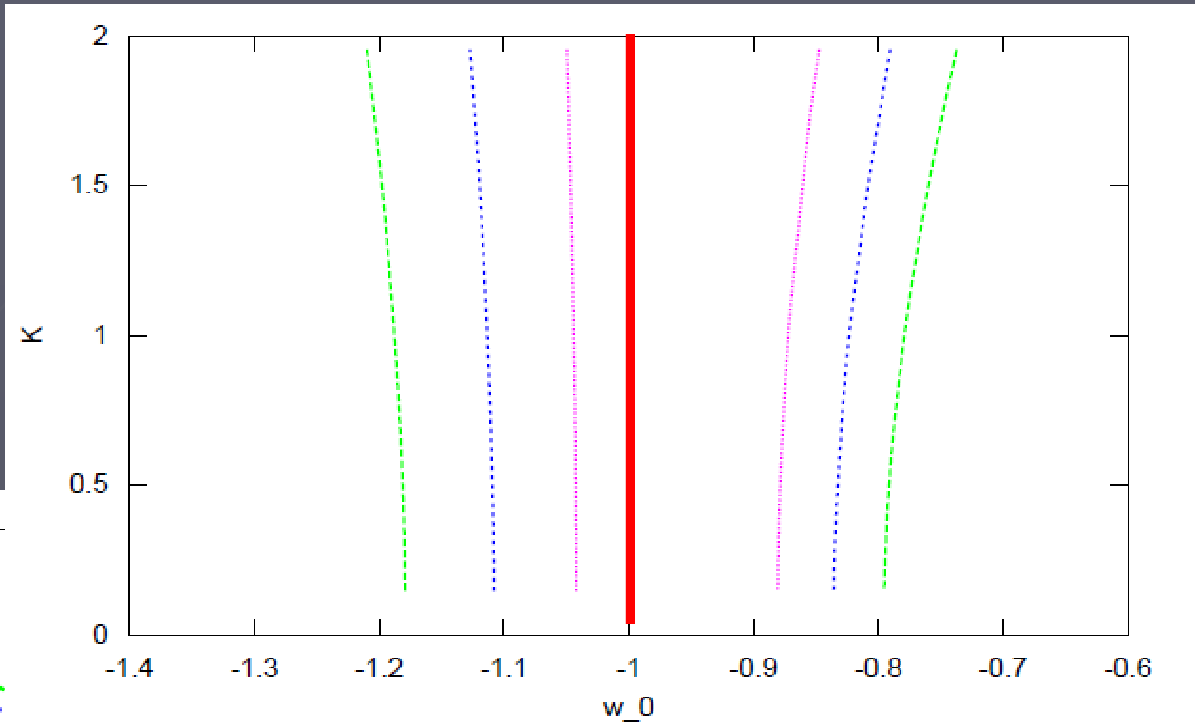

−0.9
−0.92
−0.94
−0.96
−0.98
−1
w
0.2
0.4
0.6
0.8
1
a
K=0.01,0.5,1,1.5,2

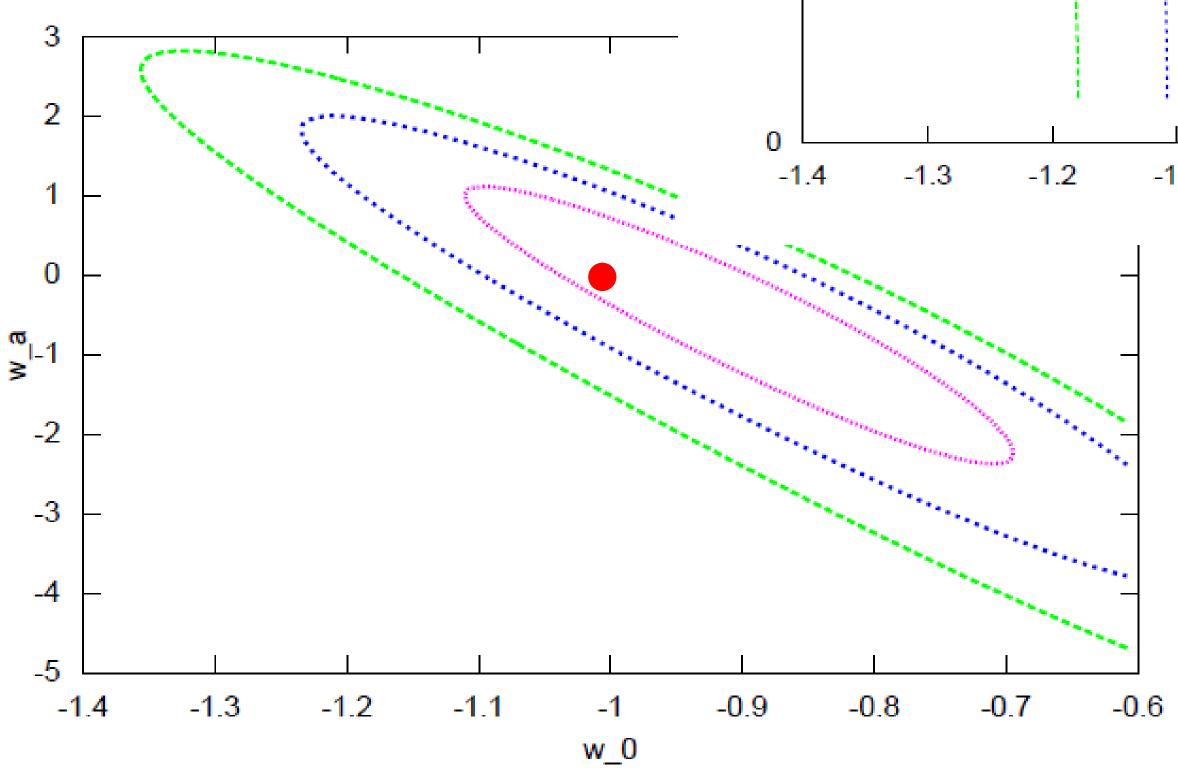
