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Non-linear perturbations from cosmological inflation

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summary:

- **non-linear perturbations offer distinctive observational signatures of physics of inflation**

gravity + inflation:

- **accelerated expansion of FLRW cosmologies**
  - **horizon, entropy, flatness...**
- **relativistic perturbations**
  - **quantum fluctuations of free scalar and tensor modes**
  - **non-linear perturbations**
    - **second-order tensor modes from scalar fluctuations**
    - **second-order density perturbations and non-Gaussian distributions**
- **inflation driven by (modified) gravity**
  - **e.g., Starobinsky (1980)**

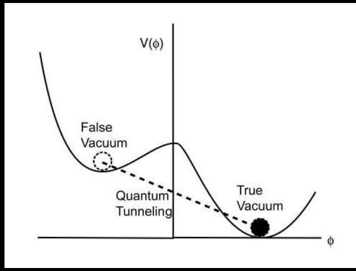
coming of age...

inflation circa 1990

extended inflation

La & Steinhardt 1989
Barrow & Maeda 1990
Steinhardt & Accetta 1990

- simple, compelling model for gravity-driven inflation...
- false vacuum + first-order transition
- + Brans-Dicke gravity



$$L = \Phi R - \frac{\omega}{\Phi}\left(\nabla\Phi\right)^2 - V_{falsevacuum}$$

- solves graceful exit problem of Guth’s old inflation
  - Φ grows during inflation, Hubble rate decreases, until first-order transition completes
- scale-free gravity (one dimensionless parameter)

dynamical solution to hierarchy problem

E. Weinberg 1989
Liddle & Wands 1992

- start near Planck scale

$$H^2 \approx \frac{M_{GUT}^4}{M_{Pl}^2(\Phi)} \approx M_{Pl}^2(\Phi) \approx M_{GUT}^2$$

- bubble nucleation rate

$$\Gamma = M_{GUT}^4 \exp\left(-S_E\right)$$

  - $S_E >> 1$ is dimensionless Euclidean action ("shape parameter")

- percolation parameter

$$P = \frac{\Gamma}{H^4} \approx \frac{M_{Pl}^4(\Phi)}{M_{GUT}^4} \exp\left(-S_E\right)$$

  - P grows as Φ grows (gravity gets weaker) and H decreases

- phase transition completes / inflation ends when $p=1$

$$\Rightarrow \quad M_{Pl} \approx M_{GUT} \exp\left(S_E/4\right) >> M_{GUT}$$

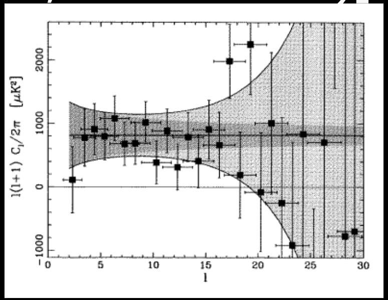
power law inflation

- power-law inflation

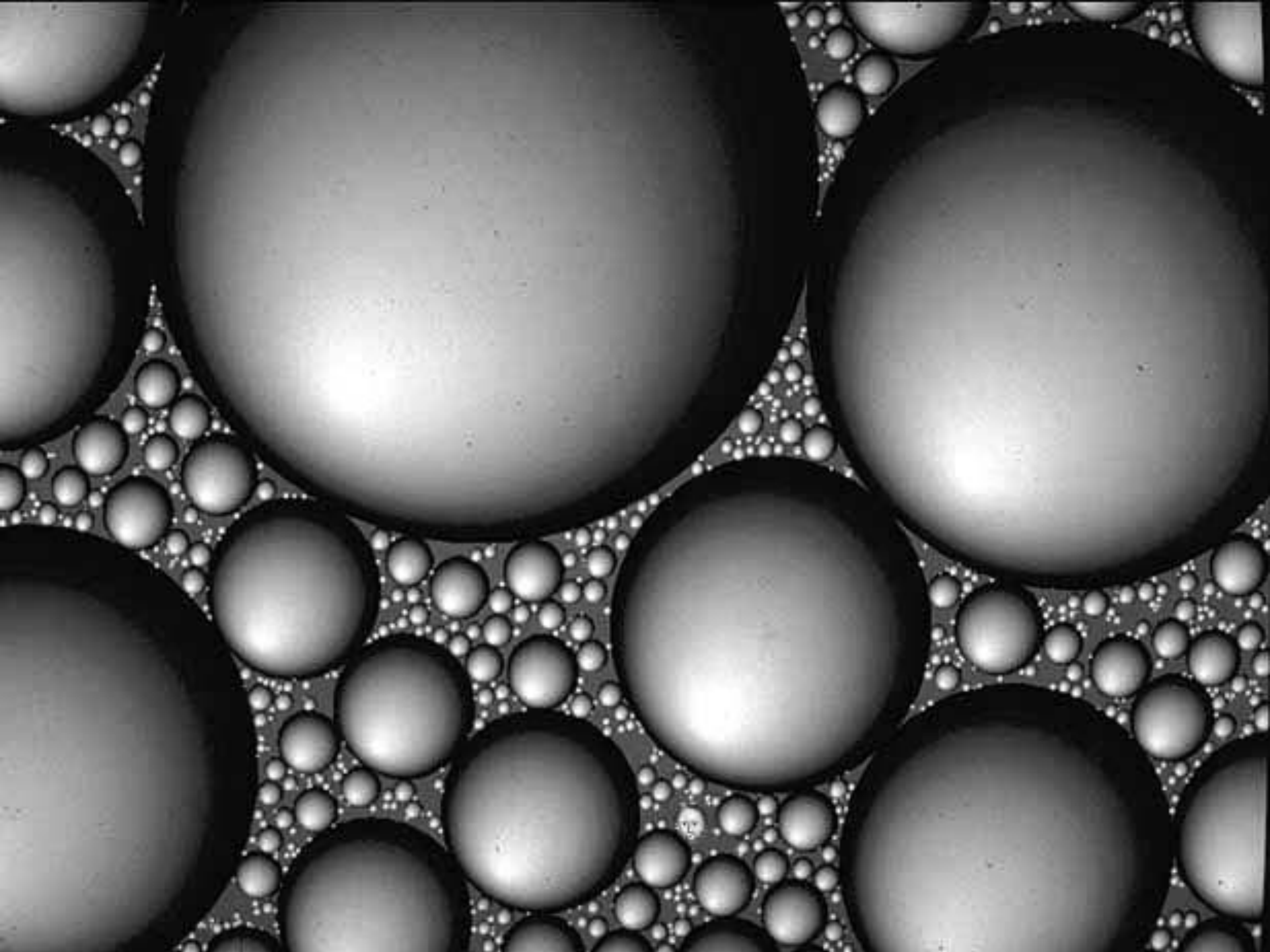
$$a \propto t^p \quad ; \quad p = \frac{2\omega + 3}{4}$$

