Black holes on gravitational instantons

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Introduction

- 5D black holes constructed on (4D) ***flat space*** and ***Taub-NUT space***, the simplest examples of

- ***Gravitational instantons***: non-singular solutions to the Euclidean Einstein equations (if $\Lambda = 0$, Riemannian ***Ricci-flat 4-manifolds***).

  E.g.: 4D flat space, self-dual Taub-NUT, Euclidean Sch/Kerr, Taub-bolt, Eguchi–Hanson, multi-Taub-NUT, Gibbons–Hawking, etc.

- ***Add a flat time*** dimension: $-\mathrm{d}t^2 + \mathrm{instanton}$ (5D space-times).

- ***Add black holes***: (vacuum) black holes on gravitational instantons.

Black holes on 4D (Euclidean) flat space

- 4D flat space $\xrightarrow{-\mathrm{d}t^2}$ 5D Minkowski space-time

- Static black hole on 4D flat space: ***5D Schwarzschild***

$$\mathrm{d}s^2 = -\left(1-\frac{2m}{r^2}\right)\mathrm{d}t^2+\left(1-\frac{2m}{r^2}\right)^{-1}\mathrm{d}r^2+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\psi^2+\cos^2\theta\,\mathrm{d}\phi^2\right).$$

Black holes on Taub-NUT

- Taub-NUT $\xrightarrow{-\mathrm{d}t^2}$ Kaluza-Klein monopole (5D pers)

- Static black hole on Taub-NUT (***Ishihara & Matsuno***):

$$\begin{aligned} \mathrm{d}s^2 \quad = \quad & -\left(1-\frac{2m}{r^2}\right)\mathrm{d}t^2+\frac{k(r)^2}{1-\frac{2m}{r^2}}\mathrm{d}r^2+\frac{k(r)r^2}{4}\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\phi^2\right) \\ & +\frac{r^2}{4}\left(\mathrm{d}\psi+\cos\theta\,\mathrm{d}\phi\right)^2 . \end{aligned}$$

Black holes on Euclidean Schwarzschild

- Static black hole on Euclidean Sch (***Emparan & Reall***):

$$\begin{aligned} \mathrm{d}s^2 &= -\frac{1+cy}{1+cx}\,\mathrm{d}t^2 + \frac{1-x}{1-y}\mathrm{d}\psi^2 \\ &\quad + \frac{2\varkappa^4\,(1-y)^2\,(1+cx)^2}{(x-y)^3}\left(\frac{\mathrm{d}x^2}{G(x)} - \frac{\mathrm{d}y^2}{G(y)} - \frac{2\,(1+x)\,(1+y)}{(x-y)}\,\mathrm{d}\phi^2\right). \end{aligned}$$

Black holes on Euclidean Kerr

- Static black hole on Euclidean Kerr:

$$
\begin{aligned}
\mathrm{d}s^2 &= -\frac{1+cy}{1+cx}\,\mathrm{d}t^2 + \frac{F(x,y)}{H(x,y)}\,(\mathrm{d}\psi+\Omega)^2 \\
&\quad + \frac{2\varkappa^4\,(1+cx)\,H(x,y)}{c^2\,(1-c)\,(1-\alpha^2)\,(1+\alpha^2)^2\,(x-y)^3}\left(\frac{\mathrm{d}x^2}{G(x)} - \frac{\mathrm{d}y^2}{G(y)} + A\,\mathrm{d}\phi^2\right),
\end{aligned}
$$

where Ω and A are defined as

$$
\begin{aligned}
\Omega &= \frac{2\alpha c^2\varkappa^2\,[1+c-(1-c)\,\alpha^2][1-c-(1+c)\,\alpha^2]}{(1+\alpha^2)} \\
&\quad \times \frac{(1+y)\,[(1-y)\,(2-c+cx)+(1-x)\,(2-c+cy)\,\alpha^2]G(x)}{(1-x)\,(x-y)\,F(x,y)}\,\mathrm{d}\phi\,, \\
A &= -\frac{2c^2\,(1-c)\left(1-\alpha^2\right)\left(1+\alpha^2\right)^2 G\,(x)\,G(y)}{(x-y)\,(1+cy)\,F(x,y)}\,.
\end{aligned}
$$

- Static black hole on Taub-bolt:

$$
\begin{aligned}
\mathrm{d}s^2 &= -\frac{1+cy}{1+cx}\,\mathrm{d}t^2 + \frac{F(x,y)}{H(x,y)}\,(\mathrm{d}\psi+\Omega)^2 \\
&\quad + \frac{2\varkappa^4\,(1-c)\,(1+cx)\,H(x,y)}{(1-\alpha^2)\,(x-y)^3}\left(\frac{\mathrm{d}x^2}{G(x)} - \frac{\mathrm{d}y^2}{G(y)} + A\,\mathrm{d}\phi^2\right),
\end{aligned}
$$

where Ω and A are defined as

$$
\begin{aligned}
\Omega &= \frac{2\alpha\,\varkappa^2[2+x+y+c\,(1+x)\,(1+y)]}{(1-\alpha^2)\,(x-y)}\,\mathrm{d}\phi\,, \\
A &= -\frac{2\,(1+x)\,(1+y)}{(1-c)\,(x-y)}\,.
\end{aligned}
$$

- Static black holes on Eguchi–Hanson have ***yet*** to be found.
- Rotating black hole on Eguchi–Hanson:

$$\begin{aligned} \mathrm{d}s^2 &= -\frac{H(y,x)}{H(x,y)}(\mathrm{d}t - \omega_\psi\,\mathrm{d}\psi - \omega_\phi\,\mathrm{d}\phi)^2 - \frac{F(x,y)}{H(y,x)}\,\mathrm{d}\psi^2 + 2\frac{J(x,y)}{H(y,x)}\,\mathrm{d}\psi\mathrm{d}\phi \\ &\quad + \frac{F(y,x)}{H(y,x)}\,\mathrm{d}\phi^2 + \frac{\varkappa^2 H(x,y)}{2(1-a^2)(1-b)^3(x-y)^2}\left(\frac{\mathrm{d}x^2}{G(x)} - \frac{\mathrm{d}y^2}{G(y)}\right), \\ \omega_\psi &= \frac{2\varkappa}{H(y,x)}\sqrt{\frac{2b(1+b)(b-c)}{(1-a^2)(1-b)}}\,(1-c)(1+y)\{2[1-b-a^2(1+bx)]^2(1-c) \\ &\qquad -a^2(1-a^2)b(1-b)(1-x)(1+cx)(1+y)\}, \\ \omega_\phi &= \frac{2\varkappa}{H(y,x)}\sqrt{\frac{2b(1+b)(b-c)}{(1-a^2)(1-b)}}\,a(1-c)(1+x)^2(1+y)[a^4(1+b)(b-c) \\ &\qquad +a^2(1-b)(-b+cb+2c)-(1-b)^2c]. \end{aligned}$$

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