from 397 Supernovae and BAO



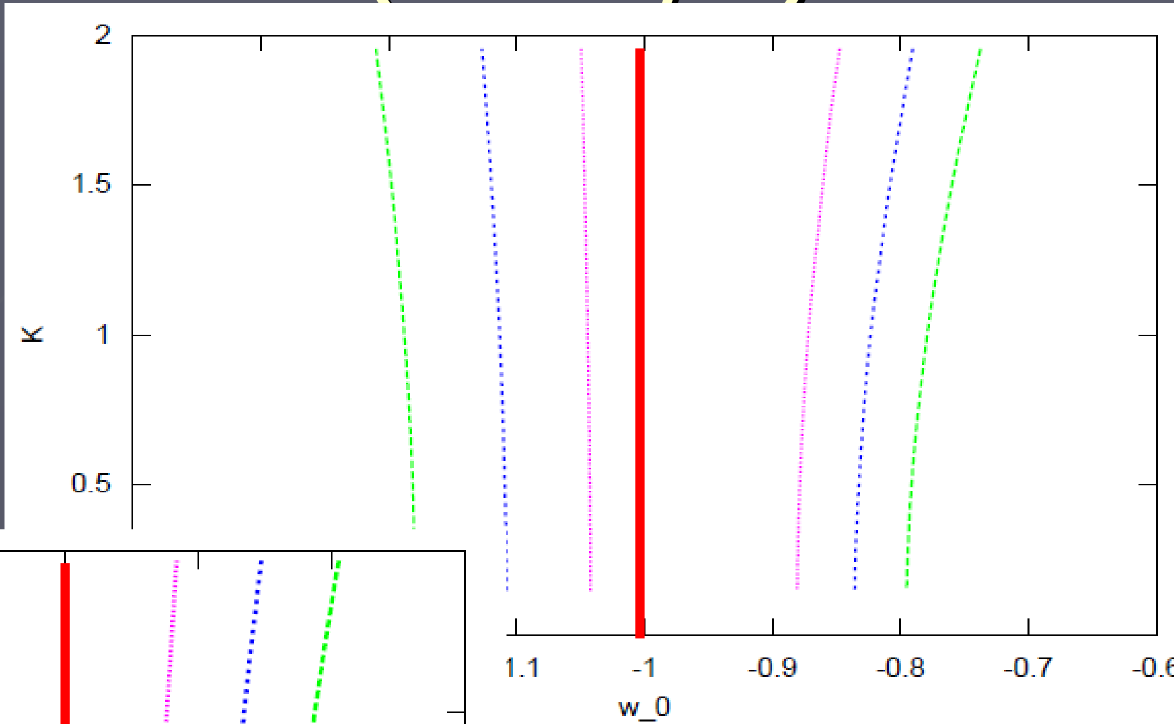
consistent with Λ: $-1.04 < w_0 < -0.86$ (1σ)