- linear perturbations
  - [conformal transform to Einstein frame (Brans 1962, Maeda 1989)]
  - reproduces scale-invariant spectrum as $\omega \rightarrow \infty$

CoBE (1994)



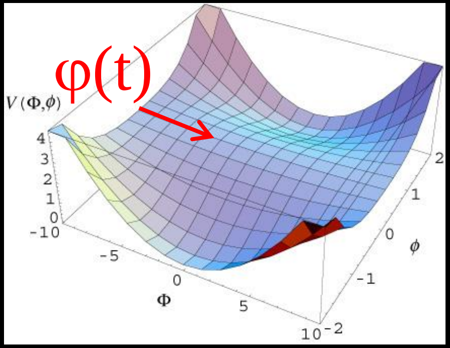
- **non-linear perturbations**
  - first-order transition leads to distribution of bubbles
  - spectrum of bubbles also becomes scale-invariant as $\omega \rightarrow \infty$
  - ***"big bubble problem"*** Weinberg (1989); Liddle & Wands; Maeda & Sakai (1992)



hybrid inflation

Linde 1993

- inflaton field changes *shape* of false vacuum potential
  - $S_E(t) \Rightarrow \Gamma(t) \sim M^4 \exp[-S_E(t)]$
  - ends by *sudden* phase transition
  - first- or second-order

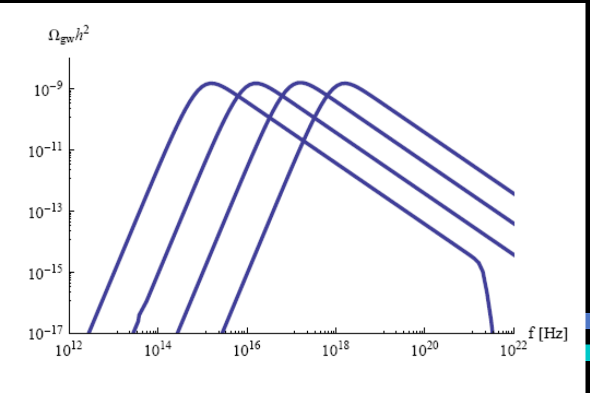


- non-linear perturbations only on small scales

Lyth; Fonseca, Sasaki & Wands 2010

+ see poster by Gong

  - inhomogeneous bubbles or tachyonic preheating
  - spectrum of relic gravitational waves on characteristic scale



Easther '09

Sources of primordial gravitational waves:

Quantum fluctuations of gravitational field

First-order phase transitions?

Preheating after inflation?

Cosmic string cusps?

Primordial density perturbations

Second-order GW from first-order density perturbations

Tomita (1967); Matarrese et al (1994); Hwang; K. Nakamura; Ananda, Clarkson & Wands (2006)

- scalar, vector and tensor modes couple at second and higher order
- tensor perturbations become gauge-dependent
- in longitudinal gauge for general FRW cosmology $w=P/\rho$, $c_s^2=dP/d\rho$

$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = S^{TT}_{ij}$$

where second-order source is transverse-tracefree part of

$$\begin{aligned} S_{ij} \ = \ & 2\Phi\partial_i\partial_j\Phi - 2\Psi\partial_i\partial_j\Phi + 4\Psi\partial_i\partial_j\Psi + \partial_i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\ & - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i(\Psi' + \mathcal{H}\Phi)\partial_j(\Psi' + \mathcal{H}\Phi) \\ & - \frac{2c_s^2}{3w\mathcal{H}}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi) \end{aligned}$$

Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290

GW from density perturbations in radiation era

Ananda, Clarkson & Wands, gr-qc/0612013

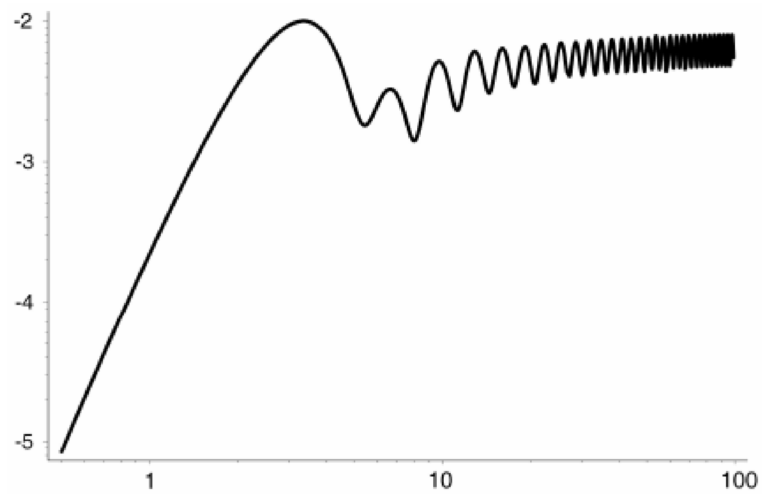
$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = S^{TT}_{ij}$$