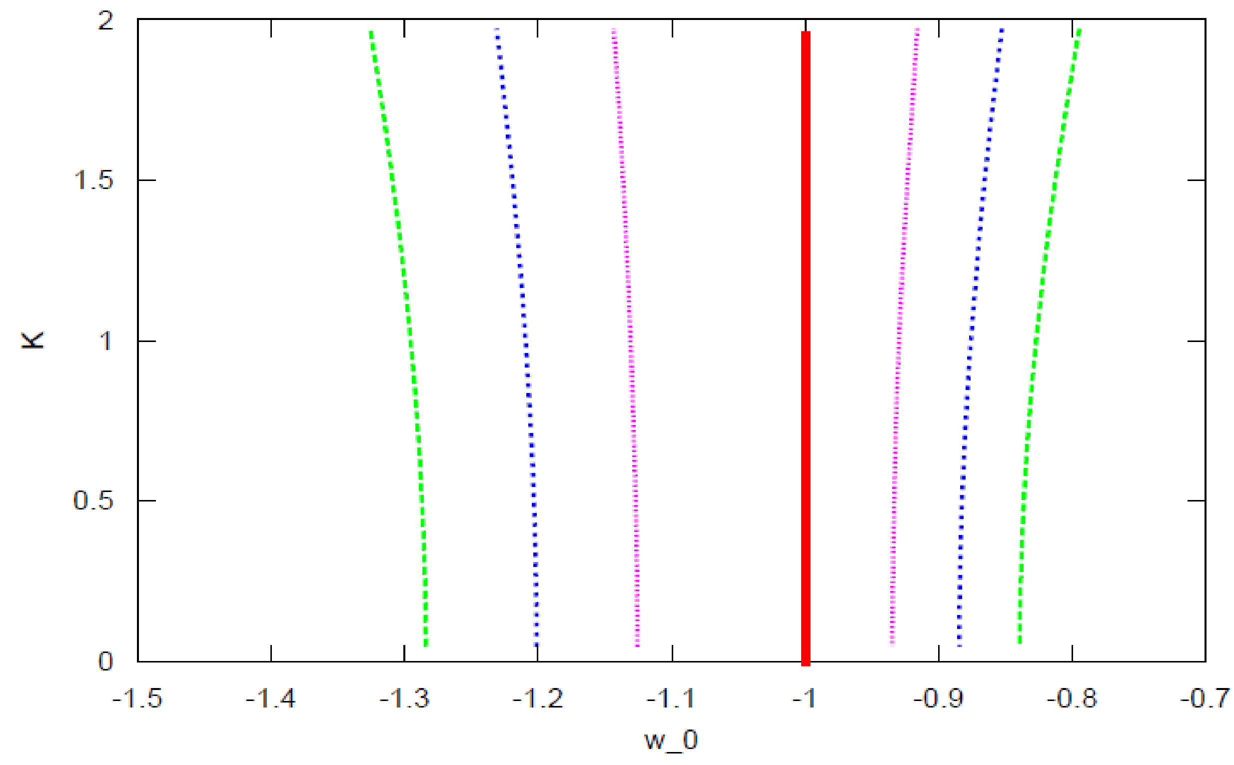
point vs. line





new BAO data(2009/7)





Still, Lambda is consistent!

extension: non-minimal coupling

non-minimally coupled scalar

(TC,Siino,Yamaguchi)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - F(\phi)R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] + S_m$$

← conformal coupling: F=1/12 ϕ^2

dilaton coupling: F=exp(-$\lambda\phi$)

simply we do not know how dark energy interacts with gravity

what is the equation of motion of slow-roll non-minimally coupled scalar field ?

equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + 6F'(\phi)\left(\dot{H} + 2H^2\right) = 0,$$

$$3H^2 = \kappa^2\left(\rho_B + \frac{1}{2}\dot{\phi}^2 + U\right) =: \kappa^2(\rho_B + \rho_\phi) =: \kappa^2\rho_{\mathrm{tot}}$$

$$U := V + 6H\left(\dot{F} + HF\right)$$

slow-roll conditions during matter/radiation era:

$$\frac{1}{2}\dot{\phi}^2 \ll U$$

equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + V'_{\text{eff}} = 0,$$
$$V'_{\text{eff}} \equiv V' + 3F'H^2(1 - 3w_B)$$

ratio:

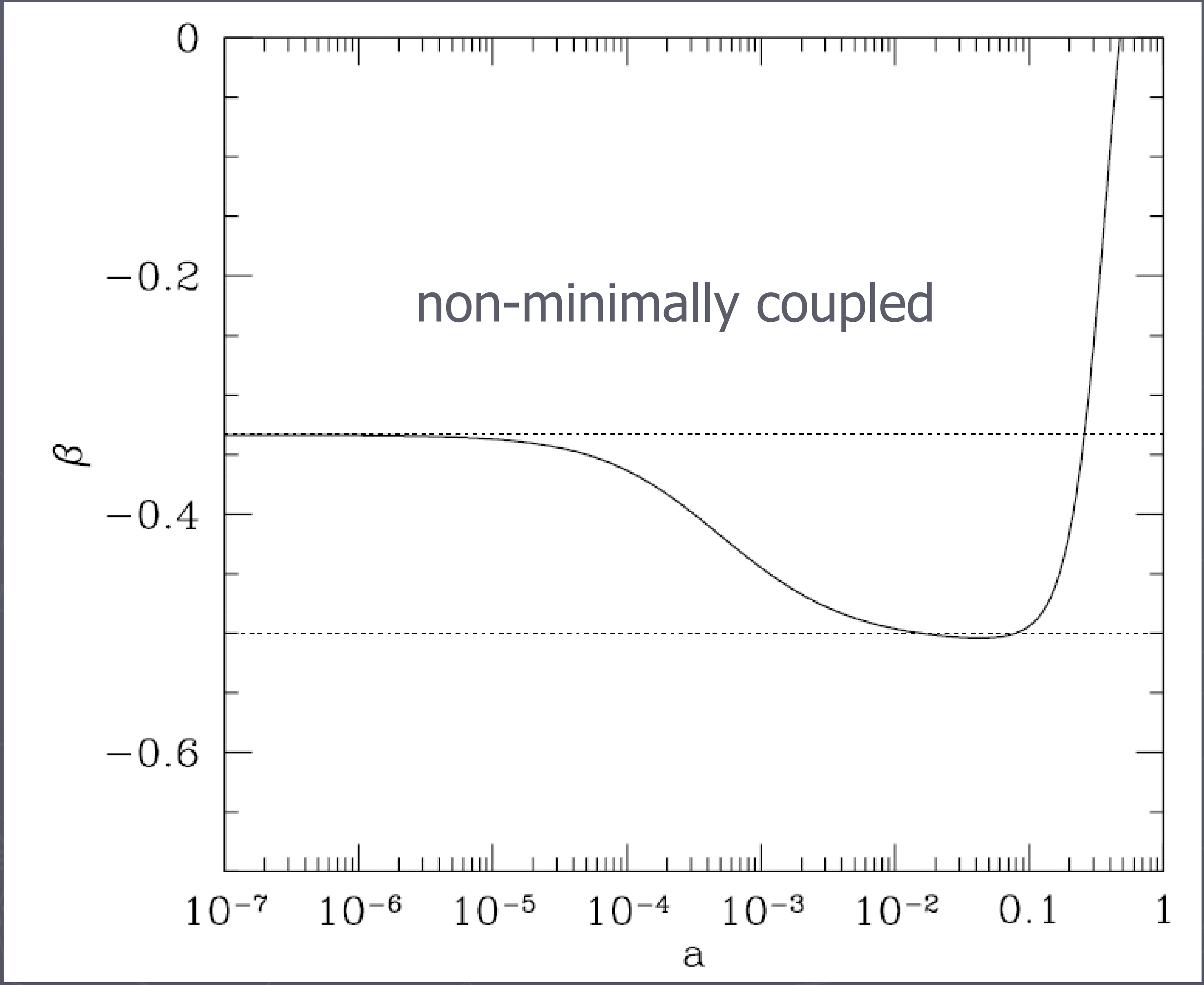
$$\beta = \frac{\ddot{\phi}}{3H\dot{\phi}} \simeq -\frac{\dot{H}}{3H^2} - \frac{V''}{9(1+\beta)H^2} - \frac{F''(1-3w_B)}{3(1+\beta)} - \frac{V'_{\text{eff}} - V'}{V'_{\text{eff}}}$$
$$= \frac{w_B - 1}{2} - \frac{V''}{9(1+\beta)H^2} - \frac{F''(1-3w_B)}{3(1+\beta)} + \frac{V'}{V'_{\text{eff}}}$$

the slow-roll conditions

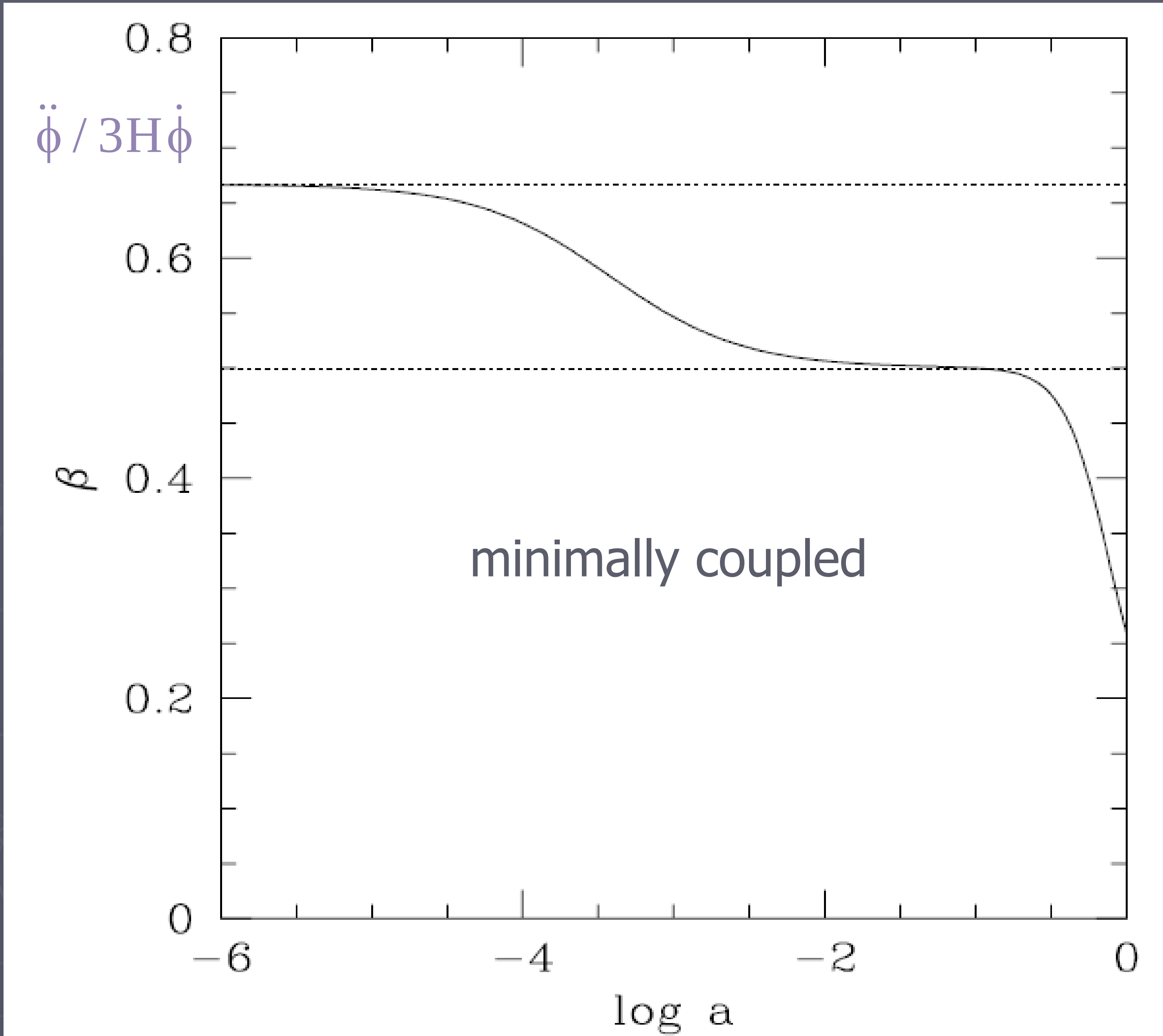
$$\eta := \frac{V''}{3H^2}; \qquad |\eta| \ll 1 \quad \text{and} \quad |F''(1-3w_B)| \ll 1 \quad \text{and} \quad \left|\frac{V'}{V'_{\text{eff}}}\right| \ll 1$$

again we have
a constant (but negative) ratio

$$\beta = \frac{w_B - 1}{2}$$



non-minimally coupled
β
0
−0.2
−0.4
−0.6
10−7
10−6
10−5
10−4
10−3
10−2
0.1
1
a



$\ddot{\phi} / 3H\dot{\phi}$
β
0.8
0.6
0.4
0.2
0
−6
−4
−2
0
log a
minimally coupled