- almost scale-invariant primordial density power spectrum

$$P_\Phi(k) = \frac{4}{9}\Delta^2_R(k) \quad \text{for } k << aH$$

- generates almost scale-invariant gravitational wave background

$$\Omega_{\mathrm{GW},0}(k) \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln k} \approx 30\Omega_{\gamma,0}\Delta^4_R(k) \quad \text{for } k >> aH$$



k/aH

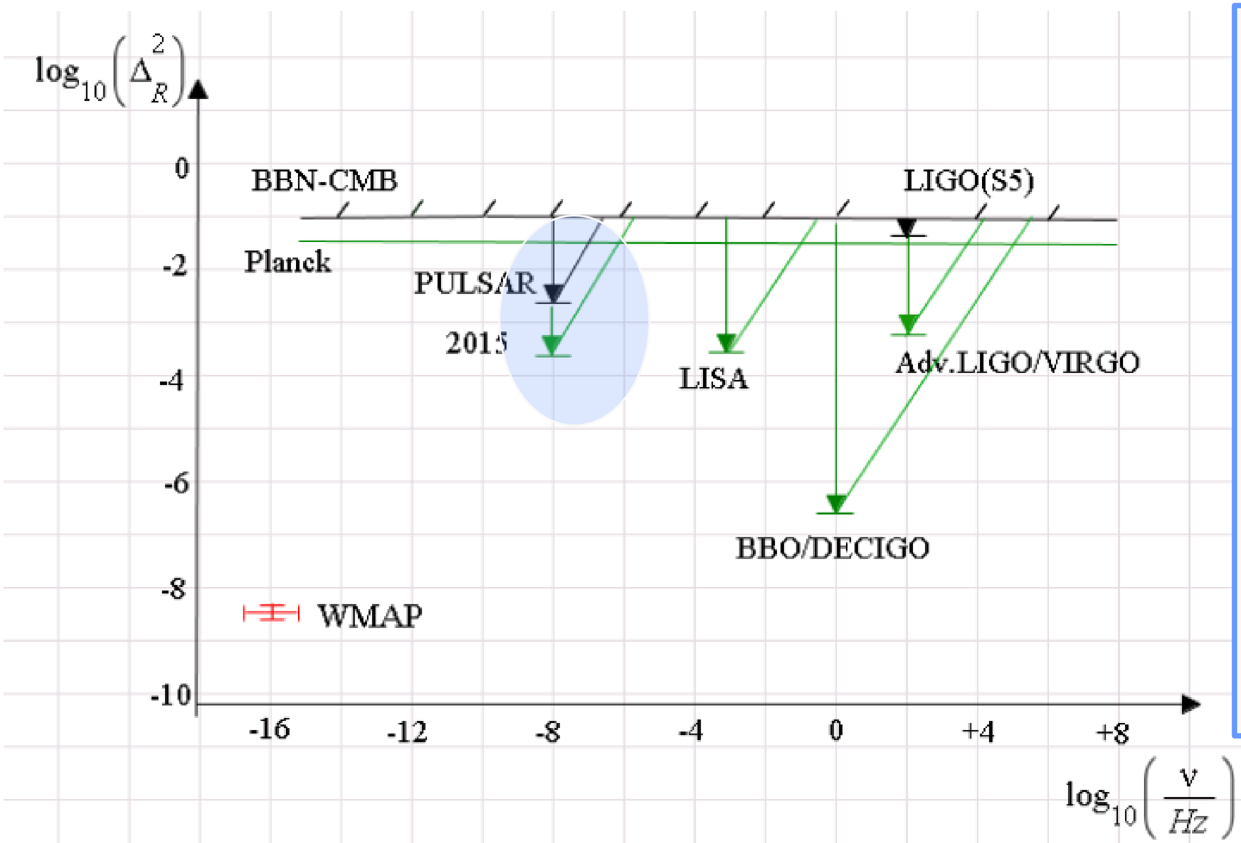
- e.g.,

$$\Omega_{\mathrm{GW},0} \approx 10^{-20} \quad \text{for } \Delta^2_R \approx 10^{-9}$$

Constraints on primordial density perturbations

Assadullahi & Wands, arXiv:0907.4073

$$\Omega_{\mathrm{GW},0}(k) \approx 30\,\Omega_{\gamma,0}\,\Delta_R^4(k)$$



- LIGO/VIRGO
  $\Delta_R^2 < 0.07\,, \quad \nu \approx 100\mathrm{Hz}$
- Advanced LIGO/VIRGO
  $\Delta_R^2 < 8\times10^{-4}$
- LISA
  $\Delta_R^2 < 3\times10^{-4}\,, \quad \nu \approx \mathrm{mHz}$
- BBO/DECIGO
  $\Delta_R^2 < 3\times10^{-7}\,, \quad \nu \approx 1\mathrm{Hz}$

- Pulsar timing data rules out intermediate mass primordial black holes
  Saito & Yokoyama, arXiv:0812.4339 (Phys Rev Lett)
  Bugaev & Klimai, arXiv:09080664

second-order density perturbations

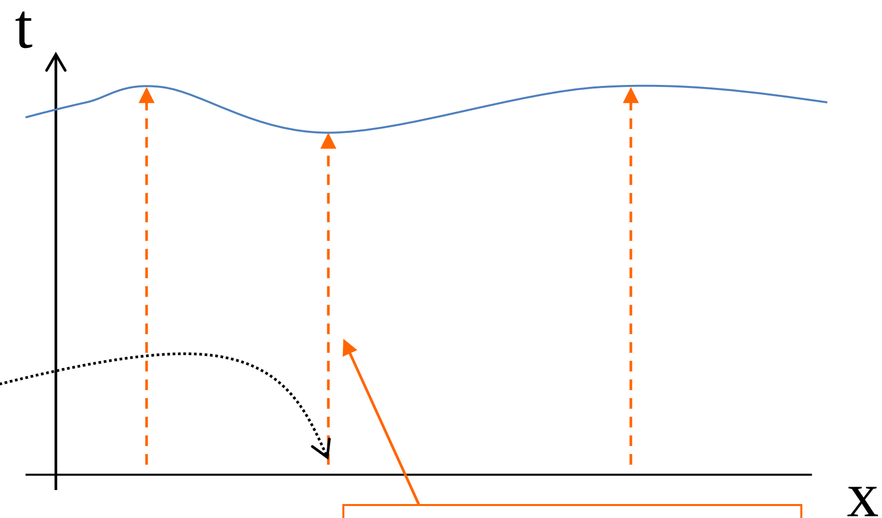
- non-linear evolution lead to **non-Gaussian distribution**
  - **non-zero bispectrum and higher-order correlators**