minimal vs. non-minimal

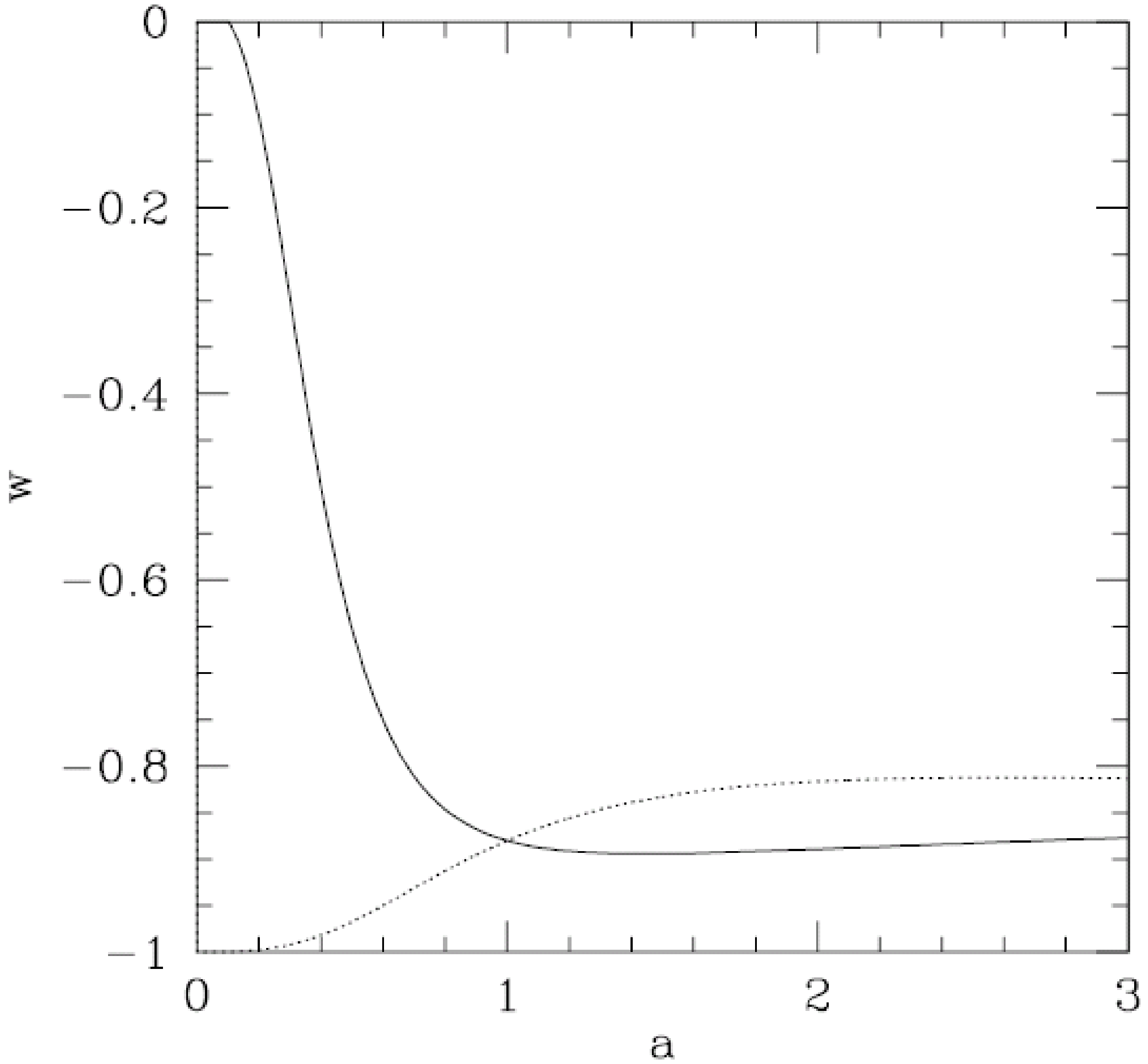
- minimal

$$\frac{\ddot{\phi}}{3H\dot{\phi}} = \frac{1+w_B}{2}$$ >0

- non-minimal

$$\frac{\ddot{\phi}}{3H\dot{\phi}} = \frac{w_B-1}{2}$$ <0

the equation state approaches asymptotically toward that of a minimally coupled scalar



solid: non-minimal w(a)

dotted :minimal w(a)

freezing model

slow-roll conditions for freezing model (TC,09)

w ≠ -1 → dϕ/dt is not negligible
→ Hubble friction is effective

$$|\beta| = \left|\frac{\ddot{\phi}}{3H\dot{\phi}}\right| \ll 1$$

so that

$$3H\dot{\phi} + V' = 0$$

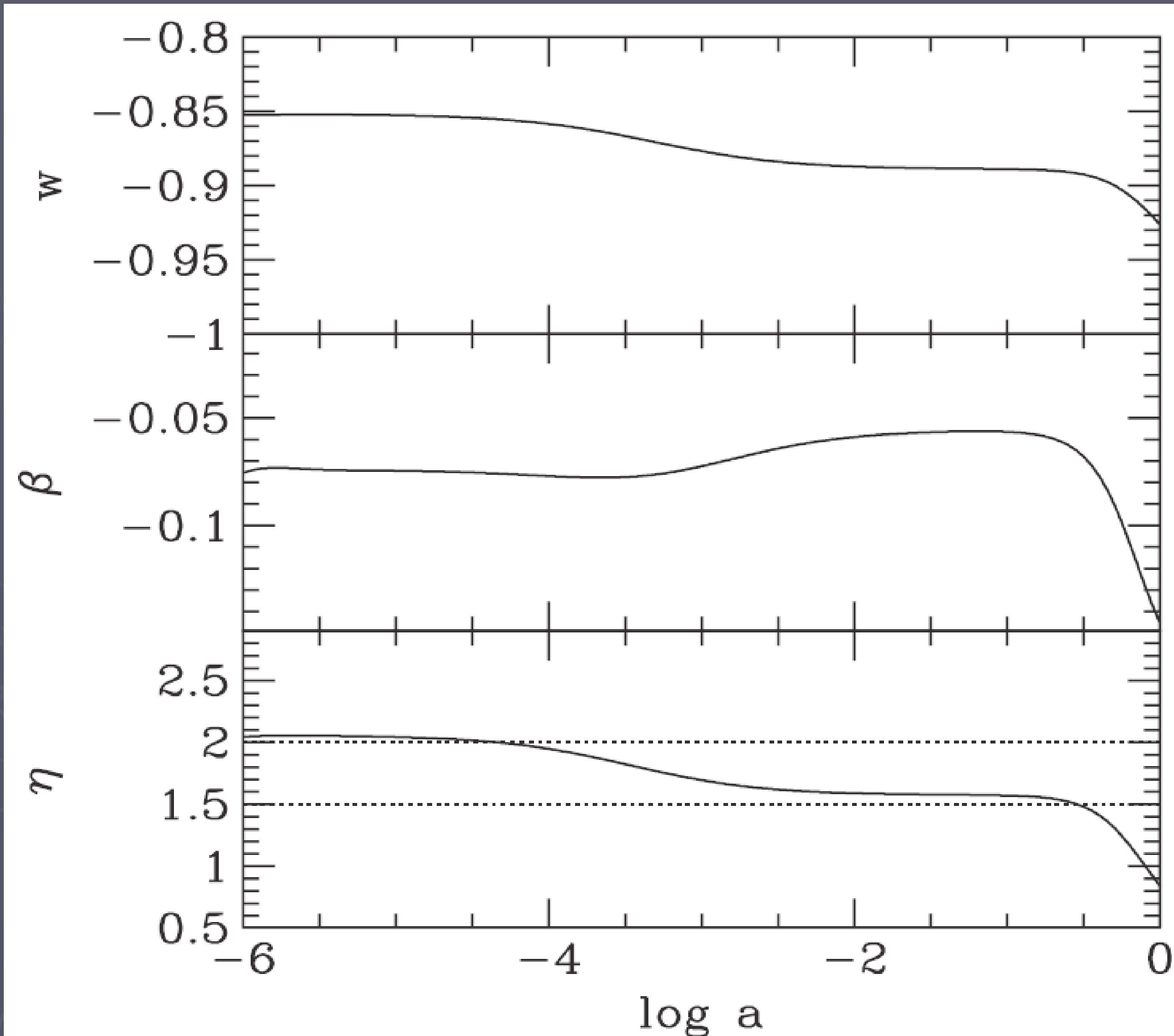
slow-roll:

$$\frac{1}{2}\dot{\phi}^2 \ll V \quad \Leftrightarrow \quad \epsilon := \frac{V'^2}{6H^2 V} \ll 1$$

from

$$\beta = \frac{\ddot{\phi}}{3H\dot{\phi}} = -\frac{V''}{9H^2} + \frac{1}{2}(1 + w_B)$$

$$\eta = \frac{V''}{3H^2} = \frac{3}{2}(1 + w_B)$$



$\eta = 3(1 + w_B)/2$

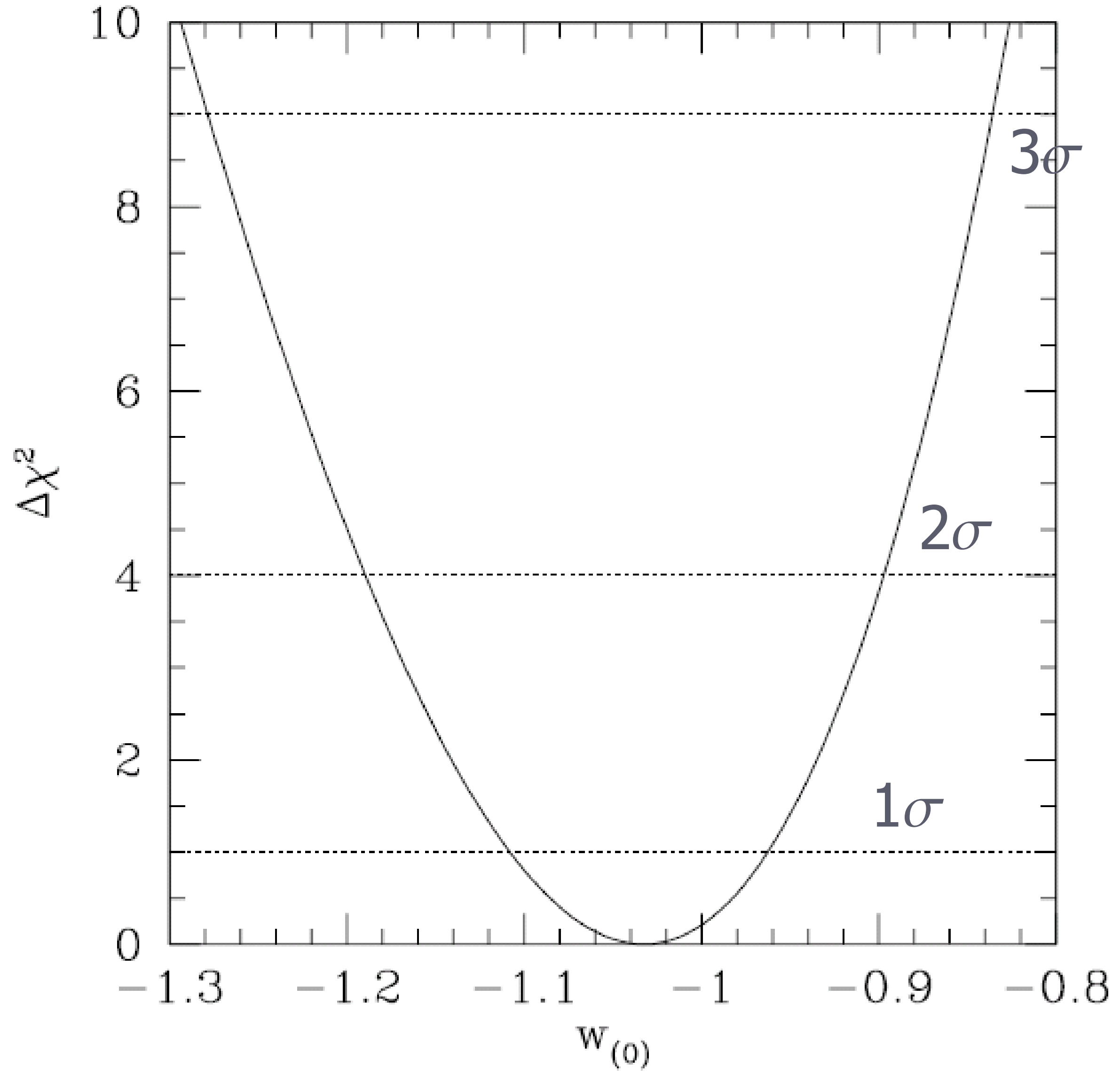
w(a) for freezing model (TC,10)