  - Local-type non-Gaussianity
    - super-Hubble evolution of Gaussian random field from multi-field inflation
  - Equilateral-type non-Gaussianity
    - sub-Hubble interactions in k-inflation/DBI inflation
  - Topological defects
    - cosmic strings from phase transitions

- **templates required to develop optimal estimators**
  - matched filtering to extract small non-Gaussian signal

the δN formalism for primordial perturbations

in radiation-dominated era
curvature perturbation ζ on
uniform-density hypersurface



during inflation
field perturbations $\delta\phi_I(x,t_i)$ on
initial spatially-flat hypersurface

$$N = \int_{initial}^{final} H\, dt$$

on large scales, neglect spatial gradients, treat as "separate universes"

$$\zeta = N(\phi_{initial}) - \bar{N} \approx \sum_I \frac{\partial N}{\partial \phi_I} \delta\phi_I$$

Starobinsky `85; Sasaki & Stewart `96
Lyth & Rodriguez '05 – works to any order

the δN formalism order by order at Hubble exit

t

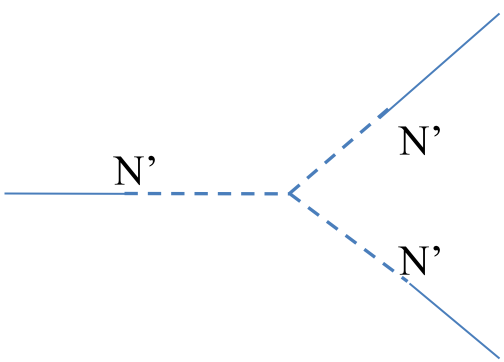
$$\delta\phi_I = \delta_1\phi_I + \frac{1}{2}\delta_2\phi_I + \ldots$$

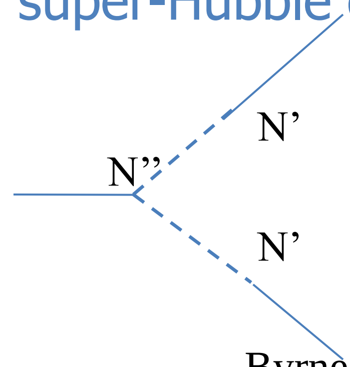
$$\zeta = \zeta_1 + \frac{1}{2}\zeta_2 + \ldots$$

$$= \left[\sum_I \frac{\partial N}{\partial \phi_I}\delta_1\phi_I\right] + \frac{1}{2}\left[\sum_I \frac{\partial N}{\partial \phi_I}\delta_2\phi_I + \sum_{I,J}\frac{\partial^2 N}{\partial\phi_I\partial\phi_I}\delta_1\phi_I\delta_1\phi_J\right] + \ldots$$

sub-Hubble quantum interactions

super-Hubble classical evolution





Byrnes, Koyama, Sasaki & DW

simplest local form of non-Gaussianity

applies to many models inflation including curvaton, modulated reheating, etc

$\zeta = \delta N(\phi)$ is local function of ***single Gaussian random field***, ϕ

$$\zeta = N'\delta\phi + \frac{1}{2}N''\delta\phi^2 + \frac{1}{6}N'''\delta\phi^3 + \ldots$$

$$\Rightarrow \quad \langle \zeta(x_1)\zeta(x_2)\rangle = N'^2\langle \delta\phi(x_1)\delta\phi(x_2)\rangle + \ldots$$

$$\langle \zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle = \frac{1}{2}N'^2N''\langle \delta\phi(x_1)\delta\phi(x_2)\delta\phi^2(x_3)\rangle + \ldots$$

$$= \frac{3}{5}f_{NL}\langle \zeta(x_2)\zeta(x_3)\rangle\langle \zeta(x_1)\zeta(x_3)\rangle + \ldots$$

where $$f_{NL} = \frac{5}{6}\frac{N''}{(N')^2}$$

- odd factors of 3/5 because (Komatsu & Spergel, 2001, used) $\Phi_1 = (3/5)\zeta_1$

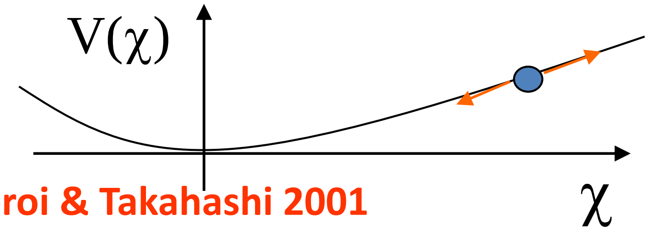
large non-Gaussianity from inflation?

- **single inflaton field**
  - adiabatic perturbations => ζ constant on large scales
  - during conventional slow-roll inflation $$f_{NL}^{\,local} \approx N''/N'^2 = \eta - 2\varepsilon \ll 1$$
  - for any* adiabatic model (Creminelli&Zaldarriaga 2004) $$f_{NL}^{\,local} = -\frac{5}{12}(n-1)$$
  - k/DBI - inflation $$f_{NL}^{\,equil} \approx 1/c_s$$
- **multi-field models**
  - typically f_{NL}~1 for slow-roll inflation
  - could be much larger from sudden transition at end of inflation ?
  - modulated reheating
  - curvaton $f_{NL}\sim 1/\Omega_{decay} \gg 1$?
  - new ekpyrotic models $|f_{NL}| \gg 1$

** but ask Sasaki-san!*

curvaton scenario:

Linde & Mukhanov 1997; Enqvist & Sloth, Lyth & Wands, Moroi & Takahashi 2001



curvaton χ = a weakly-coupled, late-decaying scalar field

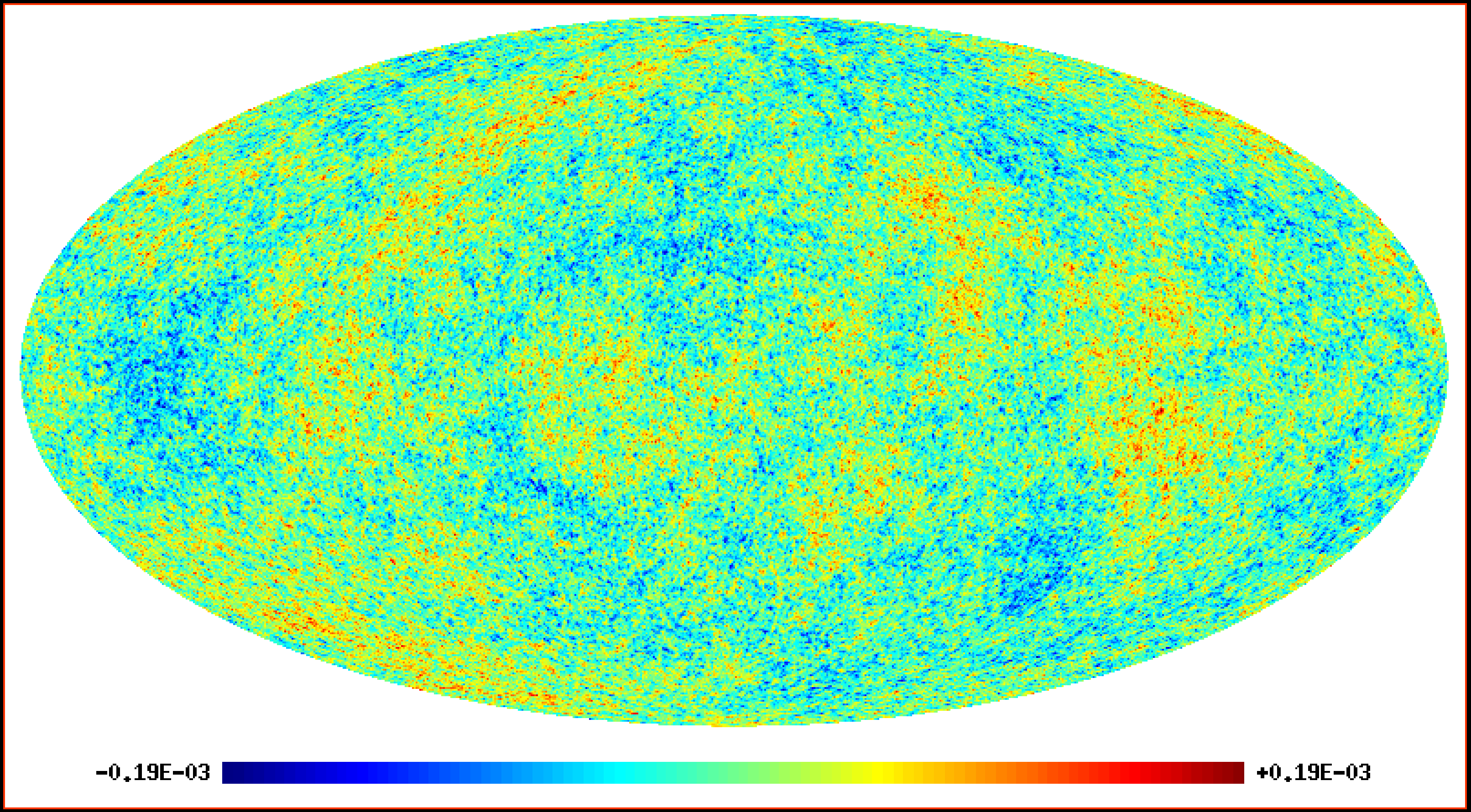
- light during inflation (m<<H) hence acquires an almost scale-invariant, *Gaussian distribution of field fluctuations* on large scales
- energy density for massive field, $\rho_\chi = m^2\chi^2/2$
- spectrum of initially isocurvature density perturbations

$$\zeta_\chi \approx \frac{1}{3}\frac{\delta\rho_\chi}{\rho_\chi} \approx \frac{1}{3}\left(\frac{2\chi\delta\chi + \delta\chi^2}{\chi^2}\right)$$

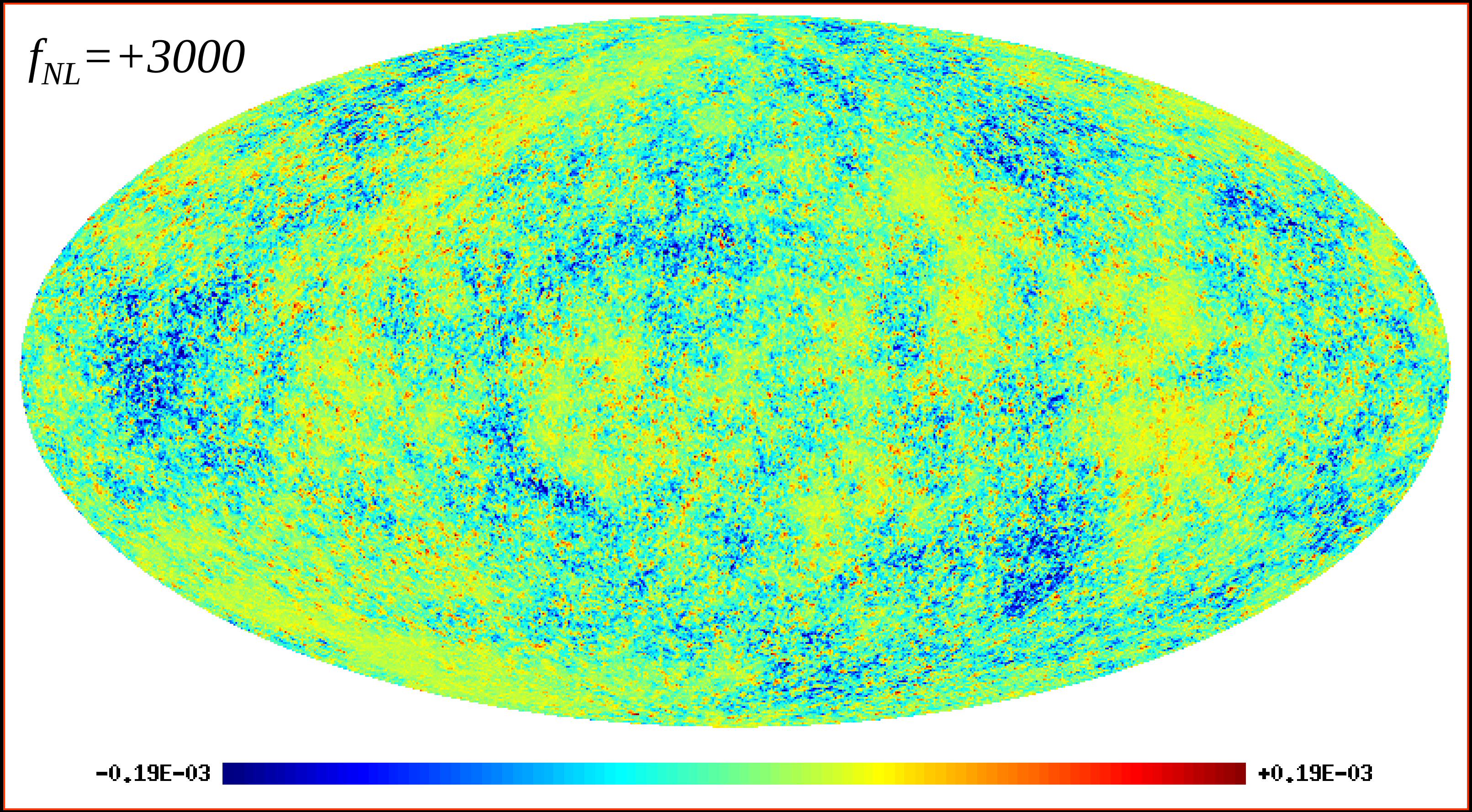
- transferred to radiation when curvaton decays with some efficiency $\approx \Omega_{\chi,decay}$

$$\zeta = \Omega_{\chi,decay}\zeta_\chi$$

$$= \zeta_G + \frac{3}{4\Omega_{\chi,decay}}\zeta_G^{\ 2} \quad \Rightarrow \quad f_{NL} = 5 \Big/ 4\Omega_{\chi,decay}$$



Liguori, Matarrese and Moscardini (2003



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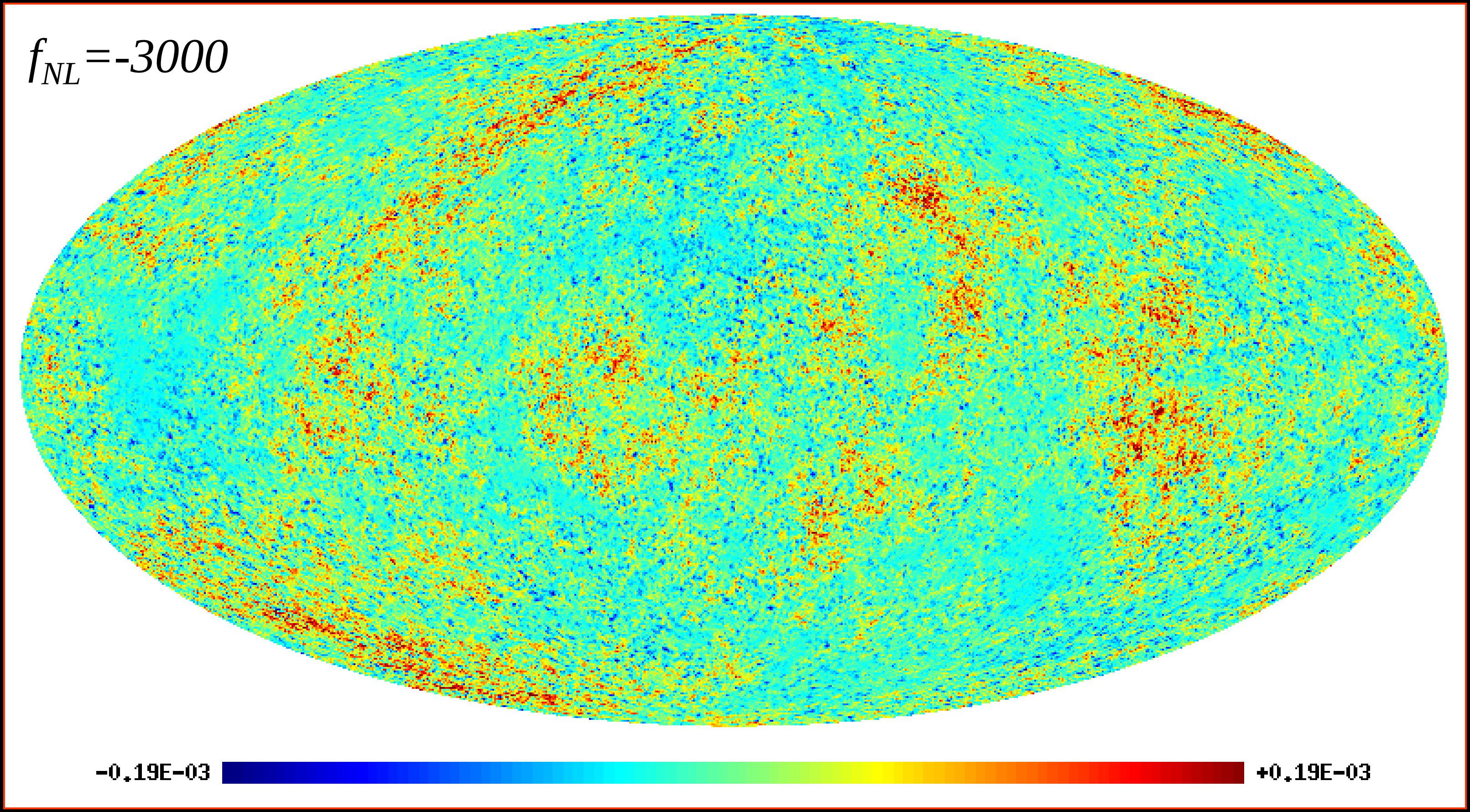