expand V(ϕ) around the scaling solution
(if $\Gamma = V''V/(V')^2 \cong$ const >1)

one parameter: $w_{(0)}$: eos during MD

$$w(a) = w_{(0)} + \frac{(1-w_{(0)}^2)w_{(0)}}{1-2w_{(0)}+4w_{(0)}^2}\Omega_\phi(a) + \frac{(1-w_{(0)}^2)w_{(0)}^2(8w_{(0)}-1)}{(1-2w_{(0)}+4w_{(0)}^2)(1-3w_{(0)}+12w_{(0)}^2)}\Omega_\phi(a)^2 + \frac{2(1-w_{(0)}^2)w_{(0)}^3(4w_{(0)}-1)(18w_{(0)}+1)}{(1-2w_{(0)}+4w_{(0)}^2)(1-3w_{(0)}+12w_{(0)}^2)(1-4w_{(0)}+24w_{(0)}^2)}\Omega_\phi(a)^3 + \ldots,$$

observational constraint (from SNIa and BAO)



$-1.11 < w_{(0)} < -0.96$ (1σ)

summary

- We have derived slow-roll conditions for scalar fields which are subdominant components of the universe
- CPL (linear) parametrization is not a good parametrization of w(a):

  For wide range of scalar field models with $w \cong -1$

  in matter era (thawing model) , be it quintessence, k-essence, or phantom,

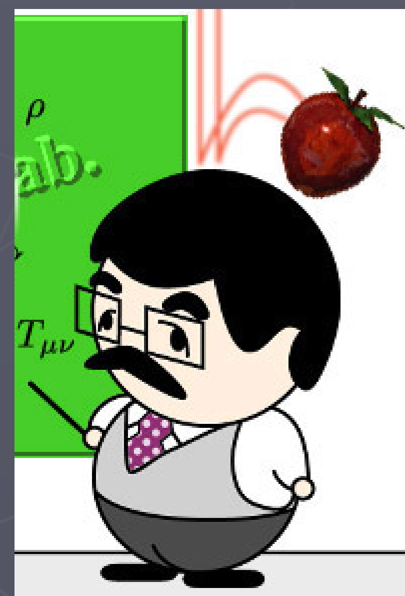
  w(a) has a universal functional form and is not well fitted by linear parametrization even if $w \cong -1$
- Λ is consistent with current SN and BAO data
- non-minimal coupling may be discriminated by the sign of the ratio $\ddot{\phi}/3H\dot{\phi}$

future

- slow-roll conditions/equation of state for other theories of gravity (f(R), DGP, cascading gravity ,etc…)

- the effect of thawing axion dynamics with large initial misalignment angle $\theta \simeq \pi$ on axion abundance

Happy birthday to Nakamura-san and Maeda-san!



use this parametrization!

$$1+w(a)=$$

$$(1+w_0)a^{3(K-1)}\left(\frac{(K-F(a))(F(a)+1)^K+(K+F(a))(F(a)-1)^K}{(K-\Omega_\phi{}^{-1/2})(\Omega_\phi{}^{-1/2}+1)^K+(K+\Omega_\phi{}^{-1/2})(\Omega_\phi{}^{-1/2}-1)^K}\right)^2$$

$$F(a)=\sqrt{1+(\Omega_\phi{}^{-1/2}-1)a^{-3}}$$