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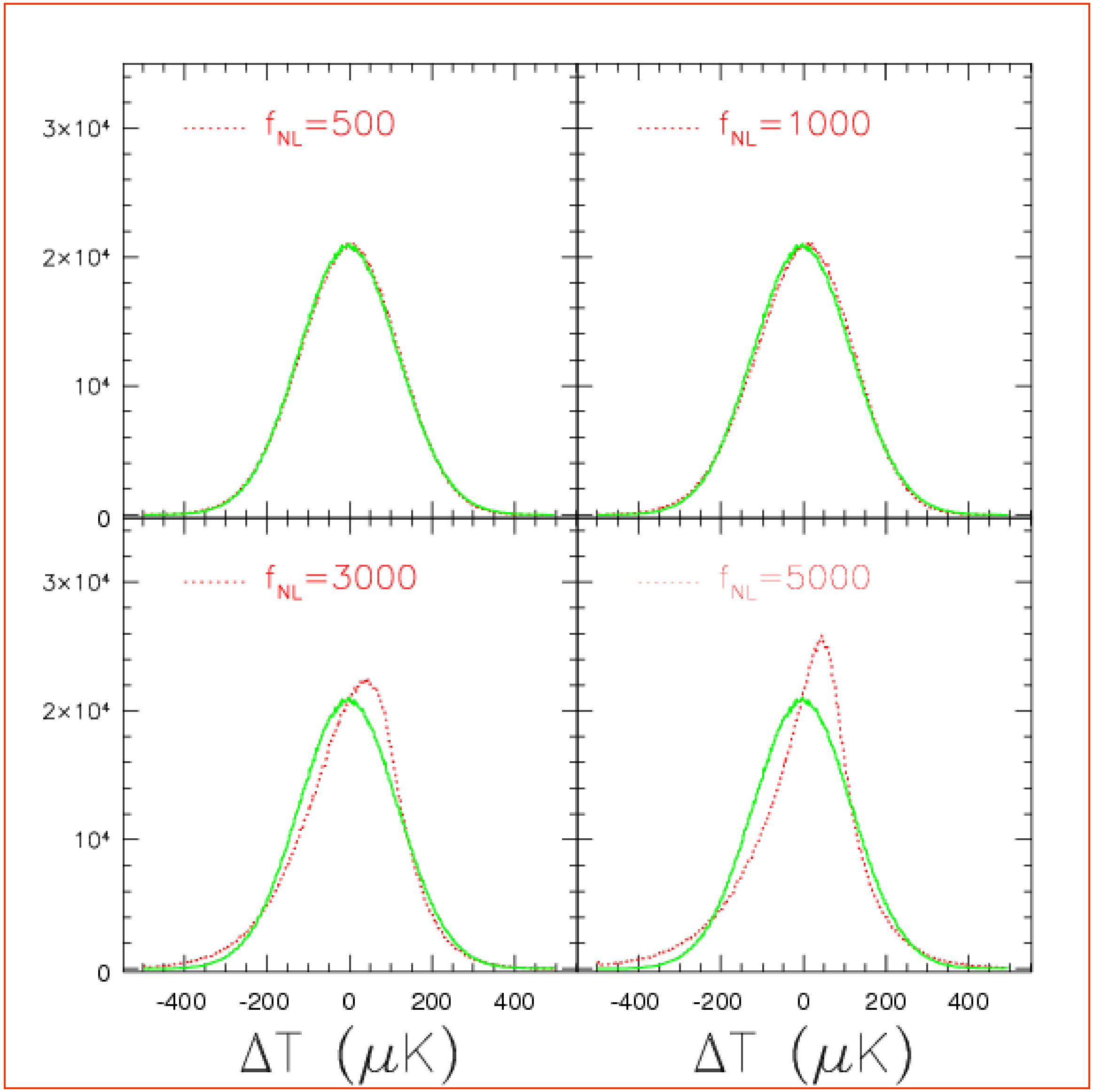


remember: $f_{NL} < 100$ implies Gaussian to better than 0.1%

evidence for local non-Gaussianity?

- $\Delta T/T \approx -\Phi/3$, so positive $f_{NL} \Rightarrow$ more cold spots in CMB

- various groups have attempted to measure this with the WMAP CMB data using estimators based on matched filtering (all 95% CL) :

  - $27 < f_{NL} < 147$ Yadav & Wandelt WMAP3 data
  - $-9 < f_{NL} < 111$ Komatsu et al WMAP5
  - $-4 < f_{NL} < 80$ Smith et al. Optimal WMAP5
  - $-10 < f_{NL} < 74$ Komatsu et al WMAP7

- Large scale structure observations have recently given independent indications due to non-local bias on large scales (Dalal et al 2007):

  - $-29 < f_{NL} < 70$ (95% CL) Slosar et al 2008
  - $27 < f_{NL} < 117$ (95% CL) Xia et al 2010 [NVSS survey of AGNs]

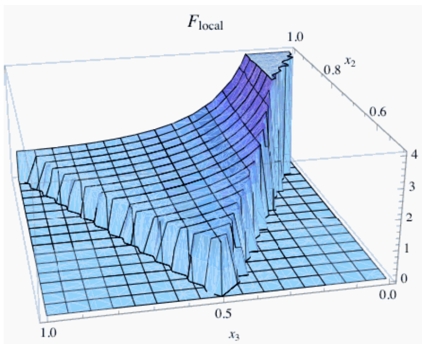
non-Gaussianity from inflation

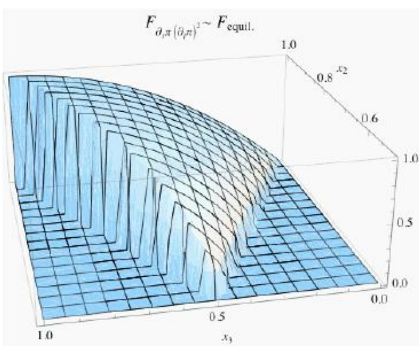
inflation

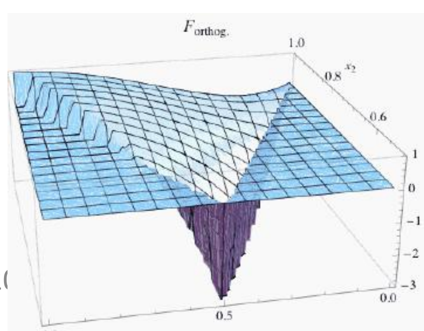
	Source of non-Gaussianity	Bispectrum type
Initial vacuum	Excited state	Folded?
Sub-Hubble evolution	Higher-derivative interactions e.g. k-inflation, DBI, ghost	Equilateral (+orthogonal?)
Hubble-exit	Features in potential	
Super-Hubble evolution	Self-interactions+gravity	Local
End of inflation	Tachyonic instability	Local
(p)Reheating	Modulated (p)reheating	Local
After inflation	Curvaton decay	Local
primordial non-Gaussianity		
Radiation + matter + last-scattering	Primary anisotropies	Local+equilateral
ISW/lensing	Secondary anisotropies	Local+equilateral

templates for primordial bispectra

$$P_\zeta(k) = \mathcal{P}(k)/k^3, \quad B_\zeta(k_1,k_2,k_3) = (6/5)\, f_{NL}(k_1,k_2,k_3)\big(P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)\big)$$







- **local type** (Komatsu&Spergel 2001)
  - local in real space (fNL=constant)
  - max for squeezed triangles: k<<k’,k’’

$$B_\zeta(k_1,k_2,k_3) = (6/5)\, f_{NL}^{local} (\mathcal{P}(k_1))^2 \left(\frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3} \right)$$

- **equilateral type** (Creminelli et al 2005)
  - peaks for k1~k2~k3

$$B_\zeta(k_1,k_2,k_3) = (6/5)\, f_{NL}^{equil} (\mathcal{P}(k_1))^2 \left(\frac{3(k_1+k_2-k_3)(k_2+k_3-k_1)(k_3+k_1-k_2)}{k_1^3 k_2^3 k_3^3} \right)$$

- **orthogonal type** (Senatore et al 2009)

$$B_\zeta(k_1,k_2,k_3) = (6/5)\, f_{NL}^{orthog} (\mathcal{P}(k_1))^2 \left(\frac{81}{k_1 k_2 k_3 (k_1+k_2+k_3)^3} \right)$$

the δN formalism order by order at Hubble exit

t

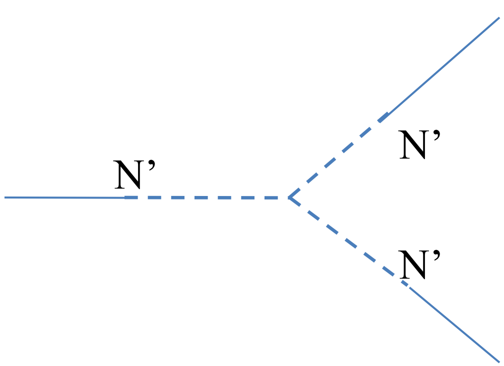
$$\delta\phi_I = \delta_1\phi_I + \frac{1}{2}\delta_2\phi_I + \ldots$$

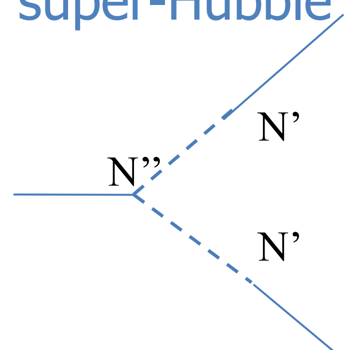
$$\zeta = \zeta_1 + \frac{1}{2}\zeta_2 + \ldots$$

$$= \left[\sum_I \frac{\partial N}{\partial \phi_I}\delta_1\phi_I\right] + \frac{1}{2}\left[\sum_I \frac{\partial N}{\partial \phi_I}\delta_2\phi_I + \sum_{I,J}\frac{\partial^2 N}{\partial\phi_I\partial\phi_I}\delta_1\phi_I\delta_1\phi_J\right] + \ldots$$

sub-Hubble quantum interactions

super-Hubble classical evolution





Byrnes, Koyama, Sasaki & DW

sub-Hubble interactions:

- requires non-minimal kinetic Lagrangian
  - k-inflation (Armendariz-Picon & Mukhanov 1999)
    - super-luminal $c_s^2>1$

$$L = P\left(\varphi, (\nabla\varphi)^2\right)$$

  - DBI brane inflation (Alishahiha, Silverstein & Tong 2004)
    - probe brane in AdS bulk
    - *$c_s^2<1$, $f_{NL} \sim 1/c_s^2$*

$$L = -f(\varphi)\sqrt{1+f^{-1}(\varphi)\left(\nabla\varphi\right)^2} + f(\varphi)$$

  - Galileon fields (Nicolis, Ratazzi & Trincherini 2009)
    - (ghost-free) DGP like scalar field with second-order equations of motion
    - *$c_s^2<1$, $f_{NL} \sim 1/c_s^2$*
    - *see posters by Kobayashi & Mizuno 2010*
  - **Galileon and DBI reunited (de Rham & Tolley 2010)**
    - brane + most general second-order gravity in AdS bulk

$$L = -\lambda + M_4{}^2 R + M_5{}^3 K_{GH} + \alpha M_5{}^3 K_{GB}$$

summary:

- **non-linear perturbations offer distinctive observational signatures of (gravitational) physics of inflation**

Happy Birthday!